Plasma current, position and shape control in tokamaks - Part 1 *(aka plasma magnetic control)*

Advanced Course on Plasma Diagnostics and Control
Ph.D. Course in Fusion Science and Engineering
12 June - Padova, Italy

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Introduction
Magnetic (control-oriented) modelling
Control engineering jargon & tools
The plasma magnetic control problem

A proposal for the magnetic control architecture
Vertical stabilization controller
Current decoupling controller
Plasma current controller
Plasma shape controller
Nonlinear validation
Current limit avoidance system

Some experimental results
Current limit avoidance at JET
ITER-like VS at EAST
MIMO shape controller at EAST
Nuclear Fusion for Dummies

Main Aim
Production of energy by means of a fusion reaction

\[ D + T \rightarrow ^4\text{He} + n \]

Plasma

- High temperature and pressure are needed
- Fully ionised gas \(\leftrightarrow\) Plasma
- Magnetic field is needed to confine the plasma
In tokamaks, magnetic control of the plasma is obtained by means of magnetic fields produced by the external active coils.

In order to obtain good performance, it is necessary to have a plasma with vertically elongated cross section ⇒ vertically unstable plasmas.

It is important to maintain adequate plasma-wall clearance during operation.
Our final objective: build a control system
Main (basic) assumptions

1. The plasma/circuits system is axisymmetric
2. The inertial effects can be neglected at the time scale of interest, since plasma mass density is low
3. The magnetic permeability $\mu$ is homogeneous, and equal to $\mu_0$ everywhere

Mass vs Massless plasma

It has been proven that neglecting plasma mass may lead to erroneous conclusion on closed-loop stability.

- M. L. Walker, D. A. Humphreys
  On feedback stabilization of the tokamak plasma vertical instability

- J. W. Helton, K. J. McGown, M. L. Walker,
  Conditions for stabilization of the tokamak plasma vertical instability using only a massless plasma analysis
Plasma model

The *input variables* are:

- The voltage applied to the active coils \( v \)
- The plasma current \( I_p \)
- The poloidal beta \( \beta_p \)
- The internal inductance \( l_i \)

\( I_p, \beta_p \text{ and } l_i \) are used to specify the current density distribution inside the plasma region.
Different model outputs can be chosen:

- fluxes and fields where the magnetic sensors are located
- currents in the active and passive circuits
- plasma radial and vertical position (1st and 2nd moment of the plasma current density)
- geometrical descriptors describing the plasma shape (gaps, x-point and strike points positions)
Lumped parameters approximation

By using finite-elements methods, nonlinear lumped parameters approximation of the PDEs model is obtained

\[ \frac{d}{dt} \left[ \mathcal{M}(y(t), \beta_p(t), l_i(t))I(t) \right] + RI(t) = U(t), \]

\[ y(t) = \mathcal{Y}(I(t), \beta_p(t), l_i(t)). \]

where:

- \( y(t) \) are the output to be controlled
- \( I(t) = [I_{PF}^T(t) \ I_e^T(t) \ I_p(t)]^T \) is the currents vector, which includes the currents in the active coils \( I_{PF}(t) \), the eddy currents in the passive structures \( I_e(t) \), and the plasma current \( I_p(t) \)
- \( U(t) = [U_{PF}^T(t) \ 0^T \ 0]^T \) is the input voltages vector
- \( \mathcal{M}(\cdot) \) is the mutual inductance nonlinear function
- \( R \) is the resistance matrix
- \( \mathcal{Y}(\cdot) \) is the output nonlinear function
Plasma linearized model

Starting from the nonlinear lumped parameters model, the following plasma linearized state space model can be easily obtained:

\[ \delta \dot{x}(t) = A \delta x(t) + B \delta u(t) + E \delta w(t), \]  
\[ \delta y(t) = C \delta l_{PF}(t) + F \delta w(t), \]  
(1)
(2)

where:

- **A**, **B**, **E**, **C** and **F** are the model matrices
- \( \delta x(t) = \begin{bmatrix} \delta I_{PF}(t) & \delta I_e(t) & \delta I_p(t) \end{bmatrix}^T \) is the state space vector
- \( \delta u(t) = \begin{bmatrix} \delta U_{PF}(t) & 0^T & 0 \end{bmatrix}^T \) are the input voltages variations
- \( \delta w(t) = \begin{bmatrix} \delta \beta_p(t) & \delta l_i(t) \end{bmatrix}^T \) are the \( \beta_p \) and \( l_i \) variations
- \( \delta y(t) \) are the output variations

The model (1)–(2) relates the variations of the PF currents to the variations of the outputs around a given equilibrium.
A linear time-invariant (LTI) continuous-time system is described by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t), \quad x(0) = x_0 \quad (3a) \\
y(t) &= Cx(t) + Du(t) \quad (3b)
\end{align*}
\]

where \( A \in \mathbb{R}^{n \times n} \), \( B \in \mathbb{R}^{n \times m} \), \( C \in \mathbb{R}^{p \times n} \) and \( D \in \mathbb{R}^{p \times m} \).

A dynamical system with single-input \((m = 1)\) and single-output \((p = 1)\) is called SISO, otherwise it is called MIMO.
Asymptotic stability of LTI systems

Asymptotic stability

This property roughly asserts that every solution of \( \dot{x}(t) = Ax(t) \) tends to zero as \( t \to \infty \).

For LTI systems the stability property is related to the system and not to a specific equilibrium

**Theorem** - System (3) is **asymptotically stable** iff \( A \) is Hurwitz, that is if every eigenvalue \( \lambda_i \) of \( A \) has strictly negative real part

\[
\Re(\lambda_i) < 0, \; \forall \; \lambda_i.
\]

**Theorem** - System (3) is **unstable** if \( A \) has at least one eigenvalue \( \bar{\lambda} \) with strictly positive real part, that is

\[
\exists \; \bar{\lambda} \text{ s.t. } \Re(\bar{\lambda}) > 0.
\]

**Theorem** - Suppose that \( A \) has all eigenvalues \( \lambda_i \) such that \( \Re(\lambda_i) \leq 0 \), then system (3) is **unstable** if there is at least one eigenvalue \( \bar{\lambda} \) such that \( \Re(\bar{\lambda}) = 0 \) which corresponds to a Jordan block with size > 1.
Equilibrium stability for nonlinear systems

For nonlinear systems the stability property is related to the specific equilibrium

**Theorem** - The equilibrium state $x_e$ corresponding to the constant input $\bar{u}$ a nonlinear system is **asymptotically stable** if all the eigenvalues of the correspondent linearized system have strictly negative real part

**Theorem** - The equilibrium state $x_e$ corresponding to the constant input $\bar{u}$ a nonlinear system is **unstable** if there exists at least one eigenvalue of the correspondent linearized system which has strictly positive real part
Transfer function of LTI systems

Given a LTI system (3) the corresponding transfer matrix from $u$ to $y$ is defined as

$$Y(s) = G(s)U(s),$$

with $s \in \mathbb{C}$. $U(s)$ and $Y(s)$ are the Laplace transforms of $u(t)$ and $y(t)$ with zero initial condition ($x(0) = 0$), and

$$G(s) = C(sI - A)^{-1}B + D.$$  \hspace{1cm} (4)

For SISO system (4) is called transfer function and it is equal to the Laplace transform of the impulsive response of system (3) with zero initial condition.
Given the transfer function $G(s)$ and the Laplace transform of the input $U(s)$ the time response of the system can be computed as the inverse transform of $G(s)U(s)$, without solving differential equations.

As an example, the step response of a system can be computed as:

$$y(t) = \mathcal{L}^{-1} \left[ \frac{1}{s} \right].$$
Poles and zeros of SISO systems

Given a SISO LTI system, its transfer function is a rational function of $s$

$$G(s) = \frac{N(s)}{D(s)} = \rho \frac{\prod_i (s - z_i)}{\prod_j (s - p_j)},$$

where $N(s)$ and $D(s)$ are polynomial in $s$, with $\text{deg}(N(s)) \leq \text{deg}(D(s))$.

We call

- $p_j$ poles of $G(s)$
- $z_i$ zeros of $G(s)$

Every pole of $G(s)$ is an eigenvalue of the system matrix $A$. However, not every eigenvalue of $A$ is a pole of $G(s)$.
When dealing with transfer functions, it is usual to resort to *Block diagrams* which permit to graphically represent the interconnections between system in a convenient way.
Series connection

\[ u \quad G_1(s) \quad G_2(s) \quad y \]

\[ u \quad G_2(s)G_1(s) \quad y \]
Parallel connection

\[ u \rightarrow G_1(s) \rightarrow + \rightarrow + \rightarrow G_2(s) \rightarrow y \]

\[ u \rightarrow G_1(s) + G_2(s) \rightarrow y \]
Feedback connection

\[
\begin{align*}
G_1(s) & \quad +/− \quad G_2(s) \\
\downarrow & \quad \downarrow \\
\frac{G_1(s)}{1 + G_1(s)G_2(s)} & \quad \Rightarrow \\
\end{align*}
\]
Stability of interconnected systems

Given two asymptotically stable LTI systems $G_1(s)$ and $G_2(s)$

- the **series** connection $G_2(s)G_1(s)$ is asymptotically stable
- the **parallel** connection $G_1(s) + G_2(s)$ is asymptotically stable
- the **feedback** connection $\frac{G_1(s)}{1 \pm G_1(s)G_2(s)}$ is not necessarily stable

**THE CURSE OF FEEDBACK!**
The plasma (axisymmetric) magnetic control in tokamaks includes the following three control problems:

- the vertical stabilization problem
- the shape and position control problem
- the plasma current control problem
Vertical stabilization problem

Objectives

► Vertically stabilize elongated plasmas in order to avoid disruptions

► Counteract the effect of disturbances (ELMs, fast disturbances modelled as VDEs, . . .)

► It does not necessarily control vertical position but it *simply* stabilizes the plasma

► The VS is the essential magnetic control system!
Consider the simplified electromechanical model with three conductive rings, two rings are kept fixed and in symmetric position with respect to the $r$ axis, while the third can freely move vertically.

If the currents in the two fixed rings are equal, the vertical position $z = 0$ is an equilibrium point for the system.
If $\text{sgn}(I_p) \neq \text{sgn}(I)$

![Diagram of stable equilibrium](image)

Stable equilibrium

Circular plasma
If $\text{sgn}(l_p) \neq \text{sgn}(l)$

Stable equilibrium

Circular plasma
If $\text{sgn}(I_p) = \text{sgn}(I)$

Unstable equilibrium

Elongated plasma
Unstable equilibrium - 2/2

If $\text{sgn}(I_p) = \text{sgn}(I)$

Unstable equilibrium

Elongated plasma
Plasma vertical instability

- The plasma vertical instability reveals itself in the linearized model, by the presence of an unstable eigenvalue in the dynamic system matrix.
- The vertical instability growth time is slowed down by the presence of the conducting structure surrounding the plasma.
- This allows to use a feedback control system to stabilize the plasma equilibrium, using for example a pair of dedicated coils.
- This feedback loop usually acts on a faster time-scale than the plasma shape control loop.
Shape and position control problem

- The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak
- The actuators are the Poloidal Field coils, that produce the magnetic field acting on the plasma
- The controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape

Objectives

- Precise control of plasma boundary
- Counteract the effect of disturbances ($\beta_p$ and $l_i$ variations)
- Manage saturation of the actuators (currents in the PF coils)
Plasma current control problem

- Plasma current can be controlled by using the current in the PF coils
- Since there is a sharing of the actuators, the problem of tracking the plasma current can be considered simultaneously with the shape control problem
- Shape control and plasma current control are compatible, since it is possible to show that generating flux that is spatially uniform across the plasma (but with a desired temporal behavior) can be used to drive the current without affecting the plasma shape.
Plasma magnetic system

Motivation

- Plasma magnetic control is one of the crucial issues to be addressed
  - is needed from day 1
  - is needed to robustly control elongated plasmas in high performance scenarios
A tokamak discharge
Magnetic control architecture - A proposal

▶ A magnetic control architecture able to operate the plasma for an entire duration of the discharge, from the initiation to plasma ramp-down

▶ Machine-agnostic architecture (aka machine independent solution)

▶ Model-based control algorithms
  
  → the design procedures relies on (validated) control-oriented models for the response of the plasma and of the surrounding conductive structures

▶ The proposal is based on the JET experience

▶ The architecture has been proposed for ITER & JT-60SA (& DEMO) and has been partially deployed at EAST (ongoing activity)

R. Ambrosino et al.
Design and nonlinear validation of the ITER magnetic control system

N. Cruz et al.,
Control-oriented tools for the design and validation of the JT-60SA magnetic control system
Introduction

Magnetic modelling
Control engineering jargon & tools
Plasma magnetic control problem

Magnetic control architecture
Vertical stabilization
Current decoupling controller
I_p controller
Shape controller
Nonlinear validation
Current allocator

Experiments
CLA @ JET
ITER-like @ EAST
MIMO shape control @ EAST

References
Four independent controllers

- Current decoupling controller
- Vertical stabilization controller
- Plasma current controller
- Plasma shape controller (+ current allocator)

The parameters of each controller can change on the base of events generated by an external supervisor

- Asynchronous events → exceptions
- Clock events → time-variant parameters
Architecture

Plasma magnetic control system

- PF Current Decoupling Controller
- PF & IC power supplies
- Tokamak/Plasma
- Magnetic Diagnostic
- Vertical Stabilization System
- Vertical stabilization
- Current decoupling controller
- $I_c$ controller
- Shape controller
- Nonlinear validation
- Current allocator
- PLANT
- PF circuit voltages
- V
- Plasma vertical speed
- currents in the VS circuits
- Scenario currents
- Plasma Current Controller
- Plasma Shape Controller
- Ip ref
- Shape descriptors
- Shape refs
The vertical stabilization controller

- The vertical stabilization controller has as input the centroid vertical speed, and the current flowing in the in-vessel circuit (a in-vessel coil set)
- It generates as output the voltage references for both the in-vessel and ex-vessel circuits

\[
U_{IC}(s) = F_{VS}(s) \cdot \left( K_V \cdot \bar{I}_{p\text{ref}} \cdot V_p(s) + K_{ic} \cdot I_{IC}(s) \right),
\]
\[
U_{EC}(s) = K_{ec} \cdot I_{IC}(s),
\]

- The vertical stabilization is achieved by the voltage applied to the in-vessel circuit
- The voltage applied to the ex-vessel circuit is used to reduce the current and the ohmic power in the in-vessel coils
- The velocity gain is scaled according to the value of \( I_p \rightarrow K_V \cdot \bar{I}_{p\text{ref}} \)

G. Ambrosino et al.
Plasma vertical stabilization in the ITER tokamak via constrained static output feedback
Architectural diagram of a plasma magnetic control system.
Current decoupling controller

- The current decoupling controller receives as input the PF circuit currents and their references, and generate in output the voltage references for the power supplies.

- The PF circuit current references are generated as a sum of three terms coming from:
  - the scenario supervisor, which provides the feedforwards needed to track the desired scenario.
  - the plasma current controller, which generates the current deviations (with respect to the nominal ones) needed to compensate errors in the tracking of the plasma current.
  - the plasma shape controller, which generates the current deviations (with respect to the nominal ones) needed to compensate errors in the tracking of the plasma shape.
1 Let $\tilde{L}_{PF} \in \mathbb{R}^{n_{PF} \times n_{PF}}$ be a modified version of the inductance matrix obtained from a plasma-less model by neglecting the effect of the passive structures. In each row of the $\tilde{L}_{PF}$ matrix all the mutual inductance terms which are less than a given percentage of the circuit self-inductance have been neglected (main aim: to reduce the control effort).

2 The time constants $\tau_{PF_i}$ for the response of the $i$-th circuit are chosen and used to construct a matrix $\Lambda \in \mathbb{R}^{n_{PF} \times n_{PF}}$, defined as:

$$\Lambda = \begin{pmatrix} 1/\tau_{PF_1} & 0 & \ldots & 0 \\ 0 & 1/\tau_{PF_2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & 1/\tau_{PF_n} \end{pmatrix}.$$
The voltages to be applied to the PF circuits are then calculated as:

\[ U_{PF}(t) = K_{PF} \cdot (I_{PF_{\text{ref}}}(t) - I_{PF}(t)) + \tilde{R}_{PF}I_{PF}(t), \]

where

- \( K_{PF} = \tilde{L}_{PF} \cdot \Lambda \),
- \( \tilde{R}_{PF} \) is the estimated resistance matrix for the PF circuits (needed to take into account the ohmic drop).

F. Maviglia et al.

Improving the performance of the JET Shape Controller

Current decoupling controller - Closed-loop transfer functions

**Figure**: Bode diagrams of the *diagonal* transfer functions.

**Figure**: Bode diagrams of the *off-diagonal* transfer functions.
Architecture

Plasma magnetic control system
The plasma current controller

- The plasma current controller has as input the plasma current and its time-varying reference, and has as output a set of coil current deviations (with respect to the nominal values).

- The output current deviations are proportional to a set of current $K_{p_{\text{curr}}}$ providing (in the absence of eddy currents) a transformer field inside the vacuum vessel, so as to reduce the coupling with the plasma shape controller.

$$\delta I_{PF}(s) = K_{p_{\text{curr}}} F_{I_p}(s) I_{pe}(s)$$

- For ITER it is important, for the plasma current, to track the reference signal during the **ramp-up** and **ramp-down** phases, the dynamic part of the controller $F_{I_p}(s)$ has been designed with a **double integral action**.
The plasma shape controller

Plasma magnetic control system
Let \( g_i \) be the abscissa along \( i \)-th control segment (\( g_i = 0 \) at the first wall).

Plasma shape control is achieved by imposing

\[
g_i^\text{ref} - g_i = 0
\]

on a sufficiently large number of control segments (gap control).

Moreover, if the plasma shape intersect the \( i \)-th control segment at \( g_i \), the following condition is satisfied

\[
\psi(g_i) = \psi_B
\]

where \( \psi_B \) is the flux at the plasma boundary.

Shape control can be achieved also by controlling to 0 the (isoflux control)

\[
\psi(g_i^\text{ref}) - \psi_B = 0
\]

- \( \psi_B = \psi_X \) for limited-to-diverted transition
- \( \psi_B = \psi_L \) for diverted-to-limited transition
Controlled plasma shape descriptors

- During the limiter phase, the controlled shape parameters are the position of the limiter point, and a set of flux differences (isoflux control)
- During the limiter/diverted transition the controlled shape parameters are the position of the X-point, and a set of flux differences (isoflux control)
- During the diverted phase the controlled variables are the plasma-wall gap errors (gap control)
Plasma shape control algorithm

- The **plasma shape controller** is based on the **eXtreme Shape Controller (XSC) approach**
- The main advantage of the XSC approach is the possibility of tracking a number of shape parameters larger than the number of active coils, minimizing a weighted steady state quadratic tracking error, when the references are constant signals

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M. Ariola and A. Pironti
Plasma shape control for the JET tokamak - An optimal output regulation approach
*IEEE Contr. Sys. Magazine, 2005*

G. Ambrosino et al.
Design and implementation of an output regulation controller for the JET tokamak
*IEEE Trans. Contr. System Tech., 2008*

R. Albanese et al.
A MIMO architecture for integrated control of plasma shape and flux expansion for the EAST tokamak
The *XSC-like* philosophy - 1/3

- The XSC-like plasma shape controller can be applied both adopting a *isoflux* or a *gap* approach.
- It relies on the current PF current controller which achieves a **good decoupling** of the PF circuits.
  - Each PF circuit can be treated as an independent SISO channel
    \[
    I_{PF_i}(s) = \frac{I_{PF_{ref},i}(s)}{1 + s\tau_{PF}}
    \]
- If \(\delta Y(s)\) are the variations of the \(n_G\) shape descriptors (e.g. fluxes differences, position of the x-point, gaps) – with \(n_G \geq n_{PF}\) – then dynamically
  \[
  \delta Y(s) = C \frac{I_{PF_{ref}}(s)}{1 + s\tau_{PF}}
  \]
  and **statically**
  \[
  \delta Y(s) = CI_{PF_{ref}}(s)
  \]
The \textit{XSC-like} philosophy - 2/3

- The currents needed to track the desired shape (in a \textit{least-mean-square} sense) are
  \[ \delta I_{PF_{ref}} = C^\dagger \delta Y \]

- It is possible to use weights both for the shape descriptors and for the currents in the PF circuits

- The controller gains can be computed using the SVD of the weighted output matrix:
  \[ \tilde{C} = QCN = USV^T \]

- The XSC minimizes the cost function
  \[ \tilde{J}_1 = \lim_{t \to +\infty} (\delta Y_{ref} - \delta Y(t))^T Q^T Q (\delta Y_{ref} - \delta Y(t)) , \]
  using \( n_{dof} < n_{PF} \) degrees of freedom, while the remaining \( n_{PF} - n_{dof} \) degrees of freedom are exploited to minimize
  \[ \tilde{J}_2 = \lim_{t \to +\infty} \delta I_{PF_N}(t)^T N^T N \delta I_{PF_N}(t) . \]

(it contributes to avoid PF current saturations)
The XSC-like philosophy - 3/3

- PI Controllers
- \( n_{G} \)
- \( n_{dof} \) \( \leq \) \( n_{PF} \) \( \leq \) \( n_{G} \)
- \( V_{M} \Sigma_{M}^{-1} U_{M}^{T} \)
- \( n_{PF} \)
- Experiments
  - CLA @ JET
  - ITER-like @ EAST
  - MIMO shape control @ EAST
- References
Plasma shape controller - Switching algorithm

- Control for limiter plasma
- Estimate position of the X point
- Control for diverted plasmas

- Control for limiter/diverted transition
- \( F \geq F_{st} \)
- \( t \geq t_{st} \)
- Generate Exception

- \( P_x \) inside a reference box

- \( (P_x \text{ active}) \text{ and } (R_{in} \text{ and } R_{out} \text{ sufficiently large}) \)

- Vertical stabilization
- Current decoupling controller
- \( I \) controller
- Shape controller
- Nonlinear validation
- Current allocator

Experiments
- CLA @ JET
- ITER-like @ EAST
- MIMO shape control @ EAST

References
Limited-to-diverted transition

- Results of nonlinear simulation of the limited-to-diverted configuration during the plasma current ramp-up
- Simulation starts at $t = 9.9 \text{ s}$ when $I_p = 3.6 \text{ MA}$, and ends at $t = 30.9 \text{ s}$ when $I_p = 7.3 \text{ MA}$
- The transition from limited to diverted plasma occurs at about $t = 11.39 \text{ s}$, and the switching between the isoflux and the gaps controller occurs at $t = 11.9 \text{ s}$
Plasma boundary snapshots
Current in the PF circuits may saturate while controlling the current and the shape

- PF currents saturations may lead to
  - loss of plasma shape control
  - pulse stop
  - high probability of disruption

A Current Limit Avoidance System (CLA) can be designed to avoid current saturations in the PF coils when the XSC is used.
The CLA uses the redundancy of the PF coils system to automatically obtain almost the same plasma shape with a different combination of currents in the PF coils.

In the presence of disturbances (e.g., variations of the internal inductance $l_i$ and of the poloidal beta $\beta_p$), it tries to avoid the current saturations by “relaxing” the plasma shape constraints.
CLA “philosophy”

- The XSC control algorithm minimizes a quadratic cost function of the plasma shape error in order to obtain at the steady state the output that best approximates the desired shape.
- The XSC algorithm **does not take into account the current limits of the actuators** ⇒ It may happen that the requested current combination is not feasible.
- The current allocation algorithm has been designed to keep the currents within their limits without degrading too much the plasma shape by finding an optimal trade-off between these two objectives.
The plant

Plant model (plasma and PF current controller)

The plant behavior around a given equilibrium is described by means of a linearized model

\[
\begin{align*}
\dot{x} &= Ax + Bu + Bd, \\
y &= Cx + Du + Dd,
\end{align*}
\]

- \( u \in \mathbb{R}^{n_{PF}} \) is the control input vector which holds the \( n_{PF} = 8 \) currents flowing in the PF coils devoted to the plasma shape control
- \( y \in \mathbb{R}^{n_{SH}} \) is the controlled outputs vector which holds the \( n_{SH} \) plasma shape descriptors controlled by the XSC (typically, at JET, it is \( n_{SH} = 32 \))
The controller model (XSC controller)

The XSC can also be modeled as a linear time-invariant system

\[
\dot{x}_c = A_c x_c + B_c u_c + B_r r, \quad (6a)
\]
\[
y_c = C_c x_c + D_c u_c + D_r r, \quad (6b)
\]

under the interconnection conditions:

\[
u_c = y, \quad (7a)
\]
\[
u = y_c. \quad (7b)
\]
Block diagram of the allocated closed-loop system

Where

\[ P(s) = C(sI - A)^{-1}B + D, \]

is the transfer matrix from \( u \) to \( y \) of (5), and

\[ P^* := \lim_{s \to 0} P(s), \]

denotes the steady-state gain.
The current allocator block

The current allocator

The allocator equations are given by

\[ \dot{x}_a = -KB_0^T \left[ \frac{I}{P^*} \right]^T (\nabla J)^T (u, \delta y), \quad (8a) \]

\[ \delta u = B_0 x_a, \quad (8b) \]

\[ \delta y = P^* B_0 x_a. \quad (8c) \]

- \( K \in \mathbb{R}^{n_a \times n_a} \) is a symmetric positive definite matrix used to specify the allocator convergence speed, and to distribute the allocation effort in the different directions.

- \( J(u^*, \delta y^*) \) is a continuously differentiable cost function that measures the trade-off between the current saturations and the control error (on the plasma shape).

- \( B_0 \in \mathbb{R}^{n_{PF} \times n_a} \) is a suitable full column rank matrix.
The CLA scenario

When designing the current allocator, **a large number of parameters must be specified** by the user once the reference plasma equilibrium has been chosen:

- the two matrices $P^*$ and $B_0$, which are strictly related to the linearized plasma model (5)
- the $K$ matrix
- the gradient of the cost function $J$ must be specified by the user. In particular, the gradient of $J$ on each channel is assumed to be piecewise linear

**Figure:** Piecewise linear function used to specify the gradient of the cost function $J$ for each allocated channel. For each channel 7 parameters must be specified.
The CLA block is inserted between the XSC and the Current Decoupling Controller.

G. De Tommasi et al.
Nonlinear dynamic allocator for optimal input/output performance trade-off: application to the JET Tokamak shape controller
*Automatica*, vol. 47, no. 5, pp. 981–987, May 2011

G. De Tommasi et al.
A Software Tool for the Design of the Current Limit Avoidance System at the JET tokamak
The CLA at JET tokamak

Figure: Shape comparison at 22.5 s. Black shape (#81710 without CLA), red shape (#81715 with CLA).

Figure: Currents in the divertor circuits. Pulse #81710 (reference pulse without CLA) and pulse #81715 (with CLA). The shared areas correspond to regions beyond the current limits enforced by the CLA parameters.
A MIMO controller for plasma shape and heat flux integrated control at EAST

Figure: Option #1 - integrated control of plasma shape and flux expansion.

Figure: Option #2 - integrated control of plasma shape and distance between null points.
The EAST architecture is *compliant* to the one proposed for ITER & DEMO.

The control algorithms deployed within the EAST PCS do not satisfy the requirements needed to easily replace the shape controller:

- **vertical stabilization is strongly coupled with plasma shape control**
- The PF Coils current controller can be improved (better decoupling)
\[ U_{\text{IC}_{\text{ref}}} (s) = \frac{1 + S \tau_1}{1 + S \tau_2} \left( K_V \cdot I_{\text{ref}} \cdot \frac{S}{1 + S \tau_Z} \cdot Z_c(s) + K_{IC} \cdot I_{IC}(s) \right) \]
Figure: EAST pulse #70799. During this pulse the ITER-like VS was enabled from $t = 2.1$ s for 1.2 s, and only $I_p$ and $r_c$ were controlled, while $z_c$ was left uncontrolled. This first test confirmed that the ITER-like VS vertically stabilized the plasma by controlling $\dot{z}_c$ and $I_{IC}$, without the need to feed back the vertical position $z_c$. 
**Figure:** EAST pulses #70799 & #71423. Tuning of the controller parameters to reduce oscillations on $z_C$. 

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**Figure:**

- **Plasma current $I_p$**
  - Observed variations in plasma current over time for EAST pulses #70799 and #71423.

- **Radial position $r_C$**
  - Changes in radial position over time for the same pulses, showing an increase.

- **Vertical position $z_C$**
  - Variations in vertical position over time, with distinct peaks and a trend analysis.

- **IC current $I_C$**
  - Time series of IC current showing stability and a comparison with a reference line.

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**Experimental results - 2/2**

The above diagrams illustrate the tuning process for EAST pulse controllers to minimize oscillations in $z_C$.
An XSC-like isoflux shape controller has been tested in 2018 at EAST

It relies on a PFC decoupling controller

**Figure:** Comparison between the SISO and MIMO shape controllers (pulses #78140 and #79289). ● control points and the target X-point position.
SISO vs MIMO isoflux shape control at EAST

Figure: Comparison between the SISO and MIMO shape controllers (pulses #78140 and #79289). The dashed black line in the last two plots represents the X-point position reference.
Model-based tuning of MIMO gains

Figure: Comparison between the two pulses #78289 and #79289, and the simulation used for the design of the controller used during pulse #79289. Oscillations were successfully reduced with respect to the reference pulse #78289.
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Magnetic control architecture
Vertical stabilization
Current decoupling controller $I_c$ controller
Shape controller
Nonlinear validation
Current allocator

Experiments
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