Plasma current, position and shape control in tokamaks - Part 2 (*aka* the vertical stabilization problem)

Advanced Course on Plasma Diagnostics and Control
Ph.D. Course in Fusion Science and Engineering
18 May - Padova, Italy

Gianmaria De Tommasi[1]
Dipartimento di Ingegneria Elettrica e delle Tecnologie dell’Informazione (DIETI)
Università degli Studi di Napoli Federico II, Napoli, Italy

Outline

The vertical stabilization problem

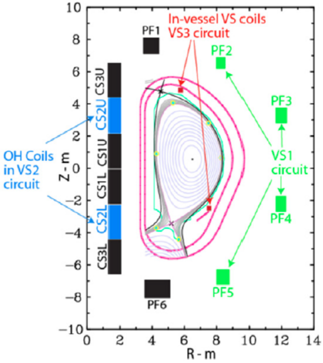
Design a VS system

- A state-space based approach
- A multi-objective optimization approach
- A input-output based approach

Vertical stabilization problem (do you remember?)

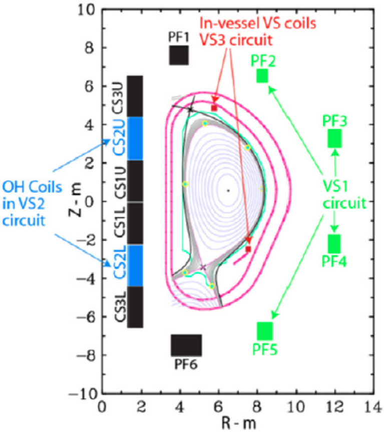
Objectives

- Vertically stabilize elongated plasmas in order to avoid disruptions
- Counteract the effect of disturbances (ELMs, fast disturbances modelled as VDEs,. . .)
- **It does not necessarily control vertical position but it *simply* stabilizes the plasma**
- **The VS is the essential magnetic control system!**



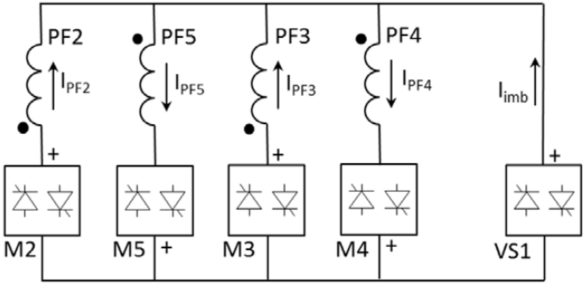
ITER PF system (as a reference)

- External superconductive coils
  - *Single coil* circuits
  - *Imbalance* circuits
- Internal copper coils



Imbalance circuits for vertical stability

The ITER *VS1* circuit



The proposed control approach

$$V_{VS3} = \mathcal{L}^{-1}\left[F_{vs}(s)\right] * (K_1 \dot{z} + K_2 I_{VS3})$$
$$V_{VS1} = K_3 I_{VS3}$$

- The vertical stabilization controller receives, as input, the centroid vertical speed, and the current flowing in the in-vessel coil ($VS3$) circuit (an in-vessel coil set)
- It generates, as output, the voltage references for $VS3$ and for the imbalance circuit ($VS1$)
- Let us first assume

$$F_{vs}(s) = 1\,,$$

which implies

$$V_{VS3} = K_1 \dot{z} + K_2 I_{VS3}$$
$$V_{VS1} = K_3 I_{VS3}$$

How to design the control gains?

$$V_{VS3} = K_1 \dot{z} + K_2 I_{VS3}$$
$$V_{VS1} = K_3 I_{VS3}$$

- The proposed approach includes (just) three gains (number)
  - the *speed* gain K_1
  - the gain on the in-vessel current K_2
  - the gain on the imbalance current K_3
- the proposed structure is rather *simple*, i.e. there are few parameters to be tuned against the operational scenario
- such a structure permits to envisage effective *adaptive* algorithms, as it is usually required in operation
- ...**but how to design these (few) gains?**...
- ...**and how to *adapt* (tune) them in real-time?**

The state-space model

$$\Sigma : \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\,, \quad \mathbf{x}(0) = \mathbf{x}_0 \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{cases}$$

where

- $\mathbf{x}(t) = \left(\mathbf{x}_{pf}^T(t)\ x_{ic}(t)\ \mathbf{x}_{ec}^T(t)\ x_{i_p}(t)\right)^T \in \mathbb{R}^{n_{PF}+n_{EC}+2}$ is the state vector
- $\mathbf{u}(t) = \left(u_{ic}(t)\ u_{imb}(t)\right)^T \in \mathbb{R}^2$ are the input voltages
- $\mathbf{y}(t) = \left(y_1(t)\ y_2(t)\right)^T = \left(x_{ic}(t)\ \dot{z}_p(t)\right)^T \in \mathbb{R}^2$ is the output vector

Modeling VDE for controller design

- A **Vertical Displacement Event (VDE)** is an uncontrolled growth of the plasma unstable vertical mode
- Although, the plasma is always vertically controlled, these uncontrolled growths can occur for different reasons:
  - fast disturbances acting on a time scale which is outside the control system bandwidth
  - delays in the control loop
  - wrong control action due to measurement noise, when plasma speed is almost zero
- VDEs represent one of the worst disturbances to be rejected by the VS system
- From the VS point of view a VDE is equivalent to a sudden and almost instantaneous change in plasma position, which causes an almost instantaneous change of the currents in $\mathbf{x}(t) \Rightarrow$
  - **i)** a VDE can be modeled as instantaneous change of the state vector
  - **ii)** the response of the plant to a VDE can be studied considering the evolution of Σ for a the initial state $\mathbf{x}(0) = \mathbf{x}_{VDE}$

Control problem

Problem

Given the plant Σ, and four positive scalars $\theta_{\min}$, $\theta_{\max}$, T, L, find a static output feedback

$$u_{ic}(t) = k_1 y_1(t) + k_2 y_2(t)\,, \quad (1a)$$

$$u_{VS}(t) = k_3 y_1(t)\,, \quad (1b)$$

such that the closed-loop system

$$\Sigma_{cl} : \begin{cases} \dot{\mathbf{x}}(t) = (\mathbf{A} + \mathbf{BKC})\mathbf{x}(t) \\ \mathbf{y}(t) = \mathbf{Cx}(t) \end{cases} ,$$

with

$$\mathbf{K} = \begin{pmatrix} k_1 & k_2 \\ k_3 & 0 \end{pmatrix} ,$$

- **i)** *is asymptotically stable;*
- **ii)** *has a decay rate $\theta_{\min} < \theta < \theta_{\max}$;*
- **iii)** *if $\mathbf{x}(0) = \mathbf{x}_{VDE}$ then it must be*

$$\|y_1\|_T = \left[\int_0^T \|y_1(t)\|^2 dt \right]^{\frac{1}{2}} < L\,. \quad (2)$$

Design parameters

- $\theta_{\min}$ and $\theta_{\max}$ permit to guarantee that the closed-loop poles belong to a given stripe of the complex plane
  - This region is chosen on the basis of desired closed-loop bandwidth
- The constraint (2) on the rms value of the on the in-vessel current is computed over an appropriate time interval of length T, whose value depends on the specification on the time required to reject the VDE disturbance
- This specification in turn takes into account limitations on voltage, current and power available on the plant.
- The parameter L is related to the thermal constraint, which limits the rms value of the current in the in-vessel coil in presence of a VDE.

Controller structure

$$\mathbf{K} = \begin{pmatrix} k_1 & k_2 \\ k_3 & 0 \end{pmatrix}$$

- The structure of **K** reflects the fact that we want to stabilize the plasma only with the in-vessel coils, while the $VS1$ circuit is employed to reduce the rms value of the current in the in-vessel coils
- The in-vessel coils response promptly to a plasma vertical displacement, being not shielded by the passive structures.
- The imbalance circuit can be effectively used to “drain” current from the in-vessel coil, in order to reduce its rms value

Design of the controller - 1/3

Theorem

Given the plant Σ *and four positive scalars* θ_{min}, θ_{max}, T, L, *then* $\mathbf{K}$ *is a solution of Problem 1 if there exist*

- *a positive definite matrix* P
- *a positive definite matrix-valued function* $\Pi(t)$
- *a positive scalar* θ

that solve the Differential Bilinear Matrix Inequality (D-BMI) feasibility problem

$$(\mathbf{A}+\mathbf{BKC})^T P + P(\mathbf{A}+\mathbf{BKC}) < -2\theta P, \quad \theta_{\text{min}} < \theta < \theta_{\text{max}} \tag{3a}$$

$$\dot{\Pi}(t) + (\mathbf{A}+\mathbf{BKC})^T \Pi(t) + \Pi(t)(\mathbf{A}+\mathbf{BKC}) + \mathbf{c}_{ic}\mathbf{c}_{ic}^T \leq 0, \quad \forall t \in [0, T] \tag{3b}$$

$$\Pi(0) < \hat{\gamma}(L)^2 \mathbf{I} \tag{3c}$$

where $\mathbf{c}_{ic}^T = \begin{pmatrix} \mathbf{0} & 1 & \mathbf{0} \end{pmatrix}$ *is the output row vector corresponding the in-vessel current.*

Design of the controller - 2/3

- Differential BMI constrains can be recasted to *standard* BMI constraints, by discretizing in time
- Optimization tools are available to solve BMI feasibility problems (e.g, `http://www.penopt.com/penbmi.html`)
- **Solving BMIs is computational demanding (NP-hard problem)**
- Offline solution of (3) on a *full order* linear model (about one hundred states) can be a problem
- Two possible *ways* to ease the solution
  1. use a reduced order model (up to 4th/5th order model)
  2. reduce the number of *DoF* when solving (3)
     - more conservative :(
     - less demanding :)

Design of the controller - 3/3

Theorem

*Given the plant Σ and four positive scalars $\theta_{\min}$, $\theta_{\max}$, T, L, then **K** is a solution of Problem 1 if there exist*

- *two positive definite matrices P_1 and P_2*
- *a positive scalar θ*

that solve the Bilinear Matrix Inequality (BMI) feasibility problem

$$(\mathbf{A}+\mathbf{BKC})^T P_1 + P_1(\mathbf{A}+\mathbf{BKC}) < -2\theta P_1\,, \quad \theta_{\min} < \theta < \theta_{\max} \tag{4a}$$

$$(\mathbf{A}+\mathbf{BKC})^T P_2 + P_2(\mathbf{A}+\mathbf{BKC}) + \mathbf{c}_{ic}\mathbf{c}_{ic}^T \leq 0\,, \tag{4b}$$

$$P_2 < \hat{\gamma}^2(L)\mathbf{I} \tag{4c}$$

- Equilibrium: $\bar{I}_p = 15$ MA, $\bar{\beta}_p = 0.1$, $\bar{l}_i = 1.0$
- VDE: 10 cm
- Given the values of the maximum allowable currents and voltages on the in-vessel coils, a reasonable compromise between control effort and closed-loop performance is to choose $T = 1\ \mathrm{s}$
- $\theta_{\text{min}} = 8$ and $\theta_{\text{max}} = 16$
- Given the thermal constraint on the ITER in-vessel coils, $L = 40\ \mathrm{kA}$

- Given the design parameters both (3) and (4) admit a solution with $\hat{\gamma} = 80$
- Reduced order models of the plant have been used
  - 2nd order model in order to solve D-BMIs (3)
  - 4th order model in order to solve BMIs (4)
- $\mathbf{K}_1 = \mathbf{K}_{DBMI} = \begin{pmatrix} 3.6 \cdot 10^{-3} & -478 \\ 3.5 \cdot 10^{-2} & 0 \end{pmatrix}, \quad \theta_1 = 9.4$
- $\mathbf{K}_2 = \mathbf{K}_{BMI} = \begin{pmatrix} 3.9 \cdot 10^{-3} & -470 \\ 3.9 \cdot 10^{-2} & 0 \end{pmatrix}, \quad \theta_2 = 8.3$

Simulations for the ITER tokamak - 3/4

- Controller validation carried out using the *full-order* model of a *different* plasma equilibrium (different linear model)
- In simulation, the model (26) has been completed adding the models of the power supplies and of the diagnostic systems (neglected in the design phase)

Simulations for the ITER tokamak - 4/4

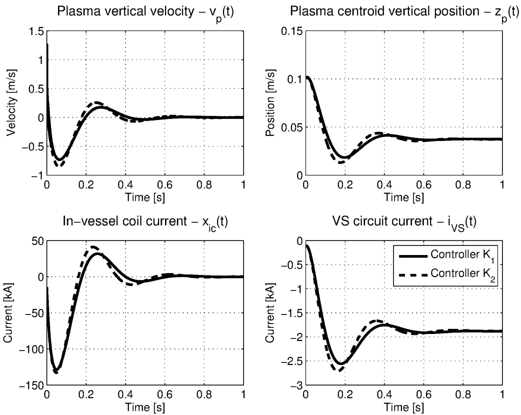
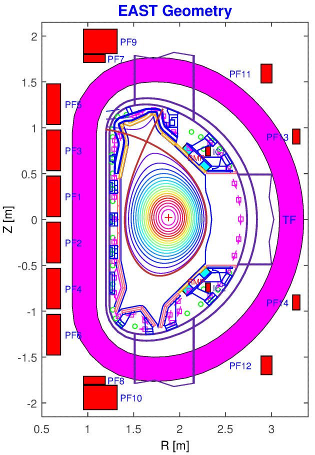
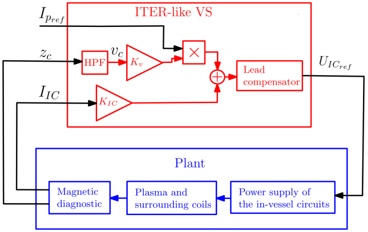


Figure: Closed-loop response to a 10 cm VDE. The solid traces show the behavior obtained with the $\mathbf{K}_1$ controller, while the dashed traces refer to the behavior obtained with $\mathbf{K}_2$.

$\|y_1\|_T = 38.92\ \mathrm{kA}$ with $\mathbf{K}_1$
$\|y_1\|_T = 39.45\ \mathrm{kA}$ with $\mathbf{K}_2$ (ITER requirement $\rightarrow$ 40 kA).

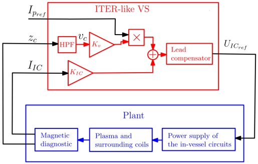
ITER-like VS for the EAST tokamak





$$U_{IC_{ref}}(s) = \frac{1+s\tau_1}{1+s\tau_2} \cdot \left(K_v \cdot \bar{I}_{p_{ref}} \cdot \frac{s}{1+s\tau_z} \cdot Z_c(s) + K_{IC} \cdot I_{IC}(s) \right)$$

SISO stability margins



- The single-input-single-output (SISO) transfer function obtained by opening the control loop in correspondence of the control output is exploited to compute the stability margins (gain and phase margins)
- Given the *i*-th plasma linearized model, it is possible to define the objective function

$$\mathcal{F}_i = c_1 \cdot (PM_t - PM(K_v, K_{IC}, \tau_1, \tau_2))^2 + c_2 \cdot (UGM_t - UGM(K_v, K_{IC}, \tau_1, \tau_2))^2 + c_3 \cdot (LGM_t - LGM(K_v, K_{IC}, \tau_1, \tau_2))^2 ,$$

- where
  - PM is the phase margin
  - UGM and LGM are the upper and lower gain margins
  - c_1, c_2 and c_3 are positive weighting coefficients
  - PM_t, UGM_t and LGM_t are the desired values (*targets*) for the stability margins

Multi-objective optimization

Given N (different) plasma equilbria, it is possible to design the VS gains by solving the following multi-objective optimization problem

$$\min_{K_v, K_{IC}, \tau_1, \tau_2} \mu$$
$$\text{s.t. } \mathcal{F}(K_v, K_{IC}, \tau_1, \tau_2) - \mu \cdot w \leq 0 \,,$$

where $\mathcal{F}$ is a vector function

$$\mathcal{F}(K_v, K_{IC}, \tau_1, \tau_2) = \left(\mathcal{F}_1(K_v, K_{IC}, \tau_1, \tau_2) \ \ldots \ \mathcal{F}_N(K_v, K_{IC}, \tau_1, \tau_2)\right)^T \,,$$

where w is a vector of weights.

The EAST case study - 1/3

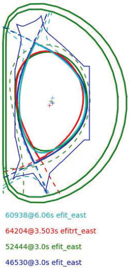


Table: Main plasma parameters of the considered EAST equilibria.

Equilibrium	Shape type	$I_{p_{eq}}$ [kA]	γ [s^{-1}]
46530	Double-null	281	137
52444	Limiter	230	92
60938	Upper single-null	374	194
64204	Lower single-null	233	512

The EAST case study - 2/3

Table: Maximum real part of the closed loop eigenvalues computed by applying to the j-th equilibrium the gains obtained with the single-objective approach for the i-th one, with $i \neq j$.

	46530	52444	60938	64204
single-objective #46530	–	-0.365	-0.088	255.99
single-objective #52444	-0.360	–	-0.358	897.01
single-objective #60938	-0.360	-0.364	–	153.57
single-objective #64204	-0.360	-0.365	-0.358	–

The EAST case study - 3/3

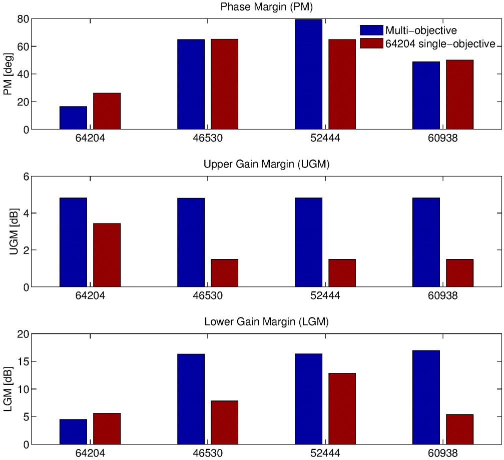


Figure: Comparison of the stability margins obtained using the multi-objective approach and by using the VS parameters obtained using a single-objective approach for the EAST pulse #64204.

Stabilizing the EAST plasma using a SISO controller - 1/2

$$\Sigma : \begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\,, \quad \mathbf{x}(0) = \mathbf{x}_0 \\ \mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \end{cases}$$

- From Σ it is possible to derive the input-output relationship between the vertical speed $V_p(s)$ and the voltage applied to the in-vessel coil $U_{IC}(s)$ (the plasma)

$$W_p(s) = \frac{V_p(s)}{U_{IC}(s)}$$

- The IC power supply is modeled as

$$U_{IC}(s) = \frac{e^{-\delta_{ps}s}}{1 + s\tau_{ps}} \cdot U_{IC_{ref}}(s)\,,$$

  with $U_{IC_{ref}}(s)$ the voltage requested by the controller, $\delta_{ps} = 550\ \mu\text{s}$, $\tau_{ps} = 100\ \mu\text{s}$

- At EAST the plasma vertical speed $V_p(s)$ is estimated by means of a derivative filter applied on $Z_p(s)$, i.e.

$$V_p(s) = \frac{s}{1 + s\tau_v} \cdot Z_p(s)\,,$$

  with $\tau_v = 1$ ms.

Stabilizing the EAST plasma using a SISO controller - 2/2

- Putting everything together we get

$$W_{plant}(s) = \frac{s}{(1+s\tau_v)(1+s\tau_{ps})} \cdot W_p(s) \cdot e^{-\delta_{ps}s} \,,$$

- The 550 μs time delay of the IC power supply can be replaced by its third order Padé approximation

$$\frac{-(s-8444)(s^2-1.34\cdot 10^4 s+8.54\cdot 10^7)}{(s+8444)(s^2+1.34\cdot 10^4 s+8.54\cdot 10^7)}$$

- **The only way to vertically stabilize EAST with a SISO stable controller (SISO strong stabilizability) is to include an integral action on the vertical speed (i.e., the vertical position z_p should be fed back**
- The reason is that the plasma unstable pole is *trapped* between two non minimum phase zeros

Parity-Interlacing-Property (PIP)

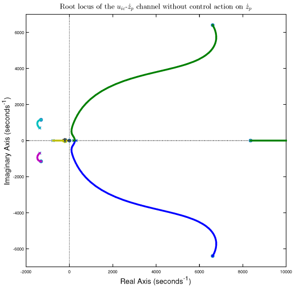
Theorem

A linear plant $W(s)$ is strongly stabilizable if and only if the number of poles of $W(s)$ between any pair of real zeros in the right-half-plane (RHP) is even.

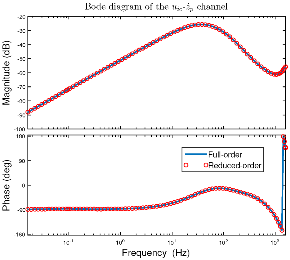
D. C. Youla, J. J. Bongiorno Jr., C. N. Lu
Single-loop feedback stabilization of linear multivariable dynamical plants
Automatica, vol. 10, no. 2, pp. 159–173, Mar. 1974

Stabilizing with a MIMO controller - 1/2

By closing the loop on $I_{IC}(s)$ we introduce another unstable pole in the $u_{ic} - \dot{z}_p$ channel



(a) Root locus of the $u_{ic} - \dot{z}_p$ channel, when the loop on the IC current is closed.



(b) Bode diagrams of the full-order and reduced-order versions of transfer function for the $u_{ic} - \dot{z}_p$ channel, when the loop on the IC current is closed.

Stabilizing with a MIMO controller - 2/2

Closing a stable controller on the vertical speed is now possible to stabilize the EAST plasma

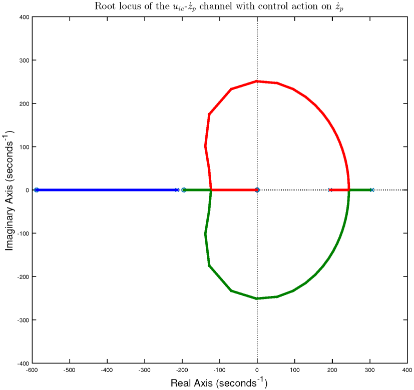


Figure: Root locus of the $u_{ic} - \dot{z}_p$ channel, when the loop on the IC current is also closed.

Conclusive remarks

- **The VS gains need to be adjusted/adapted during the pulse**
- The plasma speed gain must be scaled with I_p **EASY :)**
- The gains should be also scheduled as function of the growth rate **HARD :(**
- Whatever adaption technique is used. . .
  - gain scheduling
  - *real* adaptive control (model-based)
- . . . **an estimation of the growth rate in real-time is needed!**

References

VS design

G. Ambrosino et al.
Plasma Vertical Stabilization in the ITER Tokamak via Constrained Static Output Feedback
IEEE Transactions on Control Systems Technology, vol. 19, no. 2, pp. 376–381, Mar. 2011.

G. Ambrosino et al.
Robust vertical control of ITER plasmas via static output feedback
IEEE Multi-Conference on Systems and Control (MSC'11), Denver, Colorado, Sep. 2011, pp. 276–281.

R. Albanese et al.
ITER-like Vertical Stabilization System for the EAST Tokamak
submitted to *Nucl. Fus.*, 2017

G. De Tommasi, A. Mele, A. Pironti
Robust plasma vertical stabilization in tokamak devices via multi-objective optimization
International Conference on Optimization and Decision Science (ODS'17), Sorrento, Italy, Sep. 2017.

G. De Tommasi et al.
On plasma vertical stabilization at EAST tokamak
2017 IEEE Conference on Control Technology and Applications (IEEE CCTA'17), Kohala Coast, Hawai'i, Aug. 2017.