Plasma Current, Position and Shape Control in Tokamaks

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Tokamak nuclear fusion reactors: a control perspective

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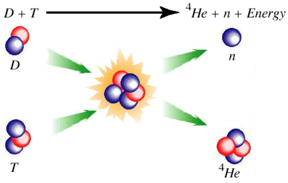
Outline

Main Aim

Production of energy by means of a fusion reaction

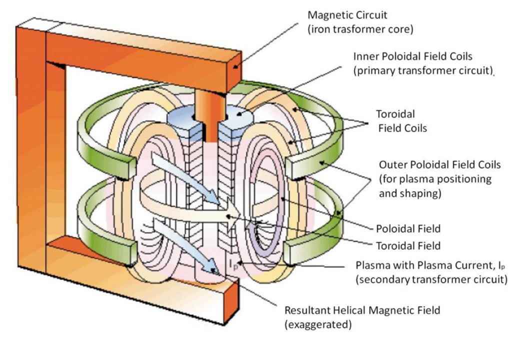
$$D + T \rightarrow \ {}^{4}\mathrm{He} + n$$



Plasma

- High temperature and pressure are needed
- Fully ionised gas $\mapsto$ Plasma
- Magnetic field is needed to confine the plasma

What is a Tokamak ?

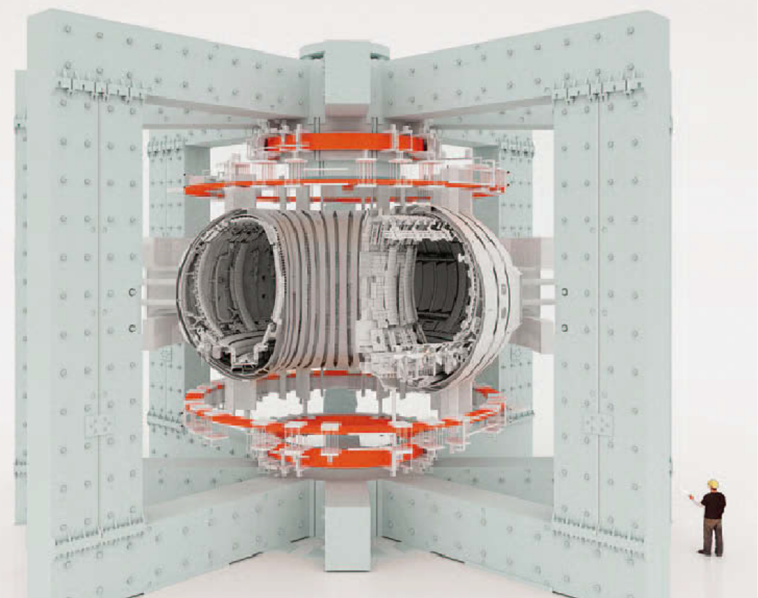


A tokamak is an electromagnetic machine containing a fully ionised gas (plasma) at about 100 million degrees within a torus shaped vacuum vessel. Poloidal and toroidal field coils, together with the plasma current, generate a spiralling magnetic field that confines the plasma.

The JET tokamak - 1

- The Joint European Torus (JET) is an example of successful European collaboration.
- JET is still the world's largest tokamak
- JET has been built in the early eighties, and it was designed to allow the exploration of the plasma regimes in proximity of break-even, the condition at which the ratio between produced fusion power and input heating power is unity
- At the time of its construction, JET was a large step in scale from existing experiments

The JET tokamak - 2



Motivation

- Plasma control is the crucial issue to be addressed in order to achieve the high performances envisaged for future tokamak devices
- Plasma magnetic **axisymmetric** control (shape and position) is an essential feature of all tokamaks
- High performance in tokamaks is achieved by plasmas with elongated poloidal cross section, which are vertically unstable
- If high performance and robustness are required, then a model-based design approach is needed

This presentation

1. focuses on plasma shape control and the vertical stabilization problems
2. presents the eXtreme Shape Controller (XSC) and the Current Limit Avoidance systems deployed at the JET tokamak

Plasma axisymmetric model - 1

Model Inputs

The *input variables* are:

- The voltage applied to the active coils v
- The plasma current I_p
- The poloidal beta β_p
- The internal inductance l_i

I_p, β_p and l_i

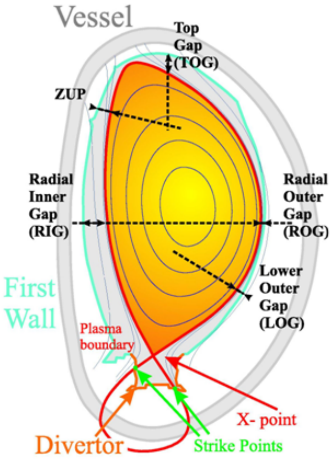
I_p, β_p and l_i are used to specify the current density distribution inside the plasma region.

Plasma axisymmetric model - 2

Model outputs

Different model outputs can be chosen:

- fluxes and fields where the magnetic sensors are located
- currents in the active and passive circuits
- plasma radial and vertical position (1st and 2nd moment of the plasma current density)
- geometrical descriptors describing the plasma shape (gaps, x-point and strike points positions)



Lumped parameters approximation

By using finite-elements methods, **nonlinear** lumped parameters approximation of the PDEs model is obtained

$$\frac{\mathrm{d}}{\mathrm{dt}}\Big[\mathcal{M}\big(\mathbf{y}(t), \beta_p(t), l_i(t)\big)\mathbf{I}(t)\Big] + \mathbf{R}\mathbf{I}(t) = \mathbf{U}(t)\,,$$

$$\mathbf{y}(t) = \mathcal{Y}\big(\mathbf{I}(t), \beta_p(t), l_i(t)\big)\,.$$

where:

- $\mathbf{y}(t)$ are the output to be controlled
- $\mathbf{I}(t) = \big[\mathbf{I}_{PF}^T(t)\ \mathbf{I}_e^T(t)\ I_p(t)\big]^T$ is the currents vector, which includes the currents in the active coils $\mathbf{I}_{PF}(t)$, the eddy currents in the passive structures $\mathbf{I}_e(t)$, and the plasma current $I_p(t)$
- $\mathbf{U}(t) = \big[\mathbf{U}_{PF}^T(t)\ \mathbf{0}^T\ 0\big]^T$ is the input voltages vector
- $\mathcal{M}(\cdot)$ is the mutual inductance nonlinear function
- $\mathbf{R}$ is the resistance matrix
- $\mathcal{Y}(\cdot)$ is the output nonlinear function

Plasma linearized model

Starting from the nonlinear lumped parameters model, the following plasma linearized state space model can be easily obtained:

$$\delta\dot{\mathbf{x}}(t) = \mathbf{A}\delta\mathbf{x}(t) + \mathbf{B}\delta\mathbf{u}(t) + \mathbf{E}\delta\dot{\mathbf{w}}(t), \qquad (1)$$

$$\delta\mathbf{y}(t) = \mathbf{C}\ \delta\mathbf{I}_{PF}(t) + \mathbf{F}\delta\mathbf{w}(t), \qquad (2)$$

where:

- **A**, **B**, **E**, **C** and **F** are the model matrices
- $\delta\mathbf{x}(t) = \left[\delta\mathbf{I}_{PF}^T(t)\ \delta\mathbf{I}_e^T(t)\ \delta I_p(t)\right]^T$ is the state space vector
- $\delta\mathbf{u}(t) = \left[\delta\mathbf{U}_{PF}^T(t)\ \mathbf{0}^T\ 0\right]^T$ are the input voltages variations
- $\delta\mathbf{w}(t) = \left[\delta\beta_p(t)\ \delta l_i(t)\right]^T$ are the β_p and l_i variations
- $\delta\mathbf{y}(t)$ are the output variations

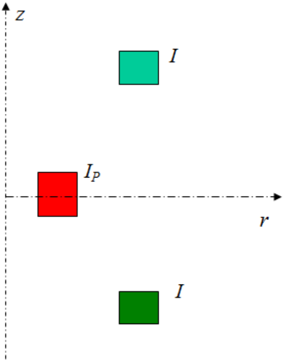
The model (1)–(2) relates the variations of the PF currents to the variations of the outputs around a given equilibrium

Objectives

- Vertically stabilize elongated plasmas in order to avoid disruptions
- Counteract the effect of disturbances (ELMs, fast disturbances modeled as VDEs,. . .)
- It does not control vertical position but it *simply* stabilizes the plasma
- The VS is the essential magnetic control system!

Simplified filamentary model

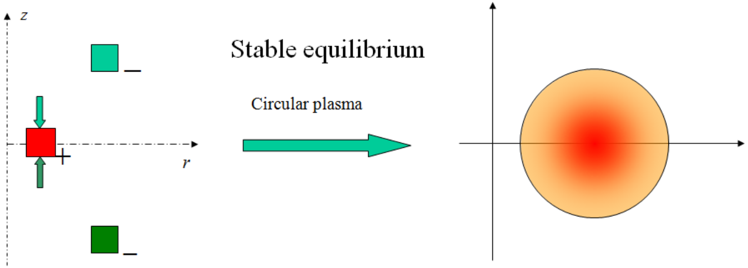
Consider the simplified electromechanical model with three conductive rings, two rings are kept fixed and in symmetric position with respect to the r axis, while the third can freely move vertically.



If the currents in the two fixed rings are equal, the vertical position $z = 0$ is an equilibrium point for the system.

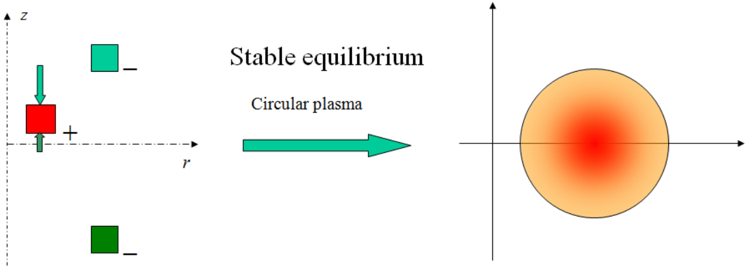
Stable equilibrium - 1

If $\text{sgn}(I_p) \neq \text{sgn}(I)$



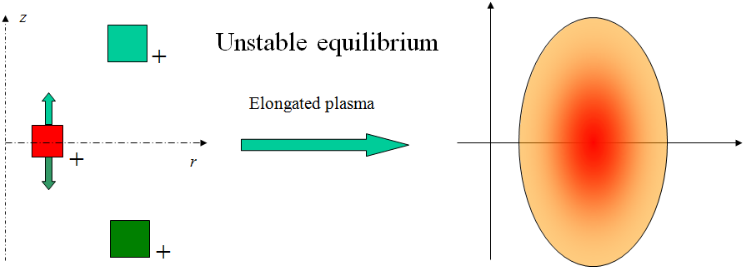
Stable equilibrium - 2

If $\mathrm{sgn}(I_p) \neq \mathrm{sgn}(I)$



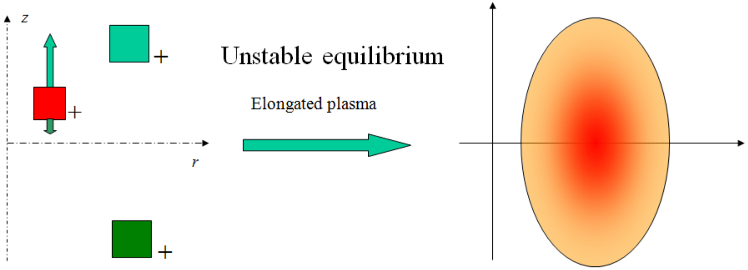
Unstable equilibrium - 1

If $\text{sgn}(I_p) = \text{sgn}(I)$



Unstable equilibrium - 2

If $\mathrm{sgn}(I_p) = \mathrm{sgn}(I)$

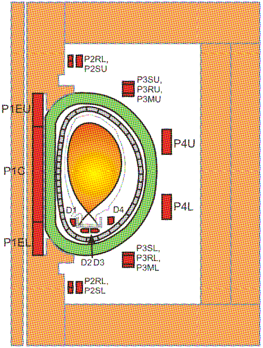


Plasma vertical instability

- The plasma vertical instability reveals itself in the linearized model, by the presence of an unstable eigenvalue in the dynamic system matrix
- The vertical instability growth time is slowed down by the presence of the conducting structure surrounding the plasma
- This allows to use a feedback control system to stabilize the plasma equilibrium, using for example a pair of dedicated coils
- This feedback loop usually acts on a faster time-scale than the plasma shape control loop

Plasma Shape Control

- The problem of controlling the plasma shape is probably the most understood and mature of all the control problems in a tokamak
- The actuators are the Poloidal Field coils, that produce the magnetic field acting on the plasma
- The controlled variables are a finite number of geometrical descriptors chosen to describe the plasma shape



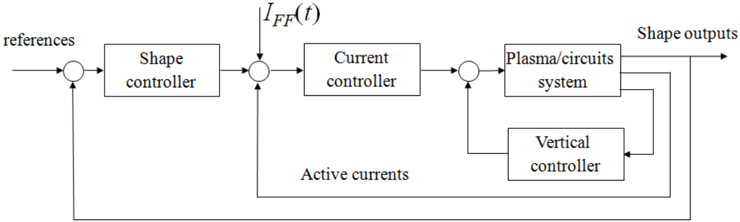
Objectives

- Precise control of plasma boundary
- Counteract the effect of disturbances (β_p and l_i variations)
- Manage saturation of the actuators (currents in the PF coils)

Plasma current control

- Plasma current can be controlled by using the current in the PF coils
- Since there is a sharing of the actuators, the problem of tracking the plasma current is often considered simultaneously with the shape control problem
- The PF coils have to generate a magnetic flux in order to drive ohmic current into the plasma
- Shape control and plasma current control are compatible, since it is possible to show that generating flux that is spatially uniform across the plasma (but with a desired temporal behavior) can be used to drive the current without affecting the plasma shape.

Control scheme



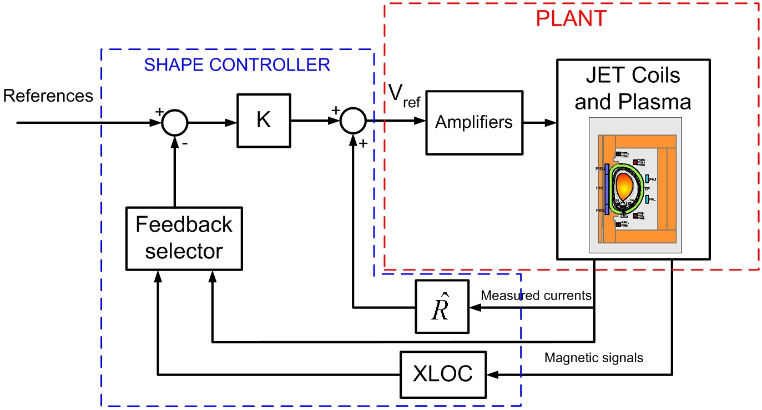
- The scenario is usually specified in terms of feed-forward currents $I_{FF}(t)$.
- It is convenient that the SC generates current references
- A PF currents controller must be designed

Plasma shape control at the JET tokamak

Two different shape controllers are available at the JET tokamak

- the *standard* Shape Controller (SC). This controller can be set in *full current control mode* (acting as a PF currents controler)
- the eXtreme Shape Controller (XSC)

JET Shape Controller - Controller Scheme



JET Shape Controller Design

Plasmaless model

$$\mathbf{V}_{PF} = \begin{bmatrix} L_1 & M_{12} & \dots & M_{1N} \\ M_{12} & L_2 & \dots & M_{2N} \\ \dots & \dots & \dots & \dots \\ M_{1N} & M_{2N} & \dots & L_N \end{bmatrix} \frac{\mathrm{d}\mathbf{I}_{PF}}{\mathrm{dt}} + \begin{bmatrix} R_1 & 0 & \dots & 0 \\ 0 & R_2 & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & R_N \end{bmatrix} \mathbf{I}_{PF}$$

Resistive compensation

$$\mathbf{V}_{PF_{ref}} = \hat{\mathbf{R}}\mathbf{I}_{PF} + \mathbf{K}(\mathbf{Y}_{ref} - \mathbf{Y})$$

Static relationship between PF coils current and controlled variables

$$\mathbf{Y} = \mathbf{T}\mathbf{I}_{PF}$$

Control Matrix

$$\mathbf{K} = \hat{\mathbf{M}}\mathbf{T}^{-1}\Lambda^{-1} \text{ with } \Lambda \text{ diagonal matrix}$$

Closed-loop system

$$\mathbf{MT}^{-1}\dot{\mathbf{Y}} + \mathbf{RI}_{PF} = \mathbf{MT}^{-1}\Lambda^{-1}(\mathbf{Y}_{ref} - \mathbf{Y}) + \mathbf{RI}_{PF} \Rightarrow$$
$$\Rightarrow \ \dot{\mathbf{Y}} = \Lambda^{-1}(\mathbf{Y}_{ref} - \mathbf{Y})$$

By a proper choice of the **T** matrix it is possible to achieve:

- current control mode
- plasma current control mode
- gap control mode

F. Sartori, G. De Tommasi, F. Piccolo
The Joint European Torus
IEEE Control Systems Magazine, April 2006

JET Shape Controller

- Each circuit is used to control a single variable (current, gap, flux)
- Up to 9 different variables can be controlled
- Since plasma current is always controlled, up to 8 gaps can be controlled

XSC "philosophy"

- To control the plasma shape in JET, in principle 8 *knobs* are available, namely the currents in the PF circuits except $P1$ which is used only to control the plasma current
- As a matter of fact, these 8 knobs do not practically guarantee 8 degrees of freedom to change the plasma shape
- Indeed there are 2 or 3 current combinations that cause small effects on the shape (depending on the considered equilibrium).
- **The design of the XSC is model-based. Different controller gains must be designed for each different plasma equilibrium, in order to achieve the desired performances**

SC in current control mode

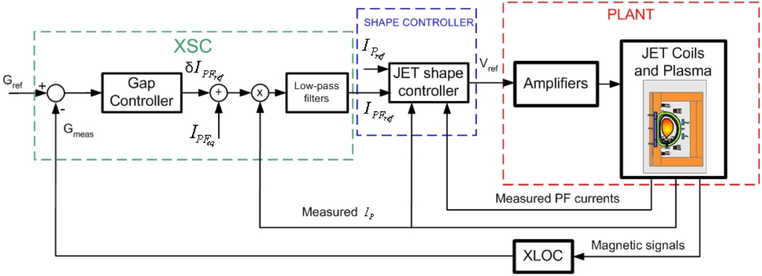
The XSC exploits the standard JET Shape Controller architecture. In particular it sets:

- the P1 circuit in *plasma current control mode*
- the other 8 PF circuits in *current control mode*

Model of the current controlled plant

$$\delta \mathbf{g}(s) = \frac{\widetilde{\mathbf{C}}}{1 + s\tau} \cdot \frac{\delta \mathbf{I}_{PF_{REF}}(s)}{I_P}$$

XSC - Controller scheme



eXtreme Shape Controller (XSC)

- The *eXtreme Shape Controller* (XSC) controls the whole plasma shape, specified as a set of 32 geometrical descriptors, calculating the PF coil current references.
- Let $\mathbf{I}_{PF_N}(t)$ be the PF currents normalized to the equilibrium plasma current, it is

$$\delta\mathbf{g}(t) = \mathbf{C}\ \delta\mathbf{I}_{PF_N}(t).$$

  It follows that the plasma boundary descriptors have the same dynamic response of the PF currents.
- The XSC design has been based on the **C** matrix. Since the number of independent control variables is less than the number of outputs to regulate, it is not possible to track a generic set of references with zero steady-state error.

$$\delta\mathbf{I}_{PF_{N_{req}}} = \mathbf{C}^{\dagger}\delta\mathbf{g}_{error}$$

eXtreme Shape Controller (XSC)

- The XSC has then been implemented introducing weight matrices both for the geometrical descriptors and for the PF coil currents.
- The determination of the controller gains is based on the Singular Value Decomposition (SVD) of the following weighted output matrix:

$$\widetilde{\mathbf{C}} = \widetilde{\mathbf{Q}}\,\mathbf{C}\,\widetilde{\mathbf{R}}^{-1} = \widetilde{\mathbf{U}}\,\widetilde{\mathbf{S}}\,\widetilde{\mathbf{V}}^{T},$$

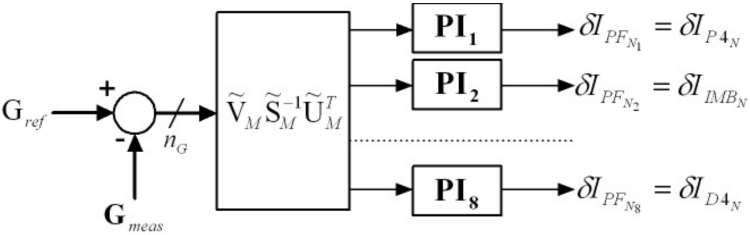
  where $\widetilde{\mathbf{Q}}$ and $\widetilde{\mathbf{R}}$ are two diagonal matrices.
- The XSC minimizes the cost function

$$\widetilde{J}_1 = \lim_{t\to+\infty} (\delta\mathbf{g}_{ref} - \delta\mathbf{g}(t))^T \widetilde{\mathbf{Q}}^T \widetilde{\mathbf{Q}} (\delta\mathbf{g}_{ref} - \delta\mathbf{g}(t)),$$

  using $\bar{n} < 8$ degrees of freedom, while the remaining $8 - \bar{n}$ degrees of freedom are exploited to minimize

$$\widetilde{J}_2 = \lim_{t\to+\infty} \delta\mathbf{I}_{PF_N}(t)^T \widetilde{\mathbf{R}}^T \widetilde{\mathbf{R}} \delta\mathbf{I}_{PF_N}(t).$$

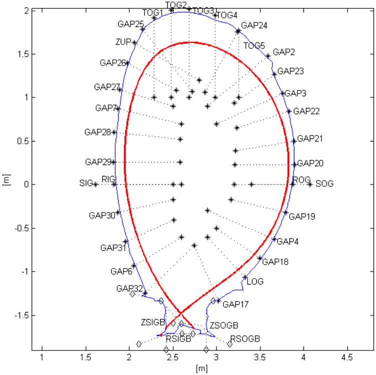
XSC - Gap controller



Shape Controller

SC

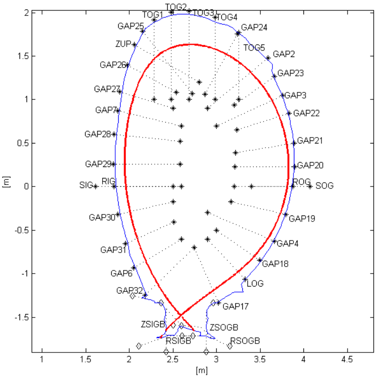
- The desired shape is achieved controlling few shape descriptors with dedicated coils (e.g. ROG with P4 and strike points with D1-D4) and by precalculating the remaining currents
- This gives a good tracking of the references on the controlled shape descriptors (e.g. ROG and strike points) but the whole shape cannot be controlled precisely
- Shape modifications due to variations of β_p and l_i are usually counteracted by the precalculated current waveforms



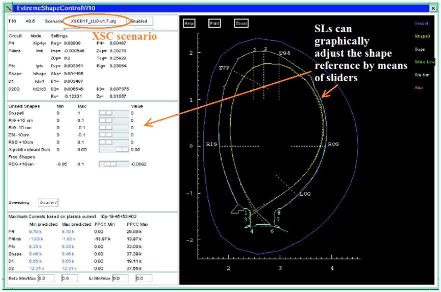
eXtreme Shape Controller

XSC

- Allows to directly specify the target shape, without specifying the PF current waveforms
- The PF current waveforms are *automatically* computed by the control algorithm as the "smallest" currents needed to minimize the error on the shape in *least mean square sense*
- The controller manages to keep the shape "constant" (in least mean square sense) even in the presence β_p and l_i variations



XSC and CLA

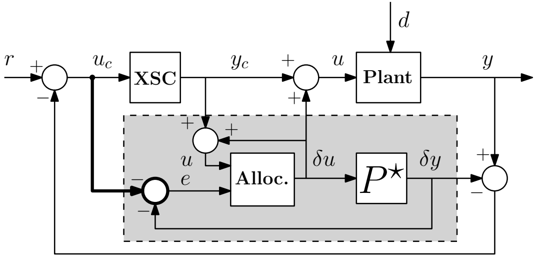


- The XSC allows the SLs to directly specify the target shape, without specifying the PF current waveforms
- The PF current waveforms are *automatically* computed by the model-based control algorithm
- The PF currents may saturate during the experiment
- The Current Limit Avoidance System (CLA) has been recently designed and implemented **to avoid current saturations in the PF coils when the XSC is used to control the plasma shape**

The Current Limit Avoidance System - 1

- The CLA uses the redundancy of the PF coils system to automatically obtain almost the same plasma shape with a different combination of currents in the PF coils
- In the presence of disturbances (e.g., variations of the internal inductance l_i and of the poloidal beta β_p), it tries to avoid the current saturations by "relaxing" the plasma shape constraints
- **Thanks to the CLA safe operations can be guaranteed**

The Current Limit Avoidance System - 2



- The proposed current allocation scheme aims keeping the value of the plant inputs (PF currents) inside a desirable region, meanwhile ensuring a small tracking error on the plasma shape *at steady state*
- $P^\star$ is the plant steady-state gain

The Current Limit Avoidance System - 3

The allocator equations are given by

$$\dot{x}_a = -KB_0^T \begin{bmatrix} I \\ P^\star \end{bmatrix}^T (\nabla J)^T \Big|_{(u\,,e)}, \tag{3a}$$

$$\delta u = B_0 x_a, \tag{3b}$$

$$\delta y = P^\star B_0 x_a \tag{3c}$$

- $J(u^\star\,, e^\star)$ is a continuously differentiable cost function that penalizes (at steady-state)
  - large PF currents
  - large plasma shape error
- The key property of the current allocator algorithm (3) is that, for each constant current request of the XSC, it has a unique globally asymptotically stable equilibrium $x_a^\star$ coinciding with the unique global minimizer $J(\cdot\,,\cdot)$

Comparison between SC and XSC

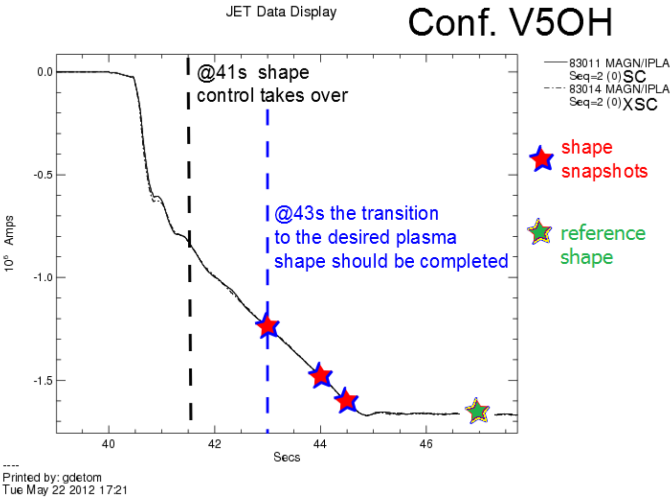
The following pulses are considered in order to compare the behavior of the two plasma shape controllers during the I_p ramp-up

- #83011 – with SC
- #83014 – with XSC

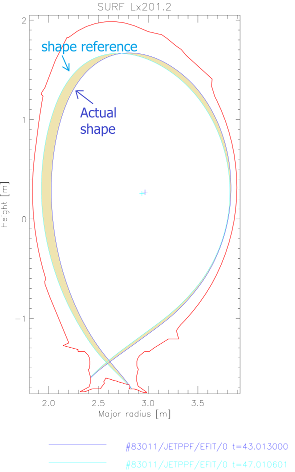
while the comparison during the I_p ramp-down is done considering the pulses

- #72203 – with SC
- #83014 – with XSC

Pulses #83011 and #83014 - I_p ramp-up



#83011 - Shape tracking during the ramp-up with SC



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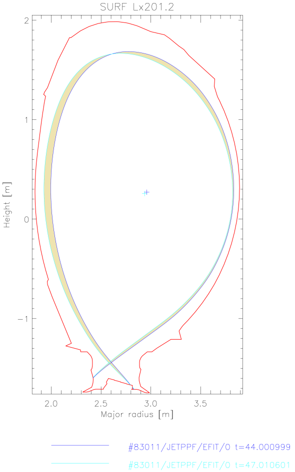

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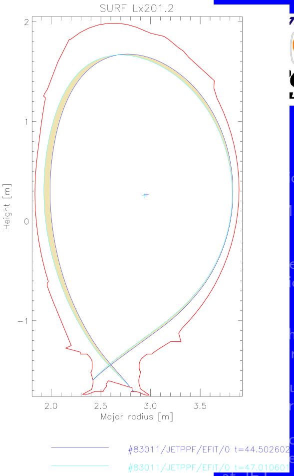
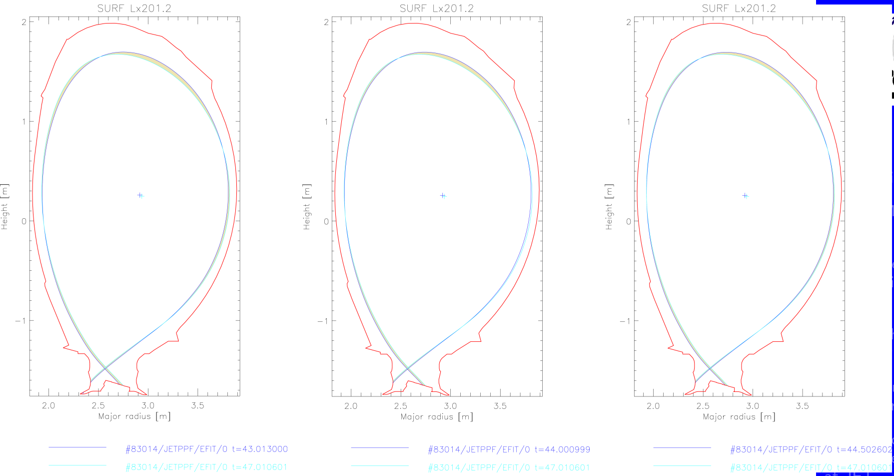


@44.5s

#83011 - Comments

- Bad shape control in the inner side.
- This is mainly due to the fact that P4 is used to control ROG, while RIG is not controlled

#83014 - Shape tracking during the ramp-up with XSC

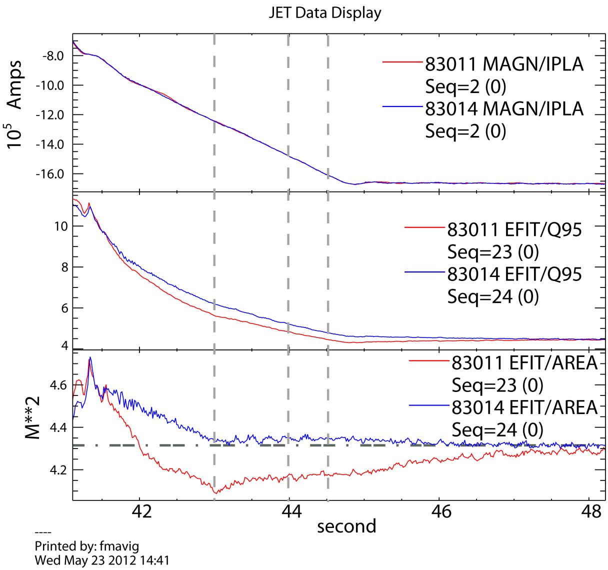


@43s @44s @44.5s

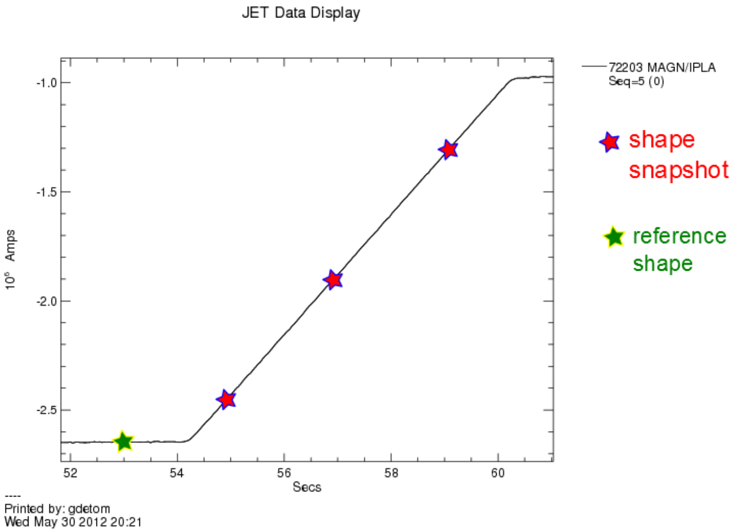
#83014 - Comments

- The biggest error in shape control is in the top outer region (remember the XSC minimizes the shape error in least mean square sense!)
- This error could be reduced by increasing the error in a different region (i.e. in the divertor region)
- Good shape tracking in both RIG and ROG regions, and good tracking of strike points and x-point position

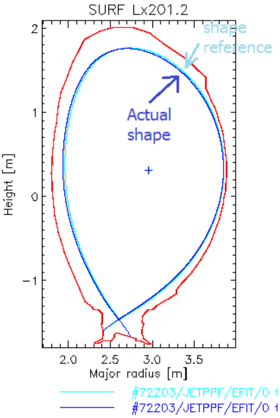
Plasma surface and q95



Pulse #72203 - I_p ramp-down with SC



#72203 - Shape tracking during the ramp-down with SC



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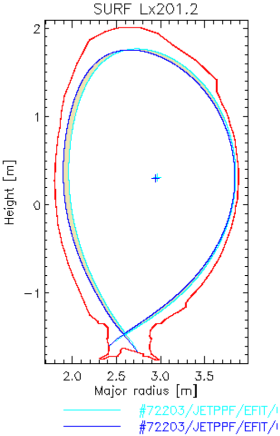

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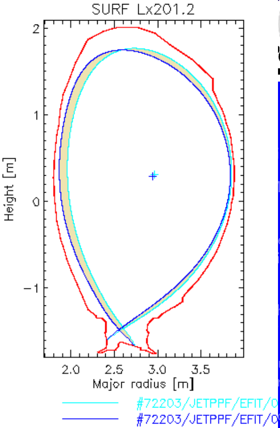

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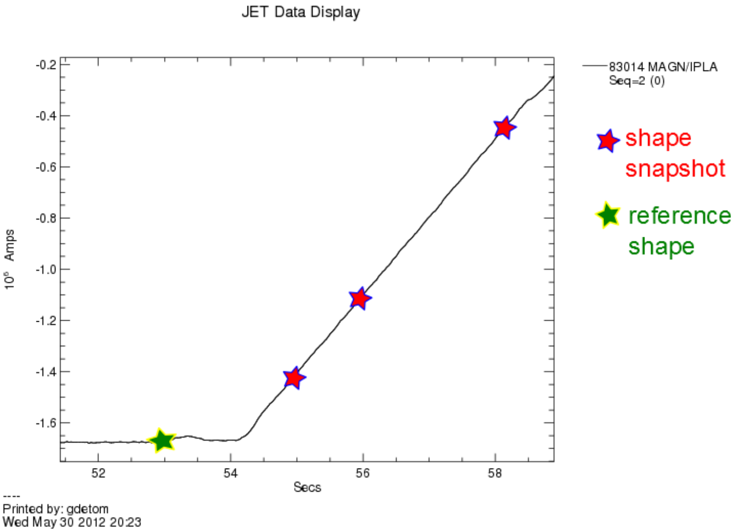
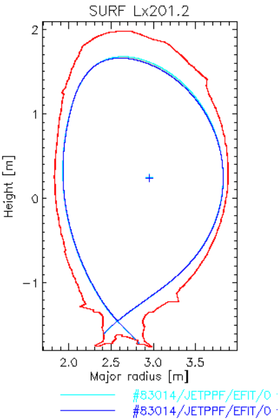
Pulse #83014 - I_p ramp-down with XSC



#83014 - Shape tracking during the ramp-down with SSC



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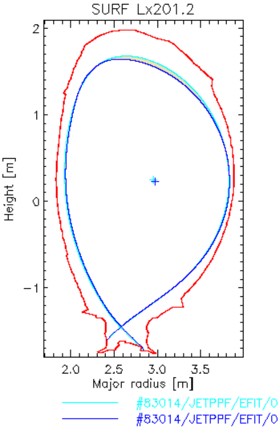

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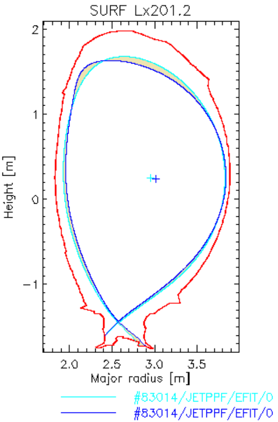

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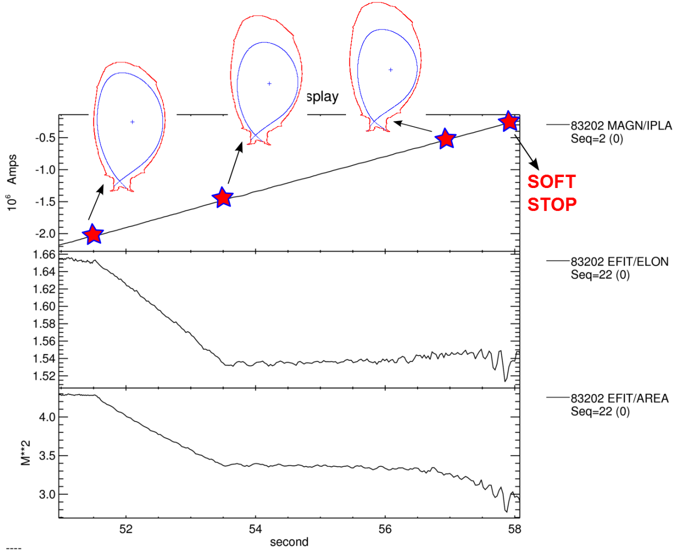
Change of elongation during the plasma current ramp-down



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Conclusions

- An overview of the three basic magnetic control problems has been given:
  - Vertical Stabilization
  - Shape Control
  - Plasma Current Control
- The solution adopted at the JET tokamak for plasma current and shape control have been introduced

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...you can have more at

- http://wpage.unina.it/detommas/ijs.html

References

Plasma magnetic modeling and control

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