A Newton method with always feasible iterates for Nonlinear MPC of walking in a multi-contact situation

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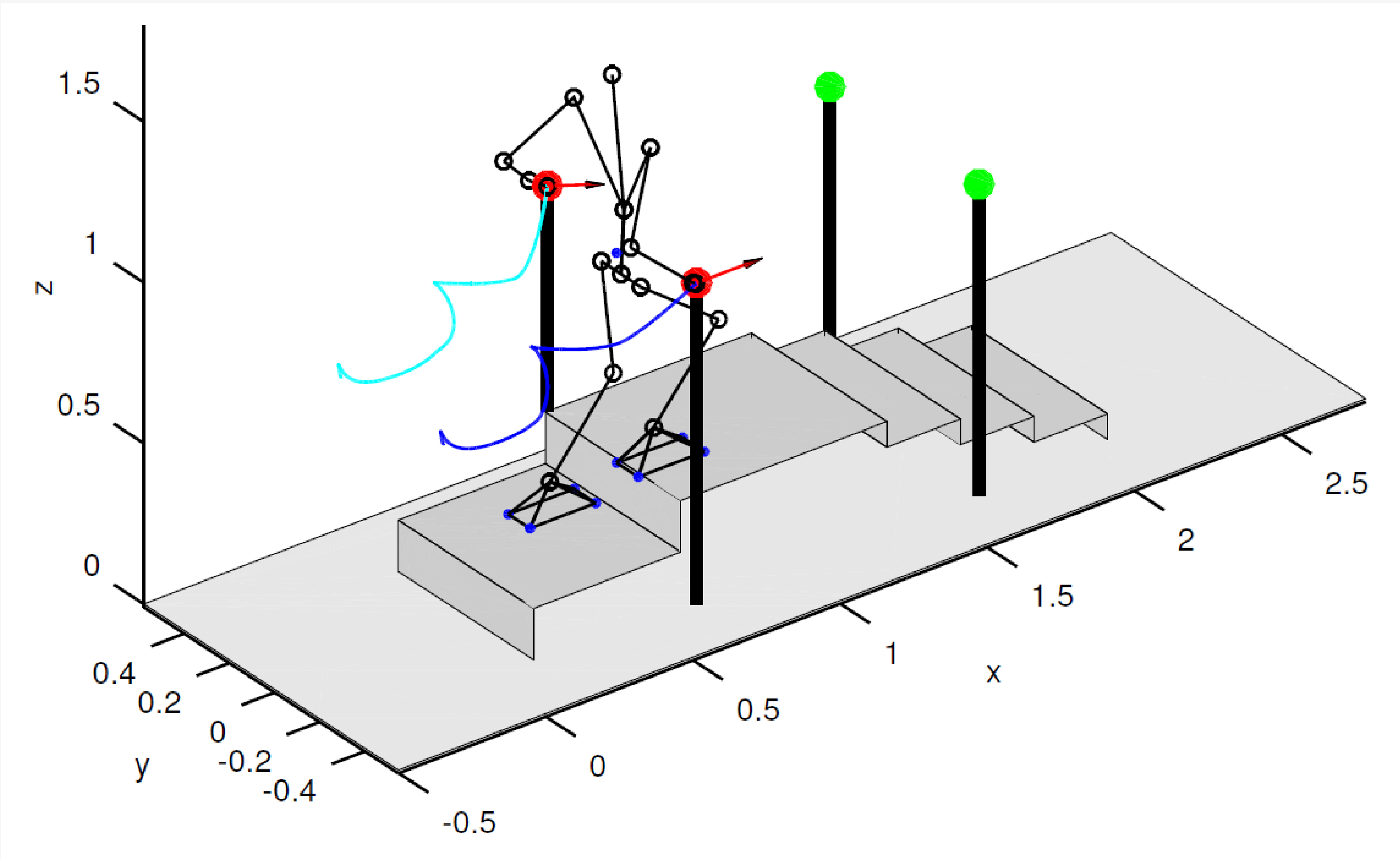
Methodology and results

- Nonlinear structure of the momentum dynamics is considered.
- Ground contact forces are separated from the other external forces and torques.
- Kinematic and dynamic feasibility constraints are enforced in each iteration of the Newton method.
- Simulations of walking up and down stairs with hand supports demonstrate the validity of the approach.



Dynamic model

- Newton-Euler equations:

$$m(\ddot{c} + g) = f_e + \sum_i f_i,$$

$$mc \times (\ddot{c} + g) + \dot{L} = n_e + c \times f_e + \sum_i s_i \times f_i,$$

- Center of pressure with external force f_e and torque: $\tilde{n}$

$$p = c^{xy} - \frac{c^z - s^z}{m(\ddot{c}^z + g^z) - f_e^z}(m\ddot{c}^{xy} - f_e^{xy}) - \frac{\Omega}{m(\ddot{c}^z + g^z) - f_e^z}\tilde{n}^{xy}.$$

Robustness to polytopic uncertainties

- The center of pressure is represented as:

$$p(\zeta_1, \zeta_2) = c^{xy} - \zeta_1\left(\ddot{c}^{xy} - \frac{f_e^{xy}}{m}\right) + \zeta_2 \Omega\, \tilde{n}^{xy},$$

- Linear with respect to ζ_1 and ζ_2,

$$\zeta_1 = \frac{m(c^z - s^z)}{m(\ddot{c}^z + g^z) - f_e^z}, \quad \zeta_2 = \frac{1}{m(\ddot{c}^z + g^z) - f_e^z}.$$

- We bound ζ_1 and ζ_2:

$$0 \le \underline{\zeta}_1 \le \zeta_1 \le \bar{\zeta}_1, \quad 0 \le \underline{\zeta}_2 \le \zeta_2 \le \bar{\zeta}_2.$$

- So we have:

$$\{p(\underline{\zeta}_1, \underline{\zeta}_2),\ p(\underline{\zeta}_1, \bar{\zeta}_2),\ p(\bar{\zeta}_1, \underline{\zeta}_2),\ p(\bar{\zeta}_1, \bar{\zeta}_2)\} \subset \mathcal{S}(s_i^{xy})$$
$$\Downarrow$$
$$p(\zeta_1, \zeta_2) \in \mathcal{S}(s_i^{xy}).$$

- Bounds are updated at each iteration from the previous iterate:

$$\underline{\zeta}_1 = \zeta_1^{(j-1)} - k\mu_1 \le \zeta_1^{(j)} \le \zeta_1^{(j-1)} + k\mu_1 = \bar{\zeta}_1$$
$$\underline{\zeta}_2 = \zeta_2^{(j-1)} - k\mu_2 \le \zeta_2^{(j)} \le \zeta_2^{(j-1)} + k\mu_2 = \bar{\zeta}_2$$

Optimal control problem

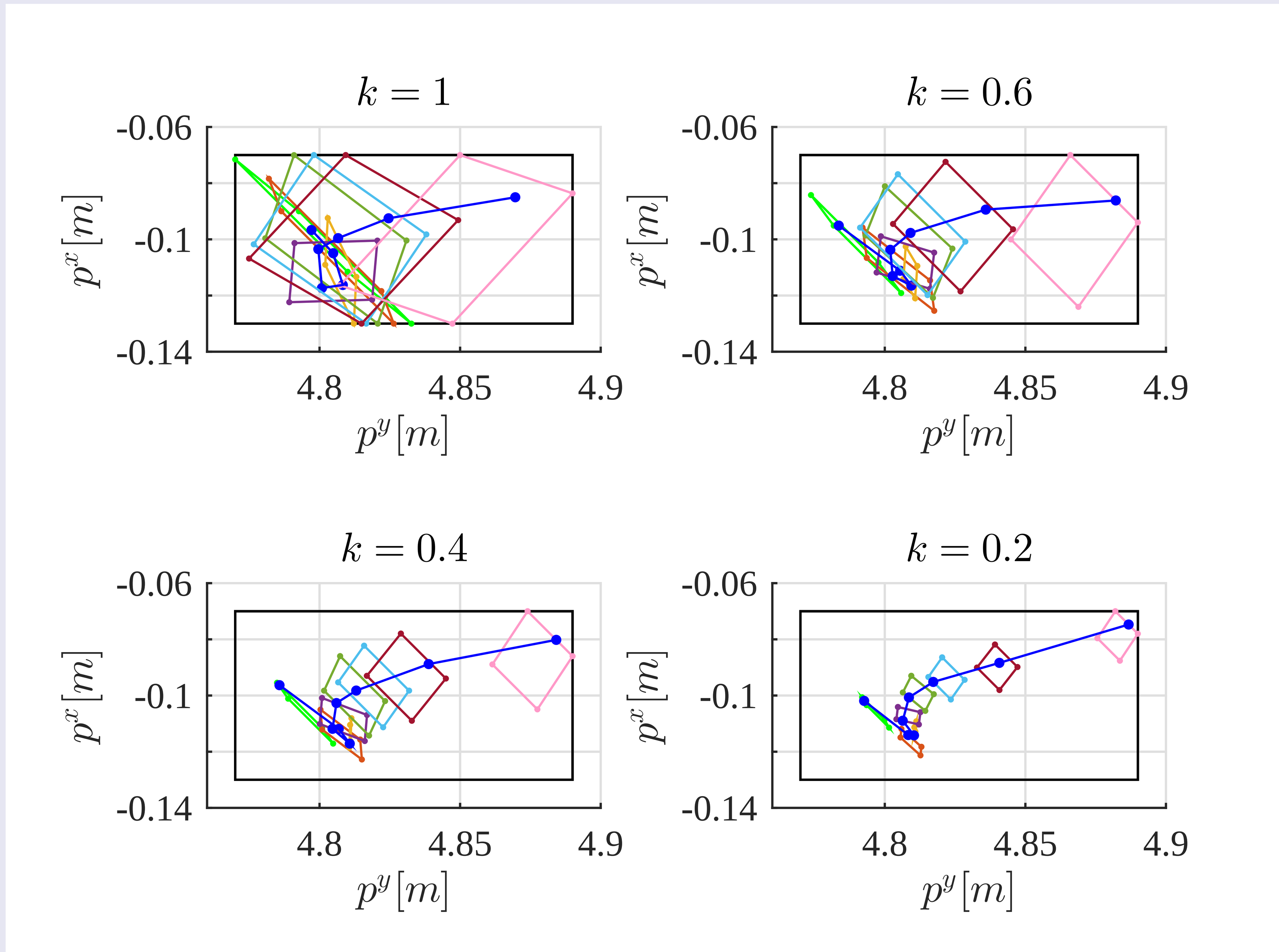
- Linear constraints:
  - (1,2) Dynamic and kinematic feasibility of motion;
  - (3,4) Simple bounds on the external wrench.
- Objective function:
  - (1) Smoothness of motion of the center of mass;
  - (2) Reference for the height of the center of mass;
  - (3) Reference for the center of pressure;
  - (4,5) Minimum external wrench.
- Proposed optimal control problem:

$$\min_{\dddot{c}, f_e, \tilde{n}} \int_0^{T_h} w_1\|\dddot{c}\|^2 + w_2\|c_s^z - \bar{c}_s^z\|^2 + w_3\|p - s_f\|^2 + w_4\|f_e\|^2 + w_5\|\tilde{n}\|^2 dt$$

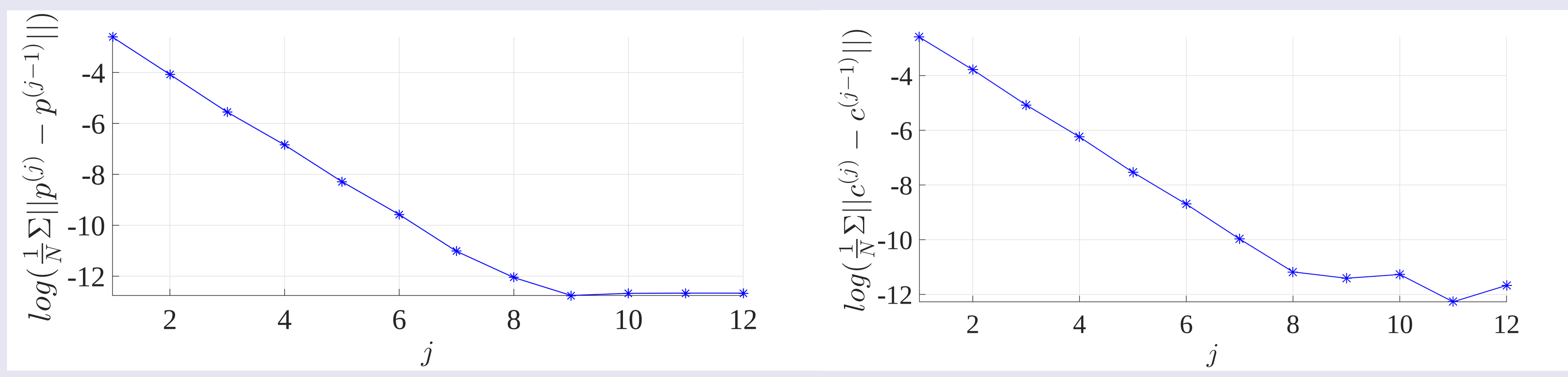
s.t.

$$(1): p \in \mathcal{S}(s_i^{xy})$$
$$(2): A(c - s_f) \le b$$
$$(3): \underline{f}_e \le f_e \le \bar{f}_e$$
$$(4): \underline{\tilde{n}} \le \tilde{n} \le \bar{\tilde{n}}$$

Simulations



- Center of pressure (blue line) always inside quadrilateral defined by four extreme points $\{p(\underline{\zeta}_1, \underline{\zeta}_2), p(\underline{\zeta}_1, \bar{\zeta}_2), p(\bar{\zeta}_1, \underline{\zeta}_2), p(\bar{\zeta}_1, \bar{\zeta}_2)\}$, which are all inside support polygon $\mathcal{S}(s_i^{xy})$. (black rectangle)
- Larger uncertainty k leads to more conservative constraints.



- Center of pressure and center of mass within 10^{-3} m of their optimal values in just 1 iteration.
- Solutions are obtained in approximately 8 ms - average time over the whole trajectory generation.

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