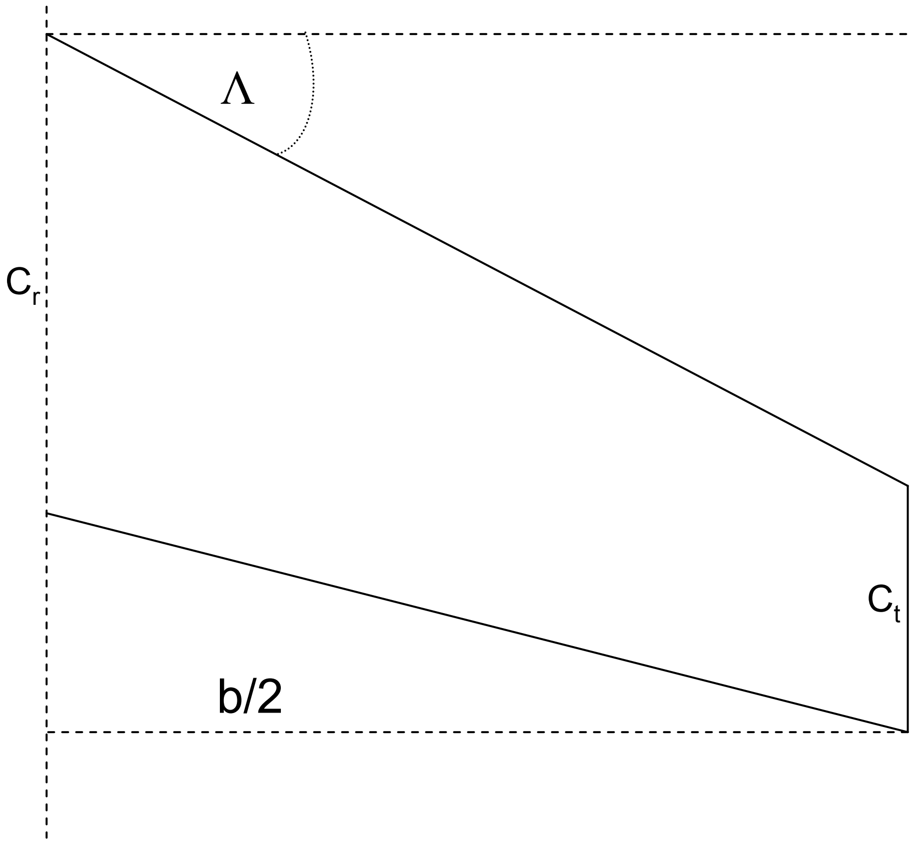
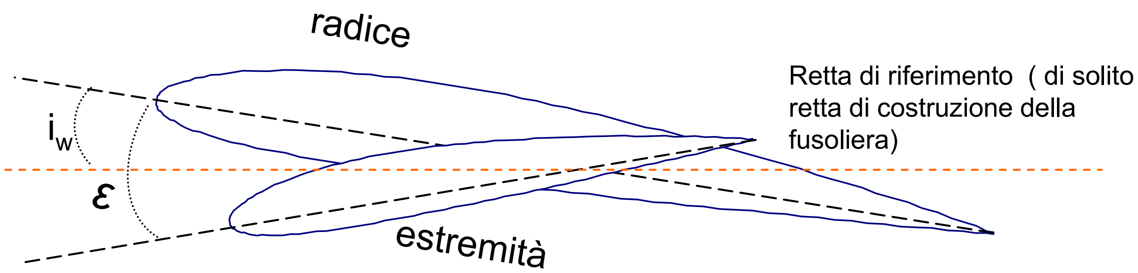
Dati





b=10 m

Cr=1.6 m

λ=0.5

Λ=15 deg

i_w=3 deg.

$e_{w=}$0.9

Profilo alla radice

$Cl_{\alpha\ 2dr}$=0.11 deg^{-1}

Cm_{02dr}=-0.07

α_{0lr}=-1.0 deg

$x_{acr=}0.25\ C_r$

Profilo all'estremità

$Cl_{\alpha 2dt}$=0.105 deg^{-1}

Cm_{02dt}=-0.08

α_{0lt}=-2.5 deg

ε_t= -3 deg

$x_{act}=0.23\ Ct$

$$\lambda = \frac{C_t}{C_r} \Rightarrow C_t = C_r \cdot \lambda = 0.64\, m$$

$$S_w = 2 \cdot \int_0^{\frac{b}{2}} C(y) \cdot dy = (C_t + C_r) \cdot \frac{b}{2} = 13.44\, m^2$$

$$AR = \frac{b^2}{S} = 10.7$$

Corda media geometrica

$$C_{mg} = \frac{2}{b} \cdot \int_0^{\frac{b}{2}} C(y) \cdot dy = \frac{S_w}{b}$$

$$C_{mg} = \frac{2}{b} \cdot \int_0^{\frac{b}{2}} C(y) \cdot dy = \frac{2}{b} \cdot \int_0^{\frac{b}{2}} C_r + \left(\frac{C_t - C_r}{b/2} \right) \cdot y \cdot dy = \frac{2}{b} \cdot \left[C_r \cdot y + \left(\frac{C_t - C_r}{b/2} \right) \cdot \frac{y^2}{2} \right]_0^{b/2} =$$

$$= C_r + \left(\frac{C_t - C_r}{2} \right) = C_r \left(\frac{1 + \lambda}{2} \right) = 1.2\, m$$

Corda media aerodinamica

$$C_{ma}=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}C(y)^2\cdot dy$$

In caso di ala trapezia

$$C_{ma}=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}C(y)^2\cdot dy=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left(C_r+\left(\frac{C_t-C_r}{b/2}\right)\cdot y\right)^2\cdot dy=$$

$$\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left(C_r{}^2+\left(\frac{C_t-C_r}{b/2}\right)^2\cdot y^2+2\cdot C_r\cdot\left(\frac{C_t-C_r}{b/2}\right)\cdot y\right)\cdot dy=$$

$$\frac{2}{b\cdot C_{mg}}\cdot\left(C_r{}^2\cdot\frac{b}{2}+\left(\frac{C_t-C_r}{b/2}\right)^2\cdot\frac{1}{3}\cdot\frac{b^3}{8}+2\cdot C_r\cdot\left(\frac{C_t-C_r}{b/2}\right)\cdot\frac{1}{2}\cdot\frac{b^2}{4}\right)=$$

$$\frac{1}{C_{mg}}\cdot\left(C_r{}^2+(C_t-C_r)^2\cdot\frac{1}{3}+C_r\cdot(C_t-C_r)\right)=$$

$$\frac{1}{C_{mg}}\cdot\left(C_r{}^2+(C_t{}^2-2\cdot C_rC_t+C_r{}^2)\cdot\frac{1}{3}+C_rC_t-C_r{}^2\right)=$$

$$\frac{1}{C_{mg}}\cdot(C_t{}^2+C_rC_t+C_r{}^2)\cdot\frac{1}{3}=\boxed{\frac{2}{3}C_r\left(\frac{\lambda^2+\lambda+1}{\lambda+1}\right)}=1.24\,m$$

Distribuzioni in apertura delle grandezze

$$C(y) = Cr + \frac{Ct - Cr}{b/2} \cdot y = 1.6m - 0.16 \cdot y$$

$$\varepsilon(y) = \varepsilon_r + \frac{\varepsilon_t - \varepsilon_t}{b/2} \cdot y = -0.6 \frac{\deg}{m} \cdot y$$

$$\alpha_{0l}(y) = \alpha_{0lr} + \frac{\alpha_{0lt} - \alpha_{0lr}}{b/2} \cdot y = -1\deg - 0.3 \frac{\deg}{m} \cdot y$$

$$Cl_\alpha(y) = Cl_{\alpha r} + \frac{Cl_{\alpha t} - Cl_{\alpha r}}{b/2} \cdot y = 0.11\deg^{-1} - 0.8 \cdot 10^{-3} \frac{\deg^{-1}}{m} y$$

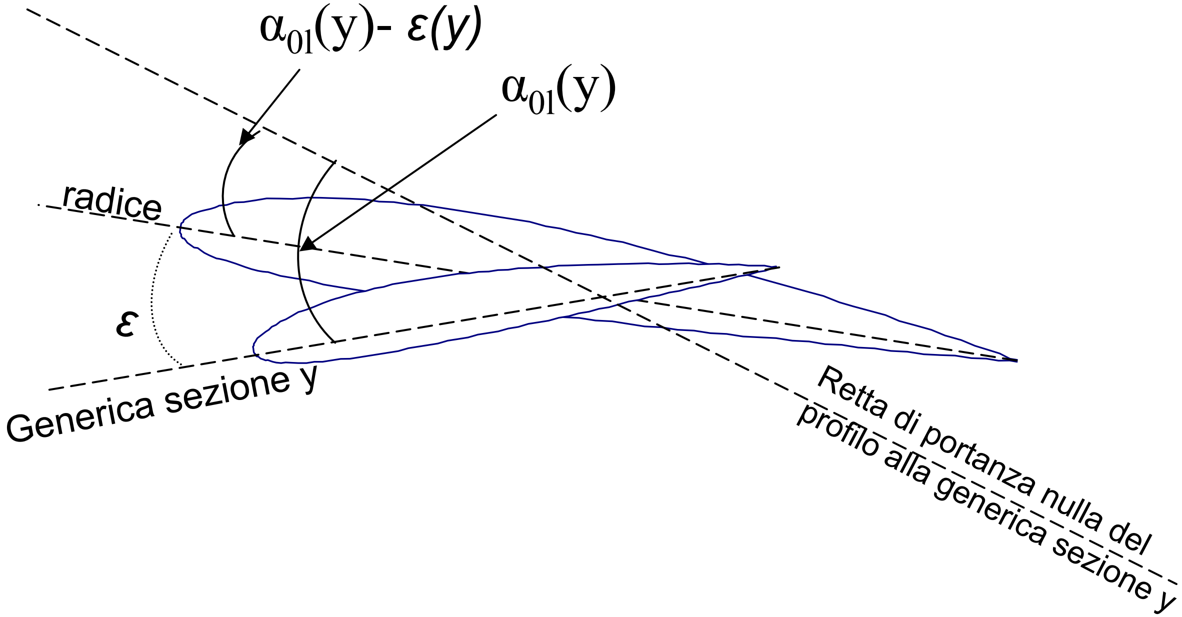
Gradiente della retta di portanza dell'ala

$$Cl_{\alpha 2d} = \frac{2}{S}\int_0^{b/2} Cl_\alpha(y)\cdot C(y)dy = 0.108\,\deg^{-1}$$

$$Cl_{\alpha w} = \frac{Cl_{\alpha 2d}}{1+\frac{Cl_{\alpha 2d}}{\pi\cdot AR\cdot e_w}} = 0.09\,\deg^{-1}$$

α_{0L} – Angolo di portanza nulla dell'ala finita

$$\alpha_{0L} = \frac{2}{S_w} \cdot \int_0^{\frac{b}{2}} \left[\alpha_{0l}(y) - \varepsilon(y)\right] \cdot C(y) \cdot dy = -0.334\,\mathrm{deg}$$



α_{0L} – *Angolo di portanza nulla dell’ala finita*

$$\alpha_{0L}=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left[\alpha_{0l}(y)-\varepsilon(y)\right]\cdot C(y)\cdot dy=$$

$$=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left[\left(\alpha_{0l\,r}+\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot y\right)-\left(\varepsilon_r+\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot y\right)\right]\left(C_r+\left(\frac{C_t-C_r}{b/2}\right)\cdot y\right)\cdot dy=$$

$$=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left[-\alpha_{0l\,r}\cdot\varepsilon_r-\alpha_{0l\,r}\cdot\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot y-\varepsilon_r\cdot\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot y-\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot y^2\right]\left(C_r+\left(\frac{C_t-C_r}{b/2}\right)\cdot y\right)\cdot dy=$$

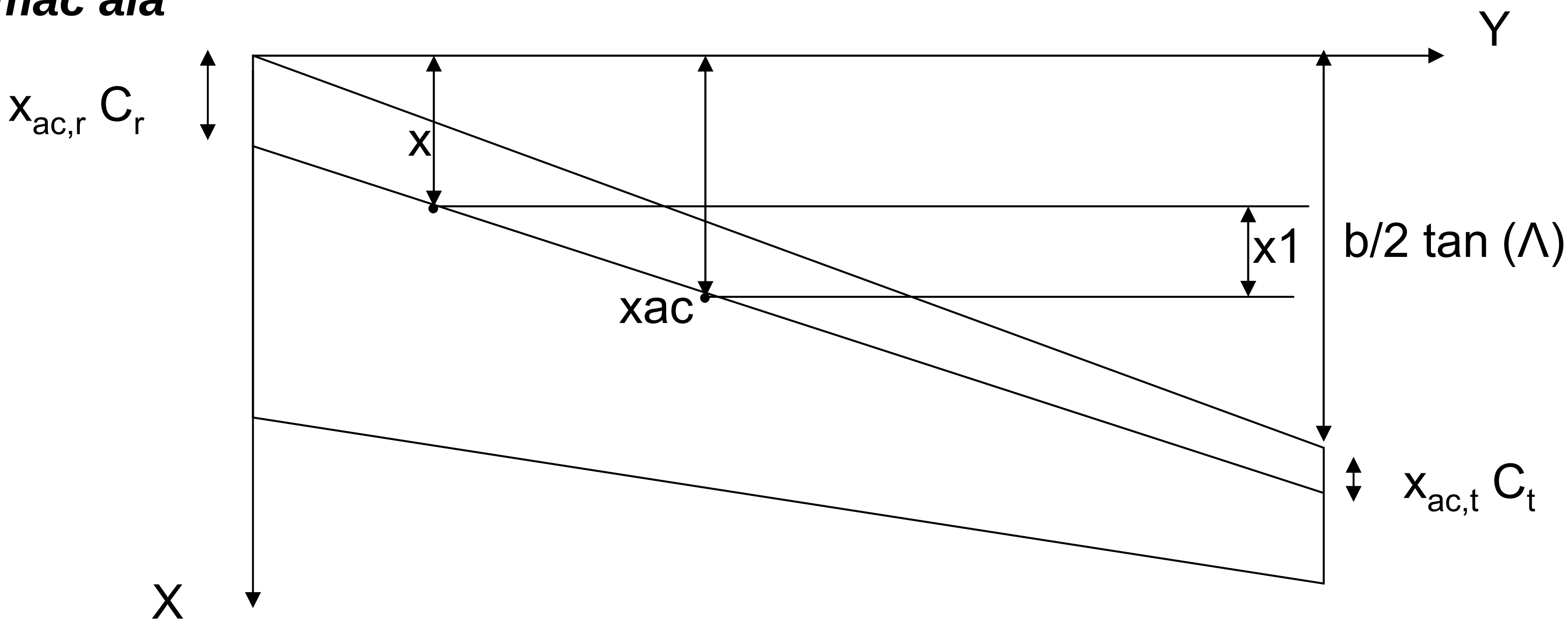
$$=\frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left\{\begin{array}{l}\left[-\alpha_{0l\,r}\cdot\varepsilon_{r\,r}\cdot C_r-\alpha_{0l\,r}\cdot\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot C_r\cdot y-\varepsilon_r\cdot\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot C_r\cdot y-\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot C_r\cdot y^2\right]+\\ \left[-\alpha_{0l\,r}\cdot\varepsilon_{r\,r}\cdot\left(\frac{C_t-C_r}{b/2}\right)\cdot y-\alpha_{0l\,r}\cdot\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot\left(\frac{C_t-C_r}{b/2}\right)\cdot y^2-\varepsilon_r\cdot\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot\left(\frac{C_t-C_r}{b/2}\right)\cdot y^2-\left(\frac{\alpha_{0l\,t}-\alpha_{0l\,r}}{b/2}\right)\cdot\left(\frac{\varepsilon_t-\varepsilon_r}{b/2}\right)\cdot\left(\frac{C_t-C_r}{b/2}\right)\cdot y^3\right]\end{array}\right\}\cdot dy=$$

$$=\frac{1}{C_{mg}\cdot b/2}\cdot\left\{\begin{array}{l}\left[-\alpha_{0l\,r}\cdot\varepsilon_{r\,r}\cdot C_r\cdot\frac{b}{2}-\alpha_{0l\,r}\cdot(\varepsilon_t-\varepsilon_r)\cdot C_r\cdot\frac{b/2}{2}-\varepsilon_r\cdot(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot C_r\cdot\frac{b/2}{2}-(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot(\varepsilon_t-\varepsilon_r)\cdot C_r\cdot\frac{b/2}{3}\right]+\\ \left[-\alpha_{0l\,r}\cdot\varepsilon_{r\,r}\cdot(C_t-C_r)\cdot\frac{b/2}{2}-\alpha_{0l\,r}\cdot(\varepsilon_t-\varepsilon_r)\cdot(C_t-C_r)\cdot\frac{b/2}{3}-\varepsilon_r\cdot(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot(C_t-C_r)\cdot\frac{b/2}{3}-(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot(\varepsilon_t-\varepsilon_r)\cdot(C_t-C_r)\cdot\frac{b/2}{4}\right]\end{array}\right\}=$$

$$=\frac{1}{C_{mg}}\cdot\left\{\begin{array}{l}\left[-\alpha_{0l\,r}\cdot\varepsilon_{r\,r}\cdot C_r-\alpha_{0l\,r}\cdot(\varepsilon_t-\varepsilon_r)\cdot C_r\cdot\frac{1}{2}-\varepsilon_r\cdot(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot C_r\cdot\frac{1}{2}-(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot(\varepsilon_t-\varepsilon_r)\cdot C_r\cdot\frac{1}{3}\right]+\\ \left[-\alpha_{0l\,r}\cdot\varepsilon_{r\,r}\cdot(C_t-C_r)\cdot\frac{1}{2}-\alpha_{0l\,r}\cdot(\varepsilon_t-\varepsilon_r)\cdot(C_t-C_r)\cdot\frac{1}{3}-\varepsilon_r\cdot(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot(C_t-C_r)\cdot\frac{1}{3}-(\alpha_{0l\,t}-\alpha_{0l\,r})\cdot(\varepsilon_t-\varepsilon_r)\cdot(C_t-C_r)\cdot\frac{1}{4}\right]\end{array}\right\}=$$

$$=-0.334\,\mathrm{deg}$$

Cmac ala



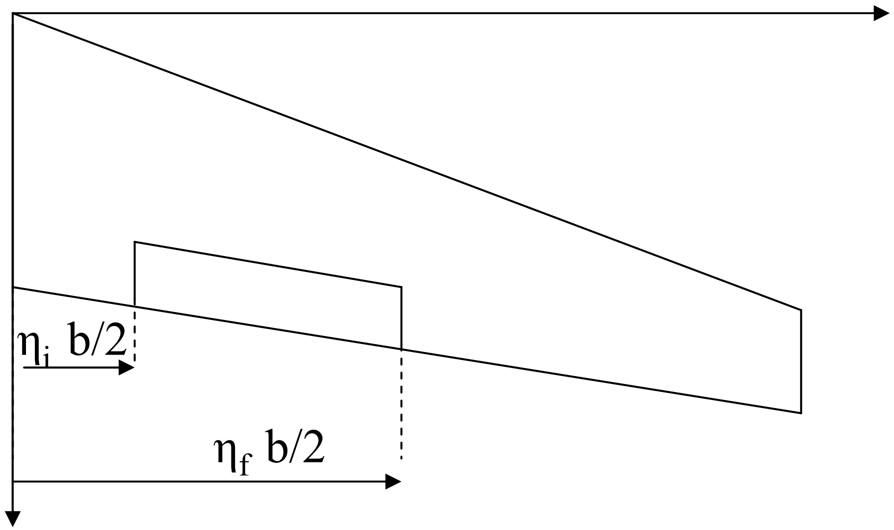
$$C_{macw} = \frac{2}{SC_{ma}} \int_0^{b/2} Cm_{0\,2d}(y) \cdot C(y)^2 \, dy + \frac{2\pi}{SC_{ma}} \cdot \int_0^{b/2} (\alpha_{0L} + \varepsilon(y) - \alpha_{0l}(y)) \cdot C(y) \cdot x1 \, dy = -0.074 + 0.006 = -0.068$$

$$x1(y) = x_{ac} - x(y)$$

$$Cm_{02D}(y) = Cm_{0r} + \frac{Cm_{0t} - Cm_{0r}}{b/2} y$$

$$x(y) = x_{ac,r} \cdot C_r + \frac{\left(b/2 \cdot \tan(\Lambda) + x_{ac,t} \cdot C_t\right) - x_{ac,r} \cdot C_r}{b/2} \cdot y$$

FLAP



$$\Delta\alpha_0 = -5\deg$$

$$\eta_i = 0.2$$

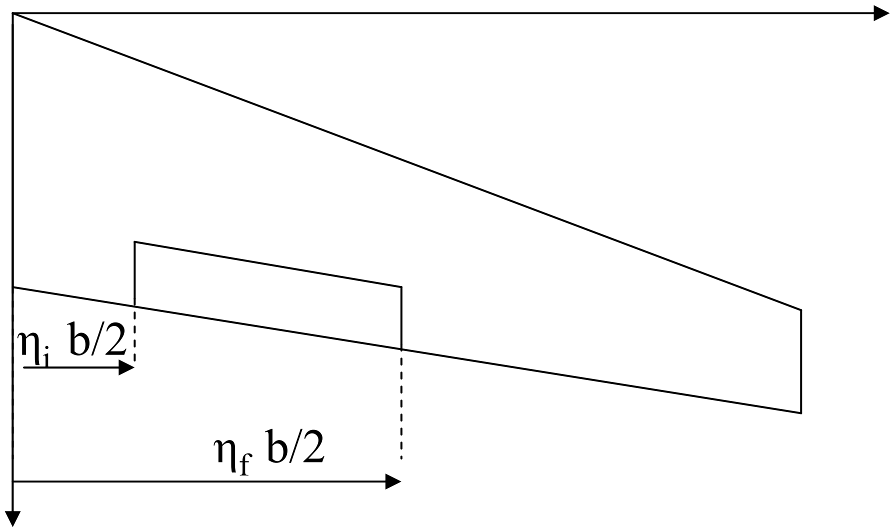
$$\eta_i = 0.6$$

$$\alpha_{0l\ flap}(y) = \alpha_{0l}(y) + \Delta\alpha_0$$

$$\alpha_{0Lw} = \frac{2}{S_w}\cdot\int_0^{\eta_i \frac{b}{2}}\left[\alpha_{0l}(y) - \varepsilon(y)\right]\cdot C(y)\cdot dy + \frac{2}{S_w}\cdot\int_{\eta_i\frac{b}{2}}^{\eta_f\frac{b}{2}}\left[\alpha_{0l}(y) + \Delta\alpha_0 - \varepsilon(y)\right]\cdot C(y)\cdot dy + \frac{2}{S_w}\cdot\int_{\eta_i\frac{b}{2}}^{\eta_f\frac{b}{2}}\left[\alpha_{0l}(y) - \varepsilon(y)\right]\cdot C(y)\cdot dy =$$

$$= \frac{2}{S_w}\cdot\int_0^{\frac{b}{2}}\left[\alpha_{0l}(y) - \varepsilon(y)\right]\cdot C(y)\cdot dy + \frac{2}{S_w}\cdot\Delta\alpha_0\int_{\eta_i\frac{b}{2}}^{\eta_f\frac{b}{2}} C(y)\cdot dy = \alpha_{0L} + \frac{S_f}{S_w}\cdot\Delta\alpha_0 = -2.467\deg$$

FLAP



$$\Delta\alpha_0 = -5\deg$$

$$\eta_i = 0.2$$

$$\eta_i = 0.6$$

$$\Delta Cm_0 = -0.1$$

$$C_{macw} = \frac{2}{SC_{ma}}\int_0^{b/2} Cm_{0\,2d}(y)\cdot C(y)^2\,dy + \frac{2\Delta Cm_0}{SC_{ma}}\int_{\eta_i\cdot b/2}^{\eta_f\cdot b/2} C(y)^2\,dy + \frac{2\pi}{SC_{ma}}\cdot\int_0^{b/2}(\alpha_{0Lw}+\varepsilon(y)-\alpha_{0l,f}(y))\cdot C(y)\cdot x1\,dy =$$

$$= \frac{2}{SC_{ma}}\int_0^{b/2} Cm_{0\,2d}(y)\cdot C(y)^2\,dy + \frac{2\Delta Cm_0}{SC_{ma}}\int_{\eta_i\cdot b/2}^{\eta_f\cdot b/2} C(y)^2\,dy + \frac{2\pi}{SC_{ma}}\cdot\int_0^{b/2}(\alpha_{0Lw}+\varepsilon(y)-\alpha_{0l}(y))\cdot C(y)\cdot x1\,dy + \frac{2\pi\cdot\Delta\alpha_0}{SC_{ma}}\cdot\int_0^{b/2} C(y)\cdot x1\,dy =$$

$$= -0.074 - 0.044 + 0.091 - 0.079 = -0.107$$

Posizione nel piano del centro aerodinamico dell'ala finita – formula esatta

$$\overline{X} = \frac{2}{C_L \cdot S_w} \cdot \int_0^{\frac{b}{2}} C_{l_a}(y) \cdot C(y) \cdot X_{ac,2D}(y) \cdot dy$$

$$\overline{Y} = \frac{2}{C_L \cdot S_w} \cdot \int_0^{\frac{b}{2}} C_{l_a}(y) \cdot C(y) \cdot y \cdot dy$$

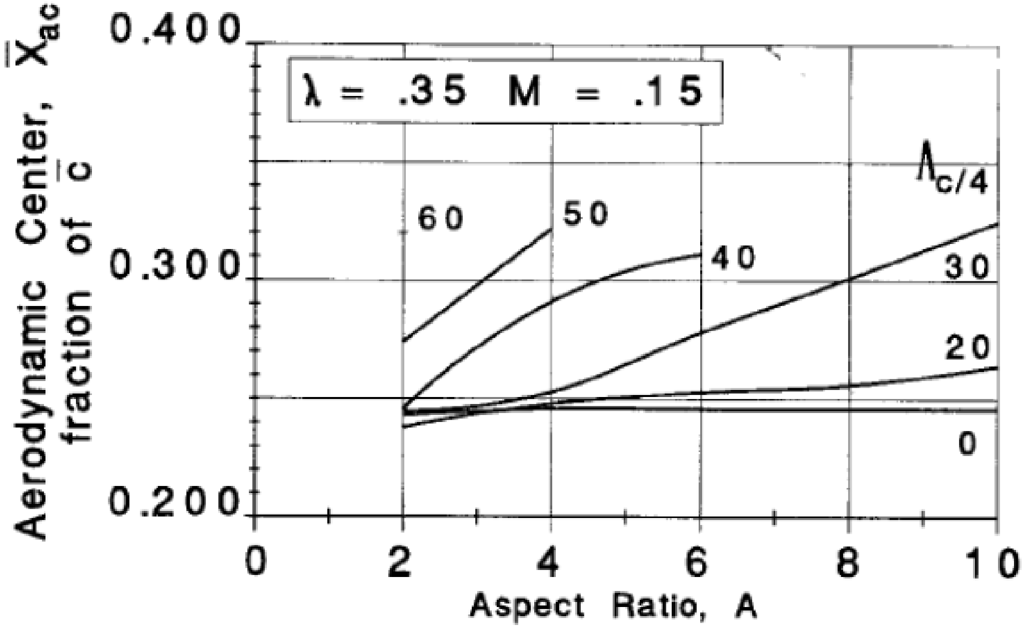


Figure 2.9 Effect of Planform Geometry on Aerodynamic Center (Subsonic)

Posizione nel piano del centro aerodinamico dell'ala finita – formula approssimata

Nota la Cma e l'espressione di C(y):

$$\overline{Y} = \ldots\{ y \Rightarrow C(y) = C_{ma} \}\ldots = \frac{C_{ma} - C_r}{C_t - C_r} \cdot \frac{b}{2} = 2.222m$$

Per cui si potrebbe scrivere trascurando gli effetti del carico addizionale:

$$\overline{X} = \left(X_{ac,r} \cdot C_r + \frac{X_{ac,t} \cdot C_t - X_{ac,r} \cdot C_r}{b/2} \cdot \overline{Y} \right) + \overline{Y} \cdot \tan(\Lambda) = 0.899m$$