



Dipartimento
Ingegneria Aerospaziale


Università di Napoli Federico II

ADAG
RESEARCH GROUP
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Corso Manovre e Stabilità

Equilibrio LONGITUDINALE

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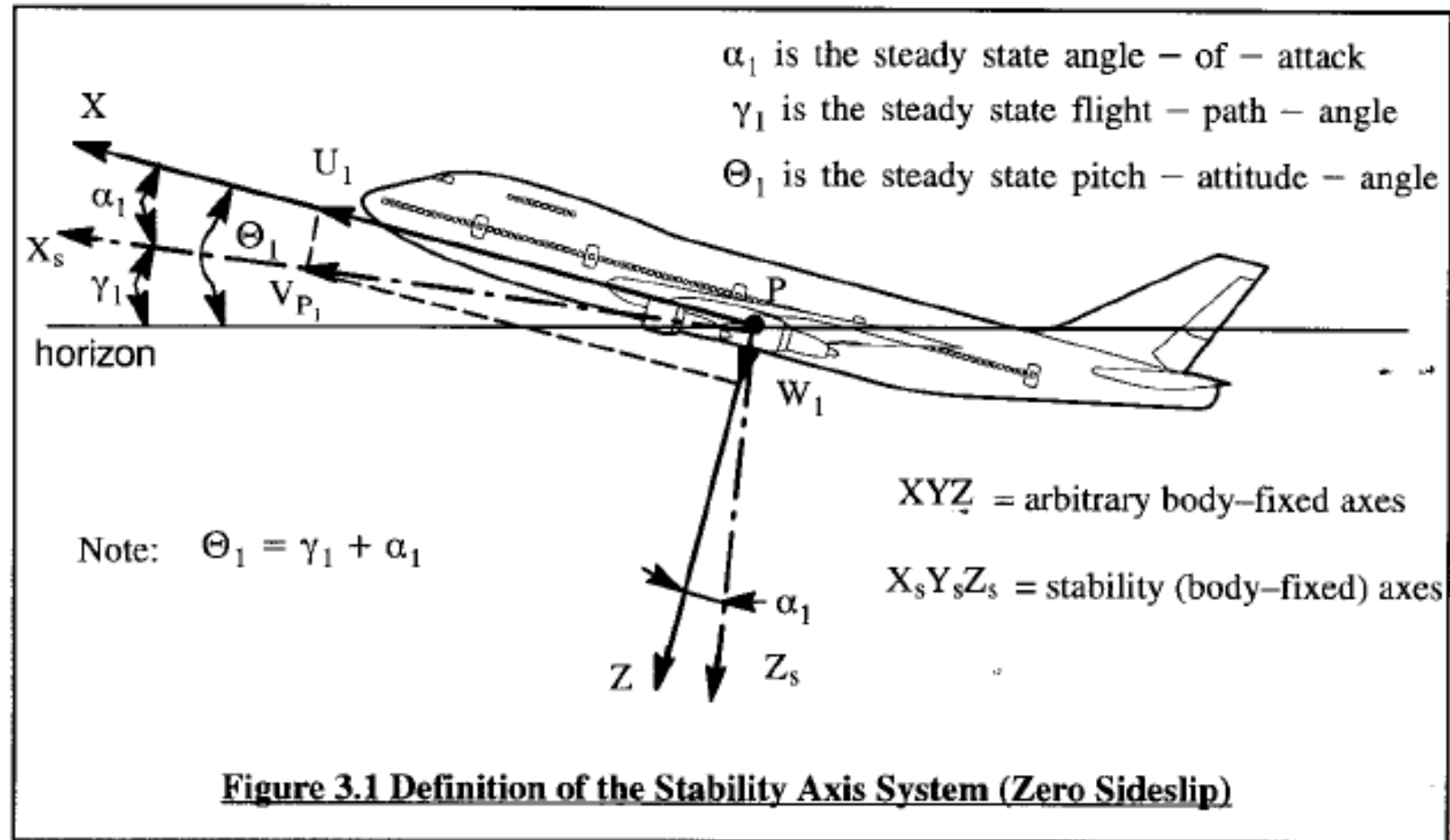


FORZE AERODINAMICHE

$$F_{A_{\text{airplane}}} = F_{A_{\text{wing}}} + F_{A_{\text{fuselage}}} + F_{A_{\text{hor. tail}}} + F_{A_{\text{vert. tail}}} + \text{etc.}$$



Forze Aerodinamiche



Forze Aerodinamiche

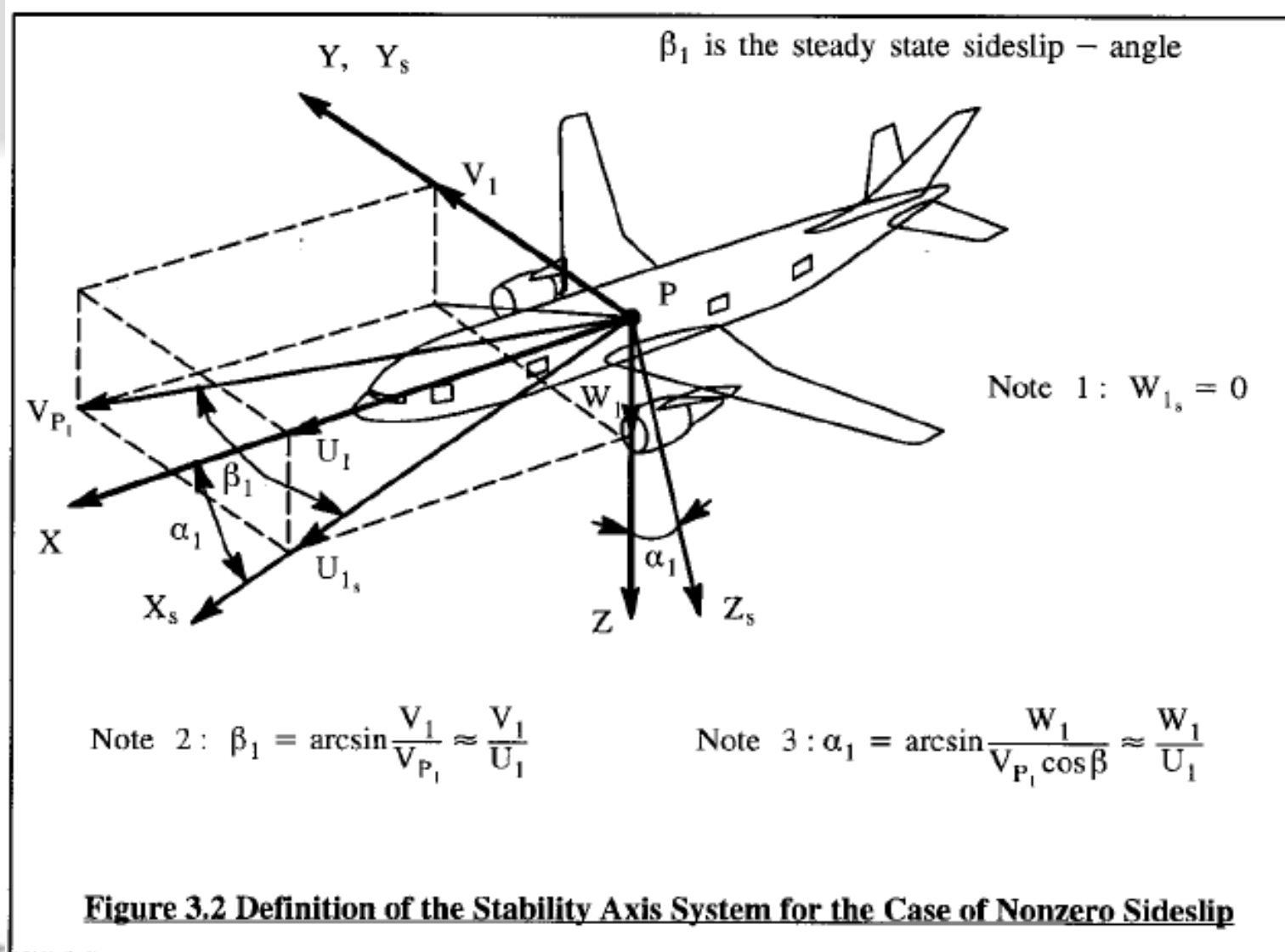
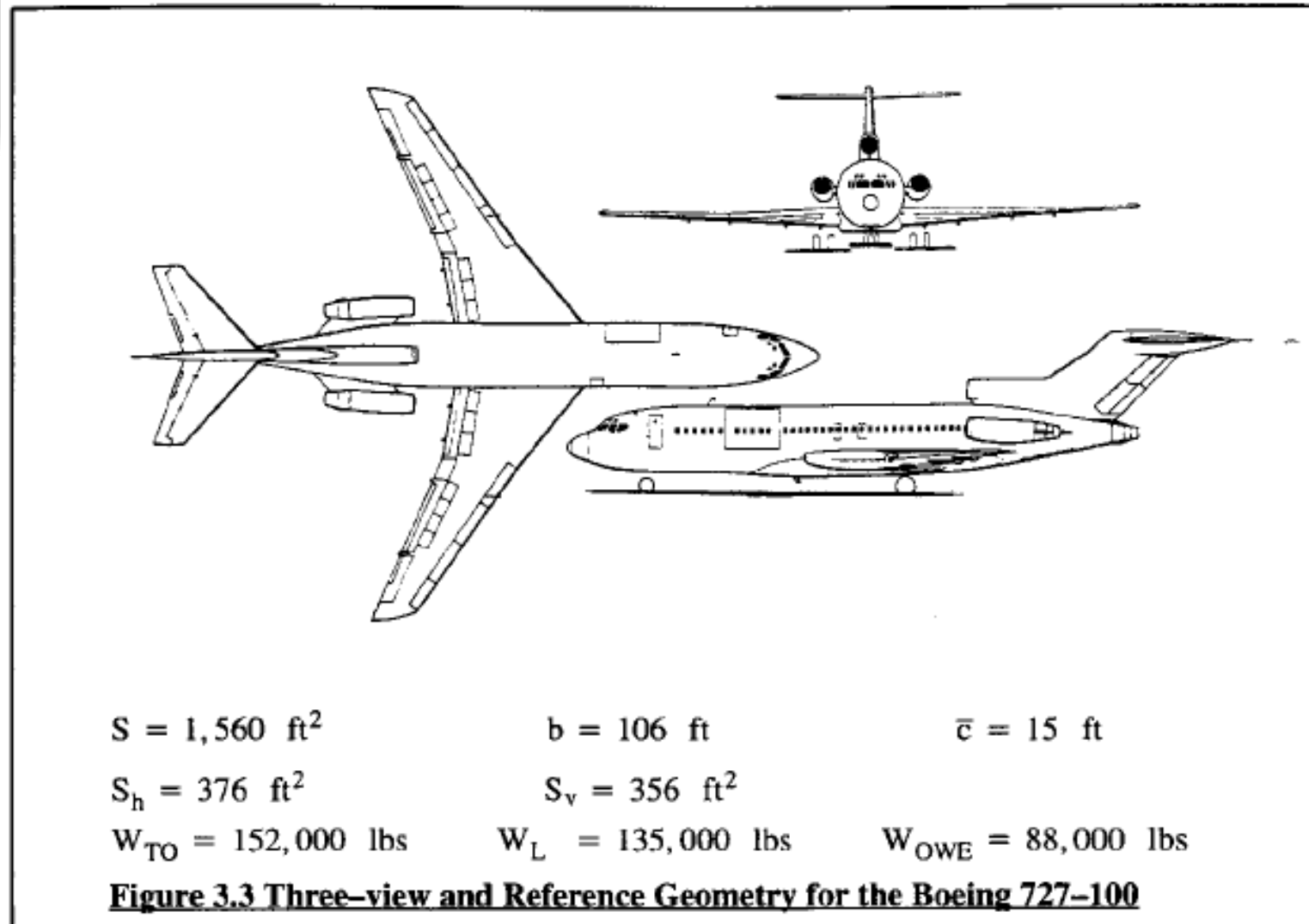


Table 3.1 Dependence of Steady State Aerodynamic Forces and Moments on Variables

Variable	all = 0	α	β	δ_a	δ_e	δ_r
F_{Ax1s}	drag at zero value for all variables	induced drag	negligible for small: β	negligible for small: δ_a	negligible for small: δ_e	negligible for small: δ_r
F_{Ay1s}	zero	negligible for small: α	side force due to: β	zero	zero	side force due to: δ_r
F_{Az1s}	lift at zero value for all variables	lift due to: α	negligible for small: β	negligible	lift due to: δ_e	negligible
L_{A1s}	zero	rolling moment due to sideslip is affected by: α	rolling moment due to: β	rolling moment due to: δ_a	zero	rolling moment due to: δ_r
M_{A1s}	pitching moment at zero value for all variables	pitching moment due to: α	negligible for small: β	negligible	pitching moment due to: δ_e	negligible
N_{A1s}	zero	yawing moment due to sideslip is affected by: α	yawing moment due to: β	yawing moment due to: δ_a	zero	yawing moment due to: δ_r



Forze Aerodinamiche



Forze Aerodinamiche

DISACCOPIAMO il Longitudinale col Latero-direzionale

Because it is assumed that little coupling between the longitudinal variables and the lateral-directional variables exists, the modelling of forces and moments will be discussed in two independent sets in the following Sub-sections:

3.1.1 through 3.1.6 Longitudinal Forces and Moments

3.1.7 through 3.1.12 Lateral-Directional Forces and Moments



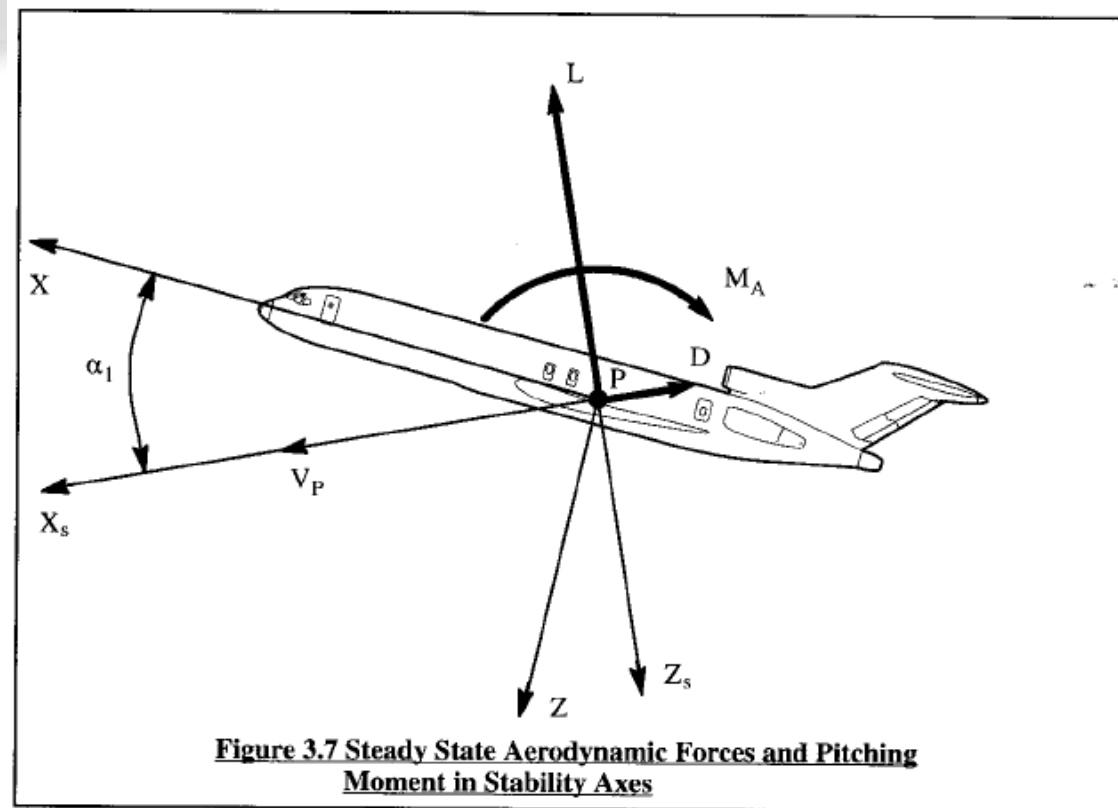
Forze Aerodinamiche

Longitudinale

$$F_{A_{x_1s}} = -D$$

$$F_{A_{z_1s}} = -L$$

$$M_{A_{1s}} = M_A$$



Forze Aerodinamiche Longitudinale - DRAG

$$D = C_D \bar{q} S$$

where: C_D is the total airplane drag coefficient.

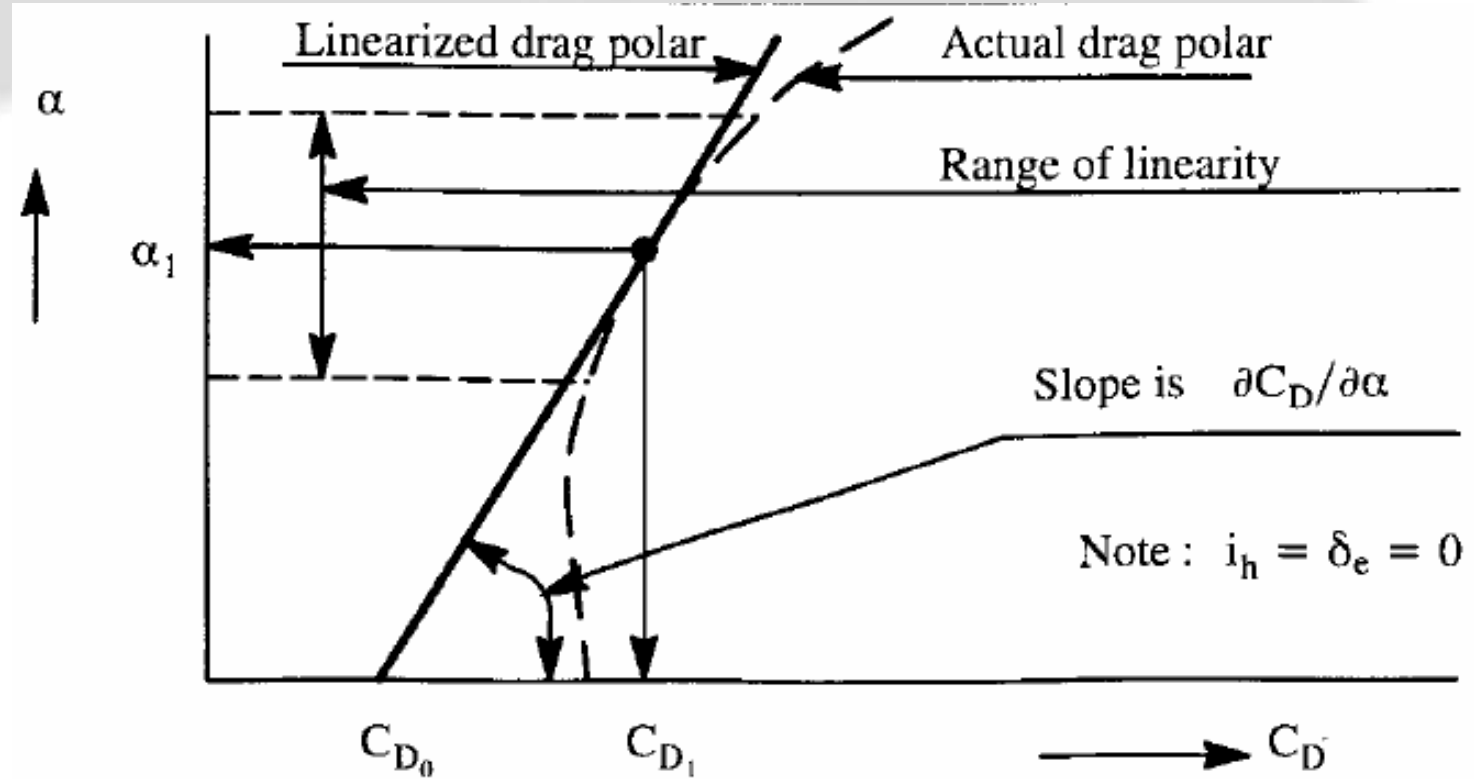
The steady state airplane drag coefficient depends on the following factors:

- * airplane wetted area
- * angle of attack, α
- * dynamic pressure, \bar{q}
- * airplane average skin friction coefficient
- * control surface deflection(s), δ_e , i_h , etc.
- * Mach number and Reynolds number

$$C_D = C_{D_0} + C_{D_\alpha} \alpha + C_{D_{i_h}} i_h + C_{D_{\delta_e}} \delta_e$$



Longitudinale - DRAG

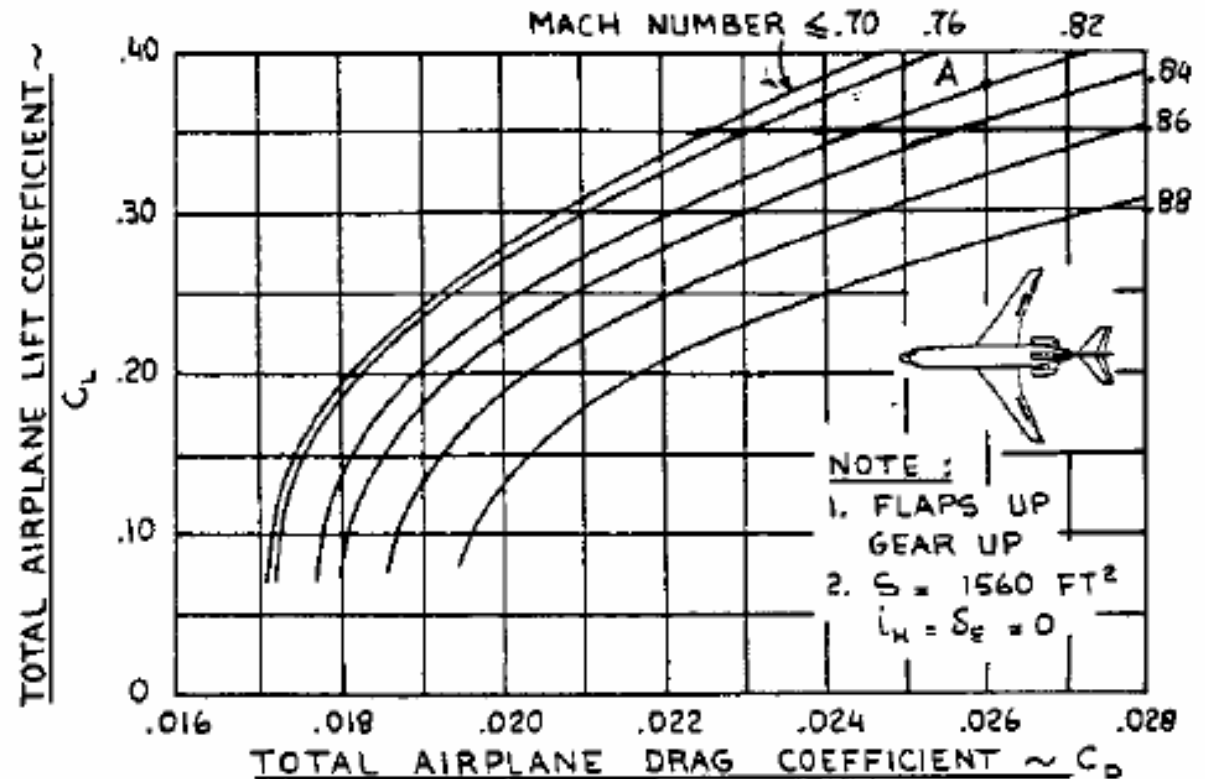


Longitudinale - DRAG

$$C_D = \bar{C}_{D_0} + \frac{C_L^2}{\pi A e}$$

POLARE PARABOLICA

Il C_{D0} è sovrassegnato per
Indicare che è il C_D a $C_L=0$
e non ad $\alpha=0$



Longitudinale - DRAG

$$C_D = \bar{C}_{D_0} + \frac{C_L^2}{\pi A e}$$

POLARE PARABOLICA

$$\bar{C}_{D_0} = f/S$$

f è l'area parassita equivalente, vera misura della resistenza di un velivolo

$$f = S_{wet} * C_{f_{eq}}$$

S_{wet} è l'area bagnata del velivolo (circa 5-6 volte S)

$C_{f_{eq}}$ è il C_f equivalente (dipende dal Re e dalla rugosità sup.)

PART I • Preliminary Considerations

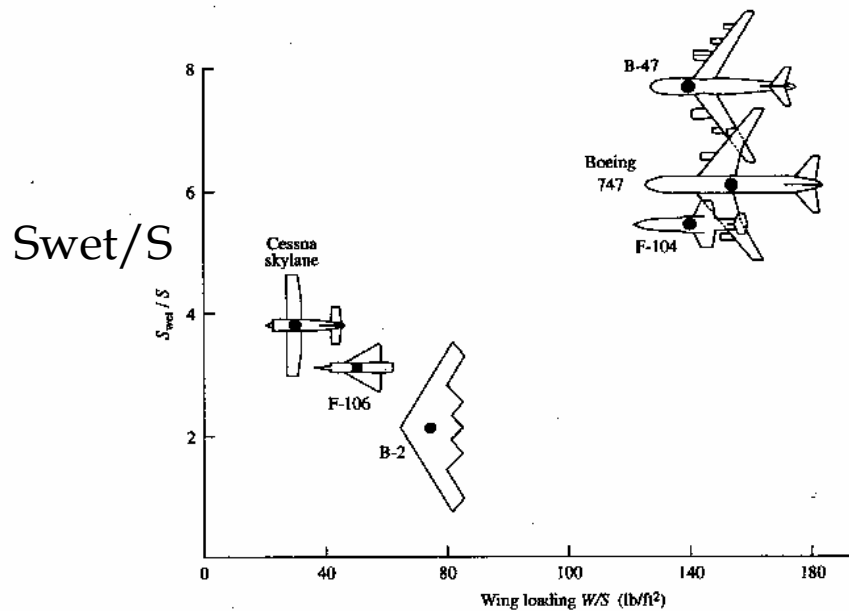
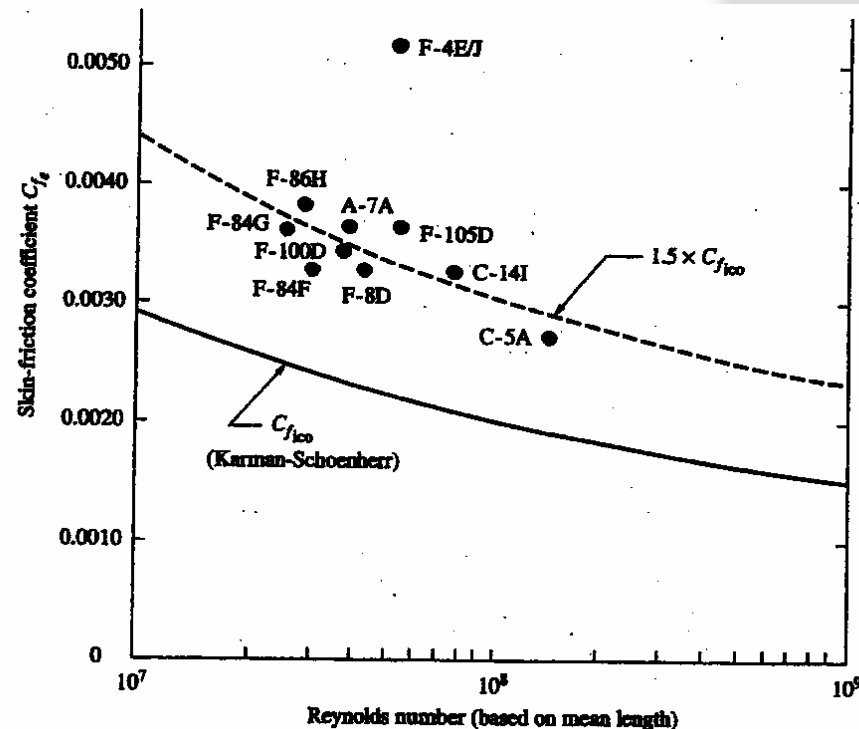


Figure 2.54 Ratio of wetted surface area to reference area for a number of different airplane configurations.



Longitudinale - DRAG

$$\bar{C}_{D_0} = f/S$$

$$f = S_{wet} * C_{f_{eq}}$$

Essendo $S_{wet} = \text{circa } 6 * S$
Viene =>

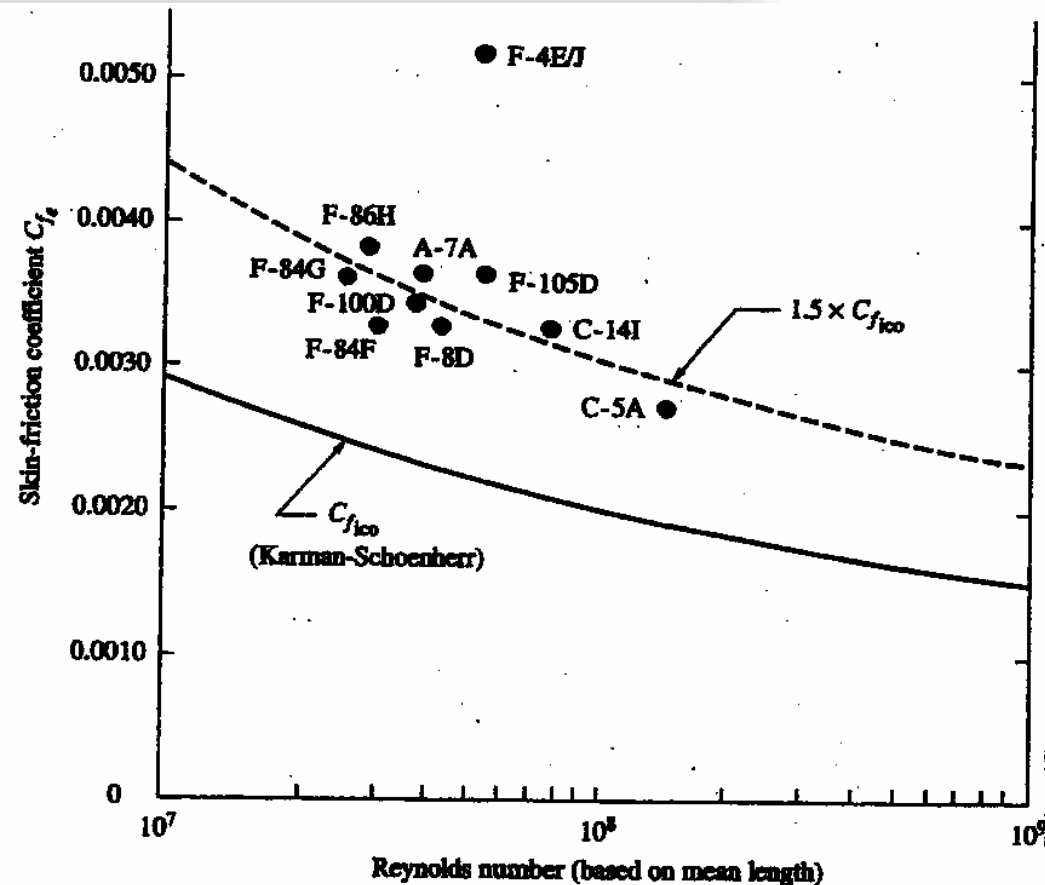
$$\bar{C}_{D_0} = f/S = 6 * C_{f_{eq}}$$

Il $C_{f_{eq}}$ vale:

- Velivolo trasp. Jet = circa 0.0030
- Velivolo turboelica = circa 0.0037
- Bimotore elica = circa 0.0045

Per cui , nel caso di vel. Trasp. Jet =>

$$\bar{C}_{D_0} = f/S = 6 * 0.0030 = 0.018$$



NB : Si parla solitamente di drag counts
1 drag count = 0.0001
Quindi il CDo in tal caso vale **180 counts**



Longitudinale - DRAG

$$C_D = C_{D_0} + C_{D_\alpha} \alpha + C_{D_{i_h}} i_h + C_{D_{\delta_e}} \delta_e$$

$$C_{D_{i_h}} = C_{D_{\delta_e}} = 0$$

$$\Rightarrow C_D = C_{D_0} + C_{D_\alpha} \alpha$$

Ma solitamente si usa l'approccio della polare parabolica :

$$C_D = \bar{C}_{D_0} + \frac{C_L^2}{\pi A e}$$

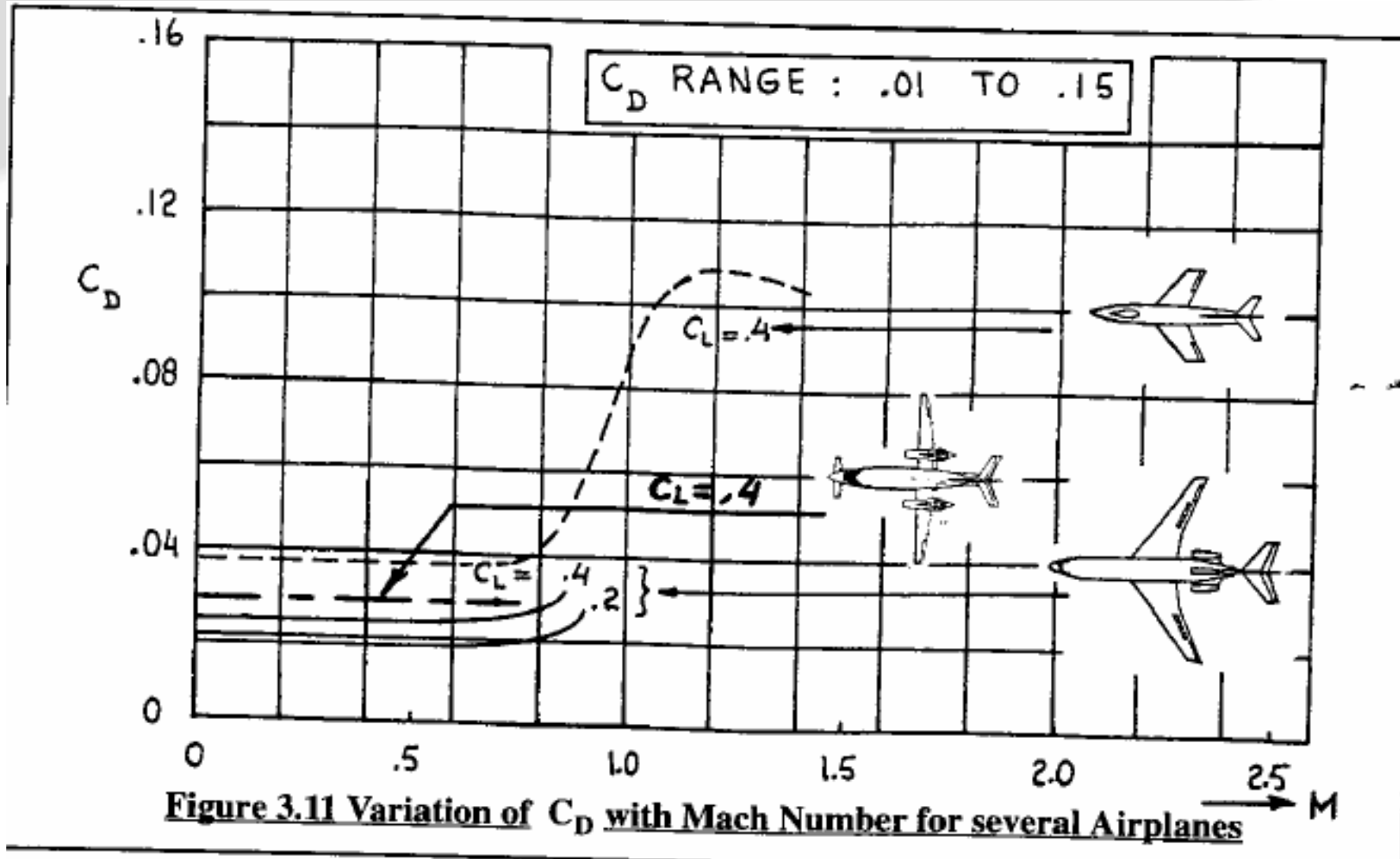
E comunque la derivata rispetto ad alfa

$$C_{D_\alpha} = (2C_{L_1} C_{L_\alpha}) / (\pi A e)$$

derivata NON COSTANTE con α !!!



Longitudinale - DRAG



Longitudinale - DRAG

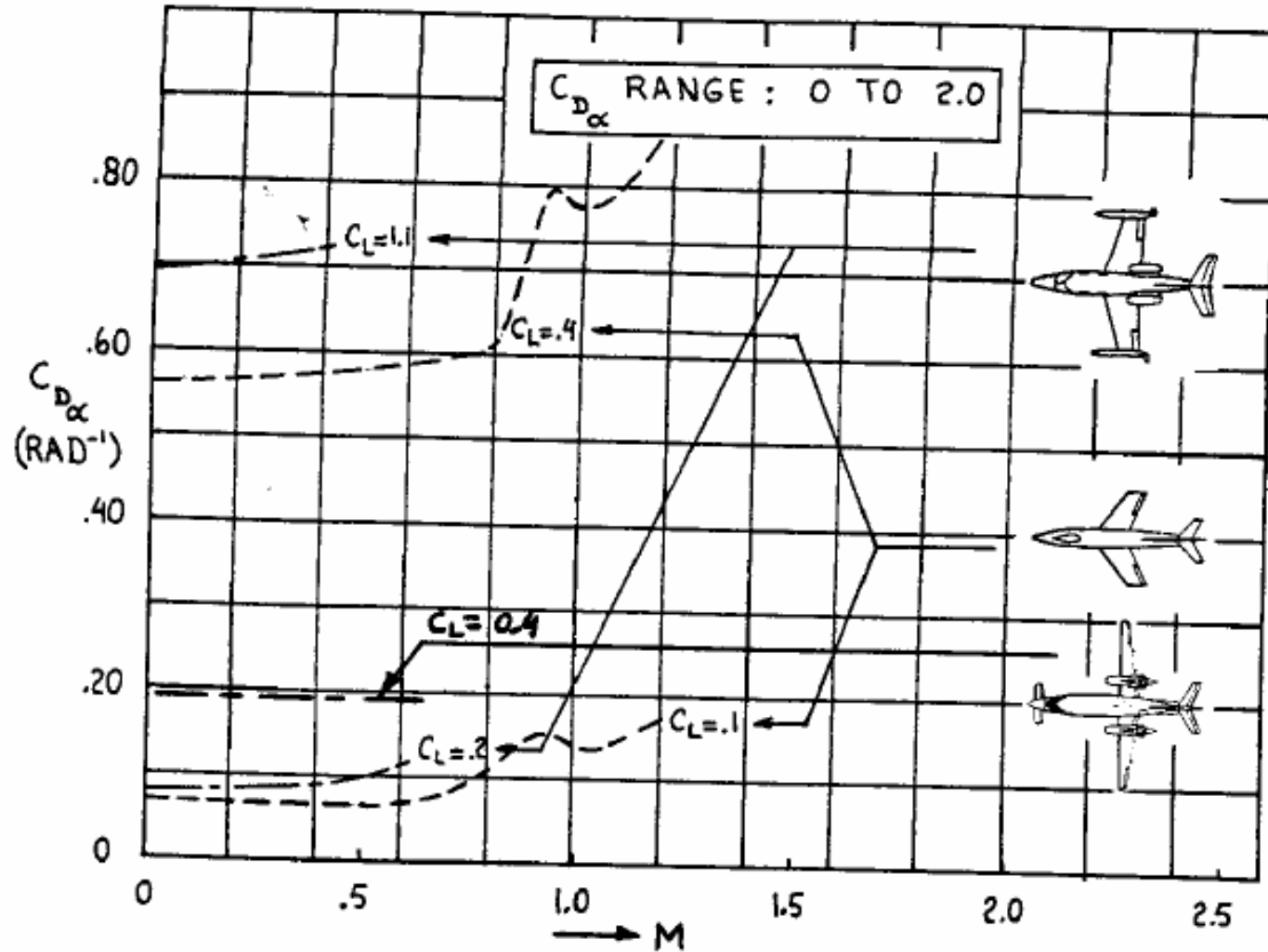


Figure 3.12 Variation of C_{D_α} with Mach Number for several Airplanes

Longitudinale - LIFT

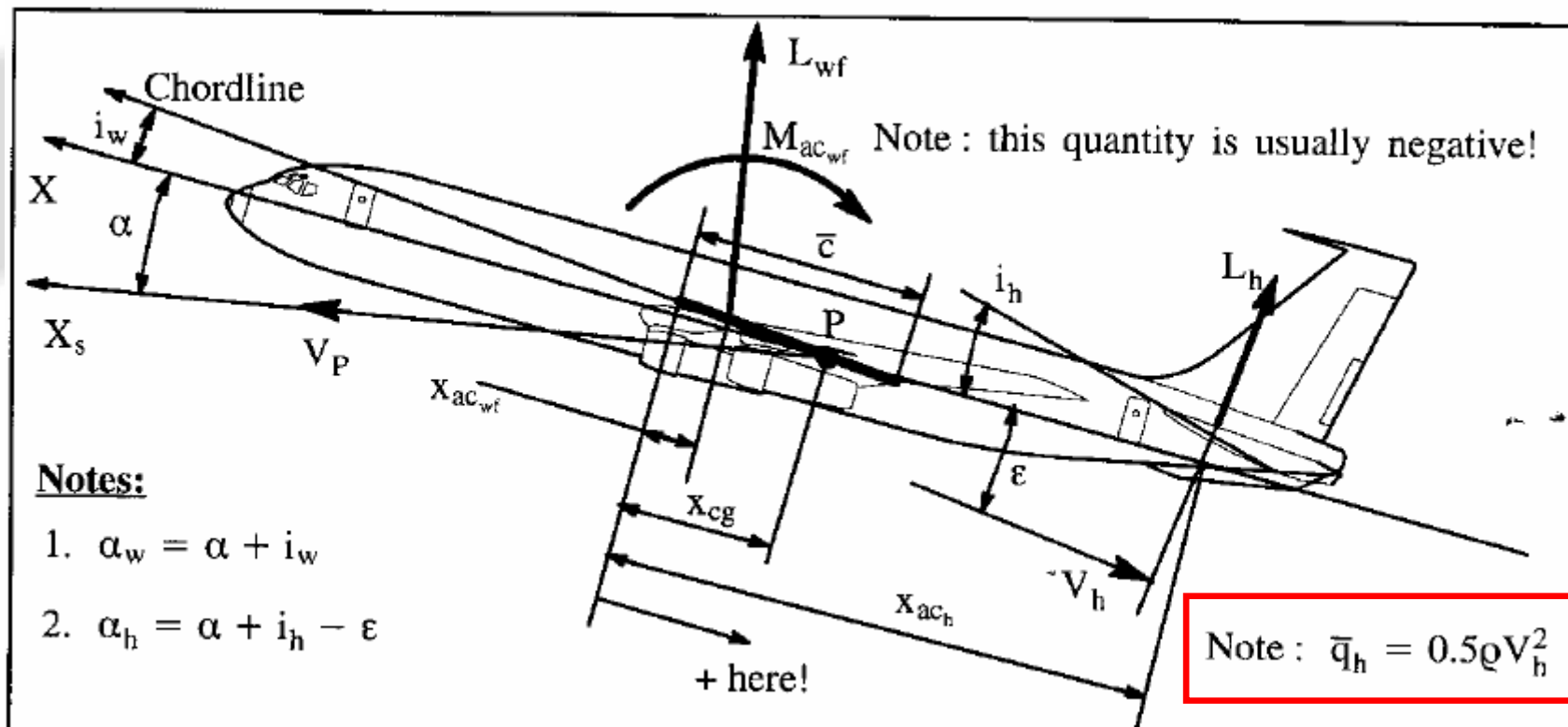
$$L = C_L \bar{q} S$$

- * angle of attack, α
- * dynamic pressure, \bar{q}
- * control surface deflection(s), δ_e , i_h , etc.
- * Mach number and Reynolds number

$$C_L = C_{L_0} + C_{L_\alpha} \alpha + C_{L_{i_h}} i_h + C_{L_{\delta_e}} \delta_e$$



Longitudinale - LIFT



$$L \approx L_{wf} + L_h \cos \epsilon \approx L_{wf} + L_h$$

Trascurando l'entità della resistenza sul piano di coda

$$C_L \bar{q} S = C_{L_{wf}} \bar{q} S + C_{L_h} \bar{q}_h S_h$$

E ponendo

$$\eta_h = \bar{q}_h / \bar{q}$$

Longitudinale - LIFT

$$C_L = C_{L_{wf}} + C_{L_h} \eta_h \frac{S_h}{S}$$

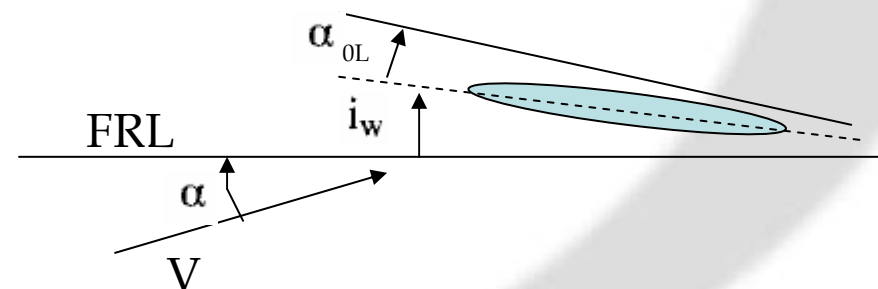
$$C_{L_{wf}} = C_{L_{\alpha_{wf}}} + C_{L_{\alpha_{wf}}} \alpha$$

La fusoliera ha un effetto piccolo (dipende dalla dimensione relativa tra ala e fusoliera)

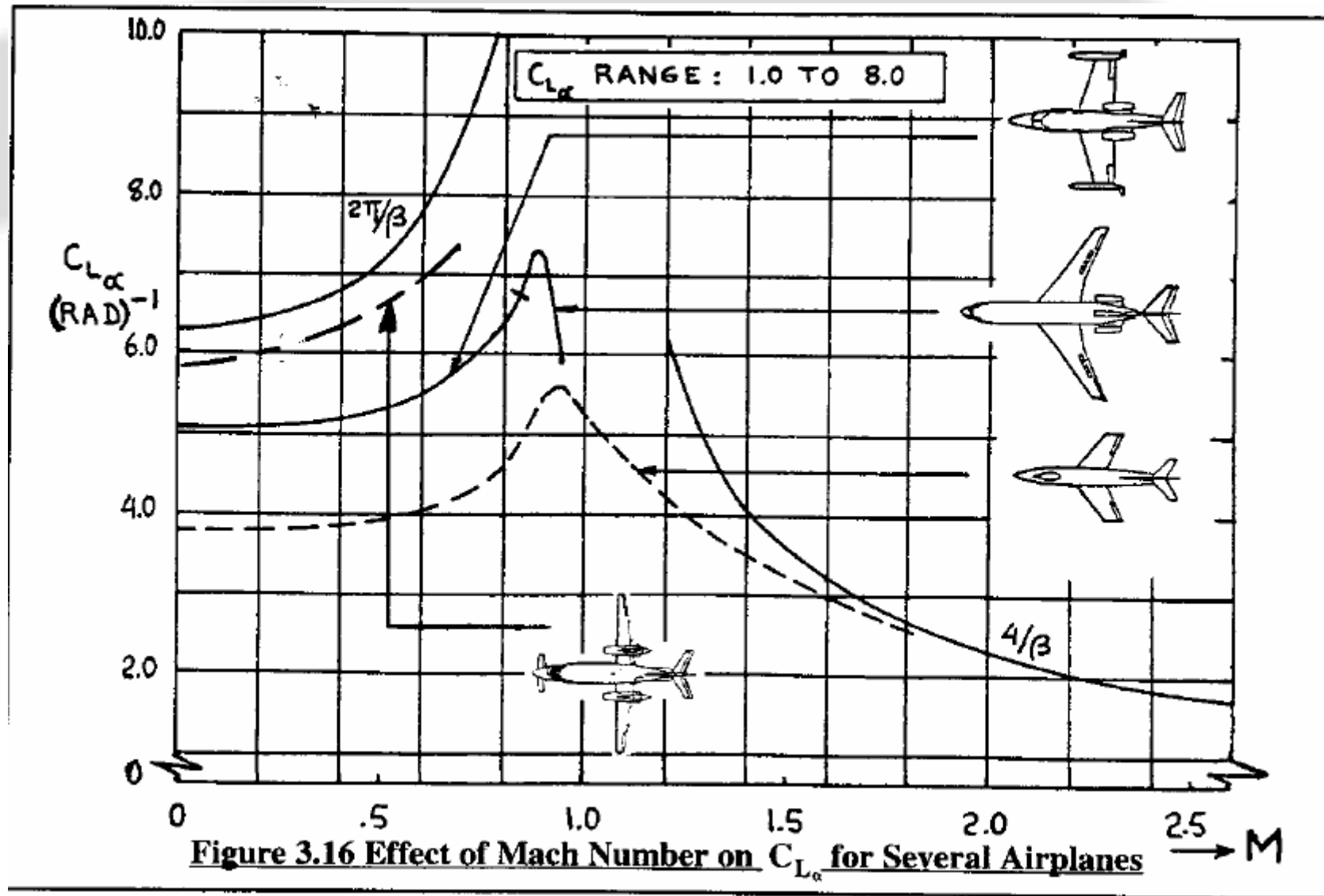
$$\Rightarrow C_{L_{\alpha_{wf}}} \approx C_{L_{\alpha_w}}$$

$$\alpha_w = \alpha + i_w$$

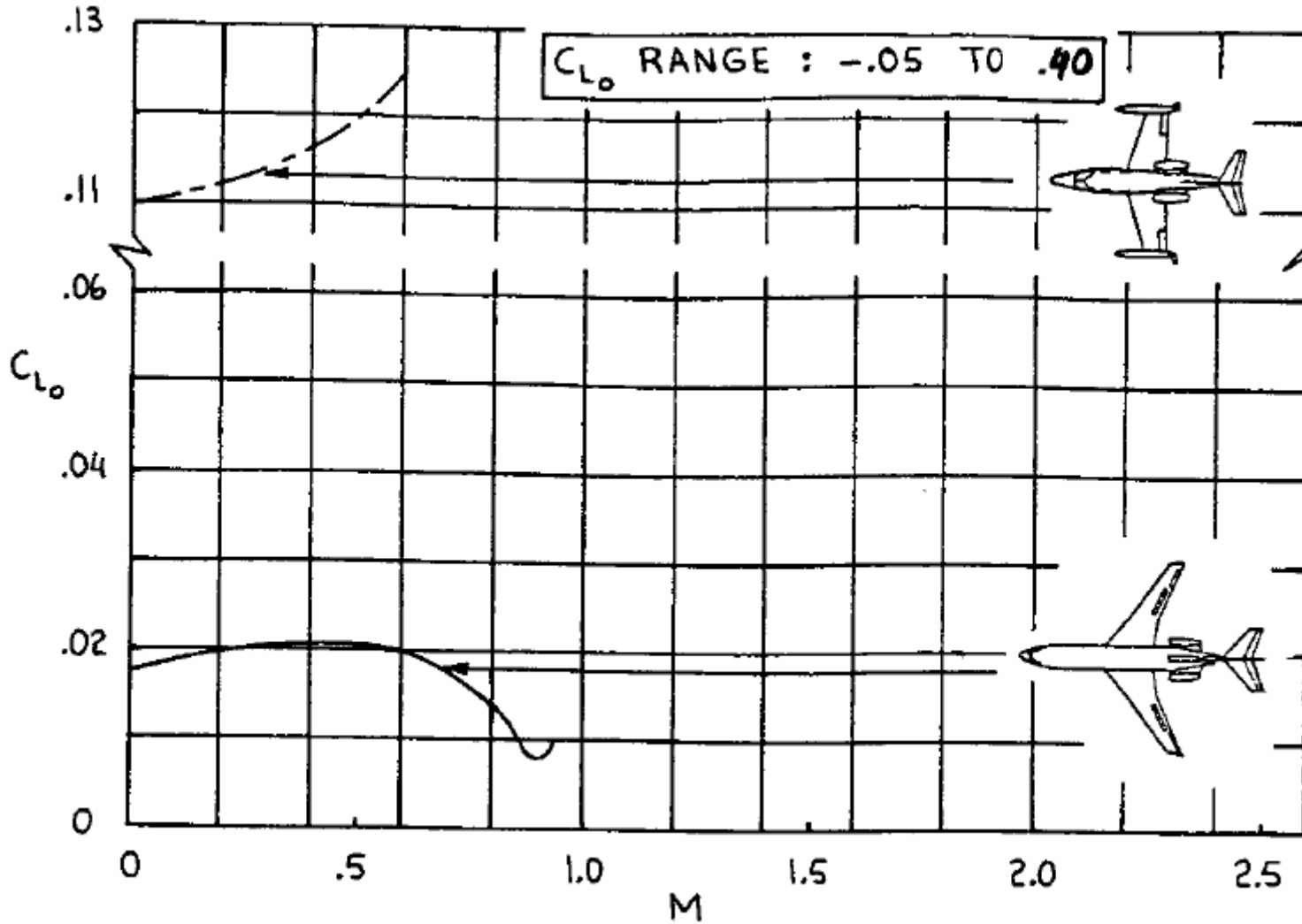
$$C_{L_{0wf}} = C_{L_{\alpha_{wf}}} * (i_w + \alpha_{0L})$$



Longitudinale - LIFT



Longitudinale - LIFT



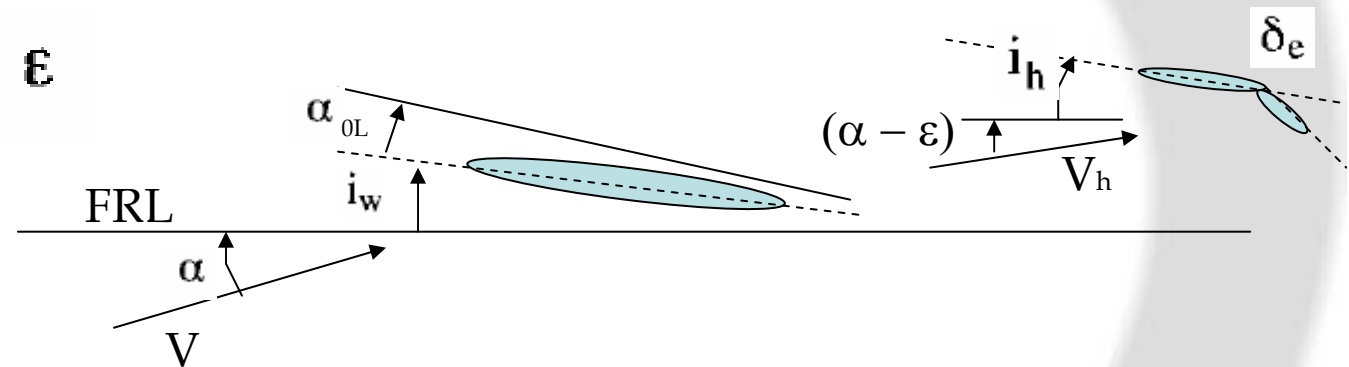
Longitudinale - LIFT

$$C_{L_h} = C_{L_{0_h}} + C_{L_{\alpha_h}} \alpha_h + C_{L_{\alpha_h}} \tau_e \delta_e$$

Il primo termine è = 0 se il profilo del piano di coda orizzontale è simmetrico. Ci sono velivoli che hanno il piano a curvatura negativa. In quel caso è < 0.

$$\alpha_h = \alpha + i_h - \epsilon$$

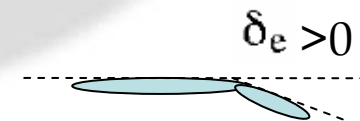
$$\epsilon = \epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha$$



$$\epsilon_0 = \frac{d\epsilon}{d\alpha} \cdot (i_w - \alpha_{0L}) = 2 \frac{C_{L_{\alpha}}}{\pi AR \cdot e_w} \cdot (i_w - \alpha_{0L}) = 2 \frac{C_{L_{0wf}}}{\pi AR \cdot e_w}$$

τ_e Indice di efficacia dell'equilibratore (tip. 0.4 - 0.5)

δ_e Deflessione dell'equilibratore (positiva se verso il basso)



Longitudinale - LIFT

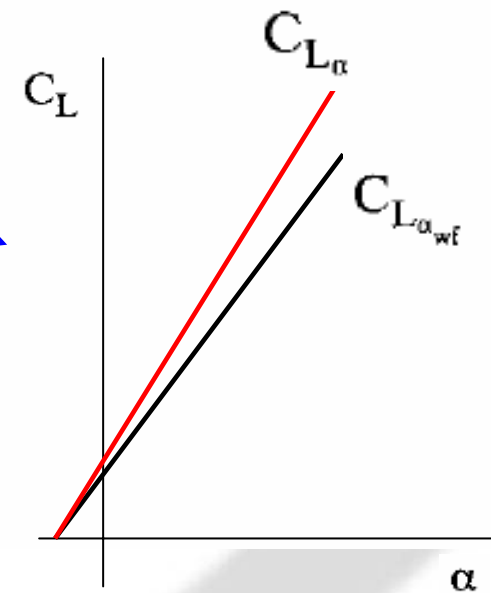
$$C_L = C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha + C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} [\alpha - (\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha) + i_h + \tau_e \delta_e] + C_{L_{0_h}}$$

$$C_{L_0} = C_{L_{0_{wf}}} - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} \epsilon_0 + C_{L_{0_h}} \approx C_{L_{0_{wf}}} \text{ in many airplanes}$$

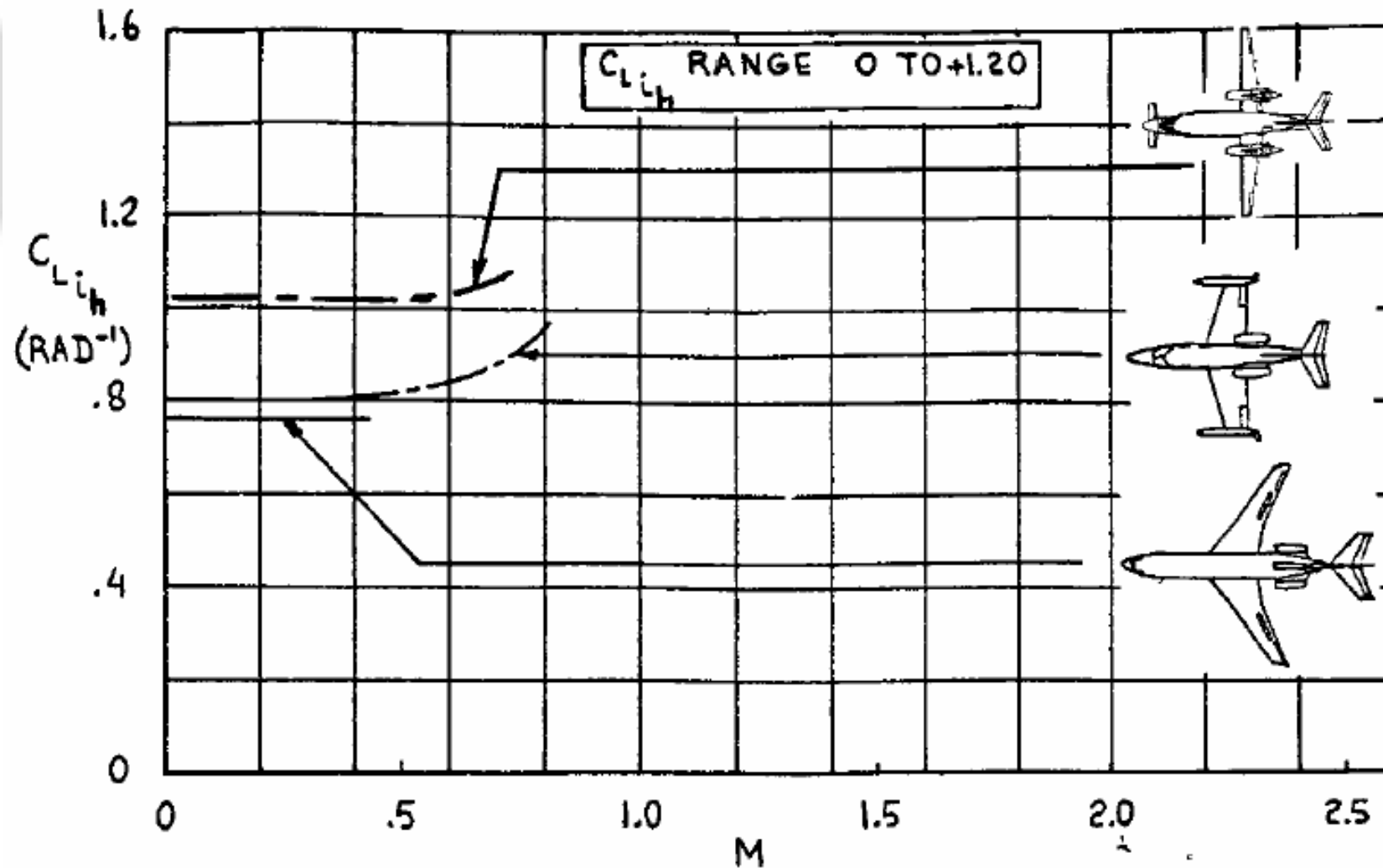
$$C_{L_\alpha} = C_{L_{\alpha_{wf}}} + C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (1 - \frac{d\epsilon}{d\alpha})$$

$$C_{L_{i_h}} = C_{L_{\alpha_h}} \eta_h \frac{S_h}{S}$$

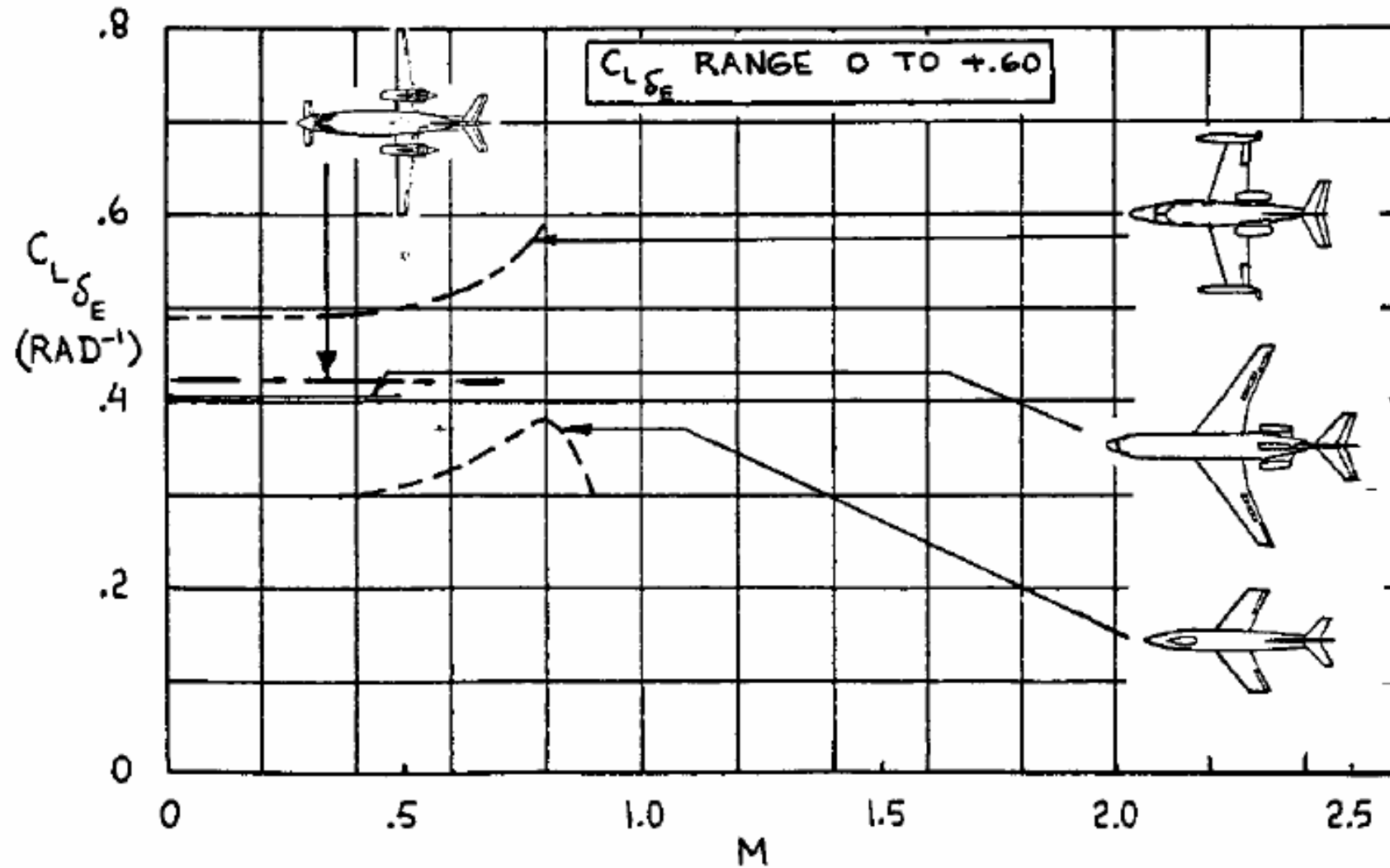
$$C_{L_{\delta_e}} = C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} \tau_e$$



Longitudinale - LIFT



Longitudinale - LIFT



Longitudinale - Momento

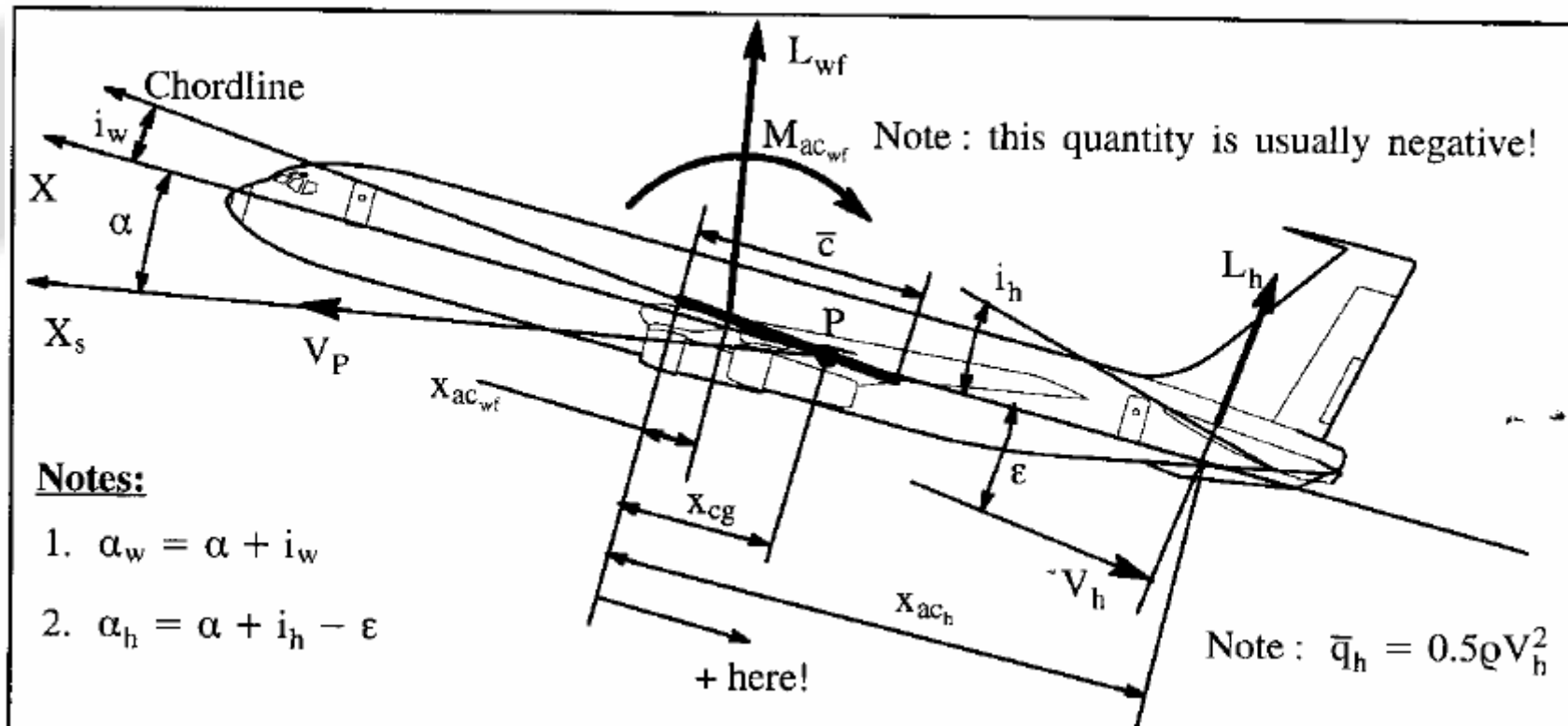
$$M_A = C_m \bar{q} S \bar{c}$$

- * angle of attack, α
- * dynamic pressure, \bar{q}
- * moment reference center (usually the center of gravity) location
- * control surface deflection(s), δ_e , i_h , etc.
- * Mach number and Reynolds number

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e$$



Longitudinale - Momento



$$M_A = M_{ac_{wf}} + L_{wf}(x_{cg} - x_{ac_{wf}}) \cos(\alpha + i_w) - L_h(x_{ac_h} - x_{cg}) \cos(\alpha + i_w - \epsilon)$$



Longitudinale - Momento

$$M_A = M_{ac_{wf}} + L_{wf}(x_{cg} - x_{ac_{wf}}) \cos(\alpha + i_w)$$

$$- L_h(x_{ac_h} - x_{cg}) \cos(\alpha + i_w - \epsilon)$$

$$C_m = C_{m_{ac_{wf}}} + C_{L_{wf}} \frac{(x_{cg} - x_{ac_{wf}})}{\bar{c}} - C_{L_h} \eta_h \frac{S_h}{S} \frac{(x_{ac_h} - x_{cg})}{\bar{c}}$$

$$C_m = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}}) (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - (\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha) + i_h + \tau_e \delta_e \right]$$

$$\bar{x}_{ac_{wf}} = \bar{x}_{ac_w} + \Delta \bar{x}_{ac_{fus}}$$

$$C_{m_{ac_{wf}}} = CM_{ac_w} + CM_{olf}$$



Longitudinale - Momento

$$C_m = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha)(\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) + i_h + \tau_e \delta_e \right]$$

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e \quad \Rightarrow \quad \boxed{C_{m_0}}$$

$$C_{m_0} = C_{m_{ac_{wf}}} + C_{L_{0_{wf}}} (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) + C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \epsilon_0 \approx$$

$$\approx C_{m_{ac_{wf}}} + C_{L_{0_{wf}}} (\bar{x}_{cg} - \bar{x}_{ac_{wf}})$$



Longitudinale - Momento

$$C_m = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha)(\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) + i_h + \tau_e \delta_e \right]$$

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e \Rightarrow C_{m_\alpha}$$

$$C_{m_\alpha} = C_{L_{\alpha_{wf}}} (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) (1 - d\epsilon/d\alpha)$$

Coefficiente volumetrico del piano di coda orizzontale $\bar{V}_h = (S_h/S)(\bar{x}_{ac_h} - \bar{x}_{cg})$

$$C_{m_\alpha} = C_{L_{\alpha_{wf}}} (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) - C_{L_{\alpha_h}} \eta_h \bar{V}_h (1 - d\epsilon/d\alpha)$$

Longitudinale - Momento

$$C_m = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha)(\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) \right] + i_h + \tau_e \delta_e$$

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e \quad \Rightarrow$$

$$C_{m_{i_h}} = - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) = - C_{L_{\alpha_h}} \eta_h \bar{V}_h$$

where : $\bar{V}_h = (S_h/S)(\bar{x}_{ac_h} - \bar{x}_{cg})$

$$\bar{V}_h = \frac{S_h}{S} \frac{(x_{ac_h} - x_{cg})}{c} = \frac{S_h}{S} \frac{l_h}{c}$$

COEFFICIENTE VOLUMETRICO

l_h è la distanza
del ac della coda dal CG

Longitudinale - Momento

$$C_m = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha)(\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) \right] + i_h + \tau_e \delta_e$$

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e \quad \Rightarrow$$

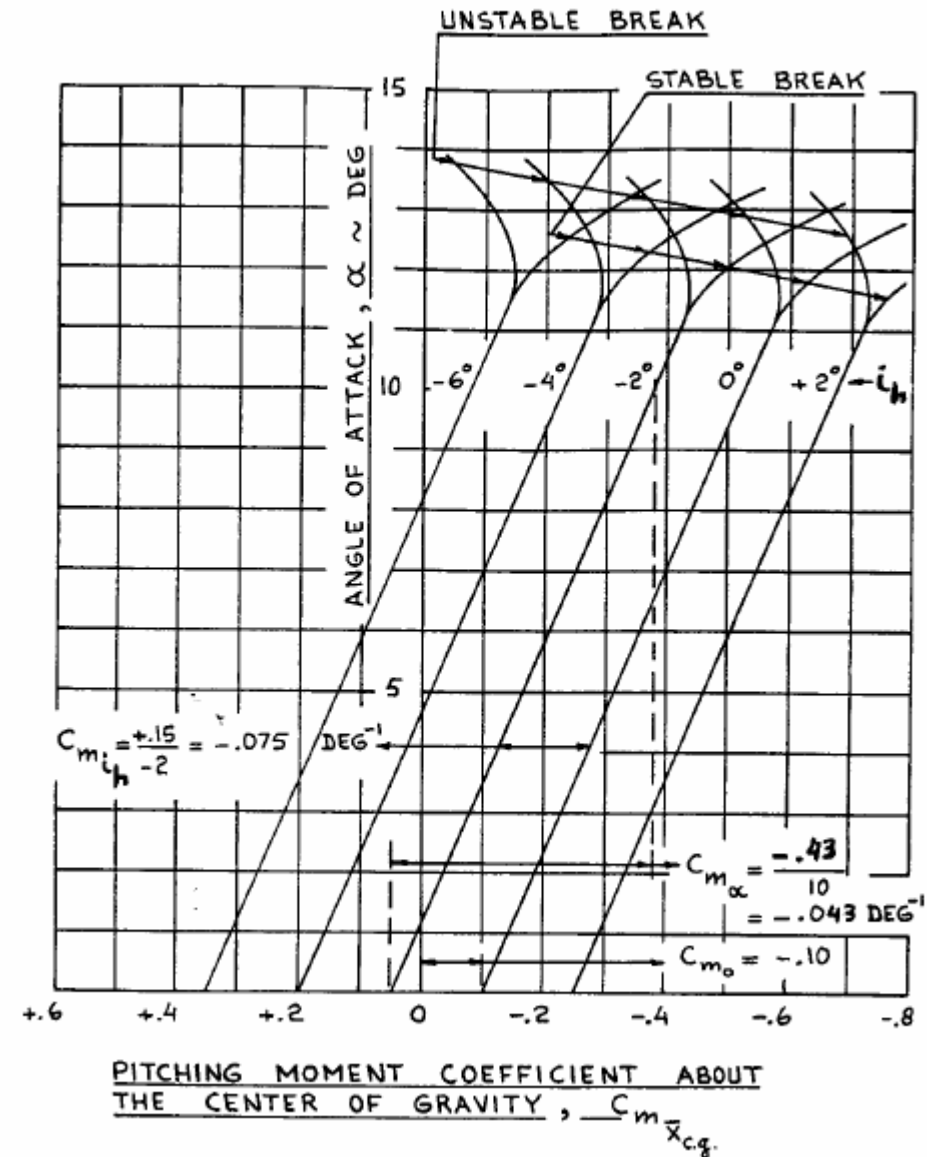
$$C_{m_{\delta_e}}$$

$$C_{m_{\delta_e}} = - C_{L_{\alpha_h}} \eta_h \bar{V}_h \tau_e$$

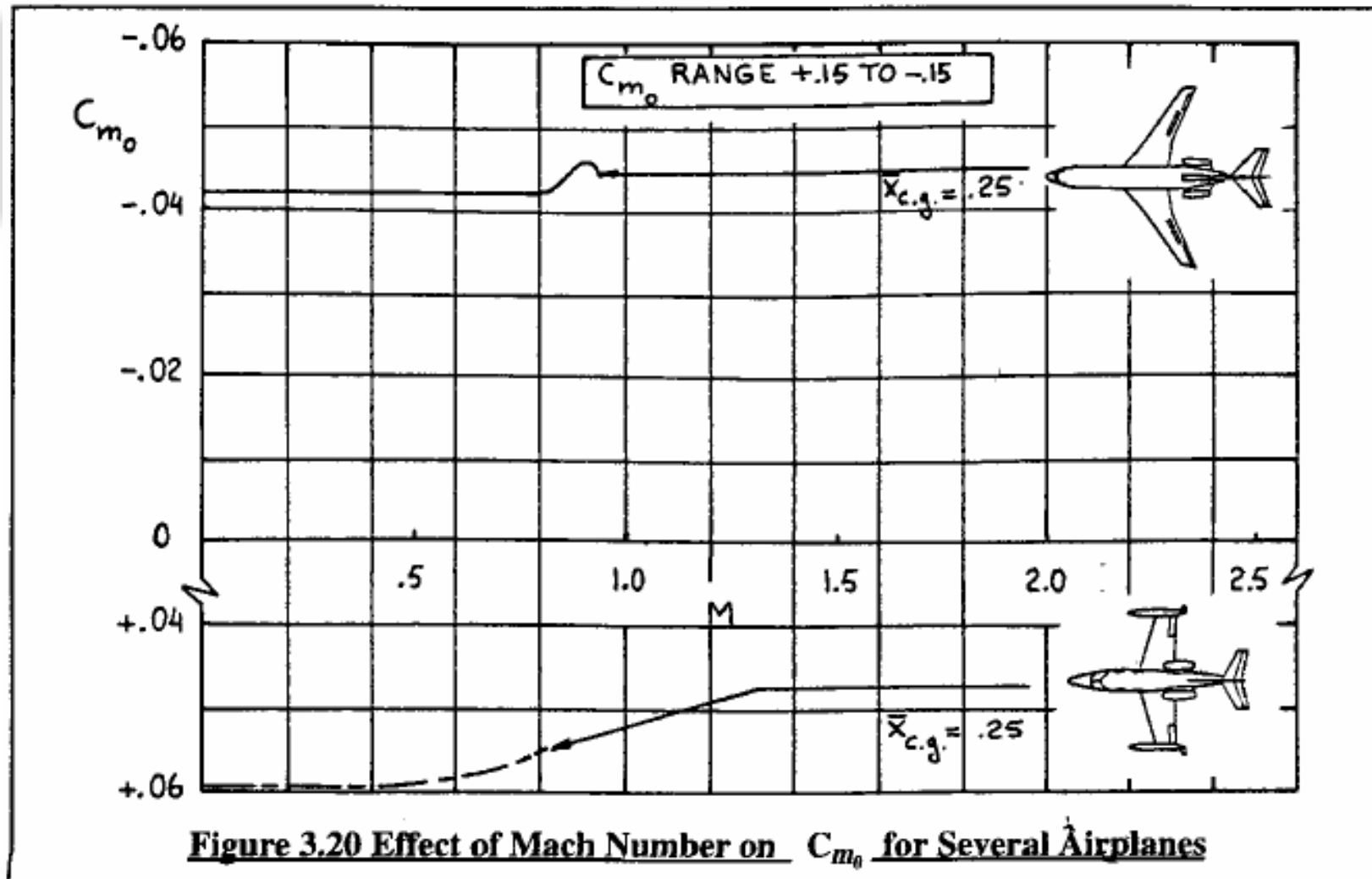
$$\bar{V}_h = (S_h/S)(\bar{x}_{ac_h} - \bar{x}_{cg})$$

COEFFICIENTE VOLUMETRICO

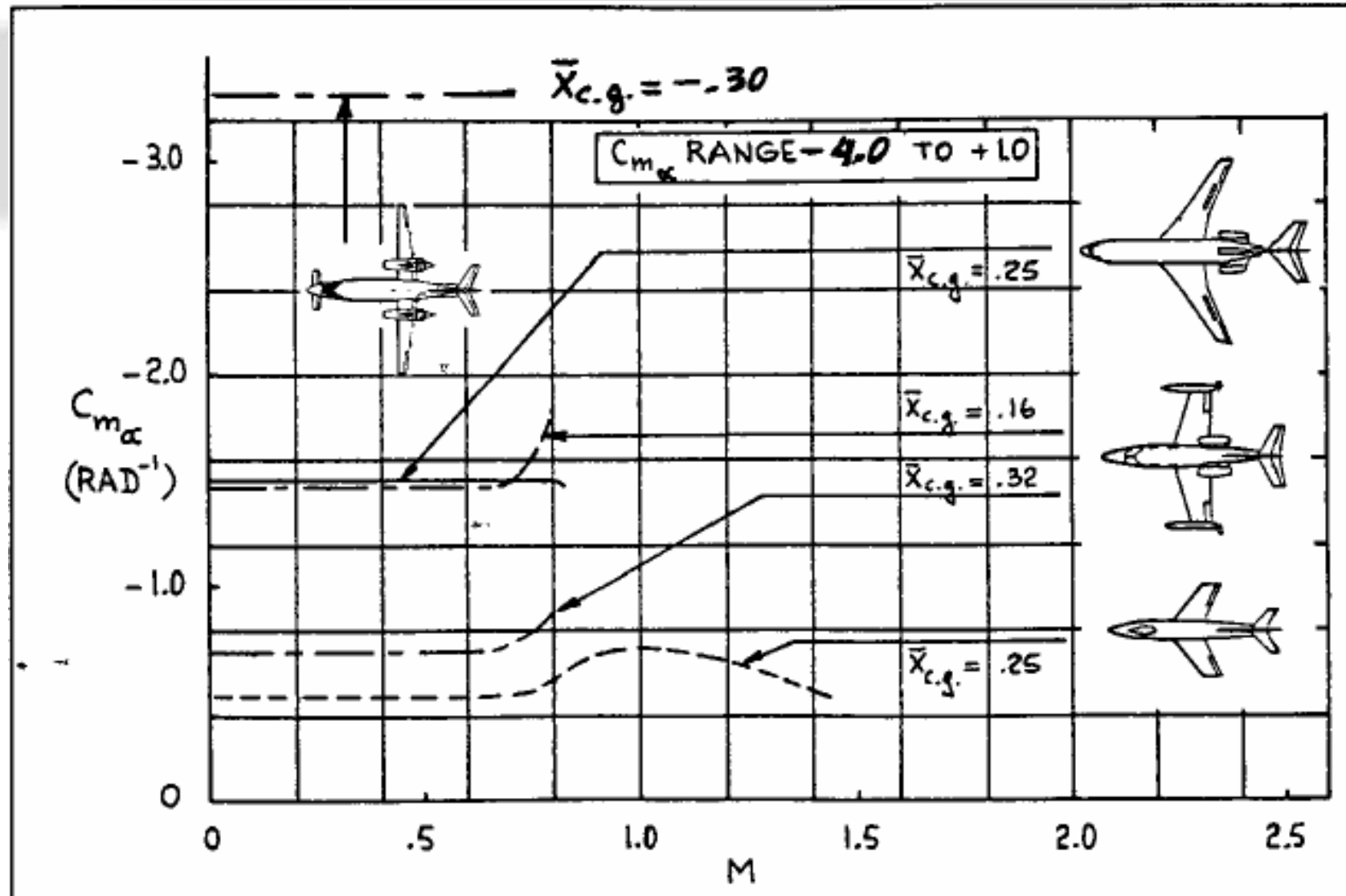




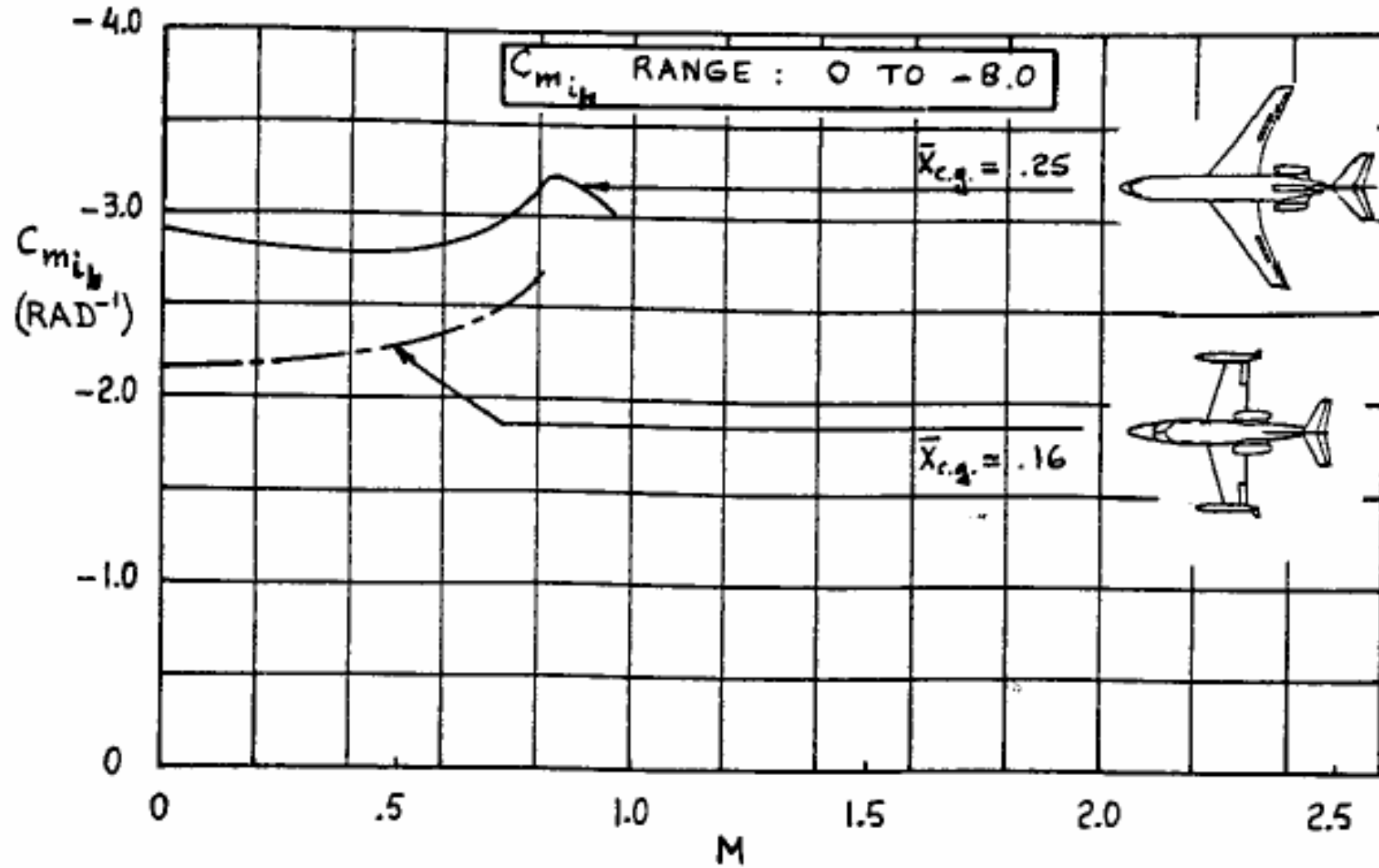
Longitudinale - Momento



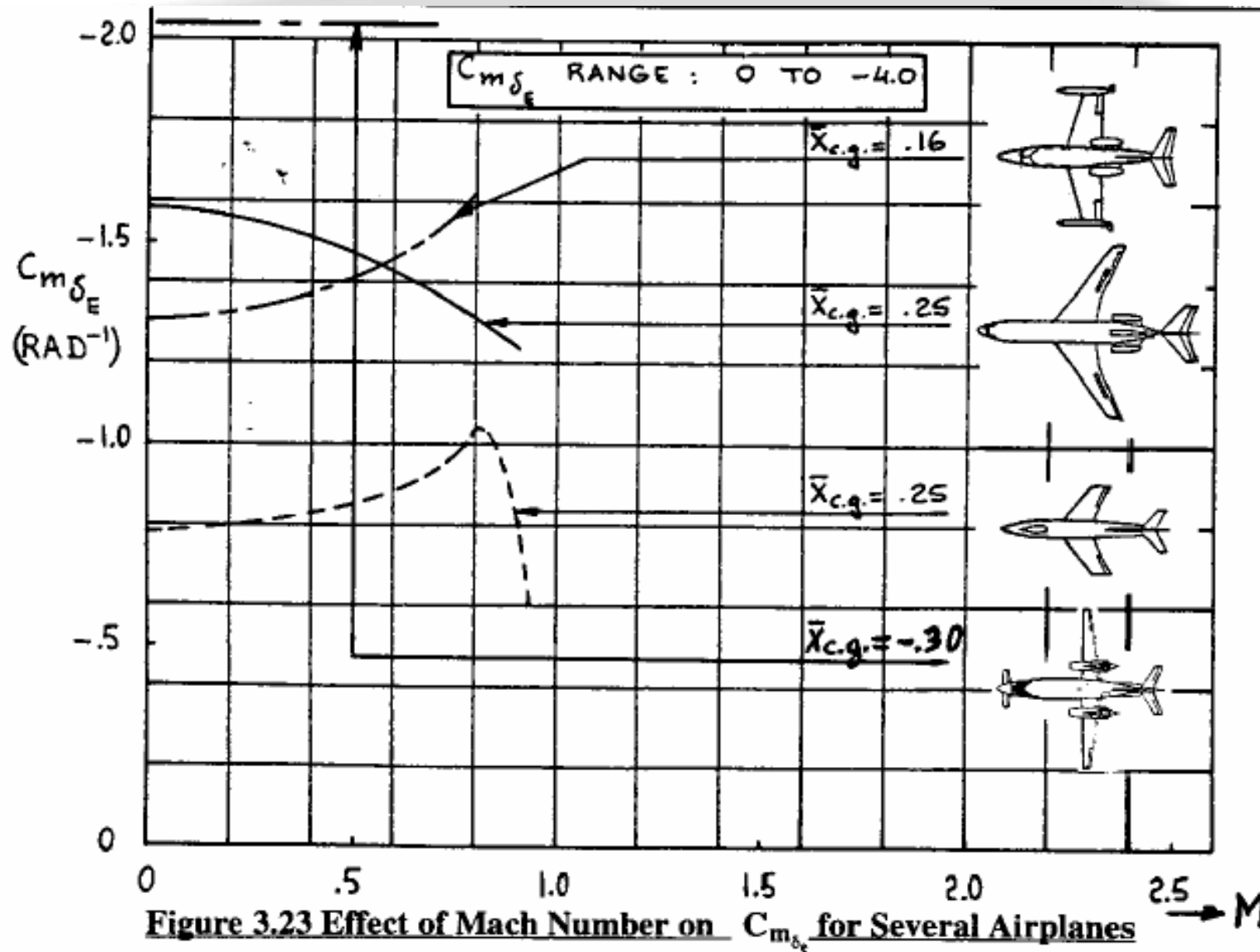
Longitudinale - Momento



Longitudinale - Momento



Longitudinale - Momento



Longitudinale - Momento

Centro aerodinamico del velivolo, detto anche

PUNTO NEUTRO a comandi bloccati => stessa definizione del ac_wing

$$C_m = C_{m_0} + C_{m_\alpha} \alpha + C_{m_{i_h}} i_h + C_{m_{\delta_e}} \delta_e$$

$$C_{m_\alpha} = C_{L_{\alpha_{wf}}} (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) (1 - d\varepsilon/d\alpha)$$

Impongo $C_{m_\alpha} = 0$ and $\bar{x}_{cg} \mapsto \bar{x}_{ac_A}$ detto anche X_N

$$X_N = \bar{x}_{ac_A} = \frac{\bar{x}_{ac_{wf}} + \frac{C_{L_{\alpha_h}}}{C_{L_{\alpha_{wf}}}} \eta_h \frac{S_h}{S} \bar{x}_{ac_h} (1 - \frac{d\varepsilon}{d\alpha})}{1 + \frac{C_{L_{\alpha_h}}}{C_{L_{\alpha_{wf}}}} \eta_h \frac{S_h}{S} (1 - \frac{d\varepsilon}{d\alpha})}$$

Longitudinale - Momento

$$X_N = \bar{X}_{ac_A} = \frac{\bar{X}_{ac_{wf}} + \frac{C_{L\alpha_h}}{C_{L\alpha_{wf}}} \eta_h \frac{S_h}{S} \bar{X}_{ac_h} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}{1 + \frac{C_{L\alpha_h}}{C_{L\alpha_{wf}}} \eta_h \frac{S_h}{S} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}$$

Facendo qualche passaggio si può vedere che :

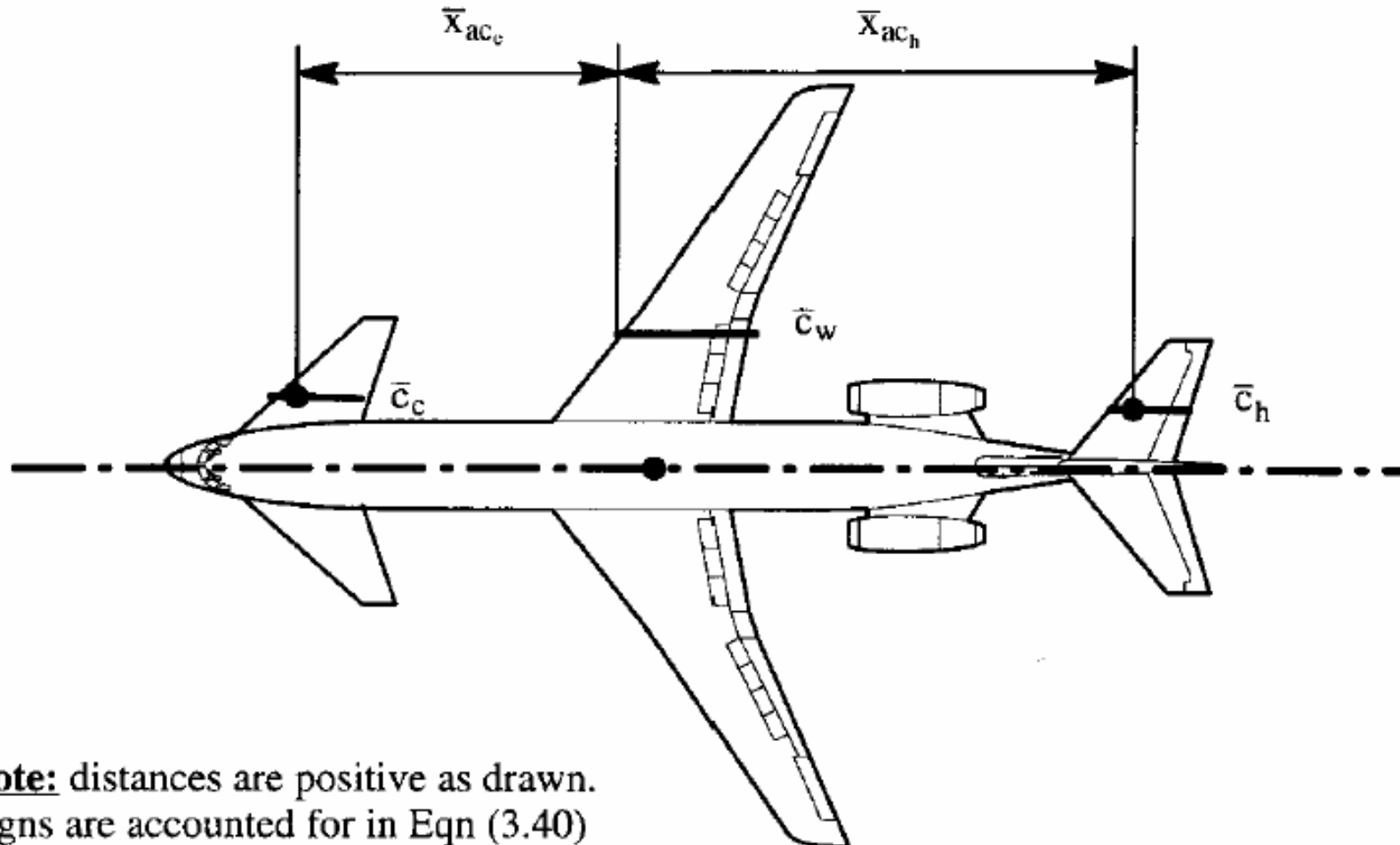
$$C_{m_\alpha} = C_{L_\alpha} (\bar{X}_{cg} - \bar{X}_{ac_A}) = C_{L_\alpha} * (X_{cg} - X_N)$$

NB : Abbiamo trascurato l'effetto sul momento delle forze propulsive !!
In velivoli ad elica monomotori (elica in prua) i momenti dovuti alle componenti che nascono fanno variare di qualche punto % $X_{ac_{wf}}$



Longitudinale - Momento

CASO PIU' GENERALE di ALA-Piano Orizz e Canard

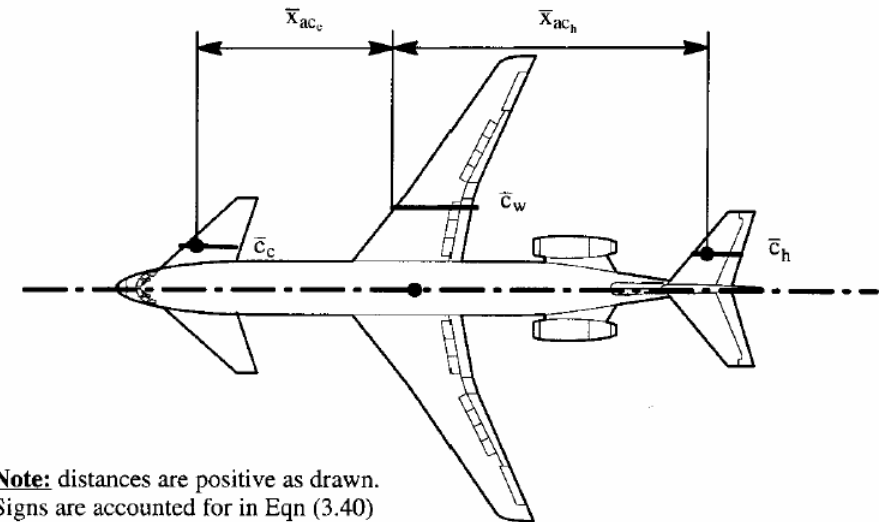


Note: distances are positive as drawn.
Signs are accounted for in Eqn (3.40)



Longitudinale - Momento

CASO PIU' GENERALE di ALA-Piano Orizz e Canard



Note: distances are positive as drawn.
Signs are accounted for in Eqn (3.40)

$$\bar{x}_{ac_A} = \frac{\bar{x}_{ac_{wf}} - \frac{C_{L\alpha_c}}{C_{L\alpha_{wf}}} \eta_c \frac{S_c}{S} \bar{x}_{ac_c} \left(1 + \frac{d\varepsilon_c}{d\alpha}\right) + \frac{C_{L\alpha_h}}{C_{L\alpha_{wf}}} \eta_h \frac{S_h}{S} \bar{x}_{ac_h} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}{1 + \frac{C_{L\alpha_c}}{C_{L\alpha_{wf}}} \eta_c \frac{S_c}{S} \left(1 + \frac{d\varepsilon_c}{d\alpha}\right) + \frac{C_{L\alpha_h}}{C_{L\alpha_{wf}}} \eta_h \frac{S_h}{S} \left(1 - \frac{d\varepsilon}{d\alpha}\right)}$$



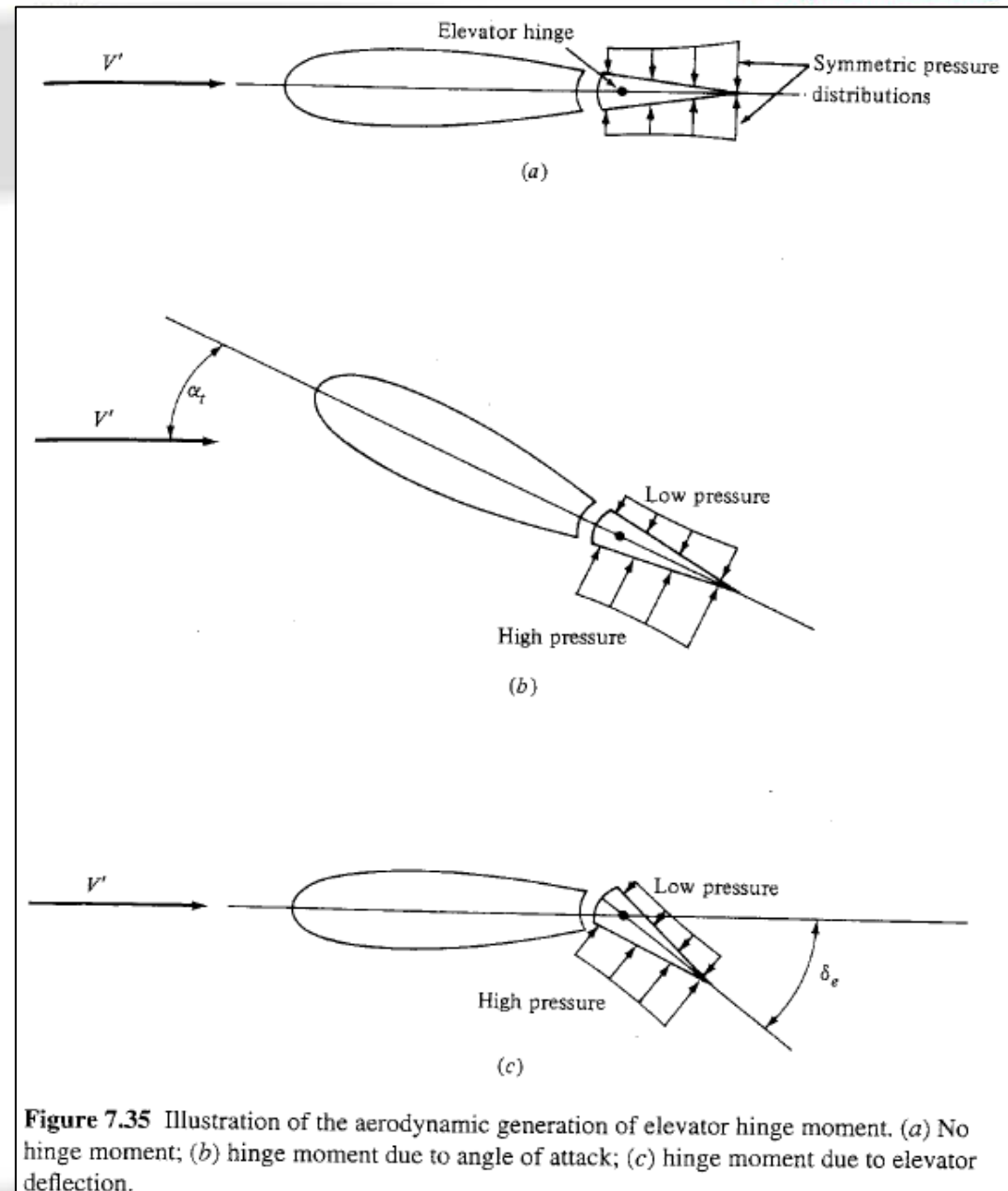
Longitudinale - Momento

COMANDI LIBERI

Momento di cerniera

$$C_{he} = \frac{H_e}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S_e c_e}$$

SI PUO' NOTARE CHE il momento (e quindi il coeff) di cerniera dovuta ad alfa e de SONO ENTRAMBI NEGATIVI (momento antiorario sulla cerniera)



Longitudinale - Momento

COMANDI LIBERI

Momento di cerniera

$$C_{he} = \frac{H_e}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S_e c_e}$$

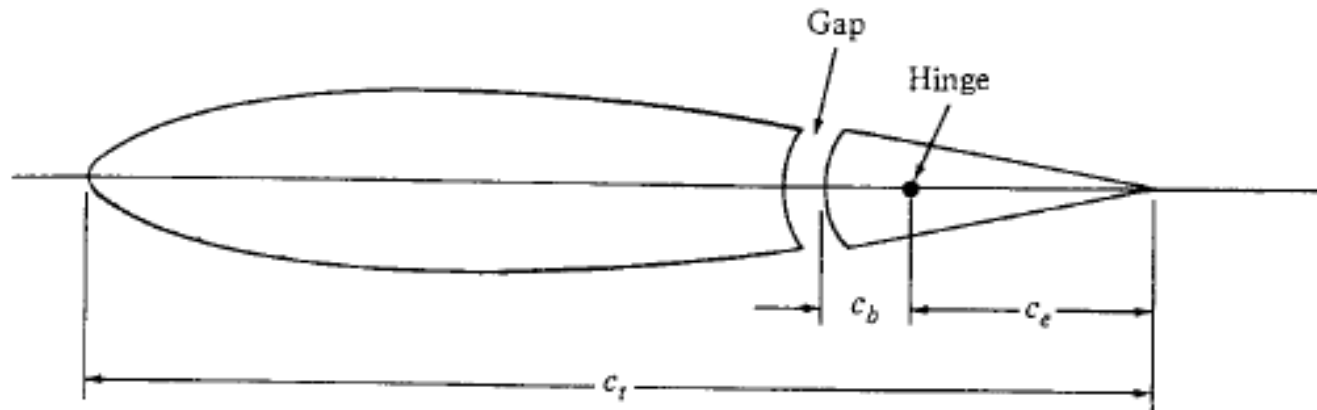


Figure 7.36 Nomenclature and geometry for hinge moment coefficient.



Longitudinale - Momento

COMANDI LIBERI

Momento di cerniera

$$C_{h_c} = \frac{H_e}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 S_e c_e}$$

$$C_{h_c} = \frac{\partial C_{h_c}}{\partial \alpha_t} \alpha_t + \frac{\partial C_{h_c}}{\partial \delta_e} \delta_e$$



Longitudinale - Momento

COMANDI LIBERI

Angolo di flottaggio (floating)

$$C_{h_e} = \frac{\partial C_{h_e}}{\partial \alpha_f} \alpha_f + \frac{\partial C_{h_e}}{\partial \delta_e} \delta_e$$

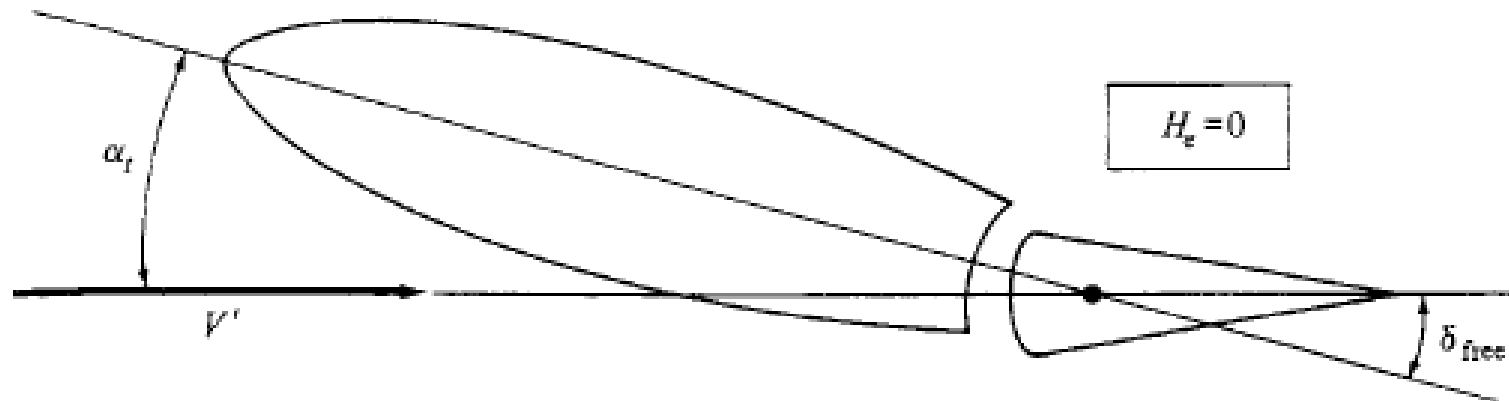


Figure 7.37 Illustration of free elevator deflection.

Thus, from Eq. (7.45),

$$C_{h_e} = 0 = \frac{\partial C_{h_e}}{\partial \alpha_f} \alpha_f + \frac{\partial C_{h_e}}{\partial \delta_e} \delta_{free} \quad (7)$$



Longitudinale - Momento

COMANDI LIBERI

Angolo di flottaggio (floating)

$$C_{h_e} = \frac{\partial C_{h_e}}{\partial \alpha_t} \alpha_t + \frac{\partial C_{h_e}}{\partial \delta_e} \delta_e$$

$$C_{h_e} = 0 = \frac{\partial C_{h_e}}{\partial \alpha_t} \alpha_t + \frac{\partial C_{h_e}}{\partial \delta_e} \delta_{\text{free}}$$

δ_{free} gives

$$\delta_{\text{free}} = - \frac{\partial C_{h_e} / \partial \alpha_t}{\partial C_{h_e} / \partial \delta_e} \alpha_t$$



Longitudinale - Momento

COMANDI LIBERI

Angolo di flottaggio (floating)

$$C_{L,t} = a_t \alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e} \delta_e$$

However, for a free elevator, $\delta_e = \delta_{\text{free}}$. Denoting the tail lift coefficient for a free elevator as $C'_{L,t}$, we see that a substitution of Eq. (7.47) into (7.36) gives

$$C'_{L,t} = a_t \alpha_t + \frac{\partial C_{L,t}}{\partial \delta_e} \delta_{\text{free}}$$

$$C'_{L,t} = a_t \alpha_t - \frac{\partial C_{L,t}}{\partial \delta_e} \frac{\partial C_{h_c} / \partial \alpha_t}{\partial C_{h_c} / \partial \delta_e} \alpha_t$$

or

$$C'_{L,t} = a_t \alpha_t F$$

(7.48)



Longitudinale - Momento

COMANDI LIBERI

Angolo di flottaggio (floating)

or

$$C'_{L,t} = a_t \alpha_t F$$

where F is the *free elevator factor*, defined as

$$F = 1 - \frac{1}{a_t} \frac{\partial C_{L,t}}{\partial \delta_e} \frac{\partial C_{h_e} / \partial \alpha_t}{\partial C_{h_e} / \partial \delta_e}$$



Longitudinale - Momento

COMANDI LIBERI

Nel caso di comandi liberi l'angolo dell'equilibratore è funzione di α_h (che a sua volta è funzione di α_{body}).

$$C_m^l = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha) (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) + i_h + \tau_e \delta_{e_{fl}} (\alpha_h) \right]$$

(1)

Quindi, rispetto al caso a comandi bloccati viene ad essere variato :

C_{m_0}

(ad $\alpha = 0$ il de di questa volta, dipendente da α NON E' NULLO)

C_{m_α}

(la dipendenza da α c'è anche nel termine dipendente da $de(\alpha)$)



Longitudinale - Momento

COMANDI LIBERI

$$C_m' = C_{m_{ac_{wf}}} + (C_{L_0_{wf}} + C_{L_{\alpha_{wf}}} \alpha)(\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - (\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha) + i_h + \tau_e \delta_{e_{fl}}(\alpha) \right] \quad (1)$$

$$C_{M_0}' = C_{M_{ac_{wf}}} + C_{L_0} (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) + C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \epsilon_0$$

$$- C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot [\tau \cdot \delta_{e_{fl}}(\alpha)] \quad (2)$$

ma $C_H = C_{H_\alpha} \cdot \alpha_h + C_{H_{\delta_e}} \cdot \delta_e$

$$\delta_{e_{fl}} = - \frac{C_{H_\alpha}}{C_{H_{\delta_e}}} \cdot (\alpha_h) \quad (3) \text{ per } \alpha = 0 \Rightarrow \delta_{e_{fl}} = - \frac{C_{H_\alpha}}{C_{H_{\delta_e}}} \cdot (-\epsilon_0) = \frac{C_{H_\alpha}}{C_{H_{\delta_e}}} \cdot \epsilon_0$$

Longitudinale - Momento

COMANDI LIBERI

$$C_{M0}' = C_{M_{ac_wf}} + C_{L_0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0 - C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left[\tau \cdot \delta_{e_{fl}} (\alpha = 0) \right] \quad (2)$$

$$C_{M0}' = C_{M_{ac_wf}} + C_{L_0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0 - C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left[\tau \cdot \frac{C_{H_\alpha}}{C_{H_\delta}} \varepsilon_0 \right] \quad (4)$$

Quindi :

$$C_{M0}' = C_{M_{ac_wf}} + C_{L_0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L_{\alpha_h}} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0 \left[1 - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \right]$$

Longitudinale - Momento

COMANDI LIBERI

$$C_{M0}' = C_{M_{ac_wf}} + C_{L0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L\alpha_h} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0 \left[1 - \tau \frac{C_{H\alpha}}{C_{H\delta}} \right]$$

$$F = \left[1 - \tau \frac{C_{H\alpha}}{C_{H\delta}} \right] \quad (6)$$

(5)

$$C_{M0}' = C_{M_{ac_wf}} + C_{L0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L\alpha_h} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0 \cdot F$$

(7)



Longitudinale - Momento

COMANDI LIBERI

Vediamo l'effetto sul CM_alfa

$$C_m = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}} \alpha) (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) + C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - (\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha) + i_h + \tau_e \delta_{e_{fl}} (\alpha_h) \right]$$

$$\delta_{e_{fl}} (\alpha_h) = -\frac{C_{H_\alpha}}{C_{H_{\delta_e}}} \cdot (\alpha_h) \quad \alpha_h = \alpha - \epsilon + i_h = \left(\alpha - \epsilon_0 - \frac{d\epsilon}{d\alpha} \alpha + i_h \right) \quad (8)$$

$$\delta_{e_{fl}} (\alpha_h) = -\frac{C_{H_\alpha}}{C_{H_{\delta_e}}} \cdot (\alpha_h) = -\frac{C_{H_\alpha}}{C_{H_{\delta_e}}} \cdot \left(\alpha - \epsilon_0 - \frac{d\epsilon}{d\alpha} \alpha + i_h \right) \quad (9)$$

La parentesi quadra diventa:

$$\left[\alpha - \epsilon_0 - \frac{d\epsilon}{d\alpha} \alpha + i_h - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \alpha + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \epsilon_0 + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \frac{d\epsilon}{d\alpha} \alpha + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} i_h \right] \quad (10)$$

COMANDI LIBERI Vediamo l'effetto sul CM_alfa

$$C_m' = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}}) (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - (\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha) + i_h + \tau_e \delta_e \right]_{ff}(\alpha_h)$$

$$\left[\alpha - \epsilon_0 - \frac{d\epsilon}{d\alpha} \alpha + i_h - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \alpha + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \epsilon_0 + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \frac{d\epsilon}{d\alpha} \alpha + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} i_h \right]$$

Isolando i termini dipendenti da alfa (alfa_body) :

(10)

$$C_{M_\alpha}' = C_{L_{\alpha_{wf}}} \cdot (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left[1 - \frac{d\epsilon}{d\alpha} - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \frac{d\epsilon}{d\alpha} \right]$$

(11)

COMANDI LIBERI

Vediamo l'effetto sul C_{M_α}

$$C_m' = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}}) (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) + C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) + i_h + \tau_e \delta_e \right]_{ff}(\alpha_h)$$

Isolando i termini dipendenti da alfa (alfa_body) :

$$C_{M_\alpha}' = C_{L_{\alpha_{wf}}} \cdot (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left[\left(1 - \frac{d\epsilon}{d\alpha} \right) \left(1 - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \right) \right] \quad (12)$$

$$C_{M_\alpha}' = C_{L_{\alpha_{wf}}} \cdot (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\epsilon}{d\alpha} \right) \cdot F$$

(13)

COMANDI LIBERI Vediamo l'effetto sul CM_α

$$C_{M_\alpha}' = C_{L_{\alpha_{wf}}} \cdot (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot F$$

(13)

Ricordo che a comandi bloccati :

$$C_{M_\alpha} = C_{L_{\alpha_{wf}}} \cdot (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) - C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

In definitiva il termine del piano di coda (che rende il C_{M_α} negativo, cioè stabile) viene ad essere ridotto per la presenza di $F (< 1)$

$$F = \left(1 - \tau \frac{C_{H_\alpha}}{C_{H_\delta}}\right) < 1$$

Infatti, tipicamente $\tau = 0.40 - 0.50$

e $\frac{C_{H_\alpha}}{C_{H_\delta}} \approx 0.70 - 0.80$ $F \approx 0.70$



COMANDI LIBERI A Comandi liberi si modifica anche il CM_{ih}

Andando a raccogliere nella equazione (10) i termini dip da ih:

$$C_{M_{ih}}' = -C_{L\alpha_h} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot F = -C_{L\alpha_h} \eta_h V_h \cdot F \quad (14)$$

Ricordo che a comandi bloccati :

$$C_{M_{ih}} = -C_{L\alpha_h} \eta_h V_h \quad (15)$$

Quindi : $C_{M_{ih}}' = C_{M_{ih}} \cdot F$

$$F = \left(1 - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \right) < 1$$

Infatti, tipicamente $\tau = 0.40 - 0.50$

e $\frac{C_{H_\alpha}}{C_{H_\delta}} \approx 0.70 - 0.80$ $F \approx 0.70$



COMANDI LIBERI Equazione

$$C_M' = C_{M_0}' + C_{M_\alpha}' \cdot \alpha + C_{M_{i_h}}' \cdot i_h$$

Infatti non ho più il termine dipendente da δ_e in quanto l'equilibratore flotta e non risulta più una variabile indipendente.



COMANDI LIBERI Termini derivate a comandi liberi (confronto con der com blocc)

$$C_{M0}' = C_{M_{ac_wf}} + C_{L0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L\alpha_h} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0 \cdot F$$

$$C_{M0} = C_{M_{ac_wf}} + C_{L0} (\bar{x}_{cg} - \bar{x}_{ac_wf}) + C_{L\alpha_h} \cdot \eta_h \frac{S_h}{S} \cdot (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \varepsilon_0$$

$$C_{M\alpha}' = C_{L\alpha_wf} \cdot (\bar{x}_{cg} - \bar{x}_{ac_wf}) - C_{L\alpha_h} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot F$$

$$C_{M\alpha} = C_{L\alpha_wf} \cdot (\bar{x}_{cg} - \bar{x}_{ac_wf}) - C_{L\alpha_h} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right)$$

$$C_{M_{ih}}' = -C_{L\alpha_h} \eta_h V_h \cdot F$$

$$C_{M_{ih}} = -C_{L\alpha_h} \eta_h V_h$$

COMANDI LIBERI

Risoluzione del problema relativo al calcolo dell'alfa di equilibrio (per dato i_h).
A comandi liberi l'equazione del momento ha una sola soluzione in alfa per dato i_h .
Infatti :

$$C_m' = C_{m_{ac_{wf}}} + (C_{L_{0_{wf}}} + C_{L_{\alpha_{wf}}}) (\bar{x}_{cg} - \bar{x}_{ac_{wf}}) +$$

$$- C_{L_{\alpha_h}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \left[\alpha - \left(\epsilon_0 + \frac{d\epsilon}{d\alpha} \alpha \right) + i_h + \tau_e \delta_e \right]_{fl(\alpha_h)}$$

$$\left[\alpha - \epsilon_0 - \frac{d\epsilon}{d\alpha} \alpha + i_h - \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \alpha + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \epsilon_0 + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} \frac{d\epsilon}{d\alpha} \alpha + \tau \frac{C_{H_\alpha}}{C_{H_\delta}} i_h \right]$$

Per dato i_h $C_M' = 0$ ha una sola soluzione in alfa. In effetti è equivalente a :

$$C_M' = C_{M_0}' + C_{M_\alpha}' \cdot \alpha + C_{M_{i_h}}' \cdot i_h = 0$$

E si può risolvere in alfa

COMANDI LIBERI

Risoluzione del problema relativo al calcolo dell'alfa di equilibrio (per dato i_h).
ALTERNATIVA (Mi permette di non calcolare anche le derivate a comandi liberi):

$$C_M' = C_{M_0} + C_{M_\alpha} \cdot \alpha + C_{M_{i_h}} \cdot i_h + C_{M_{\delta_e}} \cdot \delta_{e_{fl}}(\alpha_h) = 0$$

Esplicitato il δ_e di flottaggio come funzione di alfa:

$$\delta_{e_{fl}} = -\tau \frac{C_{H_\alpha}}{C_{H_\delta}} \cdot \left[\alpha - \varepsilon_0 - \frac{d\varepsilon}{d\alpha} \alpha + i_h \right]$$

$$\delta_{e_{fl}} = -\tau \frac{C_{H_\alpha}}{C_{H_\delta}} \cdot \left[\alpha \cdot \left(1 - \frac{d\varepsilon}{d\alpha} \right) - \varepsilon_0 + i_h \right]$$

L'equazione sopra (dove compaiono ancora le derivate di stabilità a comandi bloccati) può essere risolta in alfa e trovare quindi l'alfa body di equilibrio a comandi liberi. (VEDERE ESERCIZI SVOLTI).



MOM CERNIERA

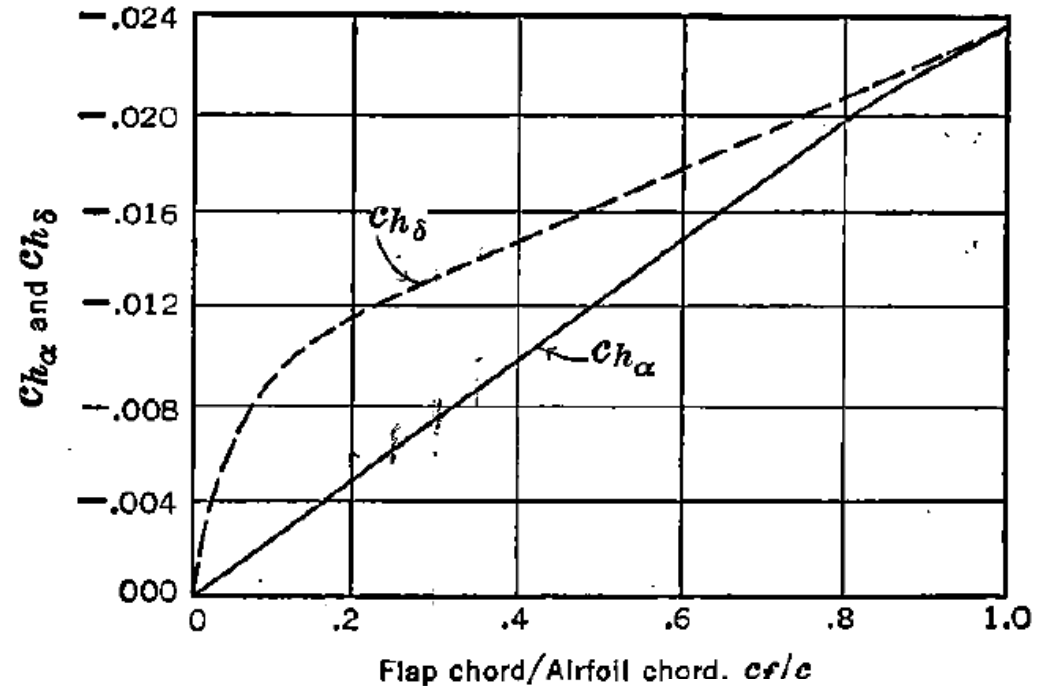
$$\frac{C_{H_\alpha}}{C_{H_\delta}} \approx 0.70 - 0.80$$

Possiamo facilmente vedere che come segno sono entrambi negativi.

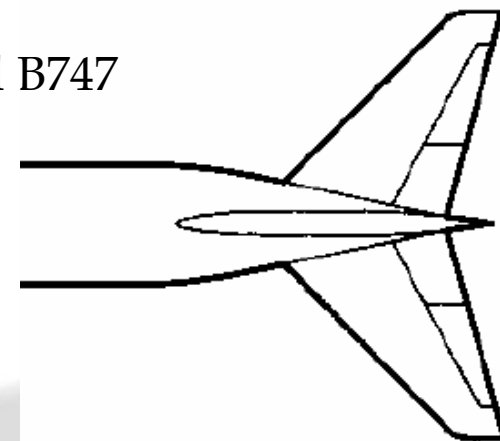
I loro valori (e quindi il rapporto) dipendono principalmente dal rapporto della corda dell'equilibratore sulla corda del piano di coda)

Tipicamente l'equilibratore si fa 0.30 - 0.35 della corda, quindi

$$C_{H_\alpha} \approx -0.008 \quad C_{H_\delta} \approx -0.013$$



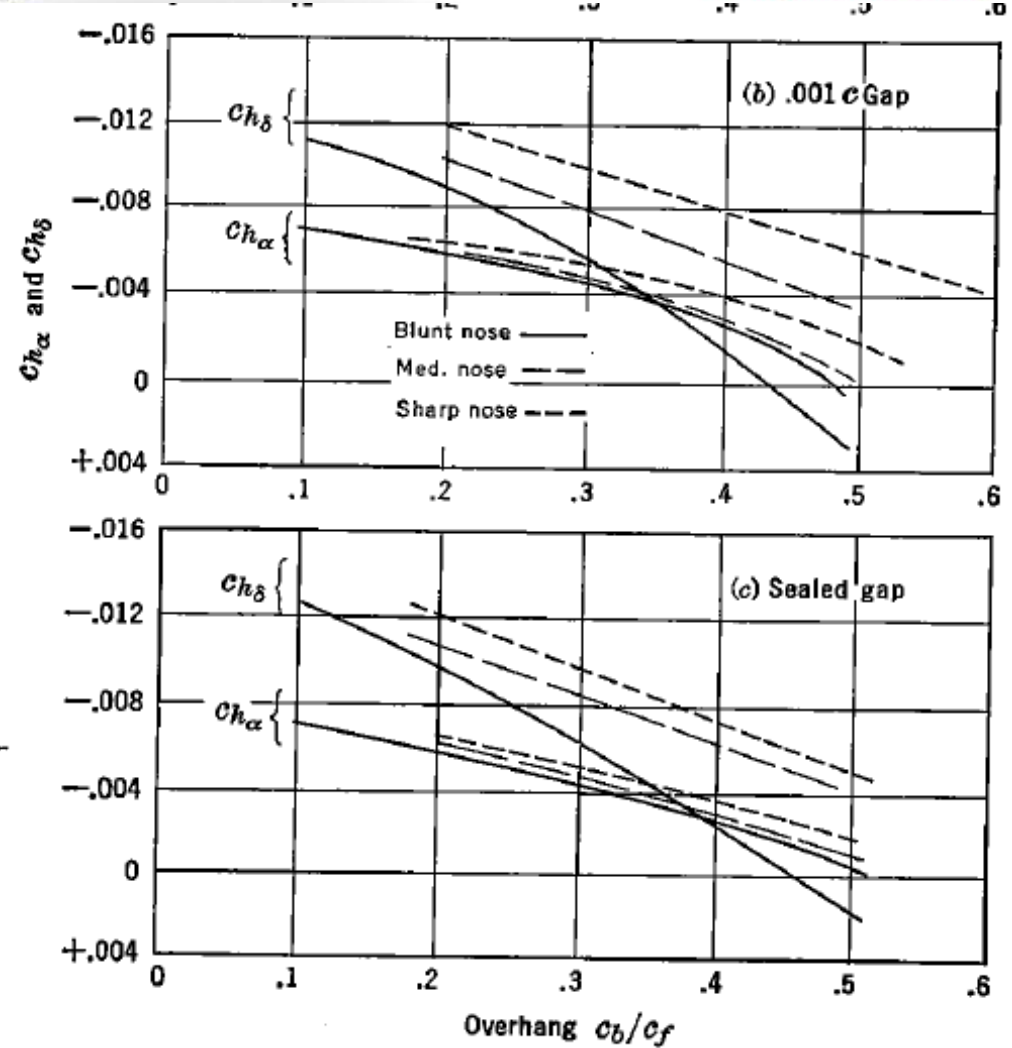
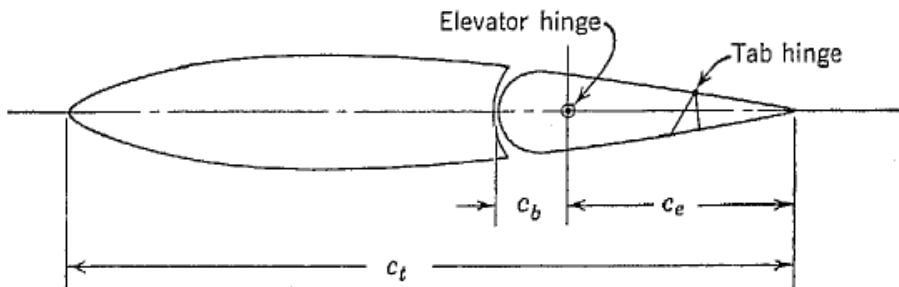
CODA del B747



MOM CERNIERA

Effetto importante è anche la posizione della cerniera.

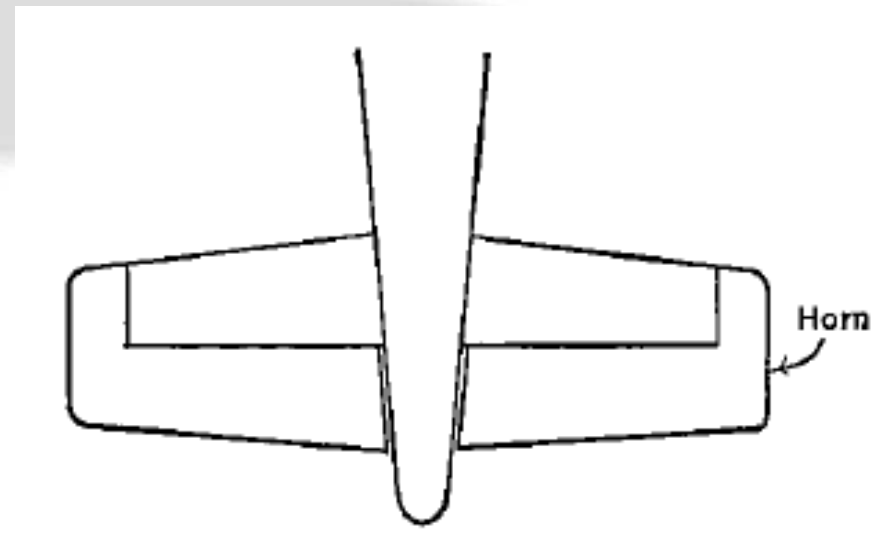
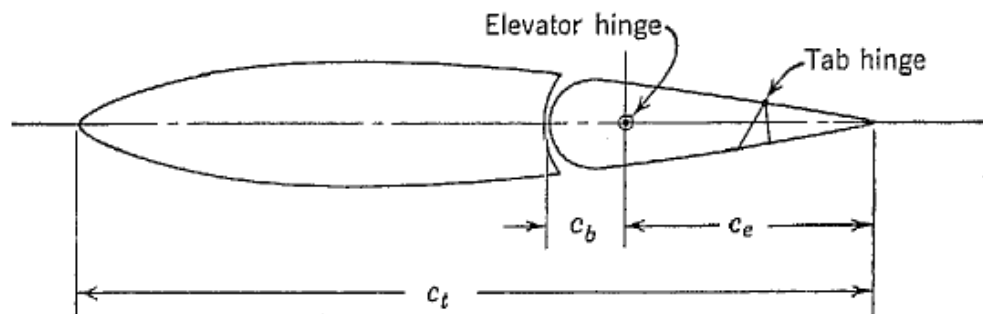
Tipicamente l'Overhang è
0.10 - 0.20



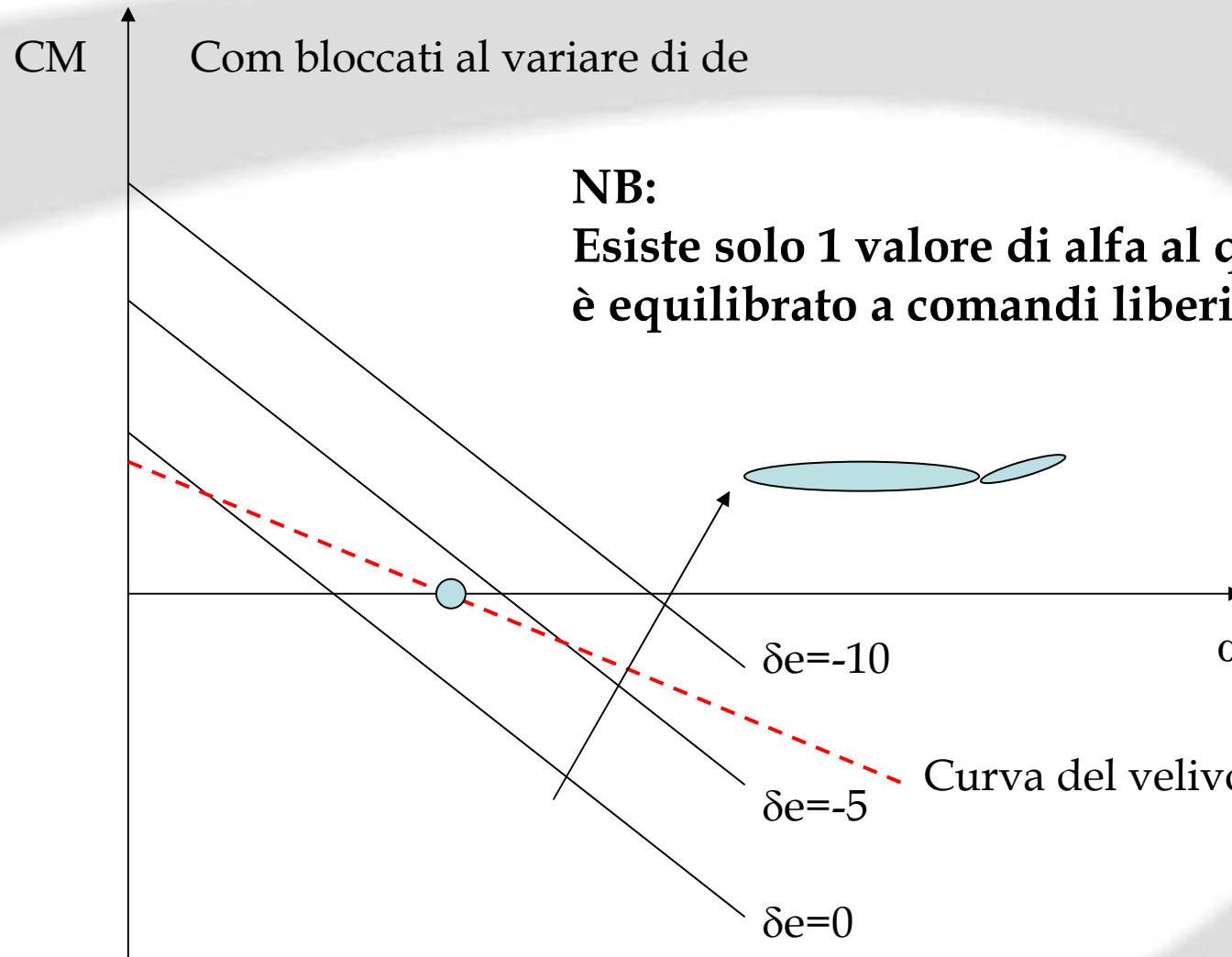
MOM CERNIERA

L'Horn fa variare sensibilmente il coeff di cerniera.

Nella zona esterna il coeff di mom cerniera diviene addirittura positivo.



COMANDI LIBERI



COMANDI LIBERI

Ovviamente si modifica quindi il margine statico di stabilità ed il punto neutro a comandi liberi:

$$\bar{x}_N = \bar{x}_{ac_wf} + \frac{C_{L\alpha_h}}{C_{L\alpha_wf}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha} \right) \cdot F$$

Ricordiamo che l'espressione (approssimata) del punto neutro di stab a comandi bloccati era:

$$\bar{x}_N = \bar{x}_{ac_wf} + \frac{C_{L\alpha_h}}{C_{L\alpha_wf}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha} \right)$$

$$\bar{x}_N = \bar{x}_{ac_wf} + \frac{C_{L\alpha_h}}{C_{L\alpha_wf}} \eta_h \cdot V_h \cdot \left(1 - \frac{d\varepsilon}{d\alpha} \right)$$

COMANDI LIBERI

Questa è una espressione approssimata

$$\bar{x}_N' = \bar{x}_{ac_wf} + \frac{C_{L\alpha_h}}{C_{L\alpha_wf}} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot F$$

Se ricaviamo C_{M_α}'

$$C_{M_\alpha}' = C_{L\alpha_wf} \cdot (\bar{x}_{cg} - \bar{x}_{ac_wf}) - C_{L\alpha_h} \eta_h \frac{S_h}{S} (\bar{x}_{ac_h} - \bar{x}_{cg}) \cdot \left(1 - \frac{d\varepsilon}{d\alpha}\right) \cdot F$$

Possiamo ricavare \bar{x}_N' in altro modo : $C_{m_\alpha} = C_{L_\alpha} (\bar{x}_{cg} - \bar{x}_{ac_A})$

$$\bar{x}_N' = \bar{x}_{cg} - \frac{C_{M_\alpha}'}{C_{L_\alpha}} = C_{L_\alpha}^* (X_{cg} - X_N)$$

COMANDI LIBERI

A rigore l'espressione corretta è :

$$\bar{x}_N' = \bar{x}_{cg} - \frac{C_{M_\alpha}'}{C_{L_\alpha}'}$$

La pendenza della retta di portanza del velivolo completo cambia tra comandi bloccati e comandi liberi

$$C_{L_\alpha}' = C_{L_{\alpha wf}} + C_{L_{\alpha h}} \frac{S_h}{S} \eta_h \left(1 - \frac{d\varepsilon}{d\alpha} \right) \cdot F \quad \text{comandi liberi}$$

$$C_{L_\alpha} = C_{L_{\alpha wf}} + C_{L_{\alpha h}} \frac{S_h}{S} \eta_h \left(1 - \frac{d\varepsilon}{d\alpha} \right) \quad \text{comandi bloccati}$$



EQUILIBRIO, Effetti Propulsivi

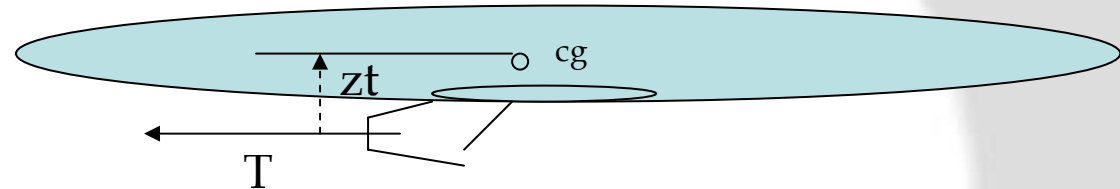
Se si inserisce anche il contributo della spinta dei motori va aggiunto all'equazione del CM il contributo dei motori.

La spinta T può essere ricavata dall'equilibrio spinta = resistenza.

Conoscendo il C_L dalla polare parabolica

Ricavo il C_D e quindi :

$$T=D=q S C_D$$



Il momento dovuto alla forza T è

$$M_t = T \cdot z_t$$

(in tal caso cabrante) , quindi positivo.

Si può stimare poi il coefficiente di momento equivalente :

Che va aggiunto nelle equazioni precedenti per ricavare i corretti valori di δ o α con inclusi gli effetti della forza propulsiva.

$$C_{M_T} = \frac{T \cdot z_T}{q \cdot S \cdot c}$$

