MODEL AIRPLANES, THE BERNOULLI EQUATION, AND THE COANDA EFFECT © 1994 by Jef Raskin

"In aerodynamics, theory is what makes the invisible plain. Trying to fly an airplane without theory is like getting into a fistfight with a poltergeist." --David Thornburg [1992]. "That we have written an equation does not remove from the flow of fluids its charm or mystery or its surprise."

--Richard Feynman [1964]

INTRODUCTION

A sound theoretical understanding of lift had been achieved within two decades of the Wright brothers' first flight (Prandtl's work was most influential'), but the most common explanation of lift seen in elementary texts and popular articles today is problematical. Here is a typical example of what is found

AIRFOIL



The common explanation, from The Way Things Work[Macaulay 1988] The reasoning--though incomplete--i s based on the Bernoulli effect, which correctly correlates the increased speed with which air moves over a surface and the lowered air pressure measured at that surface. In fact, most airplane wings do have considerably more curvature on the top than the bottom, lending credence to this explanation. But, even as a child, I found that it presented me with a puzzle: how can a plane fly inverted (upside down).

1Ludwig Prandtl (1875-1953), a German physicist, often called the "father of aerodynamics." His famous book on the theory of wings, Tragflü geltheorie, was published in 1918.

When I pressed my 6th grade science teacher on this guestion, he just got mad, denied that planes could fly inverted and tried to continue his lecture. I was very frustrated and argued until he said, "Shut up, Raskin!" I will relate what happened next later in this essay. A few years later I carried out a calculation according to a naive interpretation of the common explanation of how a wing works. Using data from a model airplane I found that the calculated lift was only 2% of that needed to fly the model. [See Appendix 1 for the calculation]. Given that Bernoulli's equation is correct (indeed, it is a form of the law of conservation of energy), I was left with my original question unanswered: where does the lift come from? In the next few sections we look at attempts to explain two related

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phenomena--what makes a spinning ball curve and how a wing's shape influences lift--and see how the common explanation of lift has led a surprising number of scientists (including some famous ones) astray. THE SPINNING BALL The path of a ball spinning around a vertical axis and moving forward through the air is deflected to the right or the left of a straight path. Experiment shows that this effect depends both on the fact it is spinning and that it is immersed in a fluid (air). Non-spinning balls or spinning balls in a vacuum qo straight. You might, before going on, want to decide for yourself which way a ball spinning counterclockwise (when seen from above) will turn. Let's see what five books say about this problem. Three are by physicists, one is a standard reference work, and the last, just for kicks, is from a book by my son's soccer coach. We'll start with physicist James Trefil, who writes [Trefil 1984], Before leaving the Bernoulli effect, I'd like to point out one more area where its consequences should be explored, and that is the somewhat unexpected activity of a baseball. Consider, if you will, the curve ball. This particular pitch is thrown so that the ball spins around an axis as it moves forward, as shown in the top in figure 11-4. Because the surface of the ball is rough, the effect of viscous forces is to create a thin layer of air which rotates with the surface. Looking at the diagram, we see that the air at the point labeled A will be moving faster than the the air at the point labeled B, because in the first case the motion of the ball's surface is added to the ball's overall velocity, while in the

second it is subtracted. The effect, then is a 'lift' force, which tends
to move the ball in the direction shown. The surface roughness is not essential.
The effect is observed no matter how smooth the ball.



Trefil's figure 11-4. It does not agree with some other sources.

Baseball aficionadoswould say that the ball curves toward third base. Trefil then shows a diagram of a fast ball, shown as deflecting downward when spinning so that the bottom of the ball is rotating forward. It is the same phenomenon with the axis of rotation shifted 90 degrees. In The Physics of Baseball, Robert K. Adair [Adair 1990] imagines a ball thrown toward home plate, so that it rotates counterclockwise as seen from above--as in Trefil's diagram. To the left of the pitcher is first base, to his right is third base. Adair writes:

We can then expect the air pressure on the third-base side of the ball, which is travelling faster through the air, to be greater than the pressure on the on the firstbase side, which is travelling more slowly, and the ball will be deflected toward first base. This is exactly the opposite of Trefil's conclusion though they agree that the side spinning forward is moving faster through the air. We have learned from these two sources that going faster through the air either increases or decreases the pressure on that side. I won't take sides in this argument as yet.

The *Encyclopedia Brittanica*[1979] gives an explanation which introduces the concept of drag into the discussion. "The drag of the side of the ball turning into the air(into the direction the ball is travelling) retards the airflow, whereas on the other side the drag speeds up the airflow. Greater pressure on the side where the airflow is slowed down forces the ball in the direction of the low-pressure region on the opposite side, where a relative increase in airflow occurs."

Now we have read that spinning the ball causes the air to move either faster or slower past the side spinning forward, and that faster moving air increases or decreases the pressure, depending on the authority you choose to follow.

Speaking of authority, it might be appropriate to turn to one of the giants of physics of this century, Richard Feynman. He takes the side of Trefil, and uses a cylinder rather than a sphere [Feynman et. al. 1964. Italics are theirs. The lift force referred to is shown pointing upwards.]:

"The flow velocity is higher on the upper side of a cylinder [shown rotating so that its top is moving in the same direction as its forward travel] than on thelower side. The pressures are therefore *lower*on the *uppers*ide than on the lower side. So when we have a combination of a circulation around a cylinder and a net horizontal flow, there is a net *vertical force*on the cylinder--it is called a *lift force*."

Now for my son's coach's book. The coach in this case is the world-class soccer player, George Lamptey. There is almost no theory given, but we can be reasonably sure that Lamptey has repeatedly tried the experiment and should therefore report the direction the ball turns correctly. He writes [Lamptey "The 19851: banana kick is more or less an off-center instep drive kick which adds a spin to the soccer ball. Kick off center to the right, the soccer ball curves to the left. Kick off center to the left, the soccer ball curves to the right... The amount the soccer ball curves depends on the speed of the spin."



Nonkicking Foot Kicking Foot

Lamptey, like Adair, has the high pressure on the side moving into the air. I will not relate more accounts, some having the ball swerve one way, some the other. Some explanations depend on the author's interpretation of the Bernoulli effect, some on viscosity, some on drag, some on turbulence. We will return to the subject of spinning balls, but we are not yet finished finding problems with the common explanation of lift.

OTHER PARADOXES The common explanation of how a wing works leads us to conclude, for example, that a wing which is somewhat concave on the bottom, often called an "undercambered" wing, will always generate *less*lift (under otherwise fixed conditions) than a flat bottomed one. This conclusion is wrong.

With an undercambered airfoil, the bottom path is longer than with the flat bottomed example above, so by the traditional explanation, this wing should generate less lift. However, it generates more. Hmmmm.

We then have to ask how a flat wing like that of a paper airplane, with no curves anywhere, can generate lift. Note that the flat wing has been drawn at a tilt, this tilt is called "angle of attack" and is necessary for the flat wing to generate lift. The topic of angle of attack will be returned to presently.

A flat wing can generate lift. This is a bit difficult to explain given the traditional mental model.

The cross-sectional shapes of wings, like those illustrated here, are called "airfoils." A very efficient airfoil for small, slow-flying models is an arched piece of thin sheet material, but it is not clear at all from the common explanation how it can generate lift at all since the top and bottom of the airfoil are the same length.

For small models, this wing cross section is very efficient, but there is almost no difference in the lengths along the top and bottom.

If the common explanation is all there were to it, then we should be making the tops of wings even curvier than they now are. Then the air would have to go even faster, and we'd get more lift. In this diagram the wiggliness is exaggerated. More realistic lumpy examples will be encountered in a few moments.

If we make the top of the wing like this, the air on top has a lot longer path to follow, so the air will go even faster than with a conventional wing. You might conclude that this kind of airfoil should have lots of lift. In fact, it is a disaster.

Enough examples. While Bernoulli's equations are correct, their proper application to aerodynamic lift proceeds quite differently than the common explanation. Applied properly or not, the equations result in no convenient visualization that links the shape of an airfoil with its lift, and reveal nothing about drag. This lack of a readily-visualized mental model, combined with the prevalence of the plausible-sounding common explanation, is probably why even some excellent physicists have been misled.

ALBERT EINSTEIN'S WING

My friend Yesso, who works for the aircraft industry (though not as a designer), came up with a proposed improved airfoil. Reasoning along the lines of the common explanation he suggested that you should get more lift from an airfoil if you restarted the top's curve part of the way along:

An extra lump for extra lift?

This is just a "reasonable" version of the lumpy airfoil that I presented above. Yesso's idea was, of course, based on the concept that a longer upper surface should give more lift. I was about to tell Yesso why his foil idea wouldn't work when I happened to talk to Jö rgen Skogh³. He told me of a humped airfoil Albert Einstein⁴ designed during WWI that was based on much the same reasoning Yesso had used [Grosz 1988].

Albert Einstein's airfoil. It had no aerodynamic virtues.

This meant that instead of telling Yesso merely that his idea wouldn't work, I could tell him that he had created a modernized version of Einstein's error! Einstein later noted, with chagrin, that he had goofed⁵. [Skogh 1993]

3Mr. Skogh worked on aircraft design for Saab in Sweden and for Lockheed in the USA

4Albert Einstein [1879-1955], a German-American physicist, was one of the greatest scientists of all time. His small error in wing design does not detract from the massive revolution his thinking brought about in physics.

5Jö rgen Skogh writes, "During the First World War Albert Einstein was for a **time hired by the LVG (Luft-Verkehrs-Gesellshaft)** as a consultant. At LVG he designed an airfoil with a pronounced mid-chord hump, an innovation intended to enhance lift. The airfoil was tested in the Gö ttingen wind tunnel and also on an actual aircraft and found, in both cases, to be a flop." In 1954 Einstein wrote "Although it is probably true that the principle of flight can be most simply explained in this [Bernoullian] way it by no means is wise to construct a wing in such a manner!" See [Grosz, 1988] for the full text. EVIDENCE FROM EXPERIMENTS

If it were the case that airfoils generate lift solely because the airflow across a surface lowers the pressure on that surface then, if the surface is curved, it does not matter whether it is straight, concave, or convex; the common explanation depends only on flow parallel to the surface. Here are some experiments that you can easily reproduce to test this idea.

1. Make a strip of writing paper about 5 cm X 25 cm. Hold it in front of your lips so that it hangs out and down making a convex upward surface. When you blow across the top of the paper, it rises. Many books attribute this to the lowering of the air pressure on top solely to the Bernoulli effect.



Now use your fingers to form the paper into a curve that it is slightly concave upward along its whole length and again blow along the top of this strip. The paper now bends downward.

2. As per the diagrams below, build a box of thin plywood or cardboard with a balsa airfoil held in place with pins that allow it to flap freely up and down. Air is introduced with a soda straw. That's one of the nice things about science. You don't have to take anybody's word for a claim, you can try it yourself!⁶In this wind tunnel the air flows only across the top of the shape.

6In some fields, e.g. the study of sub-atomic particles, you might need megabucks and a staff of thousands to build an accelerator to do an independent check, but the principle is still there.

A student friend of mine made another where a leaf blower blew on both top and bottom and he got the same results, but that design takes more effort to build and the airfoil models require leading and trailing edqe refinement. Incidentally, I tried to convince a company that makes science demonstrators to include this in their offerings. They weren't interested in it because "it didn't give the right results.""Then how does it work?" I asked. "I don't know," said the head designer. An experiment may be difficult to interpret but, unless it is fraudulent, it cannot give the wrong results.

AIRFOIL DEMONSTRATOR. These drawings are full size, but the exact size and shape aren't important. I made a number of airfoils to test. Here are drawings of the ones I made:

0
o Normal, concave
0
o recurved flat



EXPERIMENTAL RESULTS

When the straw is blown into, the normal airfoil promptly lifts off the bottom and floats up. When the blowing stops, it goes back down. This is exactly what everybody expects. Now consider the concave shape; the curve is exactly the same as the first airfoil , though turned upside down. If the common explanation were true, then, since the length along the curve is the same as with the "normal" example, you'd expect this one to rise, too. After all, the airflow along the surface must be lowering the pressure, allowing the normal ambient air pressure below to push it up. Nonetheless, the concave airfoil stays firmly down; if you hold the apparatus vertically, it will be seen to move awayfrom the airflow. an often-cited experiment which is usually taken Τn other words, as demonstrating the common explanation of lift does not do so; another effect is far stronger. The rest of the airfoils are for fun--try to anticipate the direction each will move before you put them in the apparatus. It has been noted that "progress in science comes when experiments contradict theory" [Gleick 1992] although in this case the science has been long known, and the experiment contradicts not aerodynamic theory, but the often- taught common interpretation. Nonetheless, even if science does not progress in this case, an individual's understanding of it may. Another simple experiment will lead us toward an explanation that may help to give a better feel for these aerodynamic effects.

THE COANDA EFFECT

If a stream of water is flowing along a solid surface which is curved slightly away from the stream, the water will tend to follow the surface. This is an example of the Coanda effect⁷ and is easily demonstrated by holding the back of a spoon vertically under a thin stream of water from faucet. If you hold the spoon so that it can swing, you will feel it being pulled *toward* the stream of water. The effect has limits: if you use a sphere instead of a spoon, you will find that the water will only follow a part of the way around. Further, if the surface is too sharply curved, the water will not follow but will just bend a bit and break away from the surface.



The Coanda effect works with any of our usual fluids, such as air at usual temperatures, pressures, and speeds. I make these qualifications because (to give a few examples) liquid helium, gasses at extremes of low or high pressure or temperature, and fluids at supersonic speeds often behave rather differently. Fortunately, we don't have to worry about all of those extremes with model planes.

7In the 1930's the Romanian aerodynamicist Henri-Marie Coanda(1885-1972)

observed that a stream of air (or other fluid) emerging from a nozzle tends to follow a nearby curved or flat surface, if the curvature of the surface or angle the surface makes with the stream is not too sharp.

A stream of air, such as what you'd get if you blow through a straw, goes in a straight line

A stream of air alongside a straight surface still goes in a straight line

A stream of air alongside a curved surface tends to follow the curvature of the surface. Seems natural enough.

Strangely, a stream of air alongside a curved surface that bends away from it still tends to follow the curvature of the surface. This is the Coanda effect.

Another thing we don't have to wonder about is why the Coanda effect works, we can take it as an experimentally given fact. But I hope your curiosity is you unsatisfied on this point and that will seek further. A word often used to describe the Coanda effect is to say that the airstream is "entrained" by the surface. One advantage of discussing lift and drag in terms of the Coanda effect is that we can visualize the forces involved in a rather straightforward way. The common explanation (and the methods used in serious texts on aerodynamics) are anything but clear in showing how the motion of the air is physically coupled to the wing. This is partly because much of the approach taken in the 1920s was shaped by the need for the resulting

differential equations (mostly based on the Kutta- Joukowski theorem⁸) to have closed-form solutions or to yield useful numerical results with paper-and-pencil methods. Modern approaches use computers and are based on only slightly more intuitive constructs. We will now develop an alternative way of visualizing lift that makes predicting the basic phenomena associated with it easier.

8 Discovered independently by the German mathematician M. Wilheim Kutta (1867-1944) and the Russian physicist Nikolai Joukowski (1847-1921).

A MENTAL MODEL OF HOW A WING GENERATES LIFT AND DRAG

As is typical of physicists, I have often spoken of the air moving past the wing. In aircraft wings usually move through the air. It makes no real difference, as flying a slow plane into the wind so that the plane's ground speed is zero demonstrates. So I will speak of the airplane moving or the wind moving whichever makes the point more clearly at the time. In the next illustration , it becomes convenient to look at the air from the point of view of a moving airplane

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the air molecules, attracted to the surface, are pulled down.

Think of the wing moving to the left, with the air standing still. The air moves toward the wing much as if it was attached to the wing with invisible rubber bands. It is often helpful to think of lift as the action of the rubber bands that are pulling the wing up.

Another detail is important: the air gets pulled along in the direction of the wing's motion as well. So the action is really more like the following picture.

The air is pulled forward as well as down by the motion of the wing.

If you were in a canoe and tried pulling someone in the water toward you with a rope, your canoe would move toward the person. It is classic action and reaction. You move a mass of air down and the wing moves up. This is a useful visualization of lift generated by top of the the the wing. As the diagram suggests, the wing has also spent some of its energy, necessarily, in moving the air forward. The imaginary rubber bands pull it back some. That's a way to think about the dragthat is caused by the lift the wing Lift without generates. cannot be had drag. The acceleration of the air around the sharper curvature near the front of the top of the wing also imparts a downward and forward component to the motion of the molecules of air (actually a slowing of their upward and backward motion, which is equivalent) and thus contributes to lift. The bottom of the wing is understand, explanation easier to and an is left to the reader. The experiments with the miniature wind tunnel described earlier are readily understood in terms of the Coanda effect: the downward-curved wing entrained the airflow to move downward, and a force upward is developed in reaction. The upward-curved (concave) airfoil entrained the airflow to move upwards, and a force downward was the result. The lumpy wing generates a lot of drag by moving air molecules up and down repeatedly. This eats up energy (by generating frictional heat) but doesn't create a net downward motion of the air and therefore doesn't create a net upward movement of the wing. It is easy, based on the Coanda effect, to visualize why angle of attack (the fore-and-aft tilt of the winq, as illustrated earlier) is crucially important to a symmetrical airfoil, why planes can fly inverted, why flat and thin wings work, and why Experiment 1 with its convex and concave strips of paper works as it does. What has been presented so far is by no means a physical account of lift and drag, but it does tend to give a good picture of the phenomena. We will now use this grasp to get a reasonable hold on the spinning ball problem.

WHY THE SPINNING BALL'S PATH CURVES, IN TERMS OF THE COANDA EFFECT

The Coanda effect tells us the air tends to follow the surface of the ball. Consider Trefil's side A which is rotating in the direction of flight. It is trying to entrain air with it as it spins, this action is opposed by the oncoming air. Thus, to entrain the air around the ball on this side, it must first decelerate it and then reaccelerate it in the opposite direction. On the B side, which is rotating opposite the direction of flight, the air is already moving (relative to the ball) in the same direction, and is thus more easily entrained. The air more readily follows the curvature of the B side around and acquires a velocity toward the A side. The ball therefore moves toward the B side by reaction.

It is again time for a simple experiment. It is difficult to experiment with baseballs because their weight is large compared to the aerodynamic forces on them and it is very hard to control the magnitude and direction of the spin, so let us look at a case where the ball is lighter and aerodynamic effects easier to see. I use a cheap beach ball (expensive ones are made of heavier materials and show aerodynamic effects less). Thrown with enough bottom spin (bottom moving forward) such a ball will actually*rise* in a curve as it travels forward. The lift due to spin can be so strong that it is greater than the downward force of gravity! Soon, air resistance stops both the spin and the forward motion of the ball and it falls, but not before it has shown that Trefil's explanation of how spin affects the flight of a ballis wrong. The lift due to spinning while moving through the air is usually called the "Magnus⁹effect." Some books on aerodynamics also describe the "Flettner Rotor," which is a long-since abandoned attempt to use the Magnus effect to make an efficient boat sail. Many sources besides Trefil get the effect backwards including the usually reliable Hoerner [Hoerner 1965]. College- level texts tend to get it right [Kuethe and Chow 1976; Houghton and Carruthers 1982] but, as noted above, Feynman's Lectures on Physicshas the rotation backwards.

9H. G. Magnus (1802-1870), a German physicist and chemist, demonstrated this effect in 1853.

I was relieved to see that the classicAerodynamics[von Ká rmá n 1954] gets the lift force on a spinning ball in the correct direction though the reasoning seems a bit strained.

I wish I could send this essay to the 6th grade science teacher who could not take the time to listen to my reasoning. Here's what happened: he sent me to the principal's office when I came in the next day with a balsa model plane with dead flat wings. It would fly with either side up depending on how an aluminum foil elevator adjustment was set. I used it to demonstrate that the explanation the class had been given must have been wrong, somehow. The principal, however, was informed that my offense was "flying paper airplanes in class" as though done with disruptive intent. After being warned that I was to improve my behavior, I went to my beloved math teacher who suggested that I go to the library to find out how airplanes fly--only to discover that all the books agreed with my science teacher! It was a shock to realize that my teacher and even the library books could be wrong. And it was a revelation that I could trust my own thinking in the face of such concerted opposition.

My playing with model airplanes had led me to take a major step toward intellectual independence--and a spirit of innovation that later led me to create the Macintosh computer project (and other, less-well-known inventions) as an adult.

APPENDIX 1 A QUANTITATIVE APPLICATION OF THE COMMON (INCORRECT) EXPLANATION If the pressure, in Newtons per square meter $(Nm^{-2} = kgm^{-1}s^{-2})$, on the top of a wing is notated ptop , the pressure on the bottom pbottom , the velocity (ms⁻¹) on the top of the wing v, and the velocity on the bottom v_{\text{bottom}}, and where ___ is the ^{\text{top}} density of air (approximately 1.2 kgm⁻³⁾, then the pressure difference across the the first of Bernoulli's wing is given by term equation: 1/2 (v_{top}^2) ptoppbottom= _ $v_{bottom}^2)$ A rectangular planform (top view) wing of one meter span was measured as having a length chordwise along the bottom of 0.1624 m while the length across the top was 0.1636 m. The ratio of the lengths is 1.0074. This ratio is typical for many model and full- size aircraft wings. According to the common explanation which has two adjacent molecules separated at the leading edge mysteriously meeting at the trailing edge, the average air velocities on the top and bottom are also in the ratio of 1.0074. A typical speed for a model plane of 1m span and 0.16m chord with a mass of 0.7 kα (a weight of 6.9 N) is 10 ms^{-1} is 10 ms⁻¹, so which makes $v_{top}10.074 \text{ ms}^{-1}$. Given these numbers,

find a pressure difference from the equation of about 0.9 kgm^{-1 - 2.}

The area of the wing is 0.16 ${\rm m2}^{\rm s}$

giving a total force of 0.14 N. This is not nearly enough--it misses lifting the weight of 6.9 N by a factor of about 50. We would need an air velocity difference of

about 3 ms⁻¹to lift the plane.

The calculation is, of course, an approximation since Bernoulli's equation assumes nonviscous, incompressible flow and air is both viscous and compressible. But the viscosity is small and at the speeds we are speaking of air does not compress significantly. Accounting for these details changes the outcome at most a percent or so. This treatment also ignores the second term (not shown) of the Bernoulli equation--the static pressure difference between the top and bottom of the wing due to their trivially different altitudes. Its contribution to lift is even smaller than the effects already ignored. The use of an average velocity assumes a circular arc for the top of the wing. This is not optimal but it will fly. None of these details affect the conclusion that the common explanation of how a wing generates lift--with its naï ve application of the Bernoulli equation--fails quantitatively. APPENDIX II A QUANTITATIVE APPROXIMATION OF THE LIFT GENERATED BY THE COANDA EFFECT

In normal flight the angle of attack of a wing is typically equivalent to raising the leading edge about 1/20th of the chord. If we knew the thickness of the layer of air bent by the wing we could compute the volume of air given a vertical component of motion downward. This is not an elementary question, but we can get a useful bound by looking back at the age of biplanes. As a rule of thumb, the upper and lower wings of a biplane were set at least one wing chord apart, otherwise there would be excessive interference between them. Thus the air 1/2 chord length above and below the wing is a useful approximation. Let us see if we get something close to the right amount of lift from it. In this approximation, as the wing moves forward one chord length, a mass of air 1 chord by 1 chord by the span (a volume of 0.027 m³having a mass of about 0.03 kg) is given a vertial motion 1/20 the chord length (.008 m) due to the Coanda effect. At a speed of 10 ms⁻¹the time the wing takes to move one chord length is xxx and the resultant vertical acceleration (assumed to be uniform) is yyyy.

FURTHER READING: There are many fine books and articles on the subject of model airplane aerodynamics (and many more on aerodynamics in general). Commendably accurate and readable are books and articles for modelers by Professor Martin Simons [e.g. Simons 1987]. Much can be learned from Frank Zaic's delightful, if not terribly technical, series [Zaic 1936 to Zaic 1964] (Available from the Academy of Model Aeronautics in the United States), and no treatments are more professional or useful than those of Professor Michael Selig and his colleagues [e.g. Selig et. al. 1989]. All of these authors are also well-known modelers. The other references on aerodynamics, e.g. Kuethe and Chow [1976] and

Houghton and Carruthers [1982] are graduate or upper-level undergraduate texts, they require a knowledge of physics and calculus including partial differential equations. Jones [1988] is an informal treatment by a master and Hoerner [1965] is a magnificent compendium of experimental results, but has little theory-practical designers find his work invaluable.

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