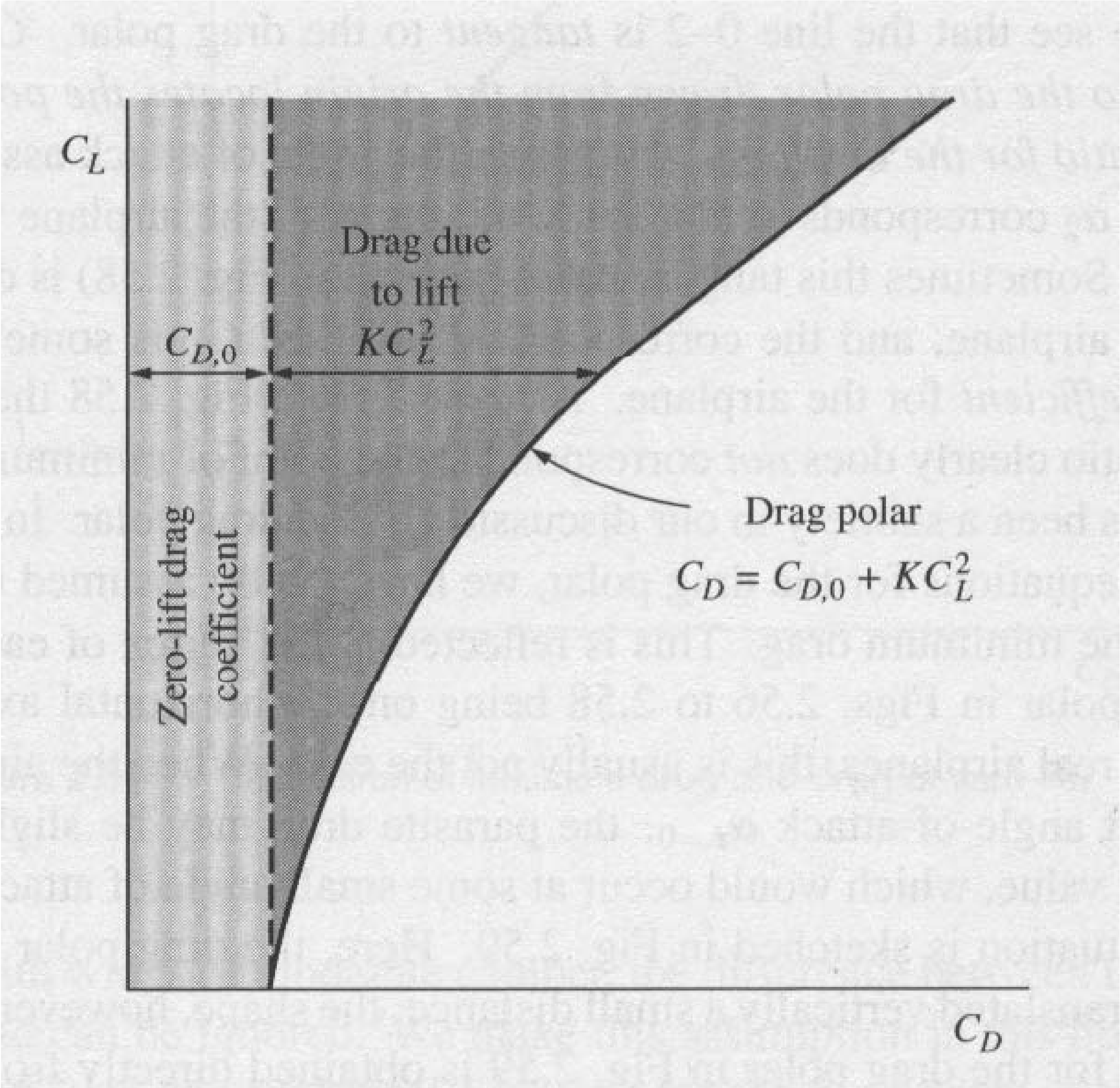
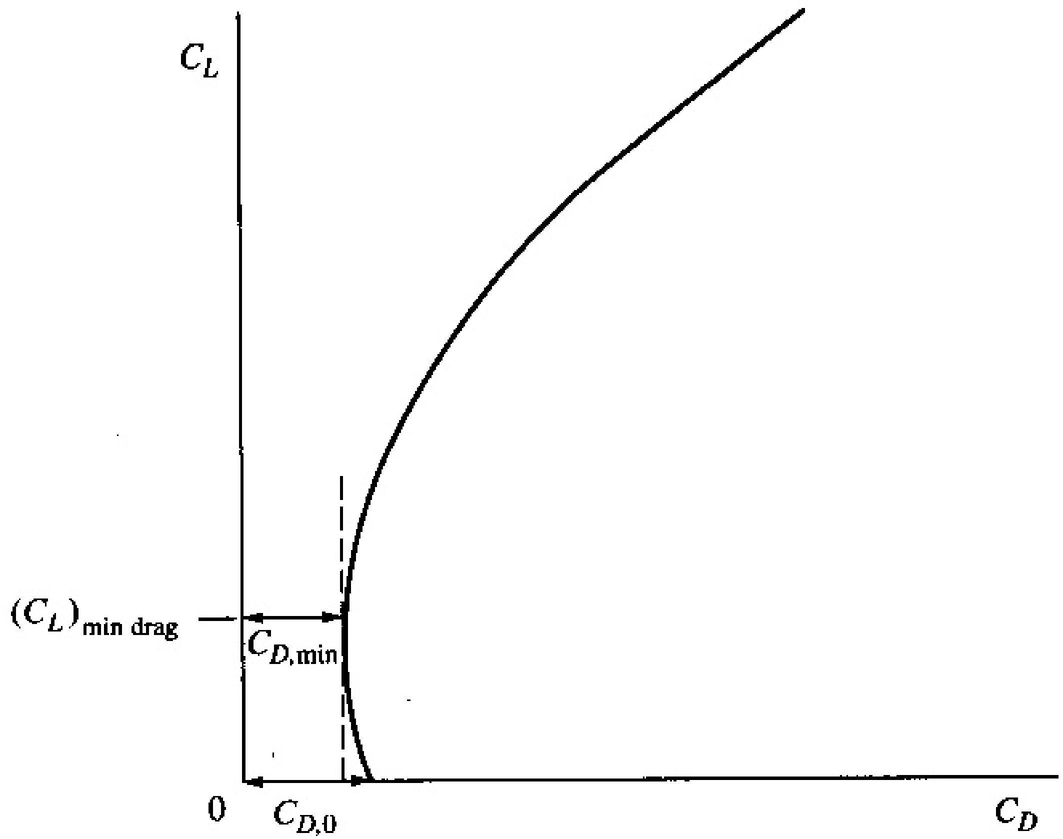
Cap.5 Polari tecniche



Cap.5 Polari tecniche

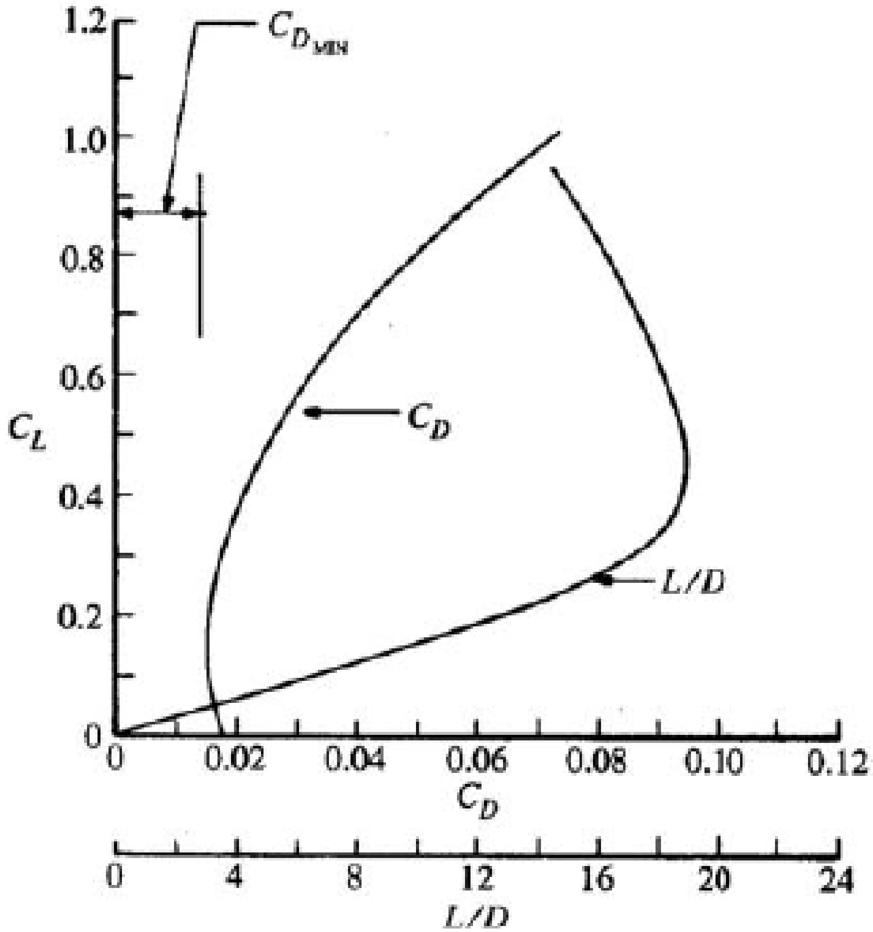
$$CD = CD_{min} + K\left(CL - CL_{min\_drag}\right)^2$$

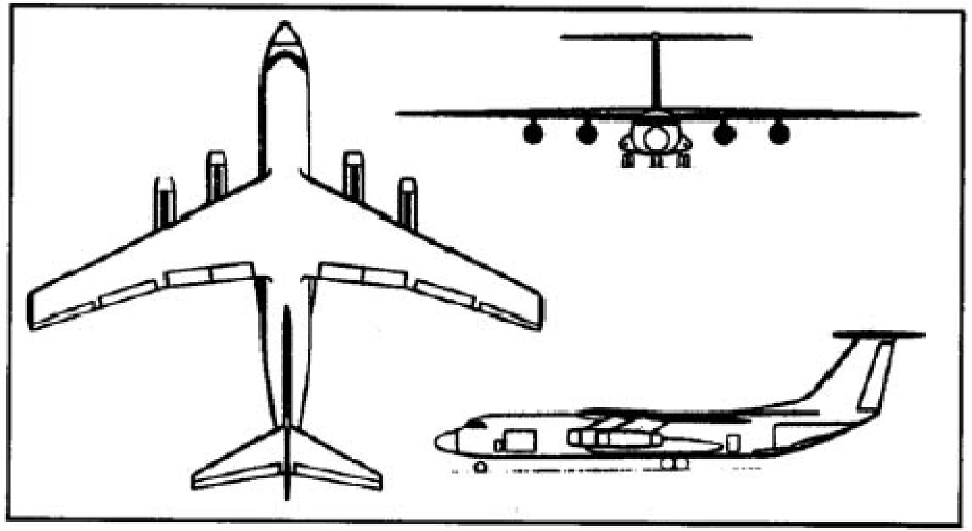
Polare parabolica ad asse spostato



Cap.5 Polari tecniche

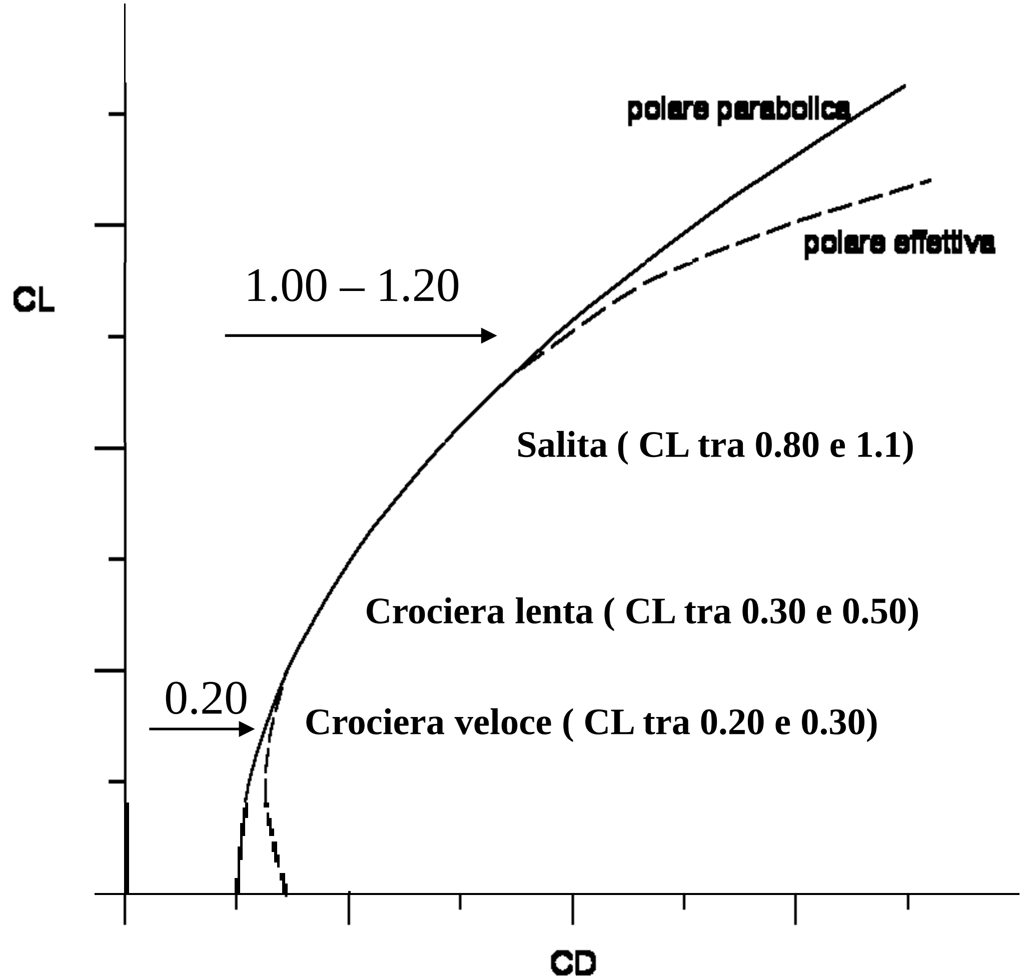
Lockheed C141 A





Cap.5 Polari tecniche

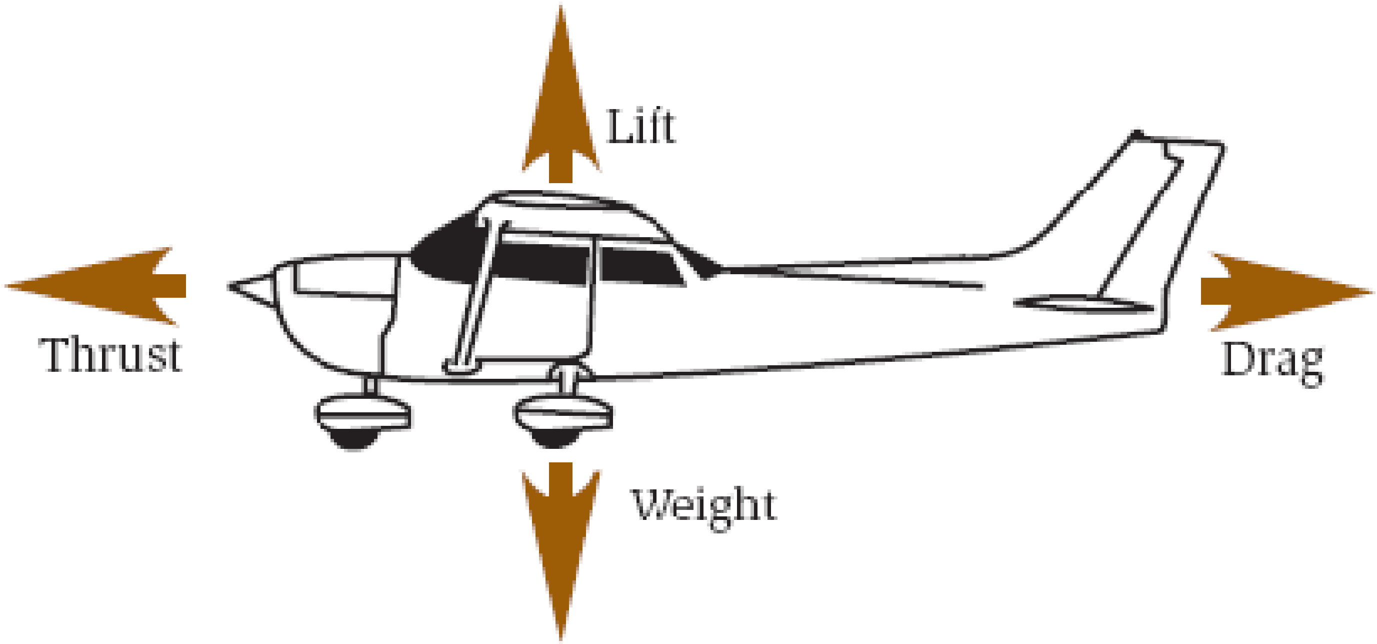
La polare parabolica
(anche ad asse non spostato)
approssima bene i regimi in cui il velivolo OPERA effettivamente



Cap.5 Polari tecniche

VOLO LIVELLATO

L=W
T=D



Cap.5 Polari tecniche

Velivolo in traiettoria curvilinea

Eq Direz parallela moto

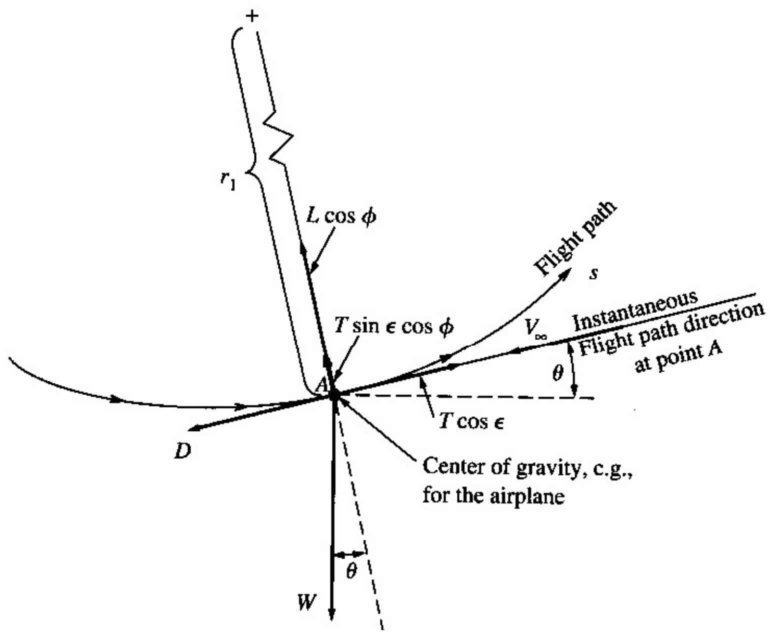
$$m \cdot a = m \cdot \frac{dV}{dt} = T \cdot \cos\varepsilon - D - W \sin\theta$$

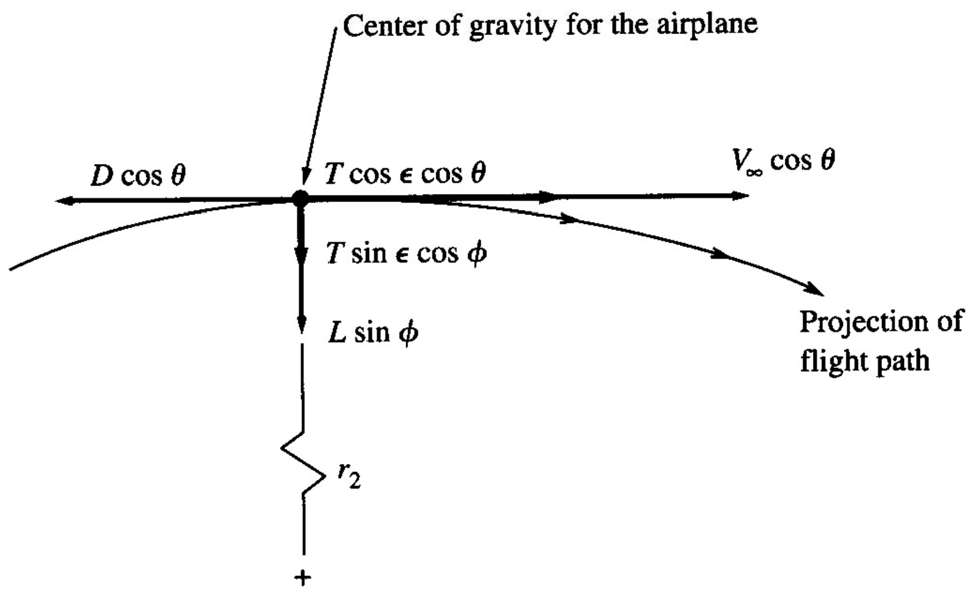
Eq Direz perpendicolare moto

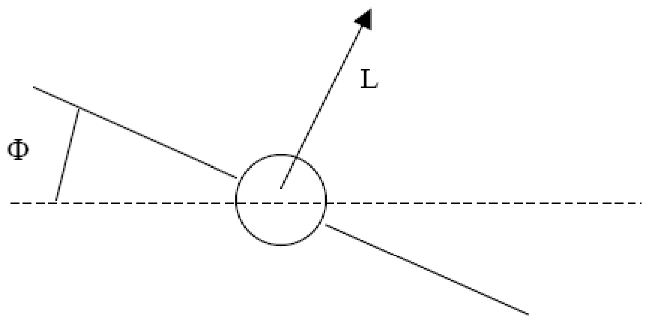
$$F = L \cdot \cos\Phi + T \cdot \sin\varepsilon \cdot \cos\Phi - W \cdot \cos\theta$$

con $\quad a = \frac{V^2}{r_1}$

$$m \cdot \frac{V^2}{r_1} = L \cdot \cos\Phi + T \cdot \sin\varepsilon \cdot \cos\Phi - W \cdot \cos$$







Cap.5 Polari tecniche

Velivolo in traiettoria curvilinea – PIANO RADIALE

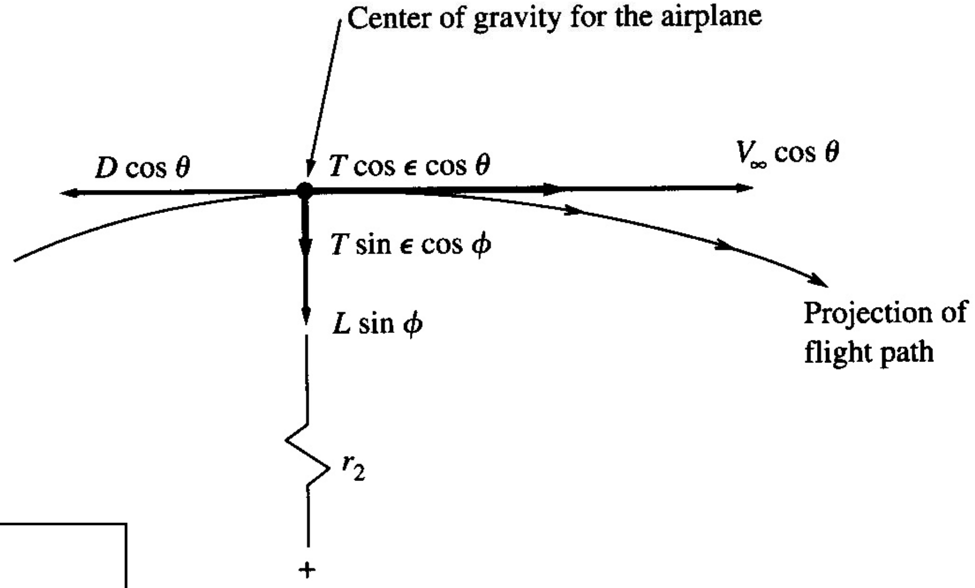
Eq forza in direzione perp. (nel piano radiale)

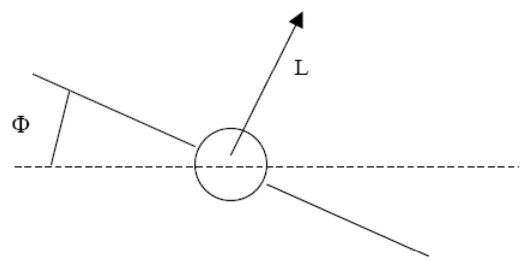
$$F_2 = L \cdot \sin\Phi + T \cdot \sin\varepsilon \cdot \sin\Phi$$

accelerazione

$$a_2 = \frac{(V \cdot \cos\theta)^2}{r_2}$$

$$\boxed{m \cdot \frac{(V \cdot \cos\theta)^2}{r_2} = L \cdot \sin\Phi + T \cdot \sin\varepsilon \cdot \sin\Phi}$$





Cap.5 Polari tecniche

Velivolo in traiettoria curvilinea – EQUAZIONI

DIR DEL MOTO $$m \cdot a = m \cdot \frac{dV}{dt} = T \cdot \cos\varepsilon - D - W \sin\theta$$

DIR. PERP. (Piano vert.) $$m \cdot \frac{V^2}{r_1} = L \cdot \cos\Phi + T \cdot \sin\varepsilon \cdot \cos\Phi - W \cdot \cos\theta$$

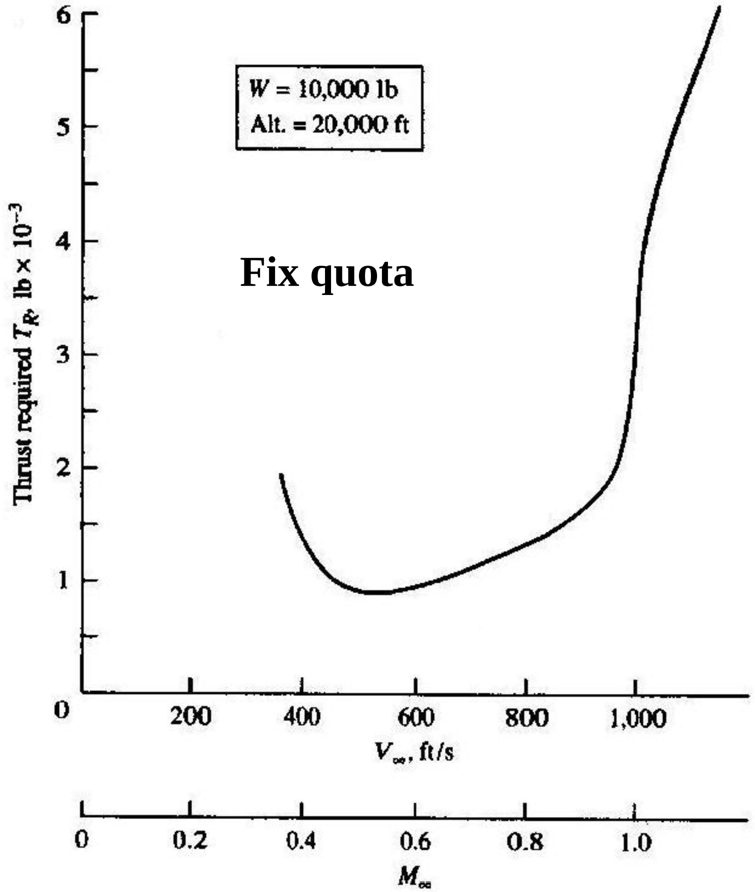
DIR. PERP. (Piano orizz.) $$m \cdot \frac{(V \cdot \cos\theta)^2}{r_2} = L \cdot \sin\Phi + T \cdot \sin\varepsilon \cdot \sin\Phi$$

Nel caso di volo livellato e non accelerato diventano
L=W
T=D

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

- La vel. Minima di sostentamento è la velocità di stallo

$$Vso = \sqrt{\frac{2}{\rho_o}\frac{W}{S}\frac{1}{CL_{MAX}}} \qquad Vs = Vso/\sqrt{\sigma}$$



$$V = \sqrt{\frac{2}{\rho}}\sqrt{\frac{W}{S}}\sqrt{\frac{1}{CL}} = \sqrt{\frac{2}{\rho_o\sigma}}\sqrt{\frac{W}{S}}\sqrt{\frac{1}{CL}}$$

$$CL = \frac{2}{\rho}\frac{W}{S}\frac{1}{V^2}$$

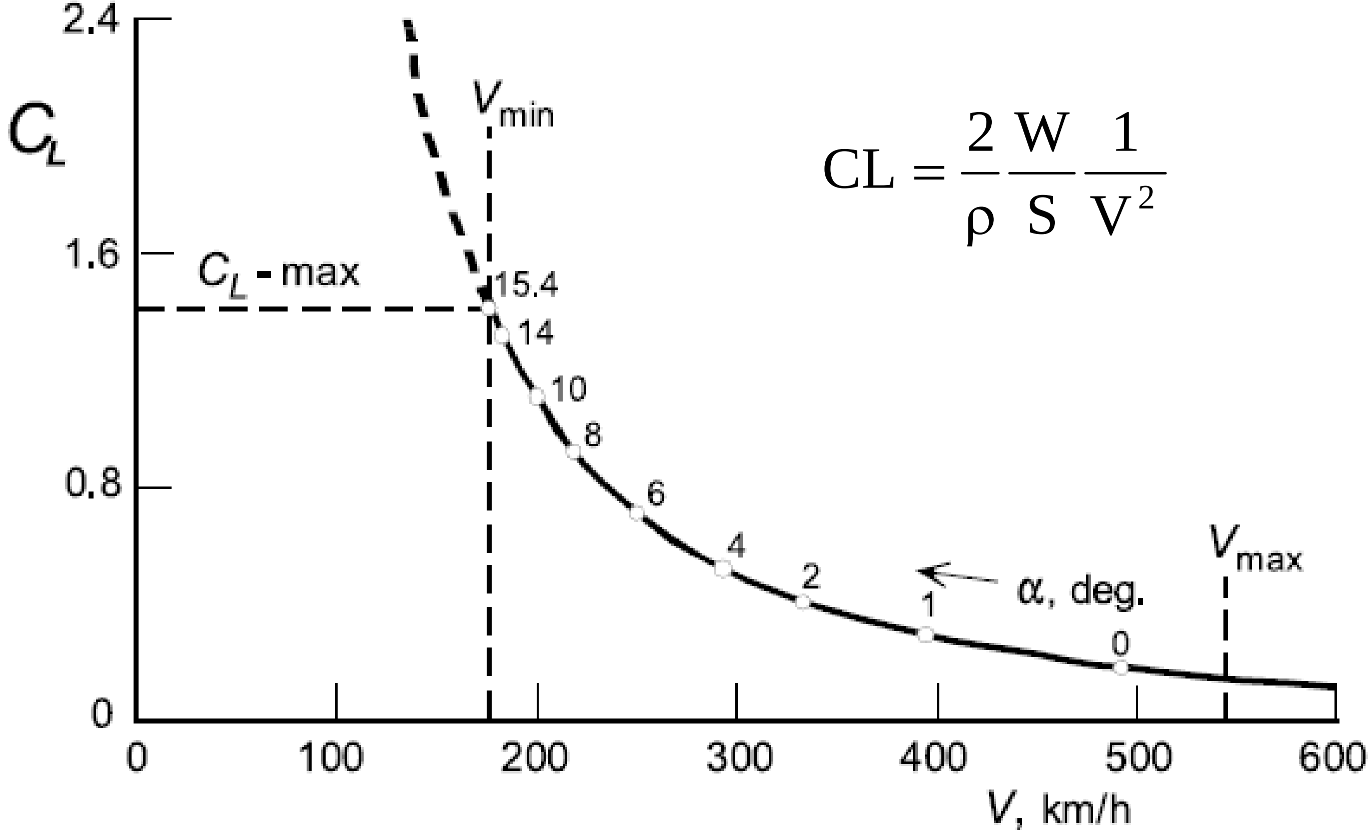
Per data quota

$$V \propto \frac{1}{\sqrt{CL}}$$

$$CL \propto \frac{1}{V^2}$$

Ricordiamo anche che CL è legato all'angolo di attacco α

Legame tra coeff. di portanza e velocità di volo
(per data:
- quota
- dati velivolo , cioè peso W e sup. alare S



$$CL = \frac{2}{\rho}\frac{W}{S}\frac{1}{V^2}$$

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

RESISTENZA

$$D = \frac{1}{2}\rho V^2 S C_D$$

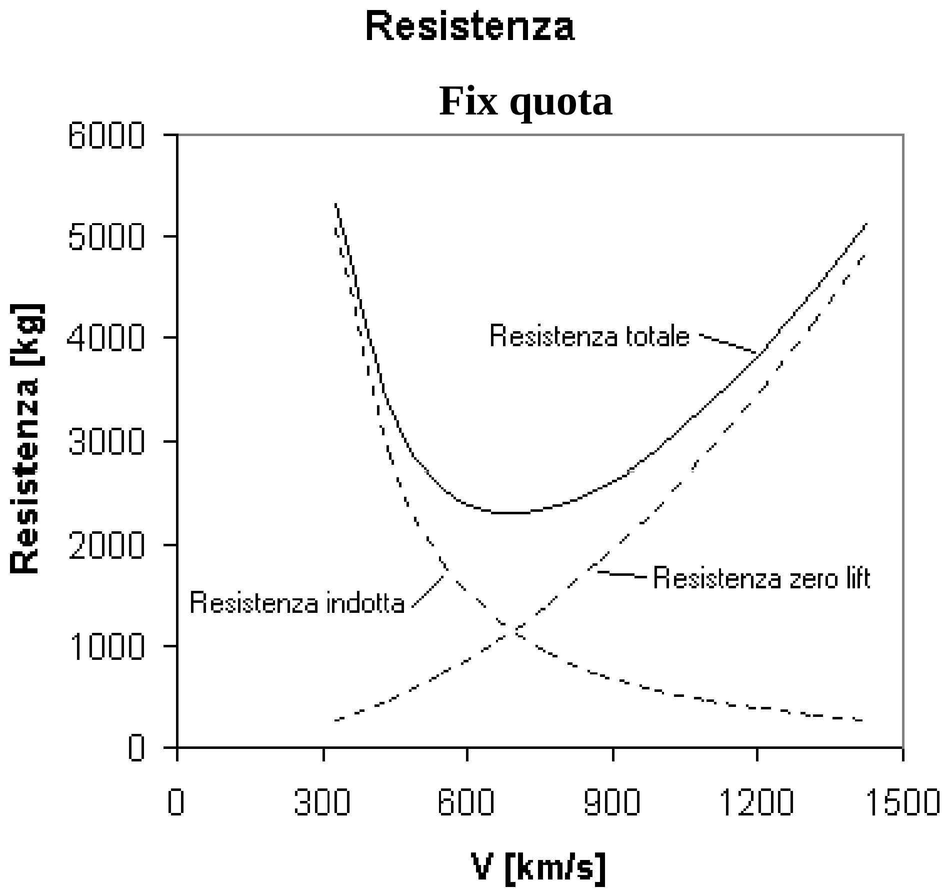
$$C_D = C_{Do} + K C_L^2$$

$$\boxed{\mathbf{D = \frac{1}{2}\rho V^2 S C_{Do} + \frac{1}{2}\rho V^2 S K C_L^2}}$$

Essendo

$$CL \propto \frac{1}{V^2}$$

Il secondo termine è una funzione $= f\left(\frac{1}{V^2}\right)$

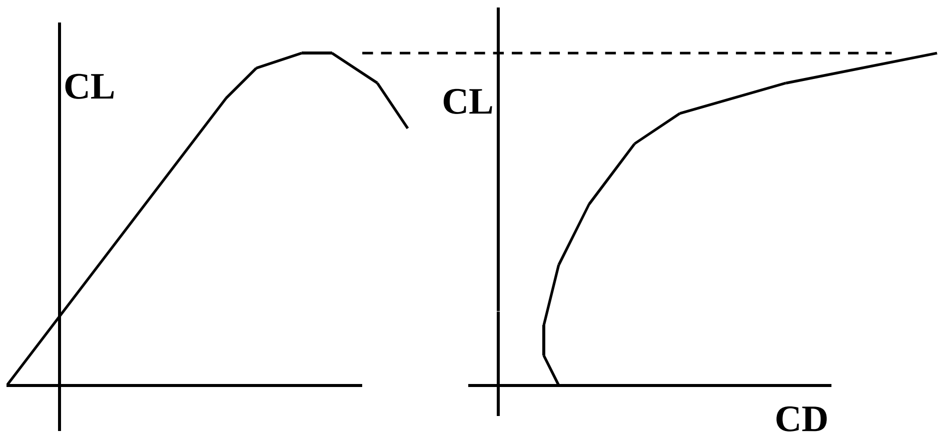


L'AEREO E' DIVERSO RISPETTO AGLI ALTRI MEZZI DI TRASPORTO
AUTO o TRENO => La resistenza aumenta all'aumentare della velocità

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

RESISTENZA

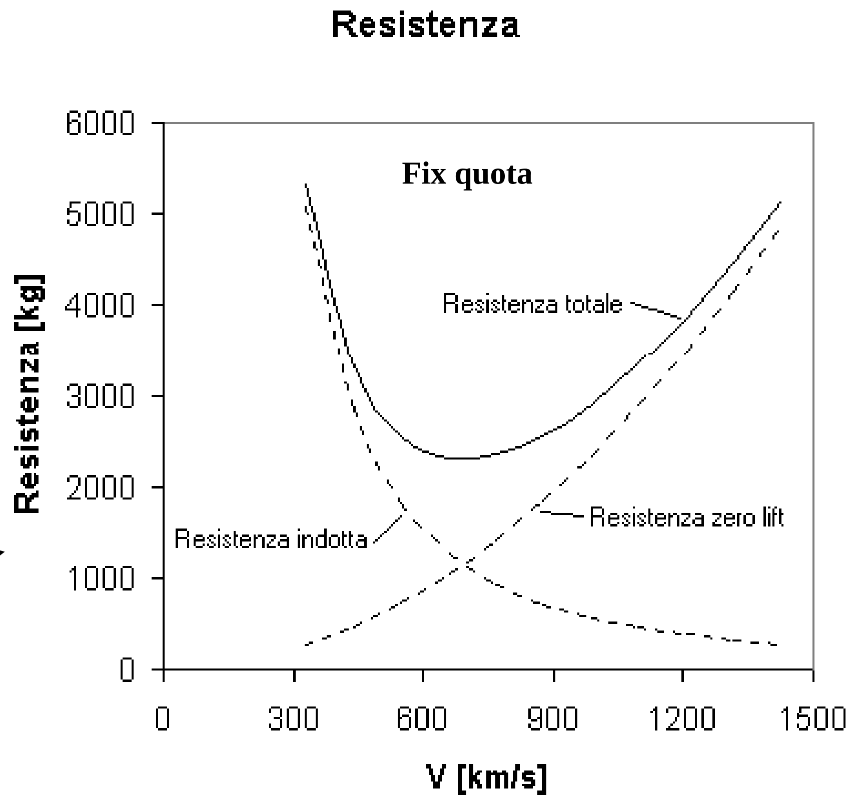
$$D = \frac{1}{2}\rho V^2 S C_{Do} + \frac{2KS}{\rho V^2}\left(\frac{W}{S}\right)^2$$



Polare (CL,CD)

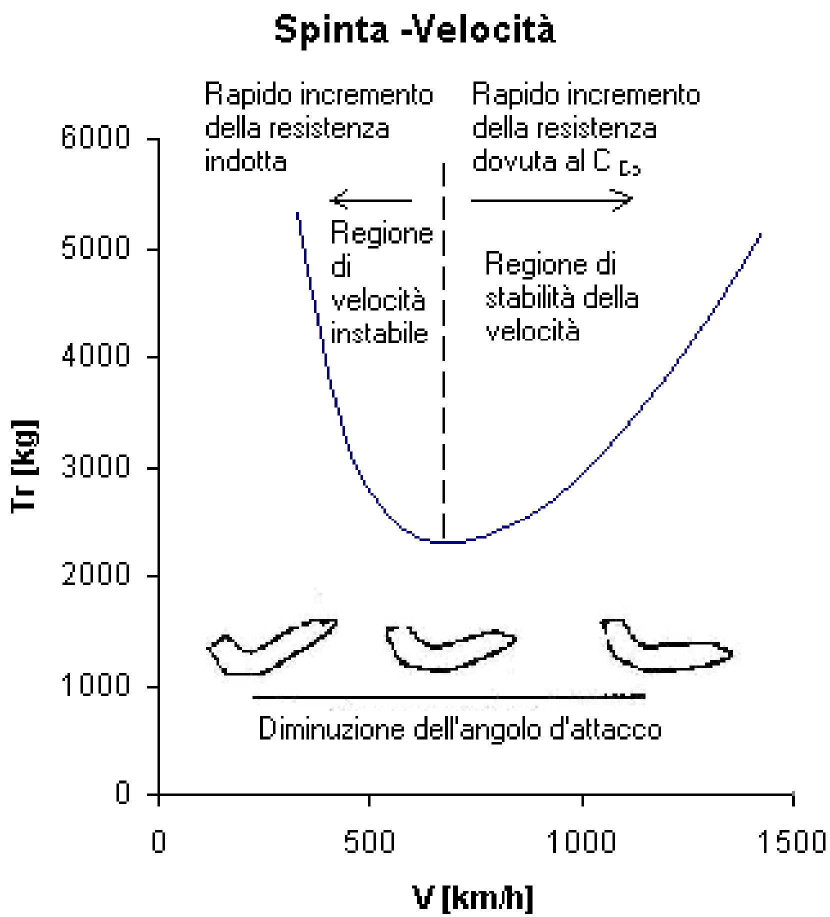
+

- Peso W
- Superficie alare S
- Quota ρ



+ Ipotesi volo livellato L=W

Cap.5 Polari tecniche – SPINTA RICH



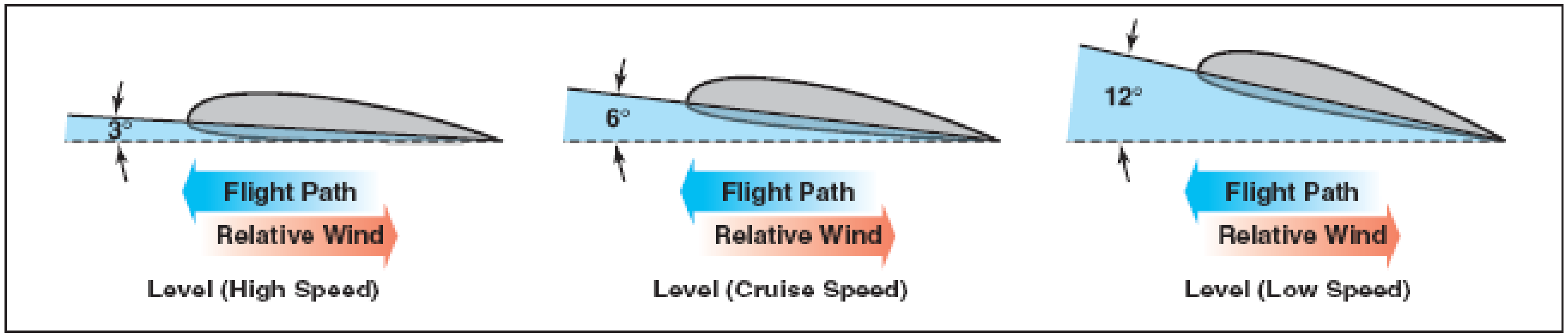
RESISTENZA

$$D = \frac{1}{2}\rho V^2 S C_D$$

$$C_D = C_{Do} + K C_L^2$$

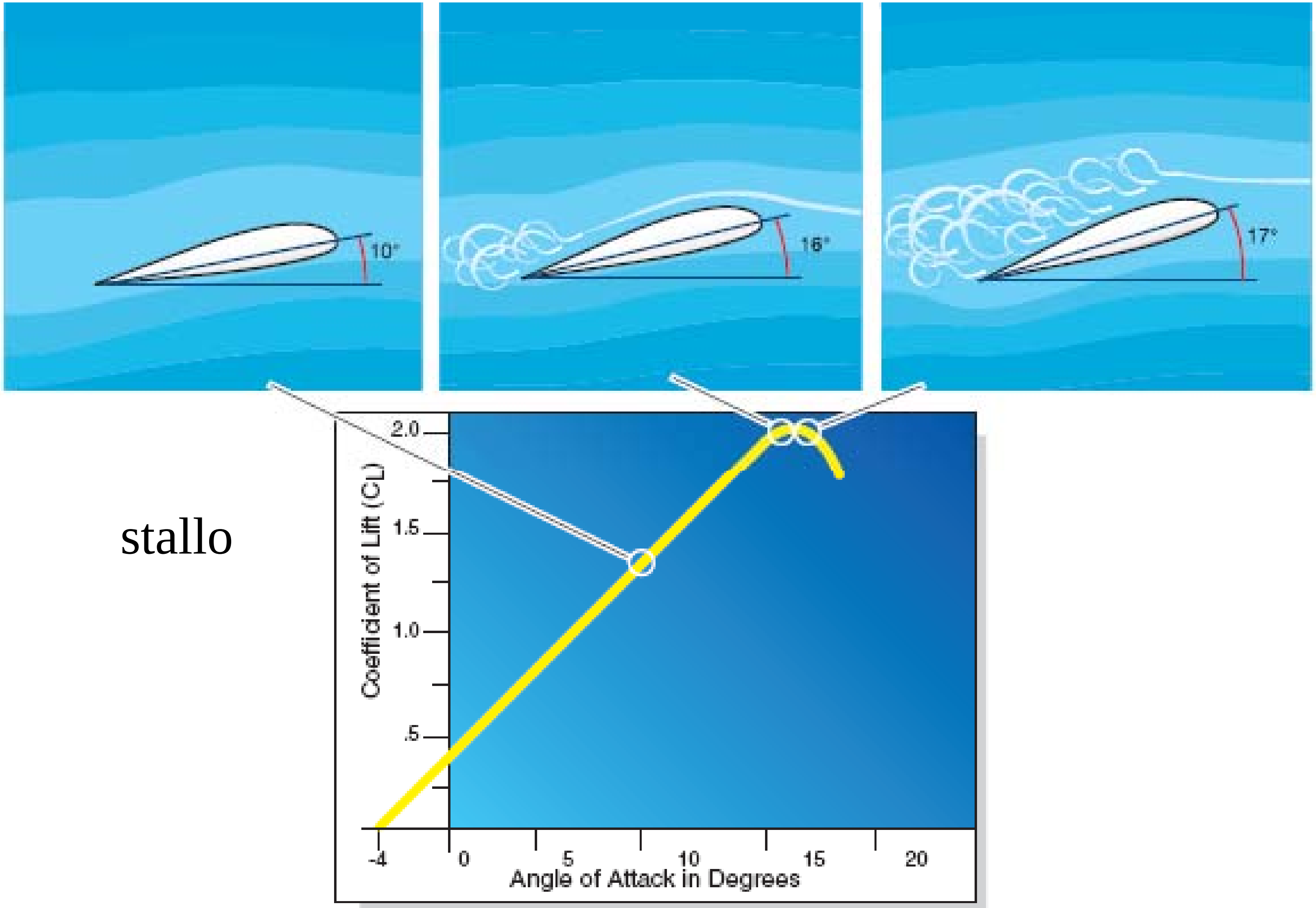
$$\boxed{\mathbf{D} = \frac{1}{2}\rho \mathbf{V}^2 \mathbf{S} \mathbf{C}_{\mathbf{Do}} + \frac{1}{2}\rho \mathbf{V}^2 \mathbf{S} \mathbf{K} \mathbf{C}_{\mathbf{L}}^2}$$

Legame tra V , CL ed α

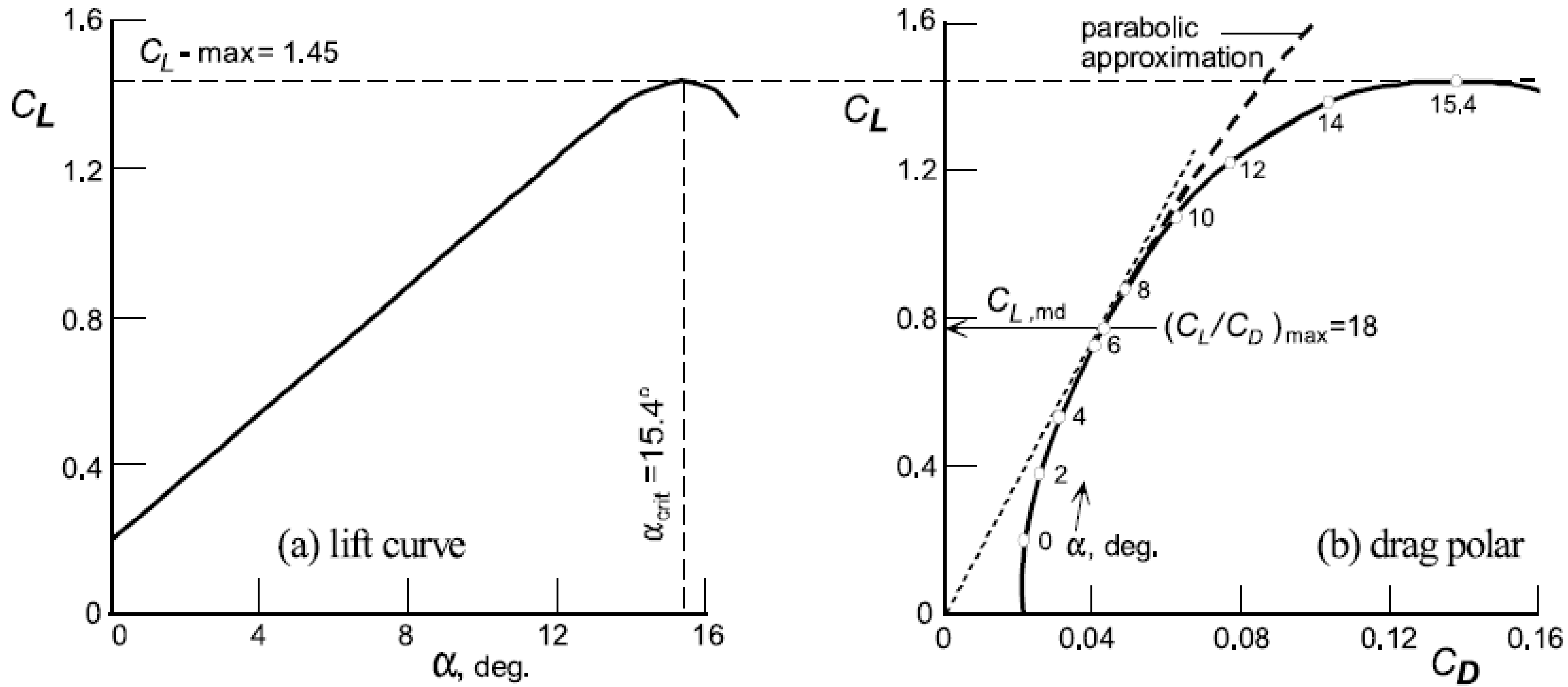


1,5

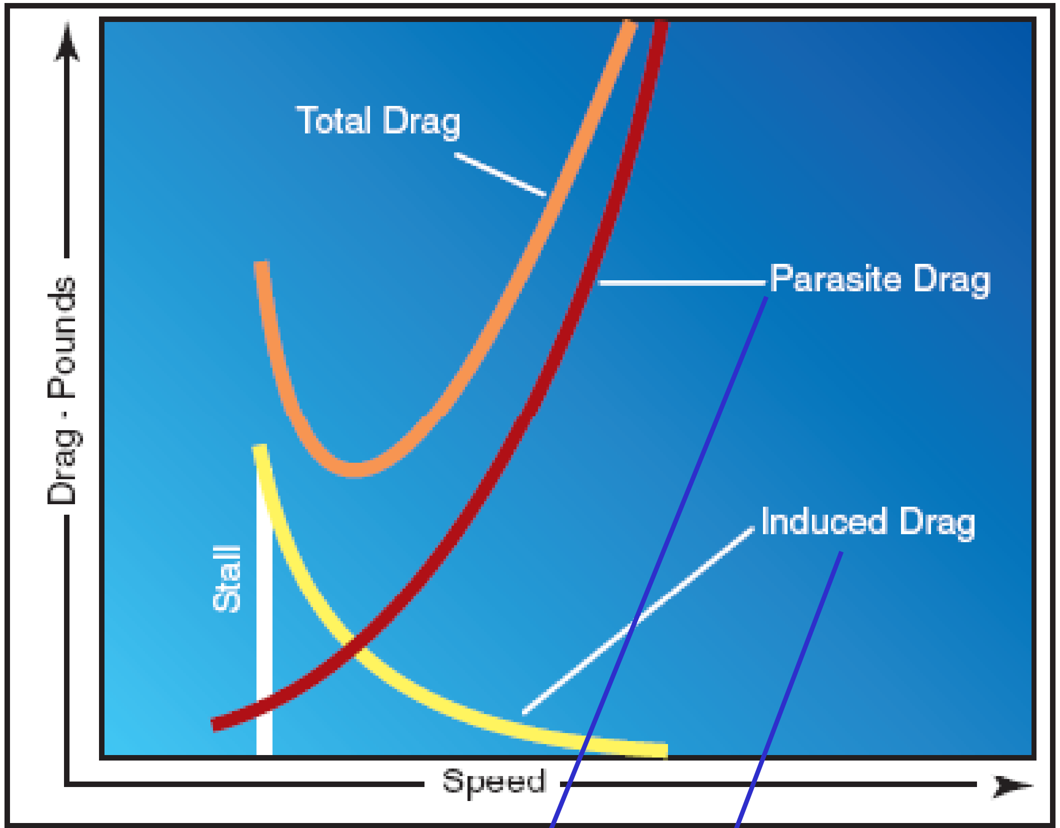
stallo


10°
16°
17°
2.0
1.5
1.0
.5
Coefficient of Lift (CL)
-4
0
5
10
15
20
Angle of Attack in Degrees

Legame tra coeff. di portanza, angolo d'attacco e coeff. di resistenza (polare del velivolo)



Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato



RESISTENZA

$$D = \frac{1}{2}\rho V^2 S C_D$$

$$C_D = C_{Do} + K C_L^2$$

$$\boxed{\mathbf{D} = \frac{1}{2}\rho \mathbf{V}^2 \mathbf{S} \mathbf{C}_{\mathbf{Do}} + \frac{1}{2}\rho \mathbf{V}^2 \mathbf{S} \mathbf{K} \mathbf{C}_{\mathbf{L}}^2}$$

Ed essendo $\quad \mathrm{CL} = \frac{2}{\rho}\frac{\mathrm{W}}{\mathrm{S}}\frac{1}{\mathrm{V}^2}$

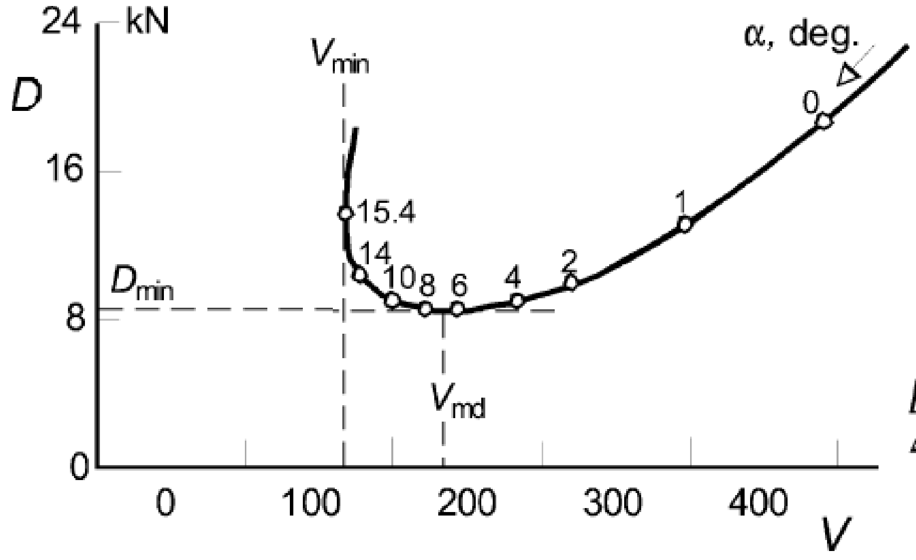
$$D = \frac{1}{2}\rho V^2 S C_{Do} + \frac{2KS}{\rho V^2}\left(\frac{W}{S}\right)^2$$

$$\boxed{\mathrm{D} = \mathrm{T}_{\mathrm{no}} = \mathrm{a}\mathrm{V}^2 + \mathrm{b}\frac{1}{\mathrm{V}^2}}$$

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

RESISTENZA

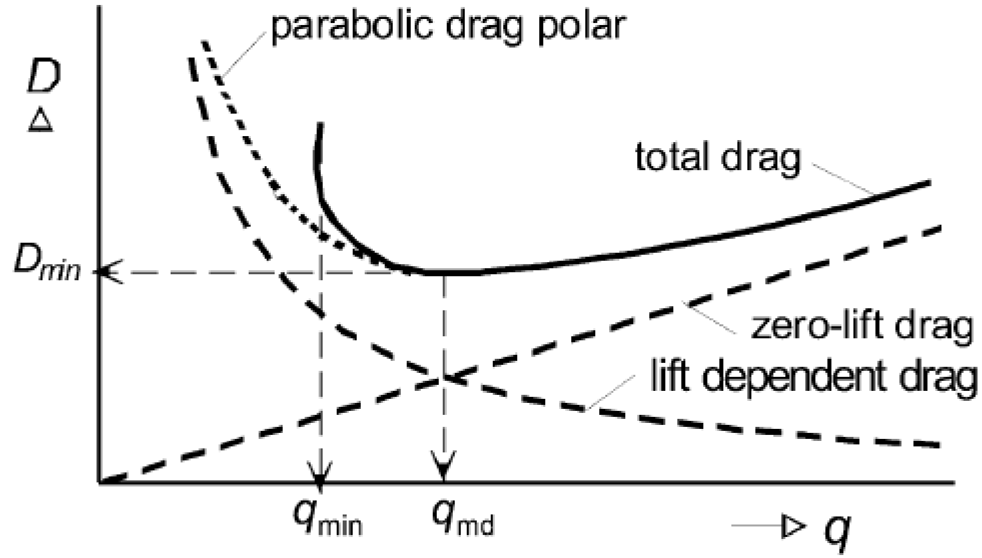
$$D = \frac{1}{2}\rho V^2 S C_{Do} + \frac{2KS}{\rho V^2}\left(\frac{W}{S}\right)^2$$



(a) Drag versus flight speed

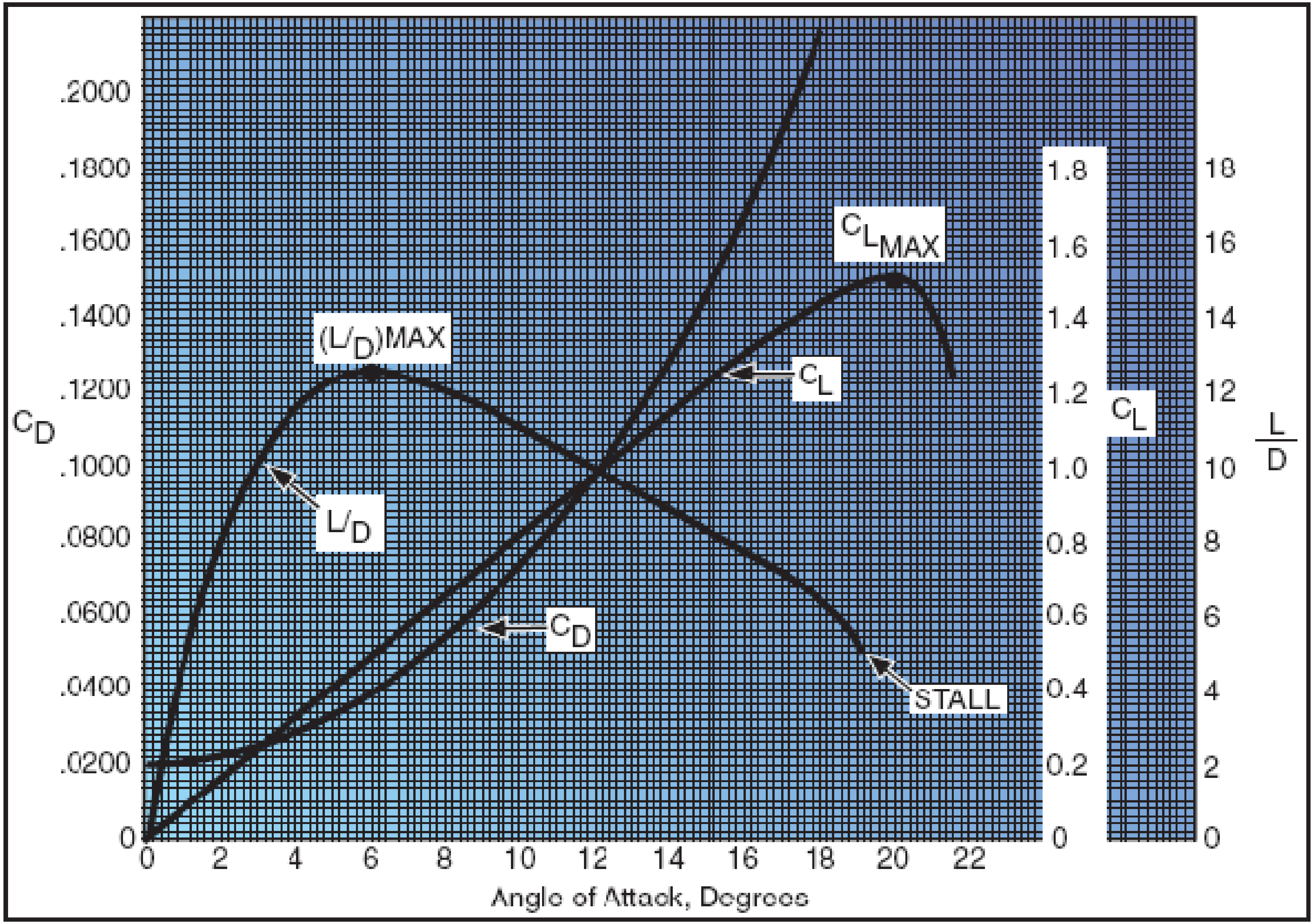
In termini di pressione dinamica

$$D = q \cdot S C_{Do} + \frac{1}{q} \cdot KS\left(\frac{W}{S}\right)^2$$

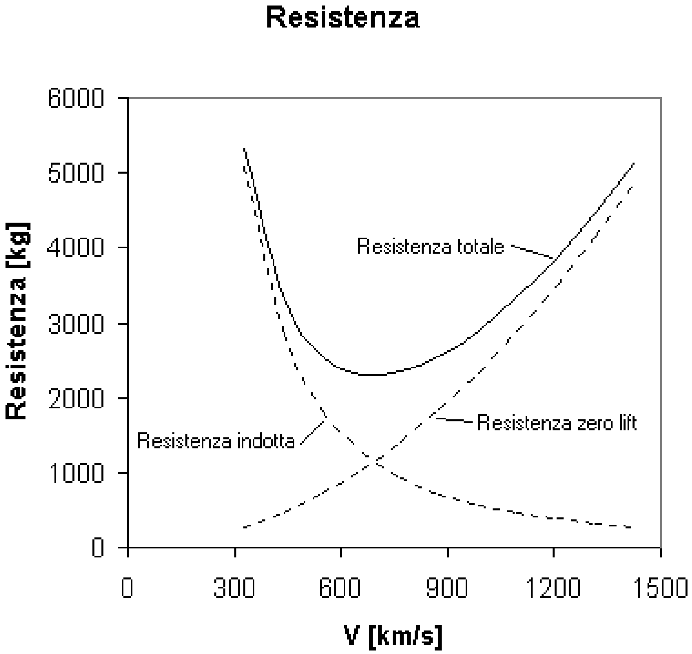


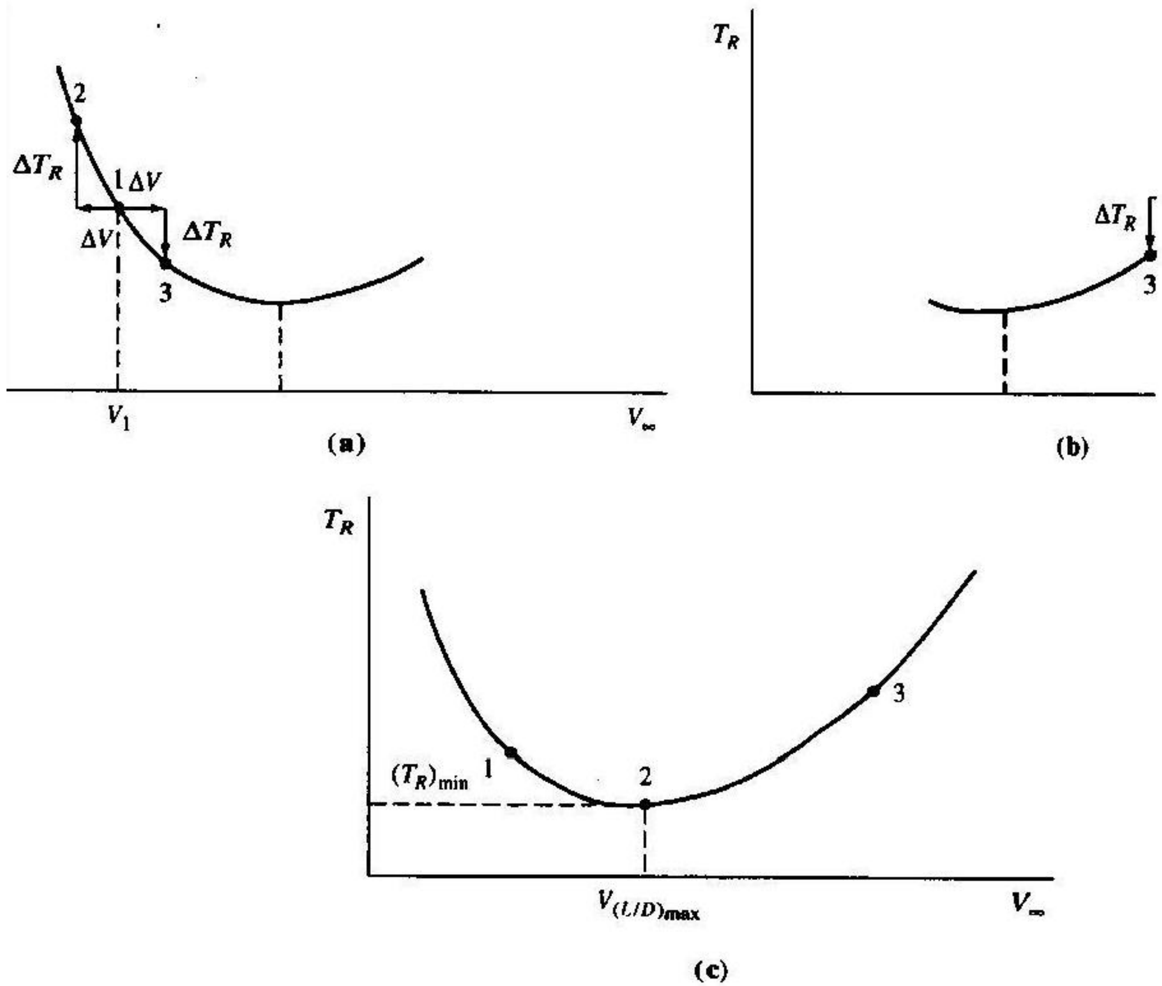
(b) Drag versus dynamic pressure

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato



Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato



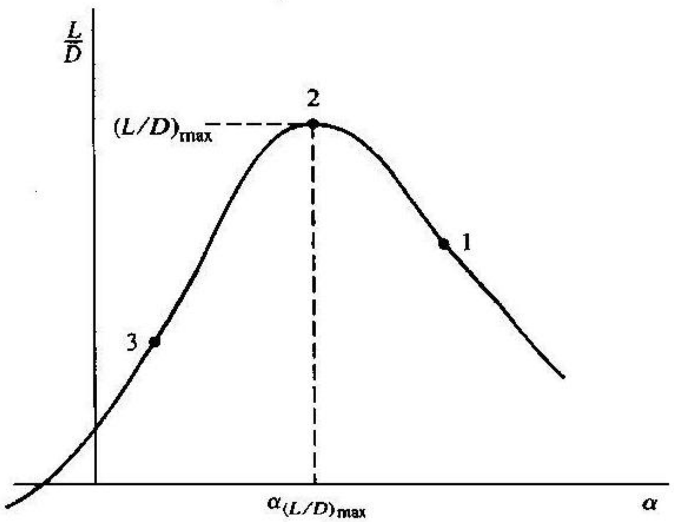


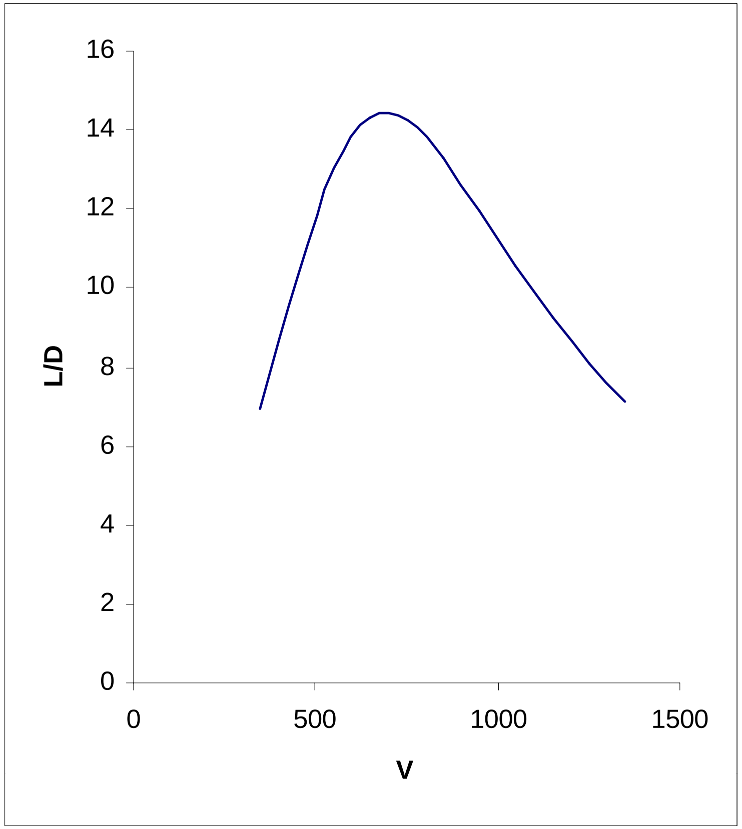
Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

Efficienza aerodinamica

$$T_R = D = \frac{D}{W} W = \frac{D}{L} W \qquad T_R = \frac{W}{L/D}$$

$$\frac{L}{D} = \frac{\frac{1}{2}\rho V^2 S C_L}{\frac{1}{2}\rho V^2 S C_D} = \frac{C_L}{C_D}$$





Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

Approccio analitico

$$D = qSC_D = qS\left(C_{Do} + KC_L^2\right)$$

$$\mathrm{L} = \mathrm{W} = \mathrm{qSC_L} = \frac{1}{2}\rho V^2 SC_L \longrightarrow \qquad C_L = \frac{2W}{\rho V^2 S}$$

$$D = \frac{1}{2}\rho V^2 S\left[C_{Do} + 4K\left(\frac{W}{\rho V^2 S}\right)^2\right]$$

$$\boxed{D = \frac{1}{2}\rho V^2 SC_{Do} + \frac{2KS}{\rho V^2}\left(\frac{W}{S}\right)^2}$$

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

Approccio analitico

$$D = \frac{1}{2}\rho V^2 S C_{Do} + \frac{2KS}{\rho V^2}\left(\frac{W}{S}\right)^2$$

$$\boxed{D = T_{no} = \frac{1}{2}\rho f V^2 + \frac{2}{\rho}\frac{W^2}{S}\frac{1}{\pi A Re}\frac{1}{V^2}}$$

$$\boxed{D = T_{no} = aV^2 + b\frac{1}{V^2}}$$

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

Approccio analitico

$$D = T_{no} = aV^2 + b\frac{1}{V^2}$$

Minima D se :   d(D)dV=0

$$2aV - 2b\frac{1}{V^3} = 0 \longrightarrow aV^2 = b\frac{1}{V^2} \longrightarrow$$ Res. Parassita=Res. Indotta

$$E_{max} = \frac{W}{D_{min}} \qquad C_{Do} = K \cdot C_L^2$$

$$CL_E = \sqrt{\frac{CDo}{K}} = \sqrt{\pi \cdot AR \cdot e \cdot CDo}$$

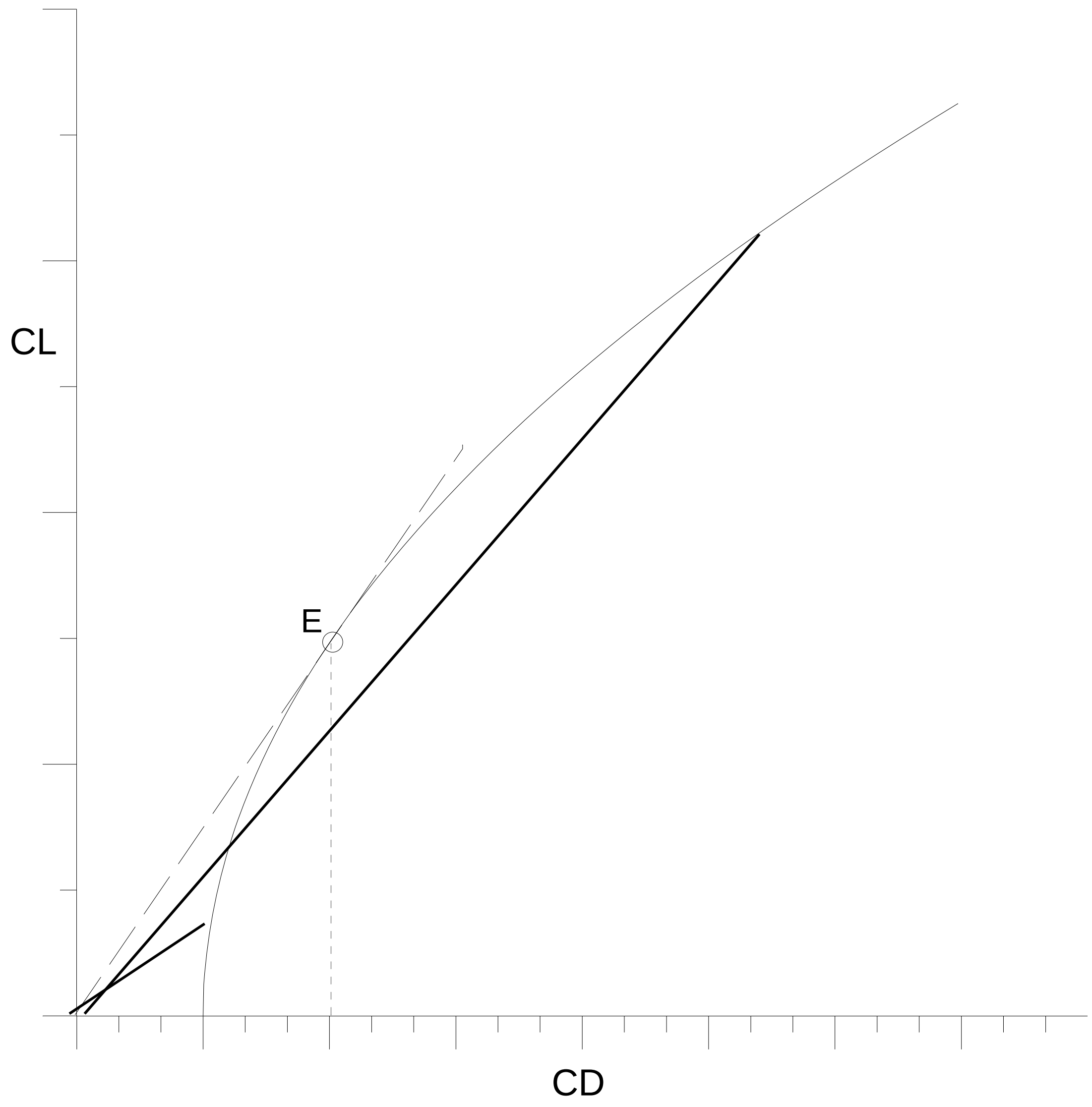
$$CD_E = CDo + KCL_E^{\ 2} = 2\,CDo$$

$$E_{MAX} = E_E = \frac{CL_E}{CD_E} = \frac{\sqrt{\pi \cdot AR \cdot e \cdot CDo}}{2 \cdot CDo} = \sqrt{\frac{\pi}{4}\frac{AR \cdot e}{CDo}}$$

$$V_E = \sqrt{\frac{2}{\rho}}\sqrt{\frac{W}{S}\sqrt{\frac{1}{CL_E}}}$$

Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

Approccio analitico

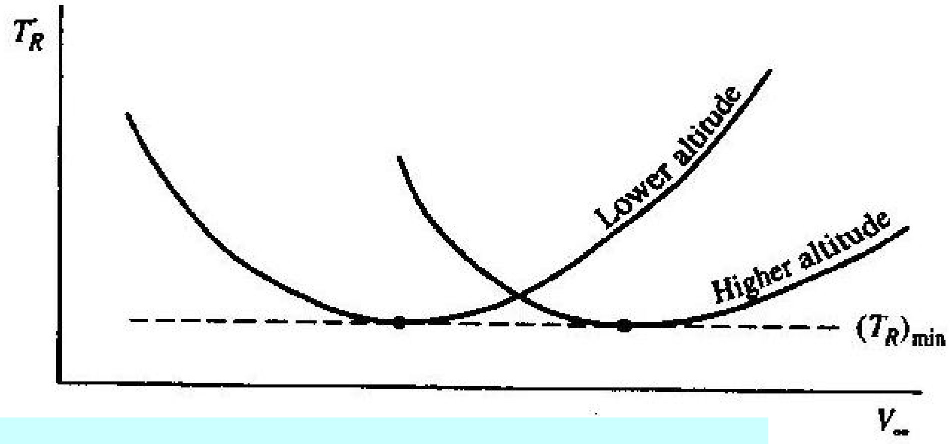


Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

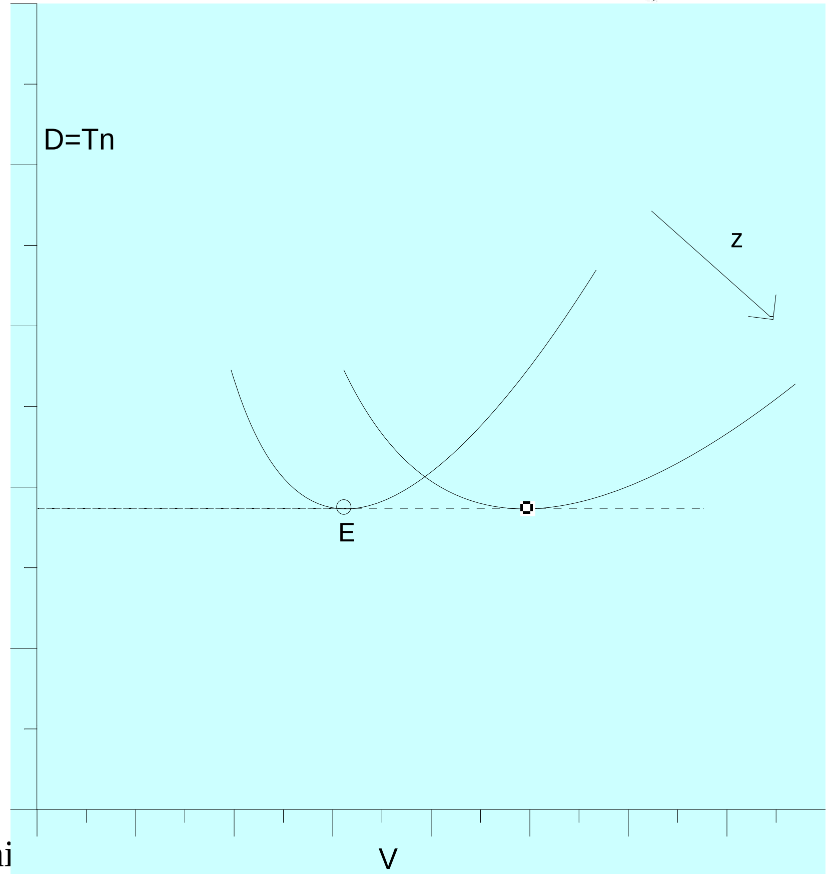
Approccio analitico

$$D_{min} = T_{no\_min} = \frac{W}{E_{MAX}}$$

$$V(z) = \frac{V_o}{\sqrt{\sigma}}$$



Influenza della quota

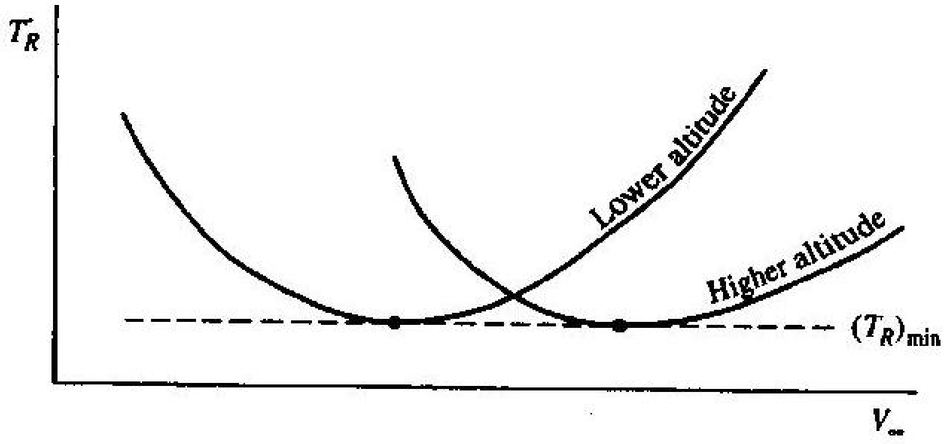


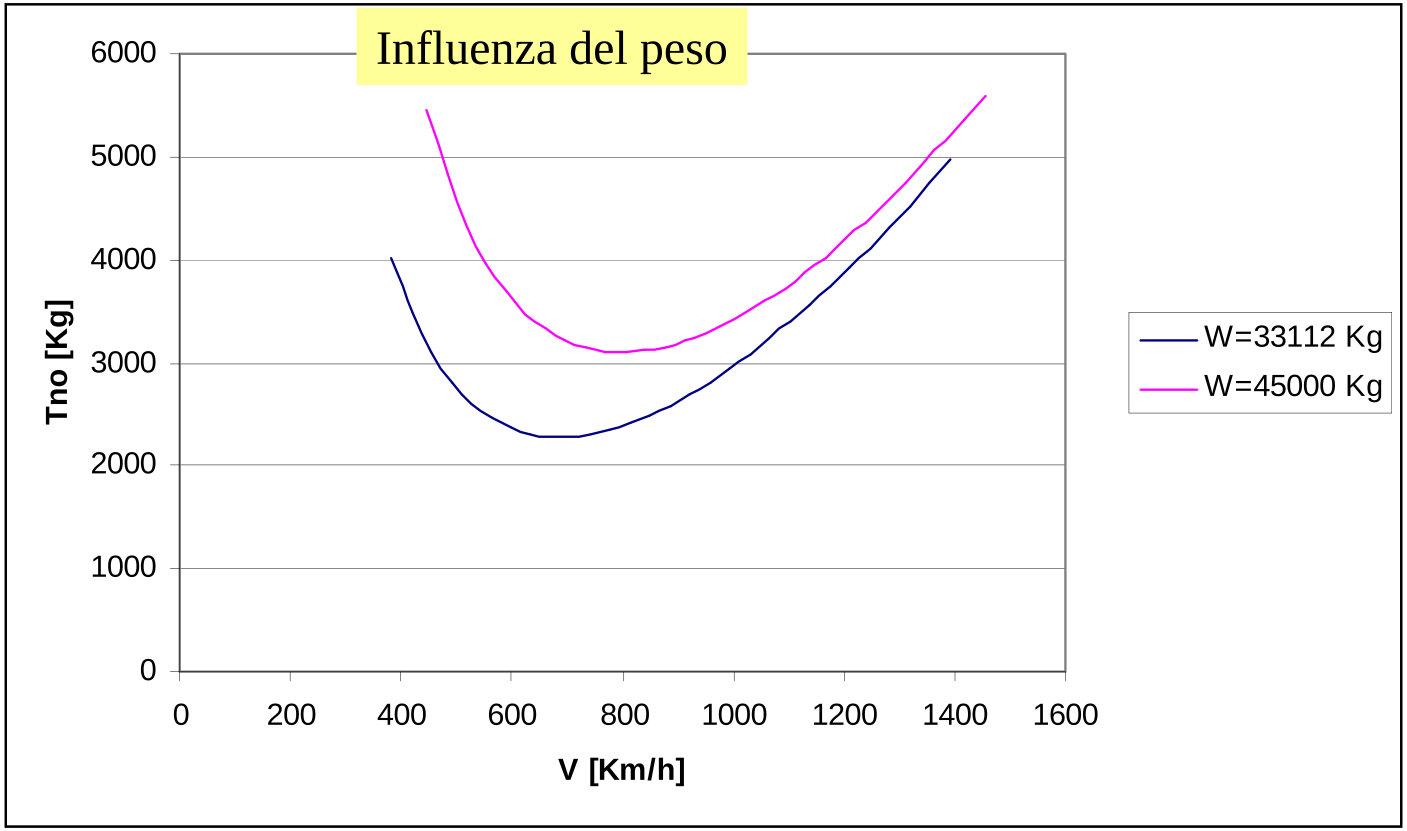
Cap.5 Polari tecniche – SPINTA RICHIESTA al volo livellato

Approccio analitico

$$D_{min} = T_{no\_min} = \frac{W}{E_{MAX}}$$

$$V(z) = \frac{V_o}{\sqrt{\sigma}}$$





Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

$$\Pi_{no} = T_{no} \cdot V = D \cdot V$$

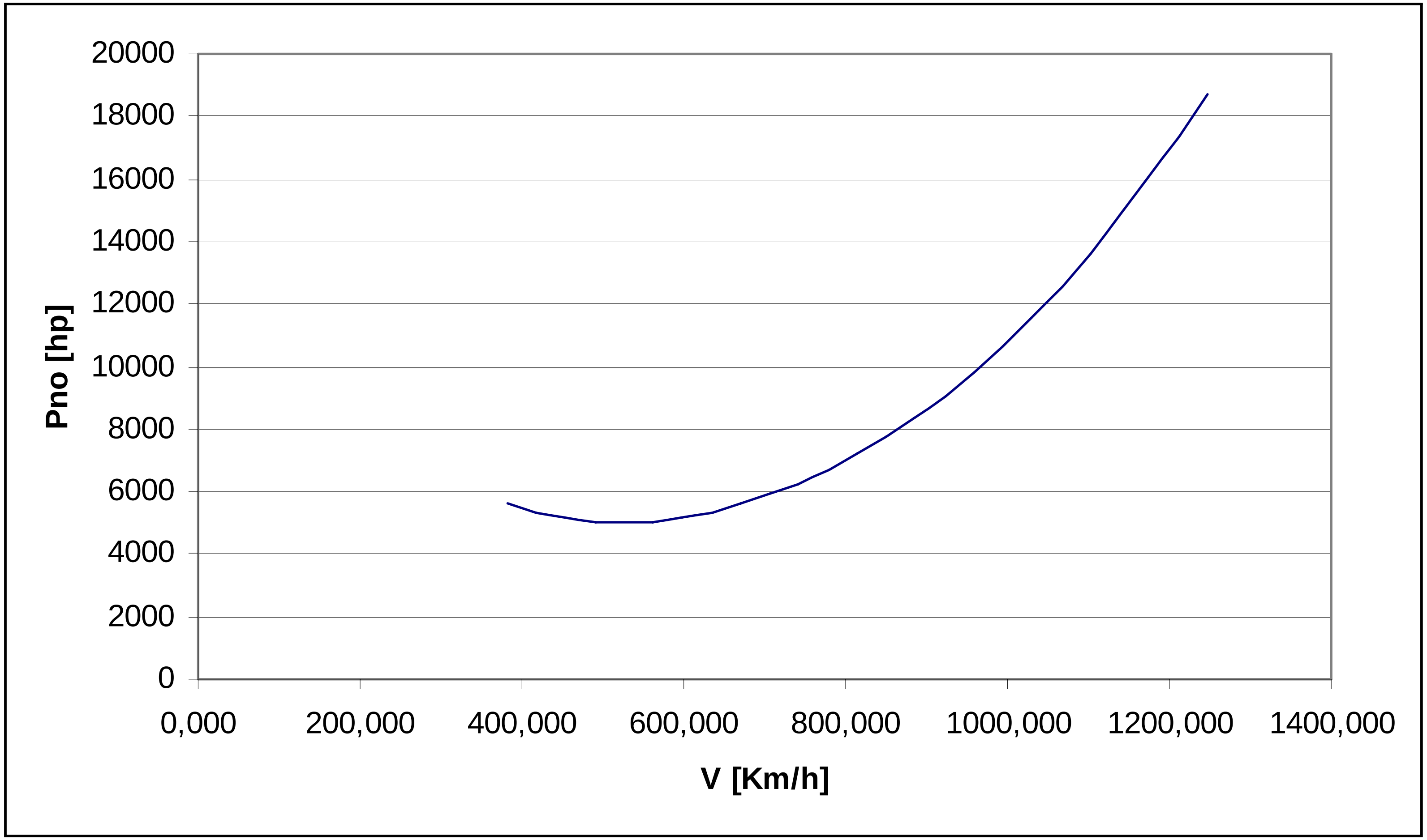
Consideriamo consideriamo sempre il velivolo con peso pari a W=33112.8 Kg , S=88.2 m2 , b=22.9 m e quota pari a 9000 m. Il CDo=0.015 e il fattore di Oswald sia 0.80.

Il calcolo inizia da un valore di V pari alla velocità di stallo a quota z=9000 m, considerando che il CLMAX del velivolo in configurazione pulita è 1.40.

V [Km/h]	CL	CD	E	D=W/E [Kg]	Pno [KW]	Pno [hp]
382,386	1,400	0,170	8,24	4019	4188	5614
418,386	1,169	0,123	9,50	3486	3974	5327
454,386	0,991	0,093	10,69	3096	3834	5139
490,386	0,851	0,072	11,78	2812	3757	5037
526,386	0,739	0,058	12,71	2606	3738	5011
562,386	0,647	0,048	13,45	2462	3772	5057
598,386	0,572	0,041	14,00	2365	3857	5170
634,386	0,509	0,035	14,35	2308	3990	5348
670,386	0,455	0,031	14,51	2283	4170	5590
706,386	0,410	0,028	14,49	2285	4398	5895
742,386	0,371	0,026	14,34	2310	4672	6263
778,386	0,338	0,024	14,06	2355	4994	6695
814,386	0,309	0,023	13,70	2417	5364	7191
850,386	0,283	0,021	13,27	2496	5783	7752
886,386	0,261	0,020	12,79	2588	6252	8381

Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

$$\Pi_{no} = T_{no} \cdot V = D \cdot V$$



Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

$$\Pi_{no} = T_{no} \cdot V = D \cdot V$$

$$\Pi_{no} = D \cdot V = \frac{1}{2}\rho \cdot V^2 \cdot S \cdot (CDo + KCL^2) \cdot V$$

$$\Pi_{no} = \frac{1}{2}\rho \cdot CDo \cdot S \cdot V^3 + \frac{1}{2}\rho \cdot S \cdot V^3 \cdot KCL^2$$

$$CL = \frac{2}{\rho}\frac{W}{S}\frac{1}{V^2}$$

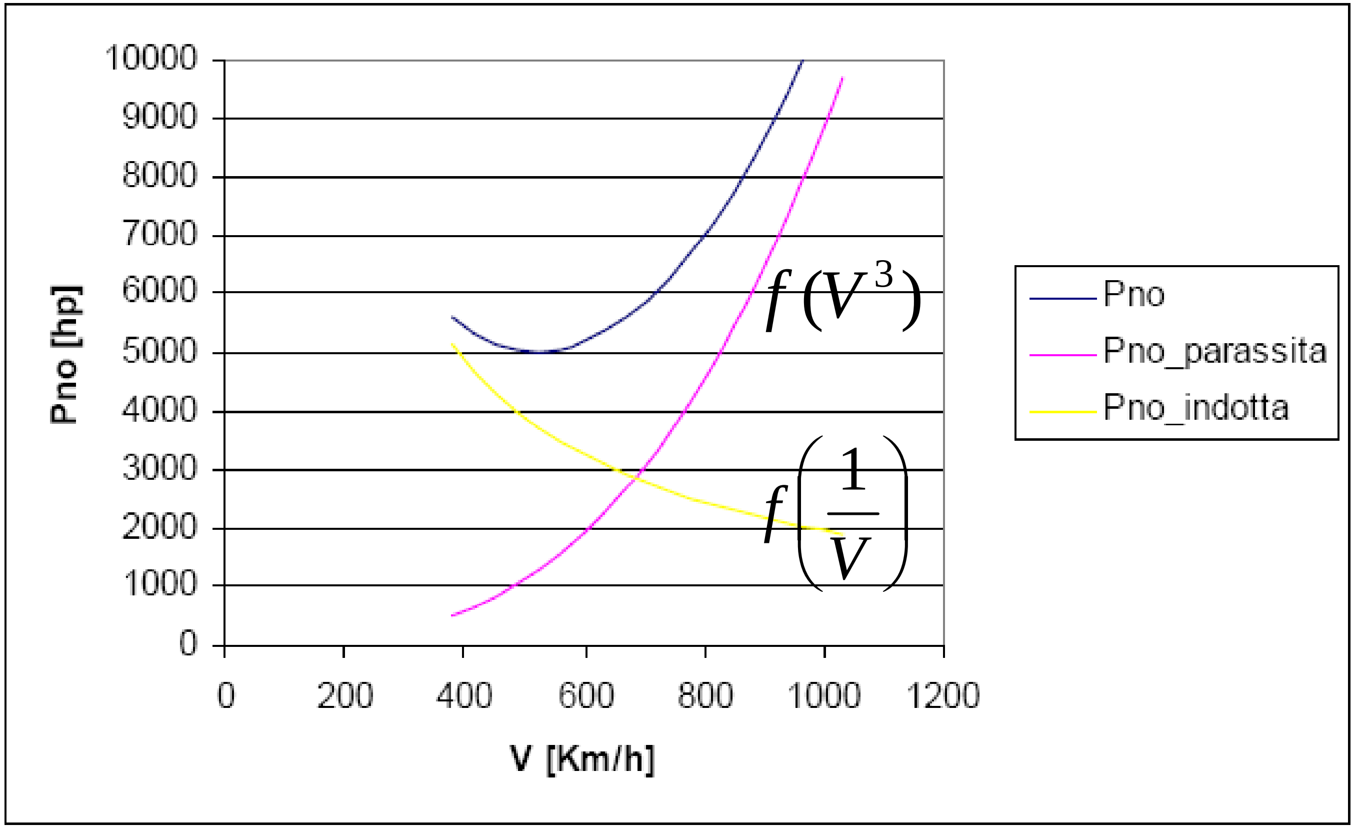
$$\boxed{\Pi_{no} = \frac{1}{2}\rho \cdot CDo \cdot S \cdot V^3 + \frac{2}{\rho} \cdot S \cdot K \cdot \left(\frac{W}{S}\right)^2 \cdot \frac{1}{V}}$$

$$\boxed{\Pi_{no} = D \cdot V = (a \cdot V^2 + \frac{b}{V^2}) \cdot V = a \cdot V^3 + \frac{b}{V}}$$

Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

$$\Pi_{no} = T_{no} \cdot V = D \cdot V$$

$$\boxed{\Pi_{no} = D \cdot V = (a \cdot V^2 + \frac{b}{V^2}) \cdot V = a \cdot V^3 + \frac{b}{V}}$$



Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

Deriviamo per trovare il minimo

$$\Pi_{no} = D \cdot V = (a \cdot V^2 + \frac{b}{V^2}) \cdot V = a \cdot V^3 + \frac{b}{V}$$

$$\frac{d\Pi_{no}}{dV} = 3 \cdot a \cdot V^2 - \frac{b}{V^2} = 0 \longrightarrow 3 \cdot a \cdot V^2 = \frac{b}{V^2} \longrightarrow 3 \cdot Do = Di$$

$$\boxed{CDi = K \cdot CL^2 = \frac{CL^2}{\pi \cdot AR \cdot e} = 3 \cdot CDo}$$

CD=4 CDo

$$CL_P = \sqrt{\frac{3 \cdot CDo}{K}} = \sqrt{3} \cdot \sqrt{\pi \cdot AR \cdot e \cdot CDo}$$

$$CL_P = \sqrt{3} \cdot CL_E = 1.73 \cdot CL_E$$

$$CD_P = 4 \cdot CDo$$

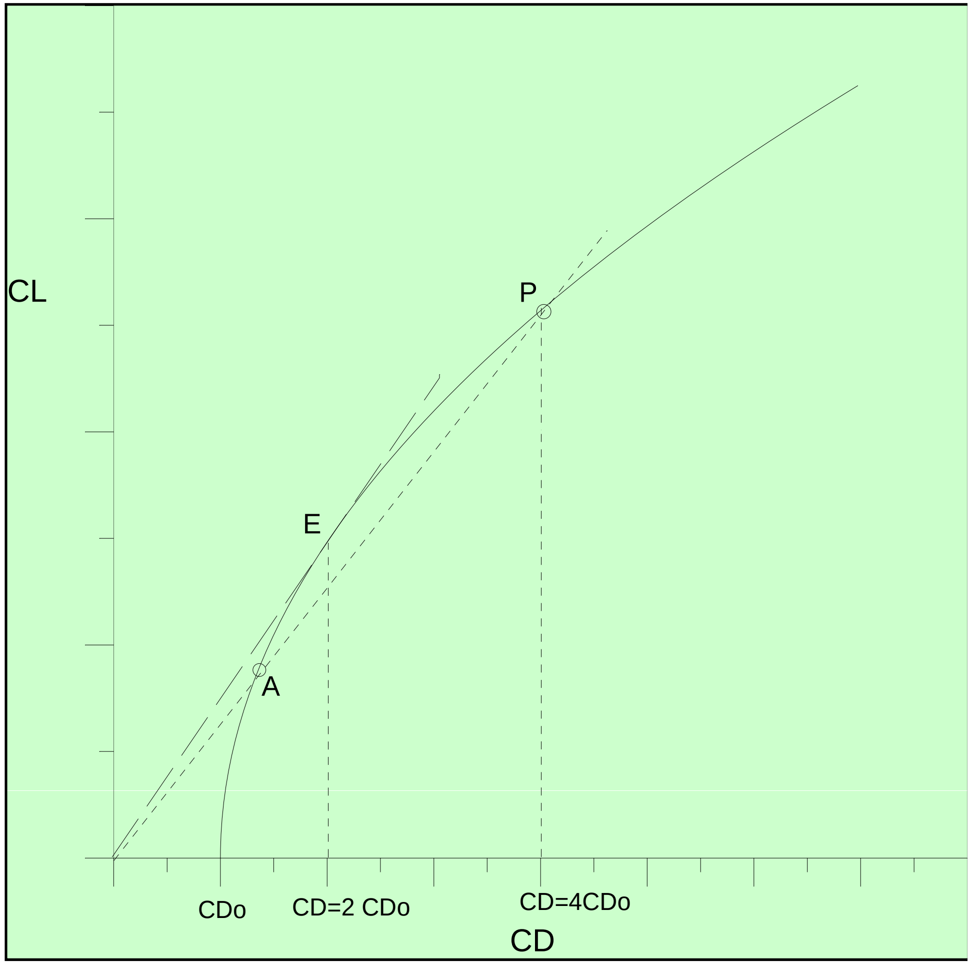
Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

PUNTO P

$$CL_P = \sqrt{3} \cdot CL_E = 1.73 \cdot CL_E$$

$$CD_P = 4 \cdot CDo$$

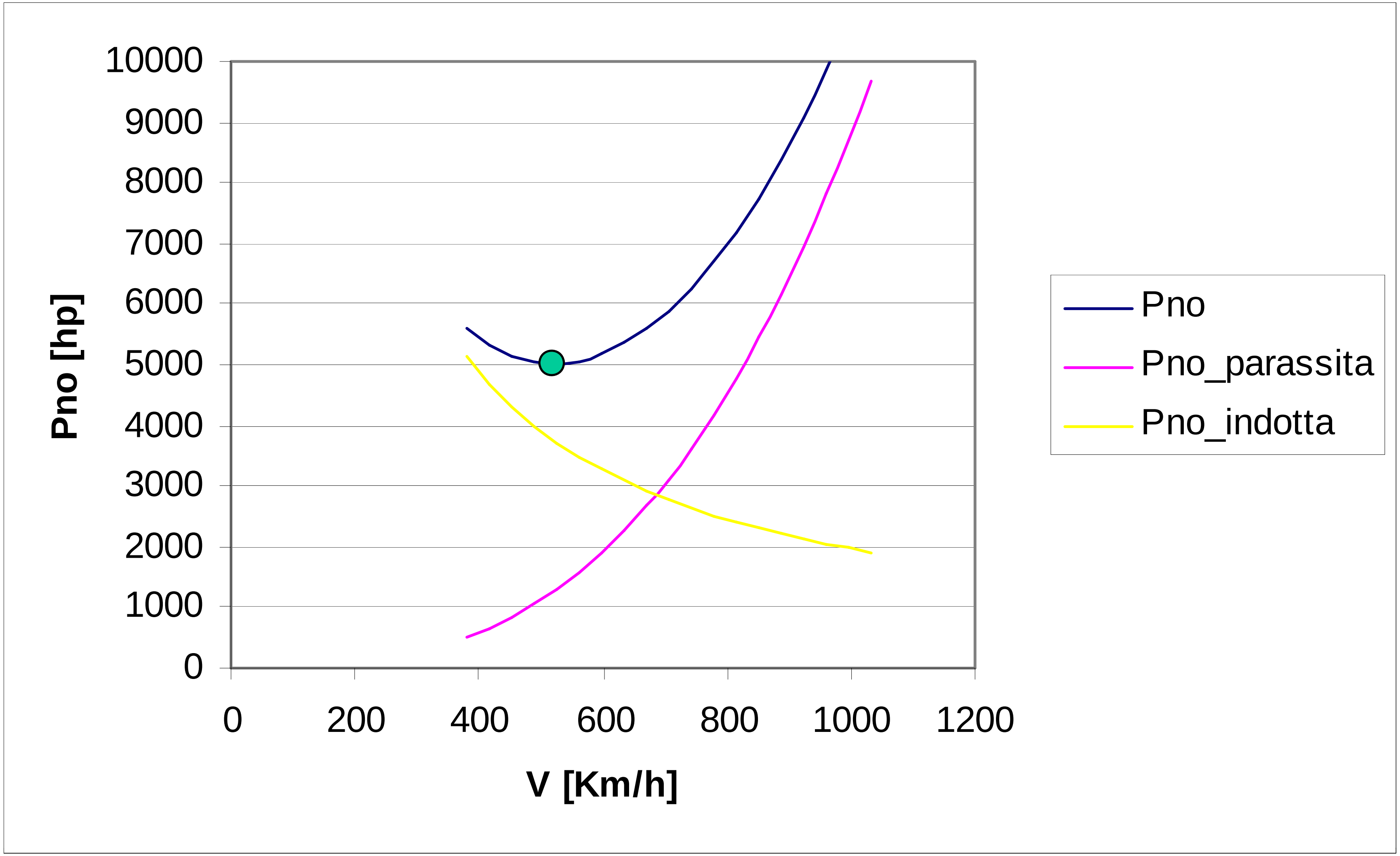
$$E_P = \frac{CL_P}{CD_P} = \frac{\sqrt{3} \cdot CL_E}{4 \cdot CDo} = \frac{\sqrt{3}}{2} \frac{CL_E}{CD_E}$$



$$V_P = \sqrt{\frac{2}{\rho}} \sqrt{\frac{W}{S}} \sqrt{\frac{1}{CL_P}} = \sqrt{\frac{2}{\rho}} \sqrt{\frac{W}{S}} \sqrt{\frac{1}{\sqrt{3} \cdot CL_E}} = \frac{V_E}{\sqrt[4]{3}} = \frac{V_E}{1.32}$$

Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

PUNTO P



Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

PUNTO P

$$D = T_{no} = \frac{W}{E} \qquad \Pi_{no} = T_{no} \cdot V = \frac{W}{E} V = \frac{W}{(E/V)}$$

$$\Pi_{no\_MIN} \Rightarrow \left(\frac{E}{V}\right)_{MAX} \qquad \Pi_{no} = \frac{W}{E} V = W \cdot \frac{CD}{CL} \cdot \sqrt{\frac{2}{\rho}} \cdot \sqrt{\frac{W}{S}} \cdot \sqrt{\frac{1}{CL}}$$

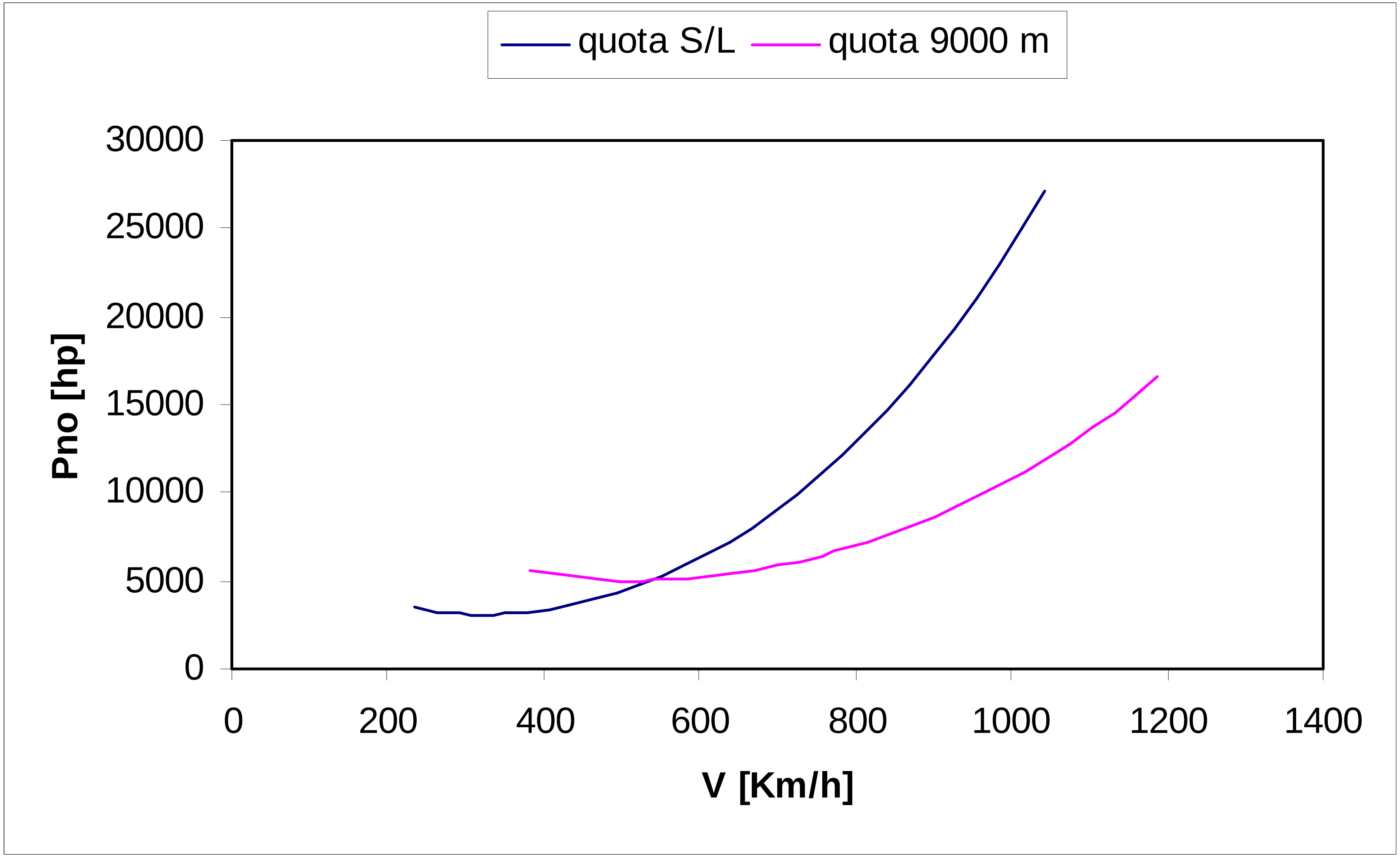
$$\Pi_{no} = \sqrt{\frac{2}{\rho_o \sigma}} \cdot \sqrt{\frac{1}{S}} \cdot W^{3/2} \cdot \frac{CD}{CL^{3/2}}$$

$$\boxed{\Pi_{no\_MIN} = \sqrt{\frac{2}{\rho_o}} \cdot \frac{1}{\sqrt{\sigma}} \cdot \frac{1}{\sqrt{S}} \cdot W^{3/2} \cdot \frac{1}{\left(\frac{CL^{3/2}}{CD}\right)_{MAX}}}$$

$$\Pi_{no\_MIN} \Rightarrow \left(\frac{CL^{3/2}}{CD}\right)_{MAX} = \left(E \cdot \sqrt{CL}\right)_{MAX}$$

Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

PUNTO P – considerazioni grafiche



Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

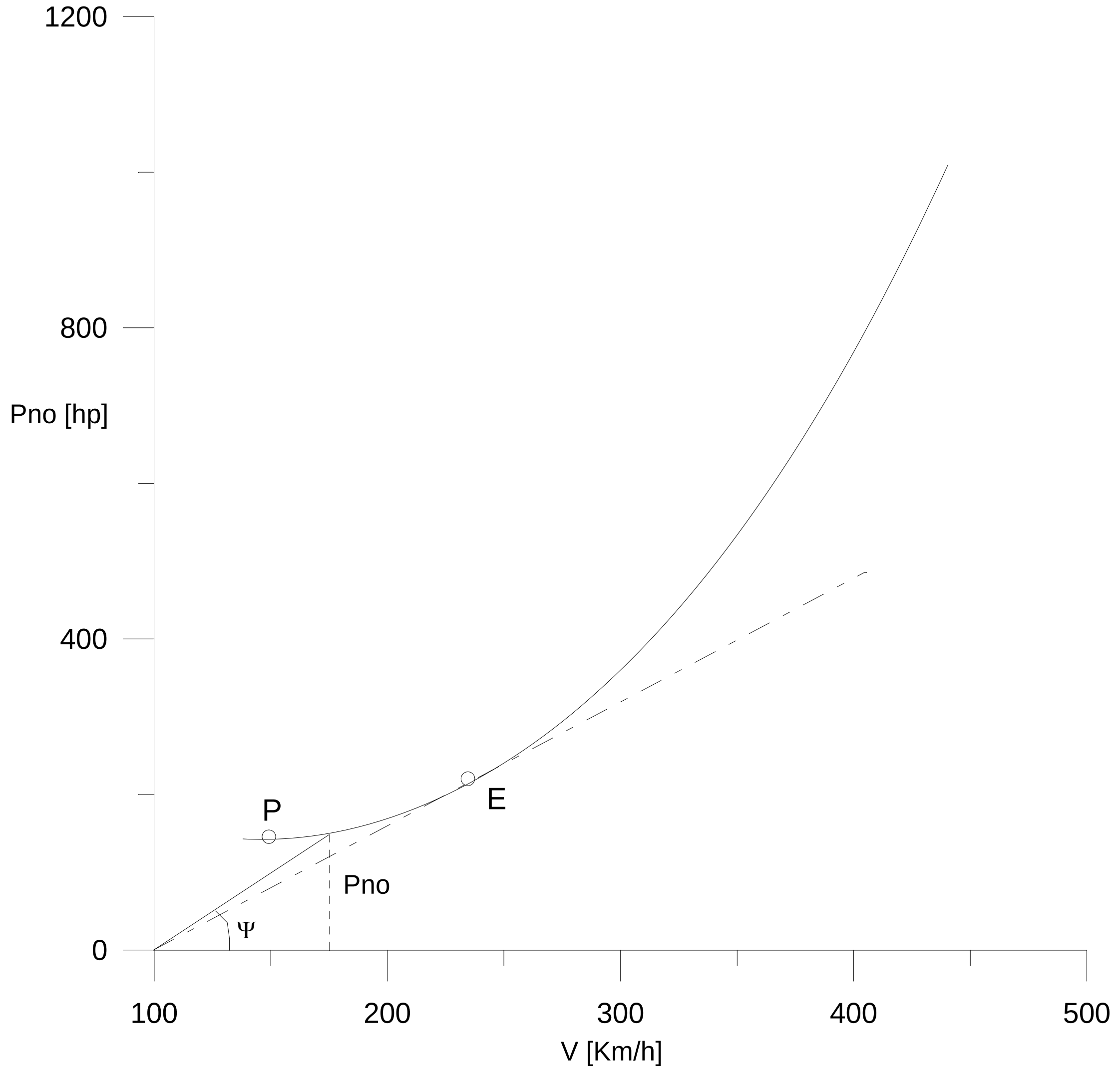
PUNTO P – considerazioni grafiche

$$\Pi_{no} = V \cdot \tan\psi$$

$$\tan\psi = T_{no} = D$$

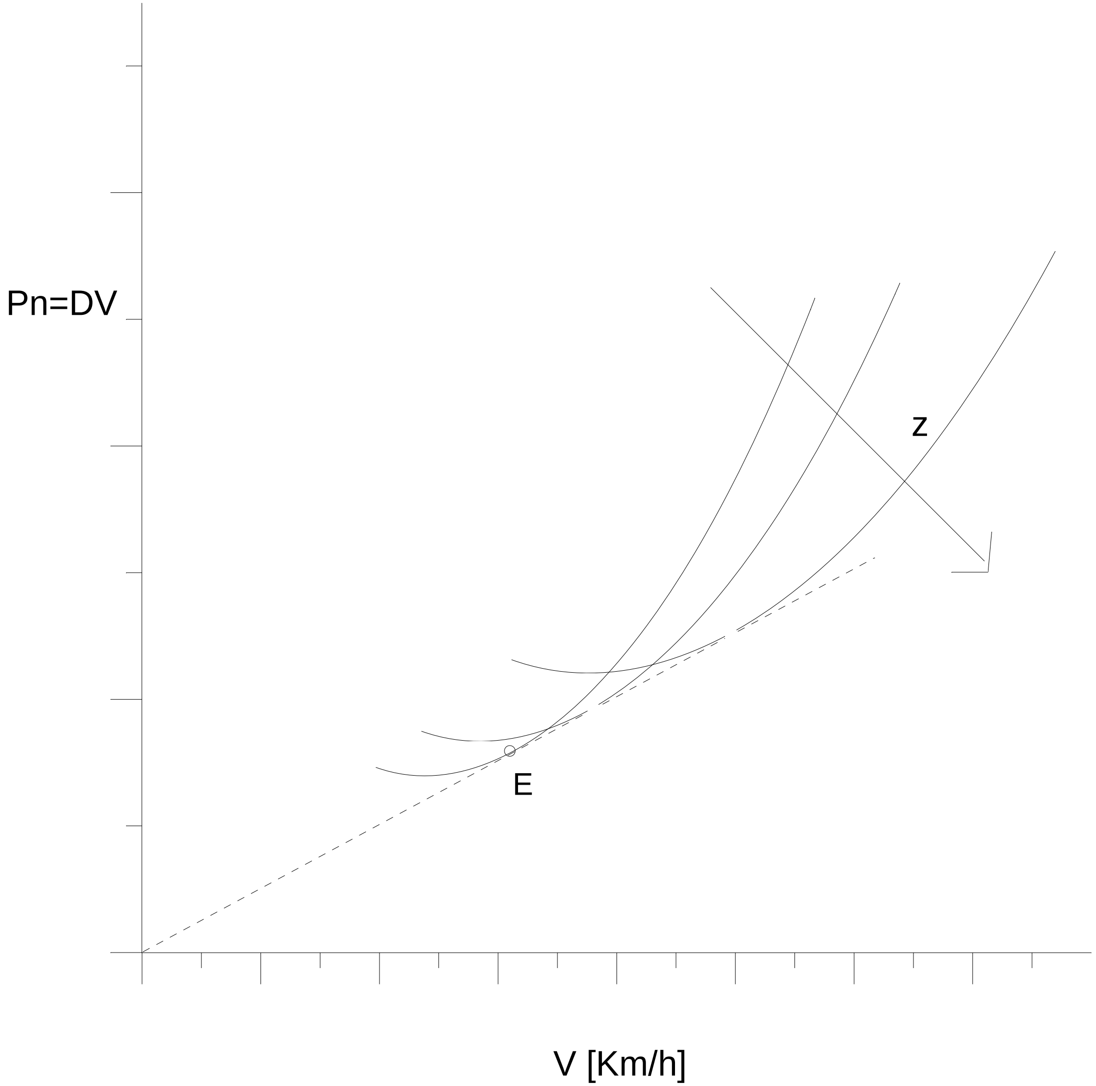
$$\psi_{MIN} = ATAN(D_{MIN}) = ATAN\left(\frac{W}{E_{MAX}}\right)$$

$$V_P = \frac{V_E}{\sqrt[4]{3}} = \frac{V_E}{1.32}$$



Cap.5 Polari tecniche – POTENZA RICHIESTA al volo livellato

PUNTO P – considerazioni grafiche

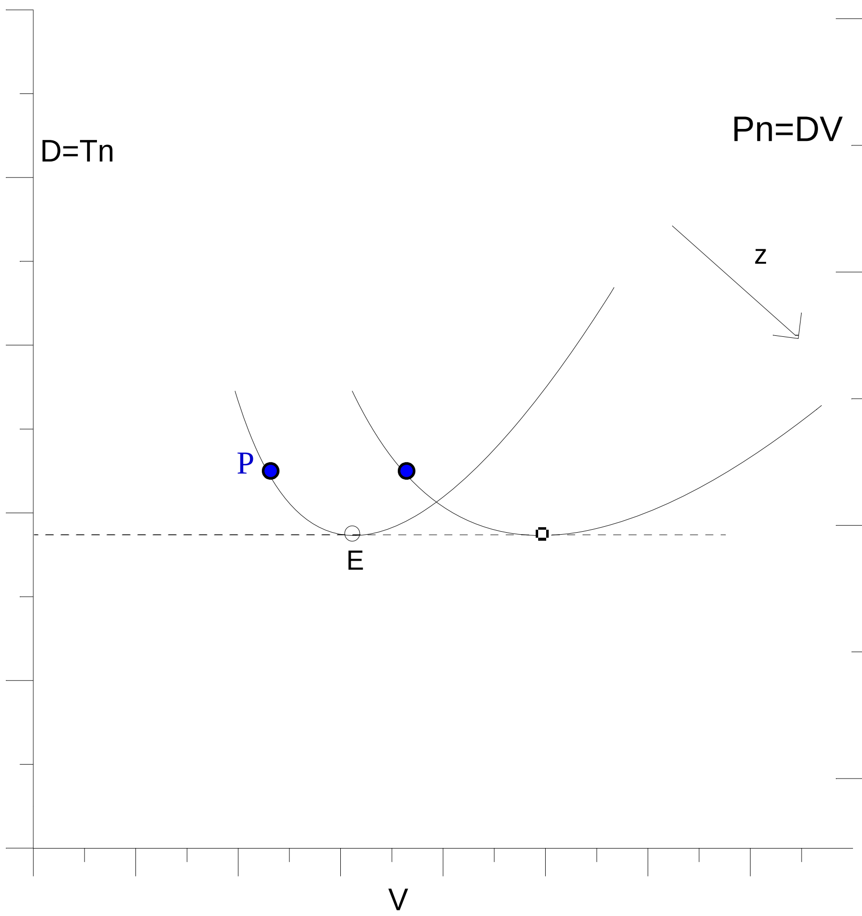


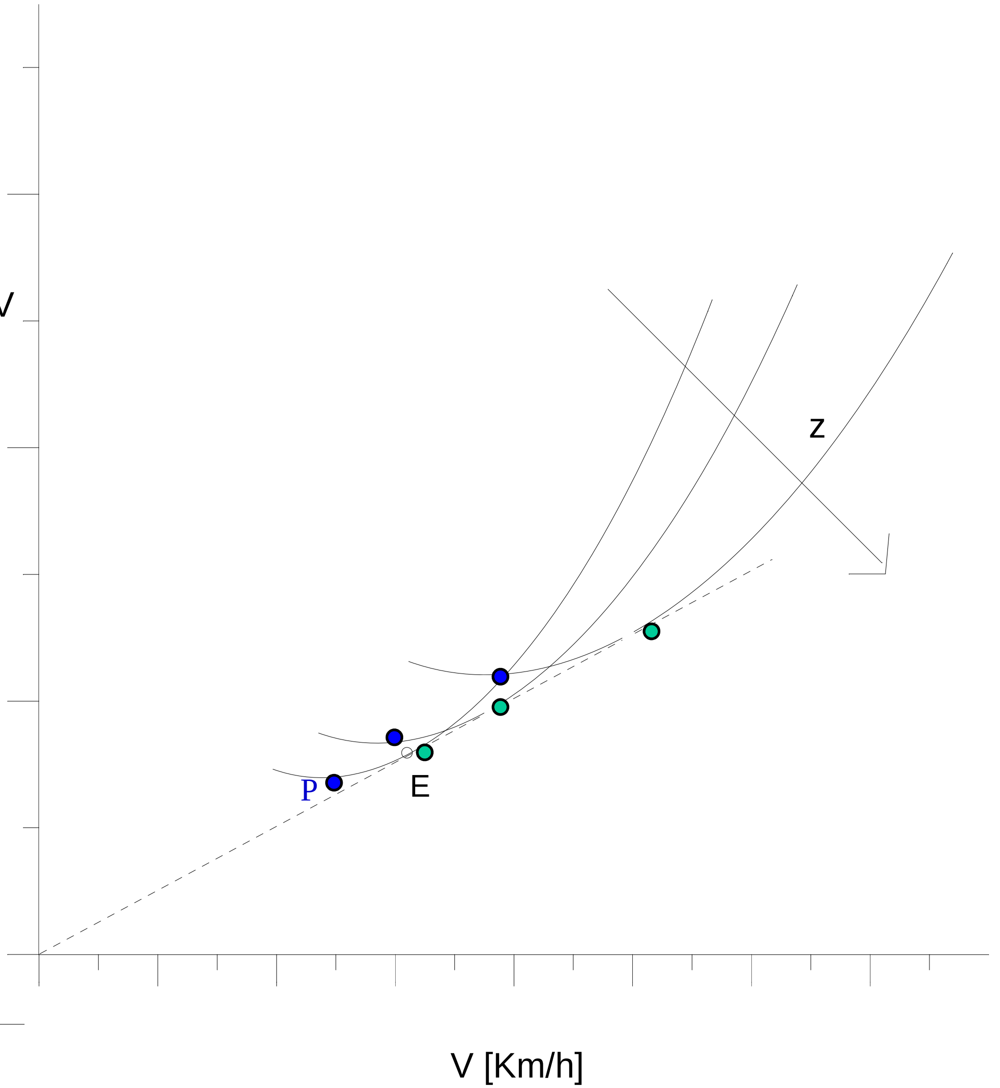
Cap.5 Polari tecniche

PUNTO E : Max Efficienza - Minima resistenza volo livellato $\left(E\right)_{MAX}$

PUNTO P : Min. Potenza volo livellato $\left(\Pi_{no}\right)_{MIN} = (D \cdot V)_{MIN}$

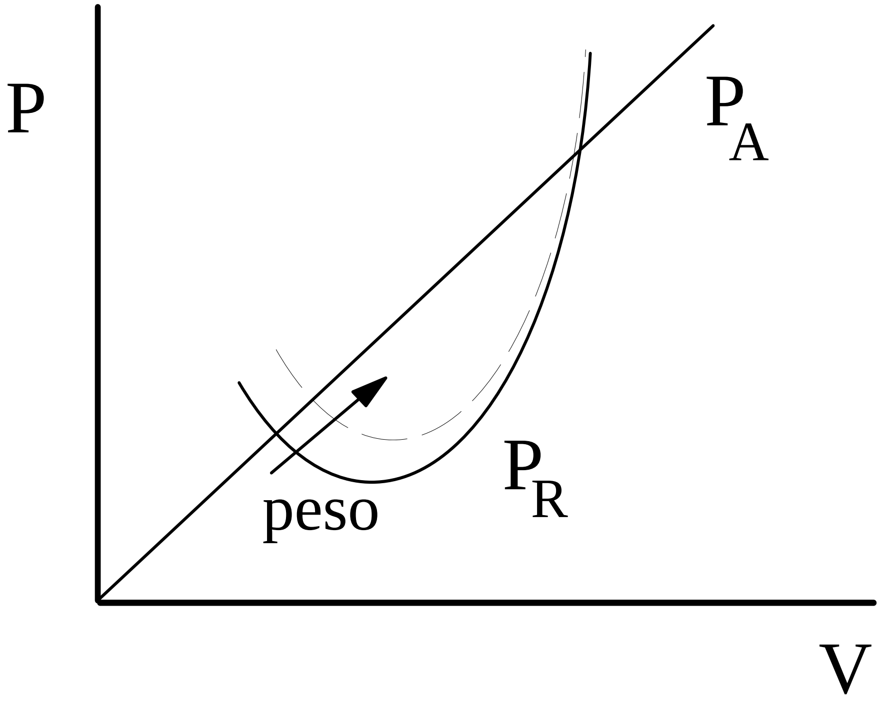
$$\left(\frac{\mathrm{E}}{\mathrm{V}}\right)_{\mathrm{MAX}} \Rightarrow \left(\mathrm{E} \cdot \sqrt{\mathrm{CL}}\right)_{\mathrm{MAX}}$$

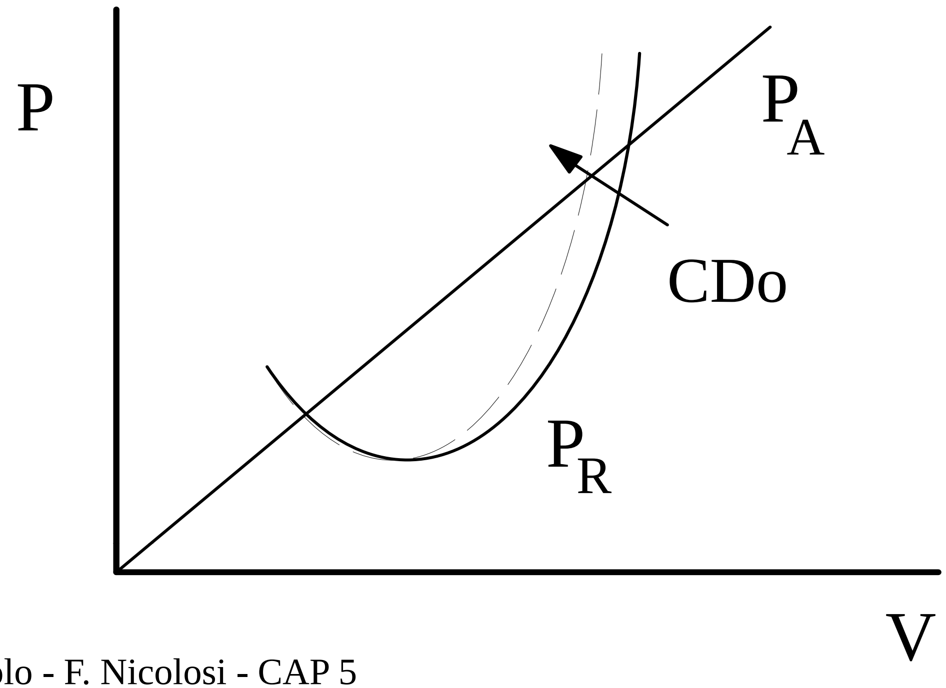




Cap.5 Polari tecniche – Influenze Peso e CDo su POT

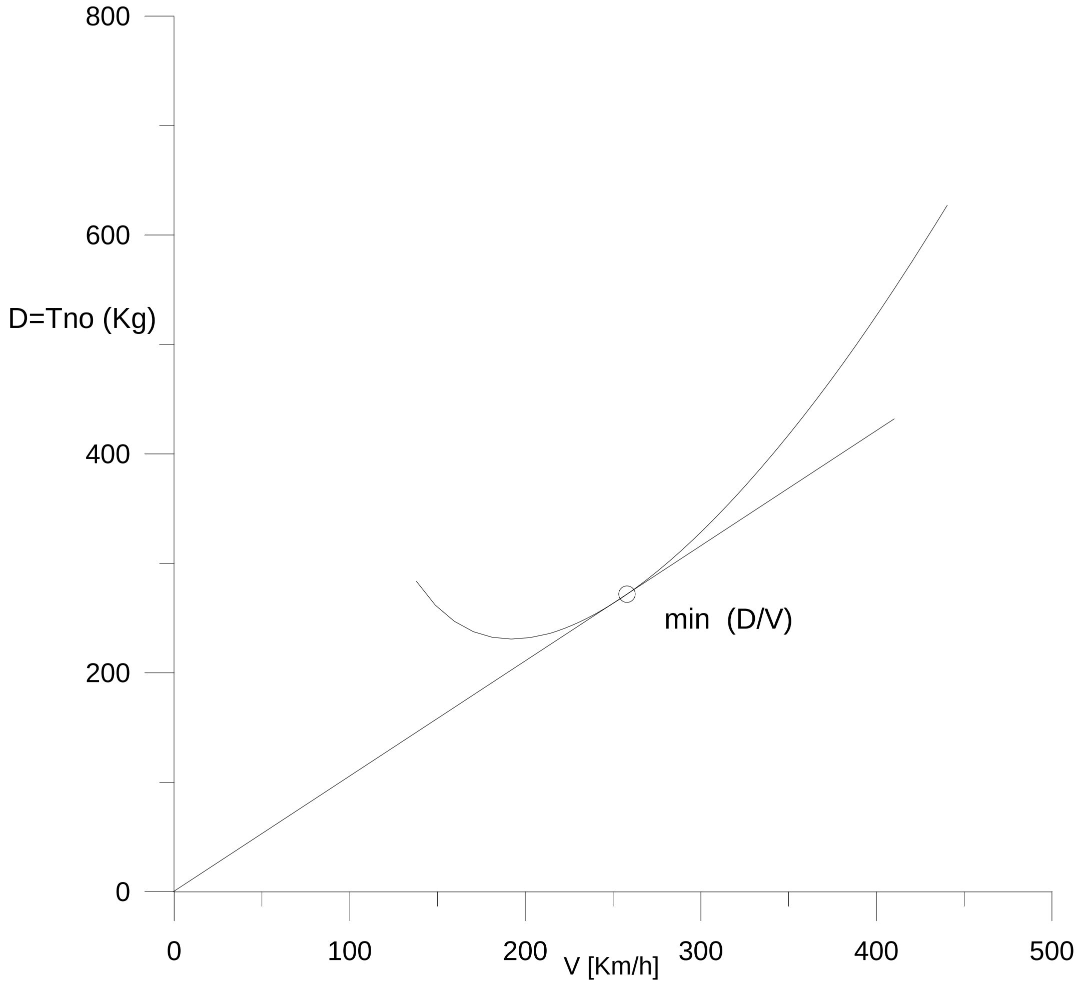
$$\Pi_{no} = \frac{1}{2}\rho \cdot CDo \cdot S \cdot V^3 + \frac{2}{\rho} \cdot S \cdot K \cdot \left(\frac{W}{S}\right)^2 \cdot \frac{1}{V}$$





Cap.5 Polari tecniche – PUNTO A

PUNTO A : $$\left(E \cdot V\right)_{MAX} \;=>\; \left(\frac{\mathrm{E}}{\sqrt{\mathrm{CL}}}\right)_{MAX} \;=>\; \left(\frac{D}{V}\right)_{MIN}$$



Cap.5 Polari tecniche – PUNTO A

PUNTO A : $\left(E \cdot V\right)_{MAX} \quad => \left(\frac{\mathrm{E}}{\sqrt{\mathrm{CL}}}\right)_{MAX} \quad => \left(\frac{D}{V}\right)_{MIN}$

$$\mathrm{T_{no}} = \mathrm{D} = \mathrm{a} \cdot \mathrm{V}^2 + \frac{\mathrm{b}}{\mathrm{V}^2}$$

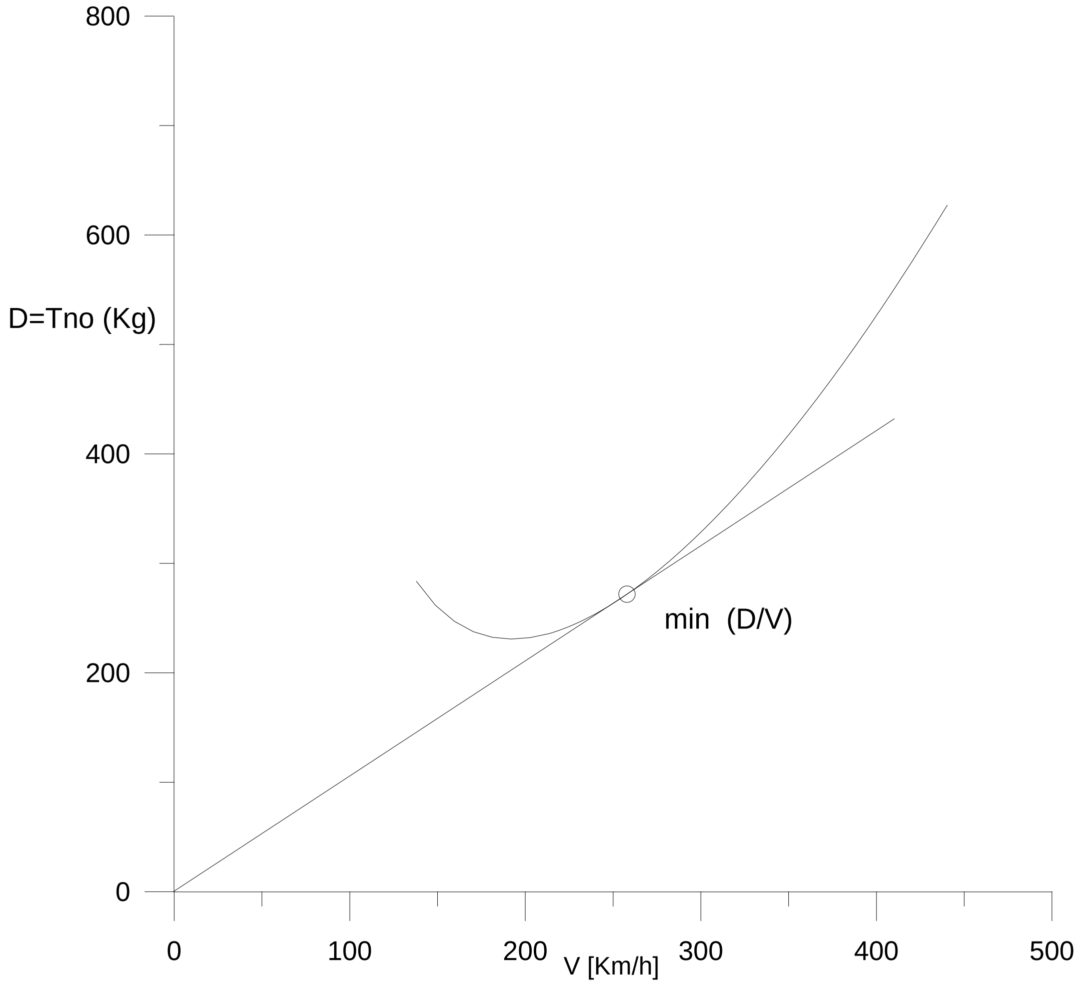
$$\frac{\mathrm{D}}{\mathrm{V}} = \mathrm{a} \cdot \mathrm{V} + \frac{\mathrm{b}}{\mathrm{V}^3}$$

$$\frac{\mathrm{d}}{\mathrm{dV}}\left(\frac{\mathrm{D}}{\mathrm{V}}\right) = \mathrm{a} - 3\frac{\mathrm{b}}{\mathrm{V}^4} = 0$$

$$\mathrm{a} = 3\frac{\mathrm{b}}{\mathrm{V}^4} \qquad \mathrm{a} \cdot \mathrm{V}^2 = 3 \cdot \frac{\mathrm{b}}{\mathrm{V}^2}$$

$$\boxed{\mathrm{Do} = 3\ \mathrm{Di}}$$

$$\mathrm{CDo} = 3 \cdot \mathrm{CDi} = 3 \cdot \mathrm{K} \cdot \mathrm{CL}^2$$



Cap.5 Polari tecniche – PUNTO A

$$\mathrm{CDo} = 3 \cdot \mathrm{CDi} = 3 \cdot \mathrm{K} \cdot \mathrm{CL}^2$$

$$\mathrm{CDi} = \frac{1}{3} \cdot \mathrm{CDo}$$

$$\mathrm{CD} = \mathrm{CDo} + \mathrm{CDi} = \frac{4}{3} \cdot \mathrm{CDo} \longrightarrow \mathrm{CD_A} = \frac{4}{3}\mathrm{CDo}$$

$$\mathrm{CL_A} = \sqrt{\frac{\mathrm{CDo}}{3 \cdot \mathrm{K}}} = \frac{1}{\sqrt{3}} \cdot \sqrt{\pi \cdot \mathrm{AR} \cdot \mathrm{e} \cdot \mathrm{CDo}}$$

$$\sqrt{\frac{2}{\rho}} \cdot \sqrt{\frac{W}{S}} \cdot \sqrt{\frac{1}{CL_A}}$$

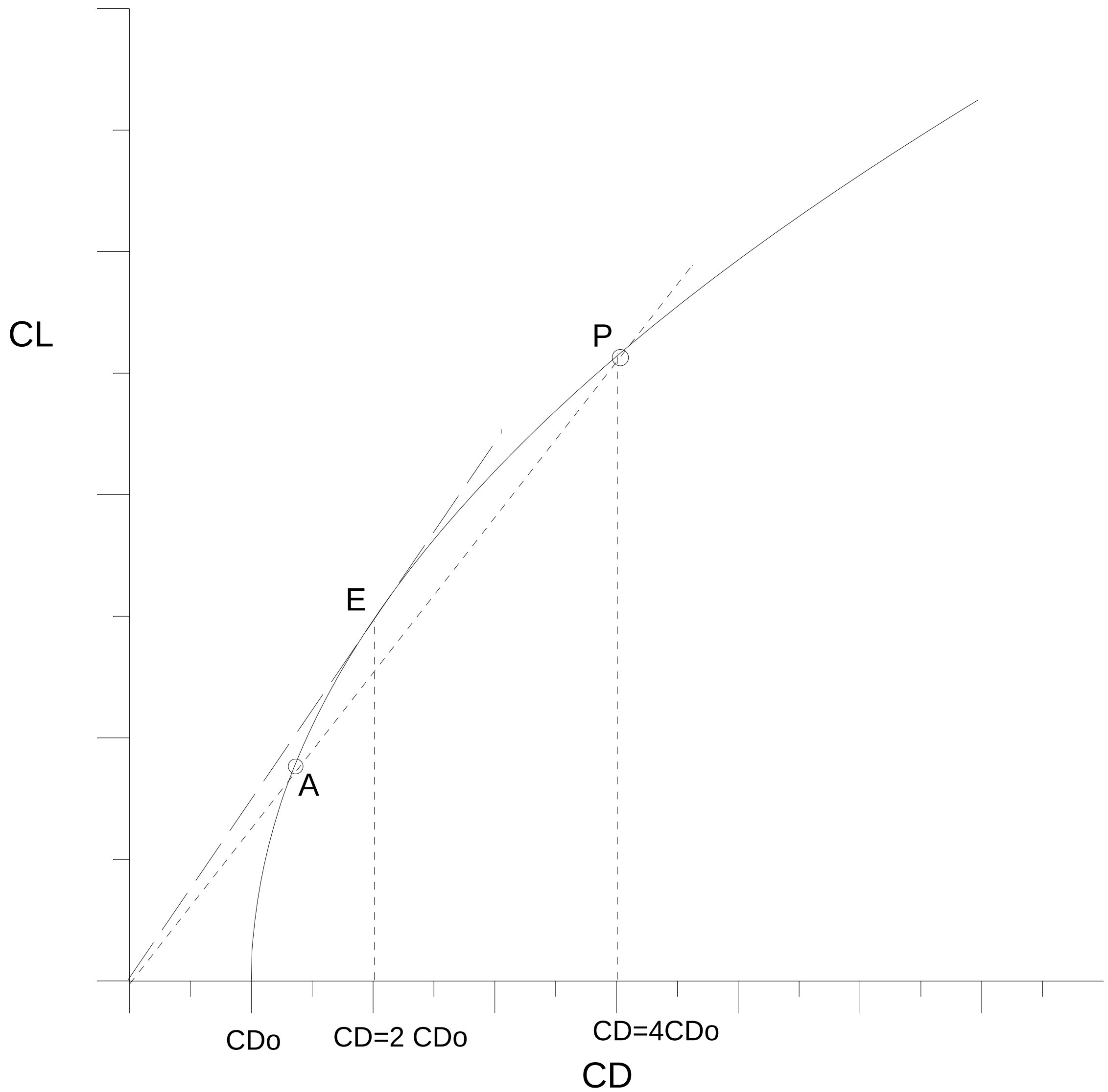
$$\mathrm{CL_A} = \frac{1}{\sqrt{3}} \cdot \mathrm{CL_E} = \frac{\mathrm{CL_E}}{1.732}$$

$$\mathrm{E_A} = \frac{\mathrm{CL_A}}{\mathrm{CD_A}} = \frac{1}{\sqrt{3}} \cdot \frac{3}{4} \cdot \frac{\mathrm{CL_E}}{\mathrm{CDo}} = \frac{1}{\sqrt{3}} \cdot \frac{3}{2} \cdot \frac{\mathrm{CL_E}}{\mathrm{CD_E}} = \frac{\sqrt{3}}{2} \cdot \mathrm{E_{MAX}}$$

$$V_A = \sqrt{\frac{2}{\rho}} \cdot \sqrt{\frac{W}{S}} \cdot \sqrt{\frac{1}{CL_A}} = \sqrt{\frac{2}{\rho}} \cdot \sqrt{\frac{W}{S}} \cdot \sqrt{\frac{\sqrt{3}}{CL_E}} = \sqrt[4]{3} \cdot V_E = 1.32\, V_E$$

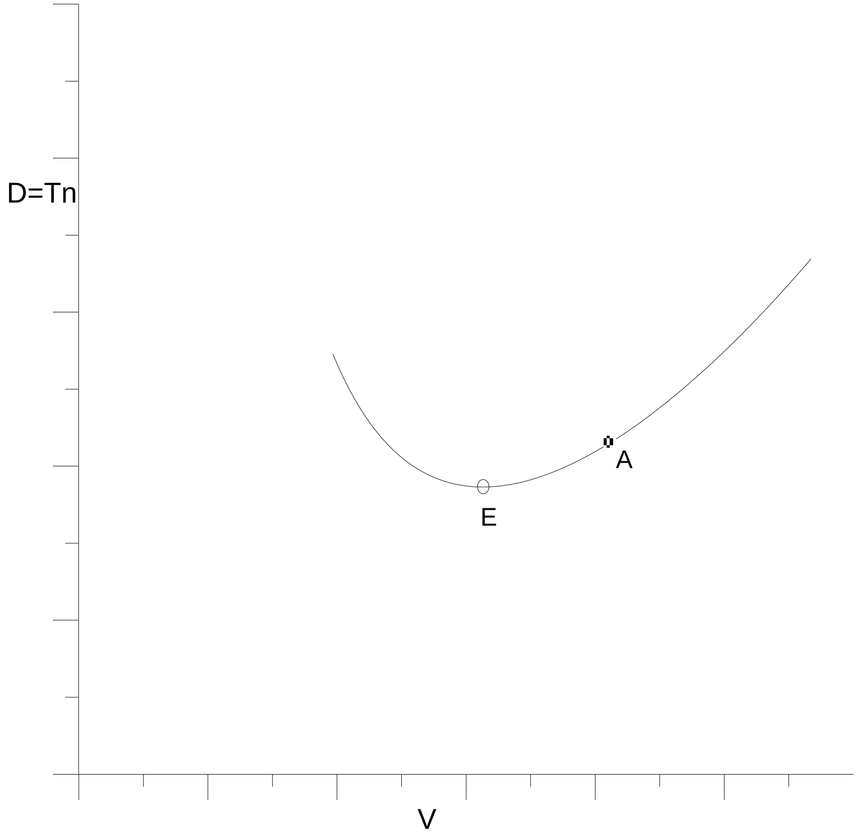
Cap.5 Polari tecniche – PUNTO A

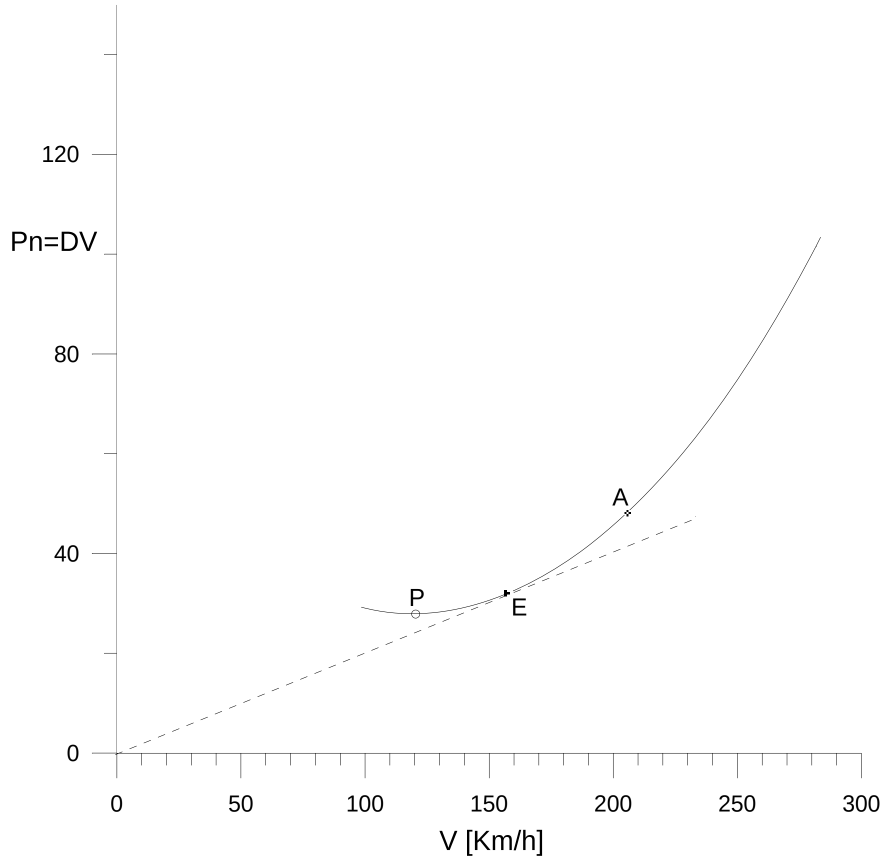
Punti A e P
=> stessa Efficienza



Cap.5 Polari tecniche – PUNTO A

$$V_A = 1.32\,V_E$$





Cap.5 Polare Parabolica – APPROCCIO ANALITICO

PUNTO E

$$\frac{L}{D}=\frac{C_L}{C_D}=\frac{C_L}{C_{Do}+KC_L^2} \qquad \frac{L}{D}=E \longrightarrow E=E_{MAX} \quad \text{se}$$

$$\frac{d(C_L/C_D)}{dC_L}=\frac{C_{Do}+KC_L^2-C_L(2KC_L)}{(C_{Do}+KC_L^2)^2}=0$$

$$C_{Do}+KC_L^2-2KC_L^2=0$$

$$C_{Do}=KC_L^2$$

$$C_L=C_{L\,E}=\sqrt{\frac{C_{Do}}{K}}=\sqrt{C_{Do}\pi A\,Re} \qquad K=\frac{1}{\pi A\,Re}$$

$$\left(\frac{L}{D}\right)_{max}=\left(\frac{C_L}{C_{Do}+KC_L^2}\right)_{max}=\frac{\sqrt{C_{Do}/K}}{C_{Do}+C_{Do}}=\frac{\sqrt{C_{Do}/K}}{2C_{Do}}$$

Cap.5 Polare Parabolica – APPROCCIO ANALITICO

PUNTO E

$$L = W = \frac{1}{2}\rho V^2 S C_L$$

$$W = \frac{1}{2}\rho V^2_{(L/D)_{max}} S \sqrt{\frac{C_{Do}}{K}}$$

$$\frac{W}{S} = \frac{1}{2}\rho V^2_{(L/D)_{max}} \sqrt{\frac{C_{Do}}{K}}$$

$$V_{(L/D)_{max}} = \left(\frac{2}{\rho} \sqrt{\frac{K}{C_{Do}}} \frac{W}{S} \right)^{1/2}$$

$$V_E = \sqrt{\frac{2}{\rho} \frac{W}{S} \frac{1}{C_{L_E}}}$$

Cap.5 Polare Parabolica – APPROCCIO ANALITICO

PUNTO P

$$P = D \cdot V \quad => \mathrm{P_{MIN}} => \left(D \cdot V\right)_{MIN} = \left(\frac{W}{E} \cdot V\right)_{MIN} \quad => \left(\frac{W}{E} \cdot \frac{1}{\sqrt{C_L}}\right)_{MIN}$$

$$\left(E \cdot \sqrt{C_L}\right)_{MAX} \quad => \left(\frac{C_L^{3/2}}{C_D}\right)_{MAX} = \left(\frac{C_L^{3/2}}{C_{Do} + KC_L^2}\right)_{MAX}$$

$$\frac{\mathbf{d}\left(\mathbf{C_L^{3/2} / C_D}\right)}{\mathbf{dC_L}} = \frac{\left(\mathbf{C_{Do} + KC_L^2}\right)\left(\frac{\mathbf{3}}{\mathbf{2}}\mathbf{C_L^{1/2}}\right) - \mathbf{C_L^{3/2}}\left(\mathbf{2KC_L}\right)}{\mathbf{C_{Do} + KC_L^2}} = \mathbf{0}$$

$$\frac{\mathbf{3}}{\mathbf{2}}\mathbf{C_{Do}} + \mathbf{KC_L^{1/2}} + \frac{\mathbf{3}}{\mathbf{2}}\mathbf{KC_L^{5/2}} - \mathbf{2KC_L^{5/2}} = \mathbf{0} \qquad \mathbf{C_{Do}} = \frac{\mathbf{1}}{\mathbf{3}}\mathbf{KC_L^2}$$

$$\mathbf{C_L} = \mathbf{C_{L_P}} = \sqrt{\mathbf{3C_{Do} / K}} = \sqrt{\mathbf{3}}\mathbf{C_{L_E}}$$

$$\left(\frac{\mathbf{C_L^{3/2}}}{\mathbf{C_D}}\right)_{\mathbf{max}} = \left(\frac{\mathbf{C_L^{3/2}}}{\mathbf{C_{Do} + KC_L^2}}\right)_{\mathbf{max}} = \frac{\left(\mathbf{3C_{Do} / K}\right)^{\mathbf{3/4}}}{\mathbf{C_{Do} + 3C_{Do}}} = \frac{\mathbf{1}}{\mathbf{4C_{Do}}}\left(\frac{\mathbf{3C_{Do}}}{\mathbf{K}}\right)^{\mathbf{3/4}}$$

Cap.5 Polare Parabolica – APPROCCIO ANALITICO

PUNTO P

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{max} = \frac{1}{4}\left(\frac{3}{KC_{Do}^{1/3}}\right)^{3/4}$$

$$L = W - \frac{1}{2}\rho V^2 S C_L$$

$$W = \frac{1}{2}\rho V_{\left(C_L^{3/2}/C_D\right)_{max}} S\sqrt{\frac{3C_{Do}}{K}}$$

$$V_{\left(C_L^{3/2}/C_D\right)_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{K}{3C_{Do}}}\frac{W}{S}\right)^{1/2}$$

$$V_{\left(C_L^{3/2}/C_D\right)_{max}} = 0.76 V_{(L/D)_{max}} = \frac{V_E}{1.32}$$

Cap.5 Polare Parabolica – APPROCCIO ANALITICO

PUNTO A

$$\left(\frac{D}{V}\right)_{MIN} = \left(\frac{W}{E}\cdot\frac{1}{V}\right)_{MIN} \Rightarrow \left(\frac{W}{E}\cdot\sqrt{C_L}\right)_{MIN}$$

$$\left(\frac{E}{\sqrt{C_L}}\right)_{MAX} \Rightarrow \left(\frac{C_L^{1/2}}{C_D}\right)_{MAX} = \left(\frac{C_L^{1/2}}{C_{Do}+KC_L^2}\right)_{MAX}$$

$$\frac{d\left(C_L^{1/2}/C_D\right)}{dC_L} = \frac{\left(C_{Do}+KC_L^2\right)\left(\frac{1}{2}C_L^{-1/2}\right) - C_L^{1/2}\left(2KC_L\right)}{C_{Do}+KC_L^2} = 0$$

$$\mathbf{C_{Do} = 3KC_L^2}$$

$$\left(\frac{\mathbf{C_L^{1/2}}}{\mathbf{C_{Do}}}\right)_{\mathbf{max}} = \frac{\mathbf{3}}{\mathbf{2}}\left(\frac{\mathbf{1}}{\mathbf{3KC_{Do}^3}}\right)^{\mathbf{1/4}}$$

$$\mathbf{V_{\left(C_L^{1/2}/C_D\right)_{max}} = \left(\frac{2}{\rho}\sqrt{\frac{3K}{C_{Do}}}\frac{W}{S}\right)^{1/2}}$$

$$\mathbf{V_{\left(C_L^{1/2}/C_D\right)_{max}} = 1.32V_{\left(L/D\right)_{max}}}$$

Cap.5 Polare Parabolica – APPROCCIO ANALITICO

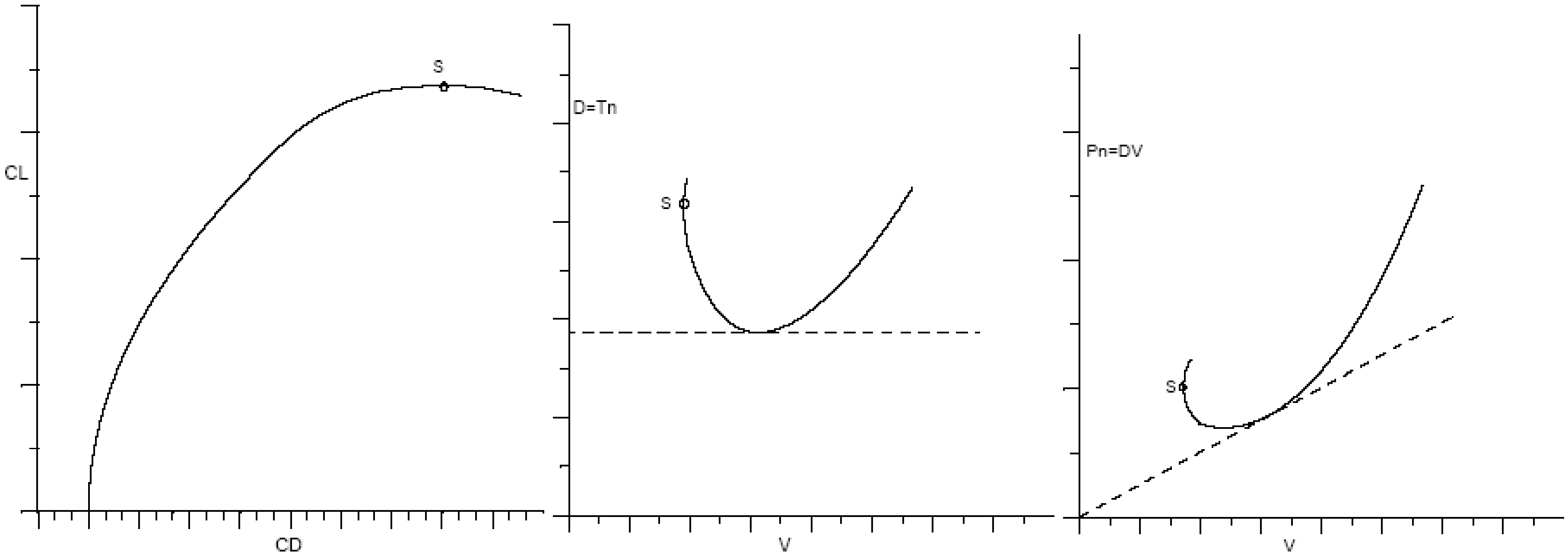
$$V_{\left(C_L^{3/2}/C_D\right)_{max}} < V_{\left(C_L/C_D\right)_{max}} < V_{\left(C_L^{1/2}/C_D\right)_{max}}$$

Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

Punto S

Stallo

Corrisponde a C_L=$C_{L\,max}$ => V=V_S =V_{min}



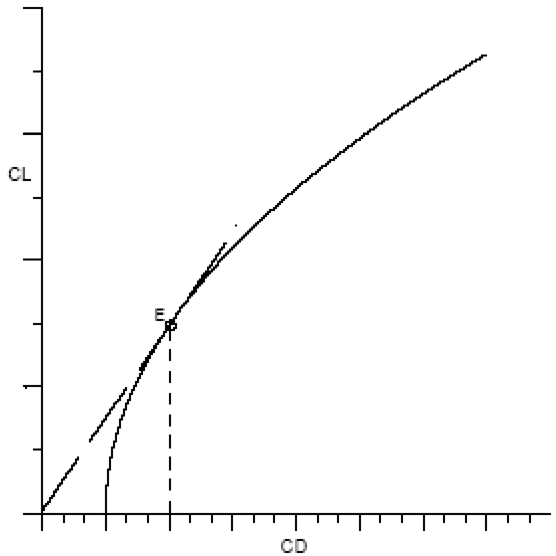
Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

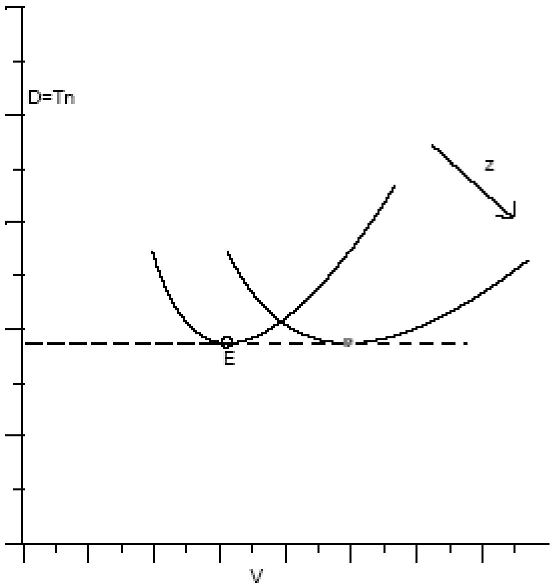
Punto E

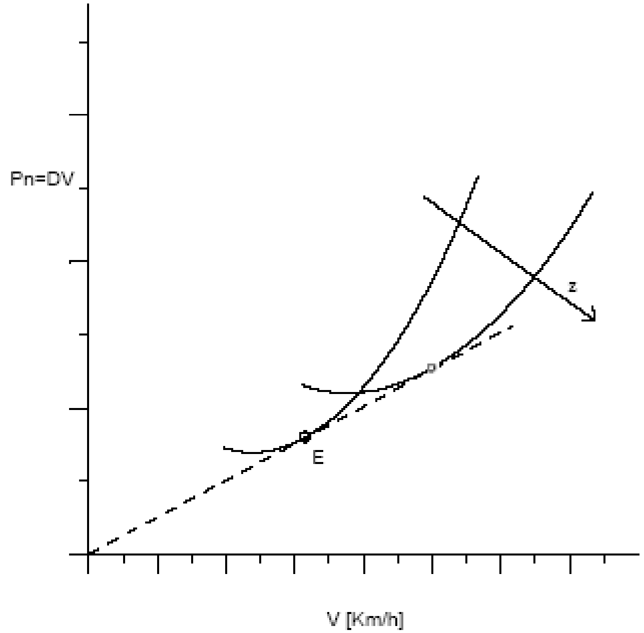
Massima efficienza

$D_{min} \Rightarrow E_{max}$ $\qquad C_{Di}=C_{Do} \Rightarrow C_D=2\,C_{Do}$

$$C_{L\,E} = \sqrt{\pi\,AR_e\,C_{Do}} \qquad E_{max} = \sqrt{\left(\frac{\pi}{4}\frac{AR_e}{C_{Do}}\right)} = \sqrt{\left(\frac{\pi}{4}\frac{b_e^{\,2}}{f}\right)} \qquad V_E = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_{L\,E}}}$$







Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

Punto P

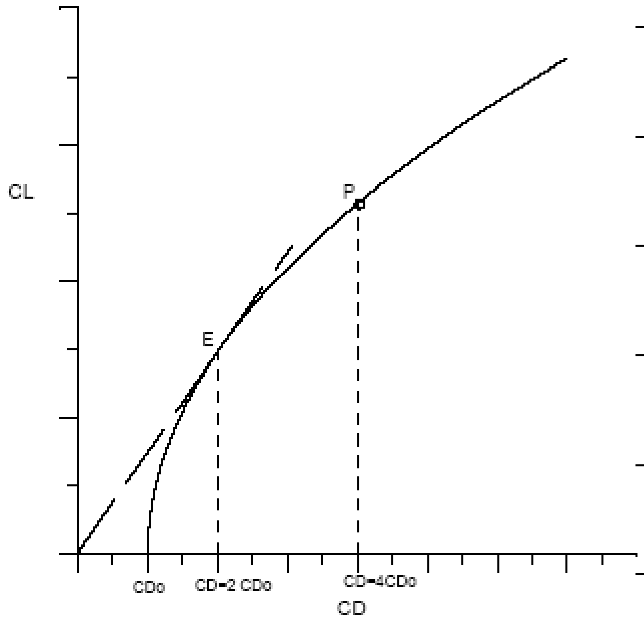
$$\Pi_{min}=(T_{no}\ V)_{min} \Rightarrow (C_D\ V^3)_{min} \Rightarrow \left(\frac{C_D}{C_L}\frac{1}{\sqrt{C_L}}\right)_{min} \Rightarrow \left(E\sqrt{C_L}\right)_{max}$$

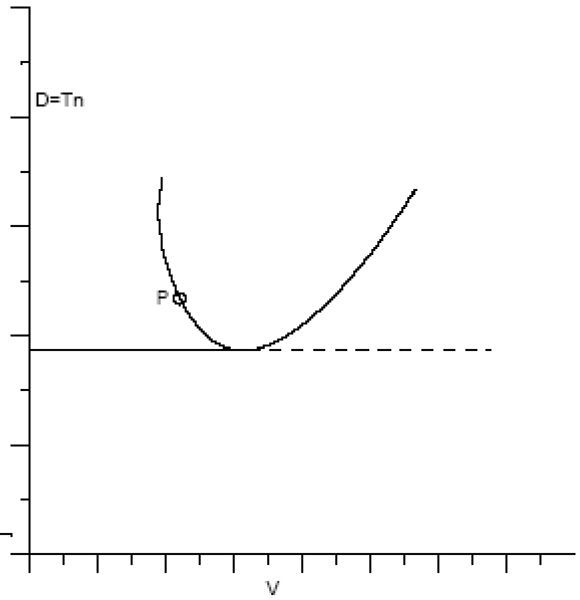
si ha : $\Pi=\sqrt{\frac{2}{\rho}}\sqrt{\frac{W^3}{S}}\frac{C_D}{C_L^{3/2}}$

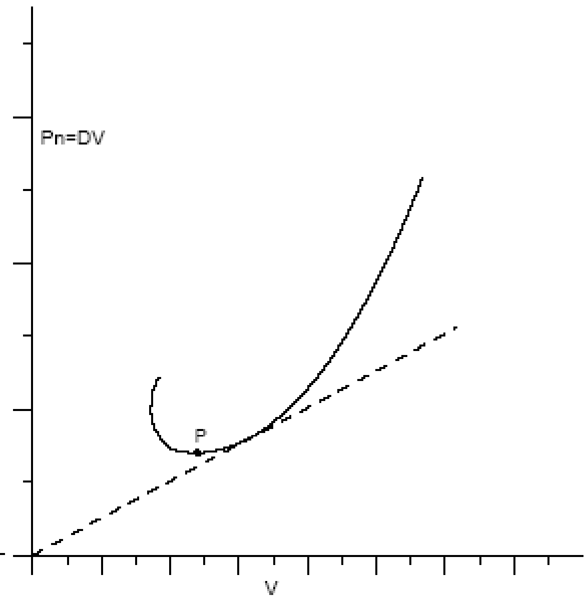
$$\Pi_{min} \Rightarrow \left(\frac{C_L^{3/2}}{C_D}\right)_{max} \Rightarrow \left(E\sqrt{C_L}\right)_{max}$$

$Di = 3\ D_o$ cioè $C_{Di} = 3\ C_{Do}$ $\qquad C_D = 4\ C_{Do}$

$$C_{L\,P} = \sqrt{3\,\pi\,AR_e\,C_{Do}} = \sqrt{3}\ C_{L\,E} = 1.732\ C_{L\,E}$$







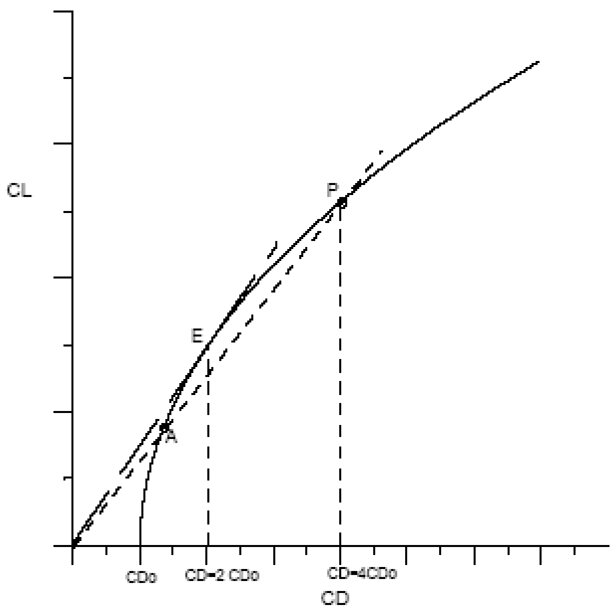
Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

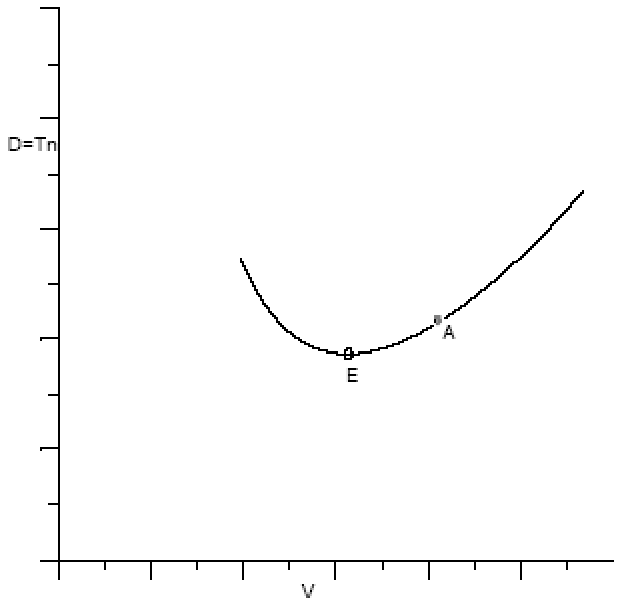
Punto A

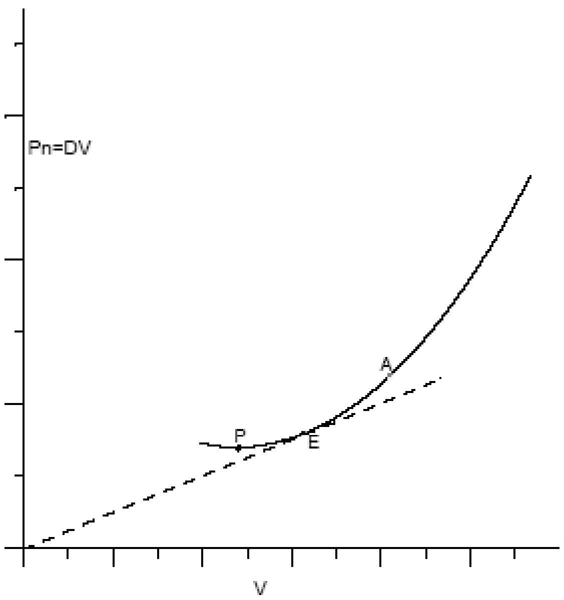
$$\left(\frac{T}{V}\right)_{min} \Rightarrow \left(C_D\, V\right)_{min} \Rightarrow \left(C_D \frac{1}{\sqrt{C_L}}\right)_{min} \Rightarrow \left(\frac{E}{\sqrt{C_L}}\right)_{max}$$

$$D_i = \frac{D_o}{3} \qquad C_{Di} = \frac{C_{Do}}{3} \qquad C_{D\,A} = C_{Do} + \frac{C_{Do}}{3} = \frac{4}{3} C_{Do}$$

$$C_{Di} = \frac{{C_L}^2}{\pi\, AR_e} = \frac{C_{Do}}{3} \qquad C_{L\,A} = \sqrt{\frac{\pi}{3} C_{Do}\, AR_e} = \frac{C_{L\,E}}{\sqrt{3}} = 0.577\, C_{L\,E}$$



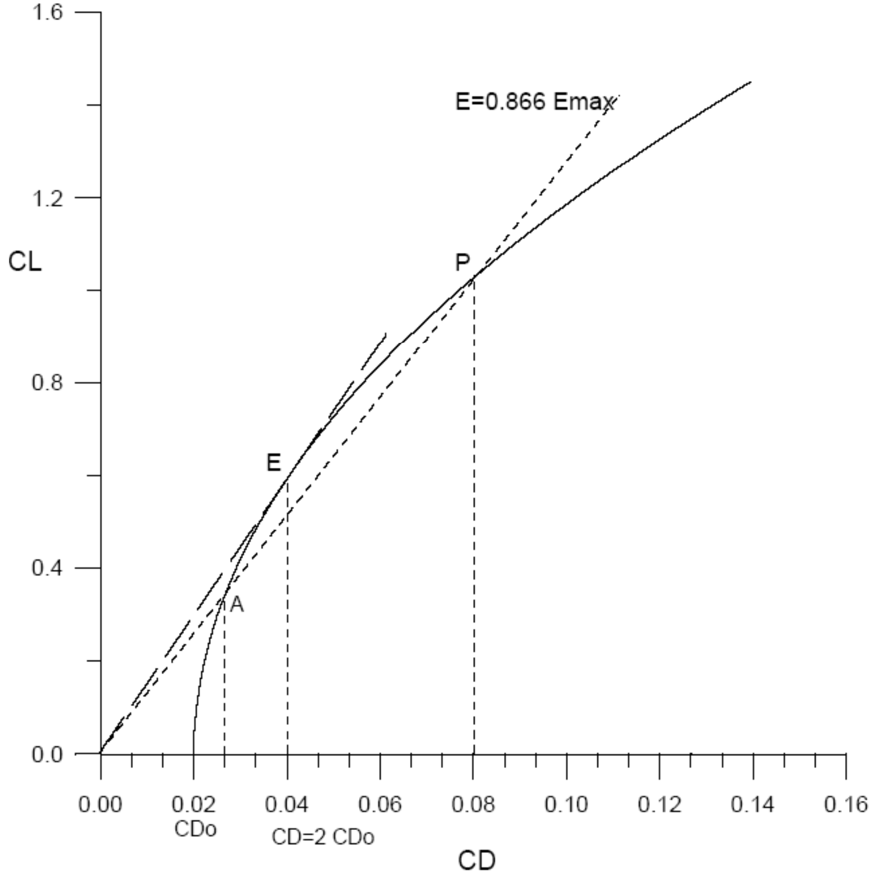




Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

PUNTO	Efficienza CL/CD	C_L	C_D
A	$E=\sqrt{\frac{3}{4}}\,E_{max} = 0.866\,E_{max}$	$C_{L_A} = \frac{C_{L_E}}{\sqrt{3}} = 0.577\,C_{L_E}$	$C_{D_A} = C_{Do} + \frac{1}{3}C_{Do} = \frac{4}{3}C_{Do}$
E	$E=E_{max}=\sqrt{\frac{\pi}{4}\frac{AR_e}{C_{Do}}}$	$C_{L_E} = \sqrt{\pi\,AR_e\,C_{Do}}$	$C_{D_E} = C_{Do} + C_{Do} = 2\,C_{Do}$
P	$E=\sqrt{\frac{3}{4}}\,E_{max} = 0.866\,E_{max}$	$C_{L_P} = \sqrt{3}\,C_{L_E} = 1.732\,C_{L_E}$	$C_{D_E} = C_{Do} + 3\,C_{Do} = 4\,C_{Do}$

Esempio di polare parabolica CD=CDo + CL^2 /(ARe)
con CDo=0.020 AR=7 e=0.80



Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

RELAZIONI DA CONSIDERARE:

$$\boxed{V_P = \frac{V_E}{\sqrt[4]{3}} = \frac{V_E}{1.32}}$$

$$\boxed{V_A = \sqrt[4]{3} \cdot V_E = 1.32 \cdot V_E}$$

$$E_A = E_P = \frac{\sqrt{3}}{2} \cdot E_E = 0.866 \cdot E_E$$

$$\boxed{D_{MIN} = D_E = \frac{W}{E_{MAX}}}$$

$$\boxed{D_A = D_P = \frac{2}{\sqrt{3}} \cdot D_E = \frac{D_E}{0.866} = 1.155 \cdot D_E}$$

$$\boxed{\Pi_P = \Pi_{MIN} = \frac{W}{E_P} \cdot V_P =}$$

$$\Pi_E = \frac{W}{E_E} \cdot V_E = \frac{\sqrt{3}}{2} \frac{W}{E_P} \cdot \sqrt[4]{3} \cdot V_P = \frac{\sqrt[4]{27}}{2} \cdot \Pi_P = 1.14 \cdot \Pi_P$$

$$\boxed{\Pi_E = 1.14 \cdot \Pi_P}$$

$$\Pi_A = \frac{W}{E_A} \cdot V_A = \frac{W}{E_P} \cdot \sqrt[4]{3} \cdot \sqrt[4]{3} \cdot V_P = \sqrt{3} \cdot \Pi_P = 1.732 \cdot \Pi_P$$

$$\boxed{\Pi_A = 1.732 \cdot \Pi_P}$$

Cap.5 Polare Parabolica – PUNTI CARATTERISTICI

TABELLA RIEPILOGATIVA :

$$E_{MAX} = \sqrt{\frac{\pi}{4}\frac{AR \cdot e}{CDo}} \qquad CL_E = \sqrt{\pi \cdot AR \cdot e \cdot CDo}$$

$$D_{MIN} = \frac{W}{E_{MAX}} \qquad \Pi_{MIN} = \Pi_P = \frac{W}{E_P} \cdot V_P$$

Punto	CD/CDo	CL/CL_E	E/E_{MAX}	V/V_E	T/T_E	Π/Π_P
P	4	$\sqrt{3} = 1.732$	$\sqrt{3}/2 = 0.866$	$1/\sqrt[4]{3} = 1/1.32$	$2/\sqrt{3}$	1
E	2	1	1	1	1	$\sqrt[4]{27}/2 = 1.14$
A	4/3	$1/\sqrt{3} = 0.577$	$\sqrt{3}/2 = 0.866$	$\sqrt[4]{3} = 1.32$	$2/\sqrt{3}$	$\sqrt{3} = 1.73$

Cap.5 Polare Parabolica – PUNTI CARATTERISTICI