$$\frac{P_{to}}{a_o W_{to}} \geq \frac{1}{\eta_p}\left(\frac{W_{to}}{\rho_o S}\right)^{\frac{1}{2}} \left\{ \frac{.136}{\left(C_{L_{max}}\right)^{\frac{1}{2}}} + \frac{2.21\, C_{D_o}^{\ 1/4}}{(\pi Ae)^{3/4}} \right\} \qquad (5\text{-}69)$$

By way of example, the following data may be applied to a light aircraft: W_{to} = 3,300 lb (1,500 kg); S = 130 sq.ft (12 m^2), hence $W_{to}/(\rho_o S)$ = .0121; $C_{L_{max}}$ = 1.8; πAe = 15.3 and C_{D_o} = .055 with flaps deflected and undercarriage down. Effective propeller efficiency: η_p = .65. Substitution of C/a ≥ .0045 into eq. 5-67 yields: $P_{to}/a_o W_{to}$ ≥ .0305, while eq. 5-69 yields $P_{to}/a_o W_{to}$ ≥ .0408. The latter being the most critical requirement, it is concluded that P_{to} must be at least 275 hp.

d. Design data.

If no better information is available, the following data may be useful in working out climb performance requirements.
Propeller efficiency during climb at sea level:
tractor propeller in fuselage nose,
fixed pitch : η_p = .61 (± .052)
constant speed: η_p = .665 (± .059)
tractor propellers, wing-mounted,
constant speed: η_p = .73 (± .058)
These data were found by applying of the present method to the performance data of a large number of aircraft. The second number gives the rms error. All figures include slipstream effects, cooling drag, power offtakes and intake losses.

For the effect of engine failure on drag, 4% may be added to C_{D_o} for the drag of feathering propellers, while the Oswald factor may be reduced by approximately 10% for wing-mounted engines.
The airplane drag polar may be estimated by the method explained in Section 5.3. For the effect of undercarriage extension, it is reasonable to take ΔC_{D_o} = .015 to .020 as a typical value. It should be noted that in most equations for climb performance small variations in C_{D_o} are of minor importance, unless the flaps are deflected. Hence, it may be assumed that the drag due to powerplant installation adds roughly 8% to C_{D_o} on turbine-powered aircraft and 12% with piston engines.

The engine thrust or power lapse rate may be determined from engine manufacturer's brochures. For jet engines the effect of M on the thrust is important, while lapse rates are very sensitive to the bypass ratio as well. Curves of $T/\delta T_{to}$ vs. M may serve the purpose. An example is shown in Fig. 6-3. In the case of turboprop engines the shaft power increases noticeably with M due to the ram effect, unless there is a structural or thermal engine limitation up to some specified altitude. If this is not the case, the following approximation may be used for a given rating and Mach number:

$$\frac{\text{power at altitude}}{\text{power at sea level}} = \sigma^n \qquad (5\text{-}70)$$

where n is generally between .7 and .8.
For naturally aspirated piston engines at constant rpm:

$$\frac{\text{full throttle power at altitude}}{\text{full throttle power at sea level}} =$$

$$= 1.132\sigma - .132 \qquad (5\text{-}71)$$

Supercharged engines maintain constant power up to the rated altitude. Above this the power decreases linearly with σ in the same way as in eq. 5-71. With many supercharged engines a cruising power of 65% to 75% of rated power can be maintained up to the cruising altitude.

5.4.4. Stalling and minimum flight speeds

In establishing low speed performance, operational flight speeds must have a specified safety margin relative to the stalling speed, in order to provide the pilot with some measure of freedom to maneuver and in order to avoid stalling due to vertical gusts.
The required field length being roughly proportional to the kinetic energy at the screen height, and hence to $(\text{velocity})^2$, a low stalling speed provides a powerful method of obtaining good field performance.
On the other hand, a decrease in stalling speed generally entails a cost penalty as a more sophisticated flap system must be developed or the wing loading decreased, or

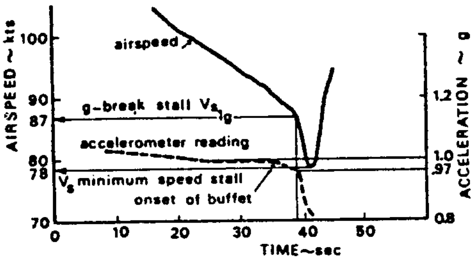

Fig. 5-15. Time history of the airspeed during a stall maneuver

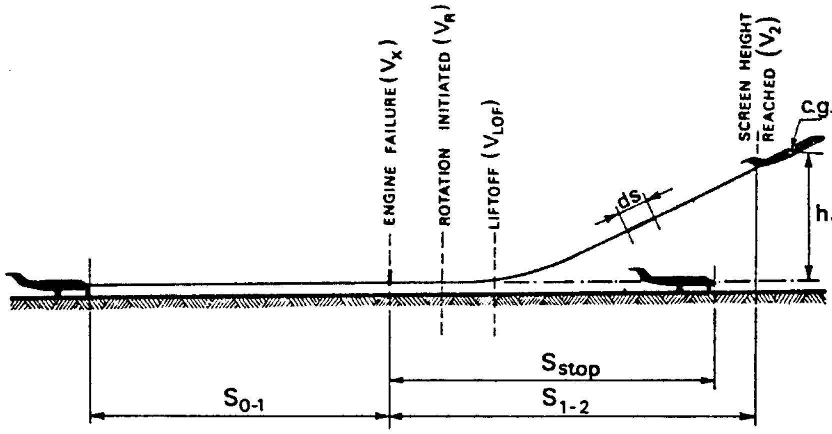

Fig. 5-16. Phases during the takeoff with engine failure

both.

With regard to the definition of stalling speeds, reference is made to Fig. 5-15, which depicts a time history of the airspeed during a stalling maneuver. In principle, several flight procedures can be chosen for this - for example, flight at constant longitudinal deceleration, constant flight path angle (or horizontal flight) or constant normal acceleration. The FAR certification procedure is a stall maneuver at constant dV/dt. Several tests are carried out at different values of dV/dt and the stalling speed is defined by interpolation at dV/dt = -1 kt/s.

In approaching the stall, the normal load factor initially remains approximately constant until a break occurs, indicating that wing stalling is progressing rapidly. The corresponding airspeed is referred to as the 1-g stalling speed or g-break stalling speed V_{S-1g}. Immediately after the g-break the airspeed continue V_S to decrease, and the sink speed increases rapidly until the pilot takes corrective action by pitching down the nose, resulting in a positive dV/dt. The minimum airspeed measured in this procedure, V_S, is noticeably lower than V_{S-1g}. The FAR-25 regulations allow V_S to be used in scheduling various reference speeds for definition of the performance, whereas the British requirements do not allow the stalling speed to be less than 94% of V_{S-1g}.

In view of this reasoning, it must be noted that a definition of C_L-max according to

$$C_{L_{max}} = \frac{W/S}{\frac{1}{2}\rho V_S^2} \qquad (5\text{-}70)$$

does not result in a "physical" C_L-max as obtained in the wind tunnel, but produces a value which may be some 10 to 20% higher. Alternatively, when calculating the stalling speed from C_L-max obtained by theoretical methods or wind tunnel experiments, it is generally appropriate to assume an apparent C_L-max increment of, say, 13%. All values of C_L-max mentioned in the following sections are corrected in this manner.

Occasionally a limit is imposed on the stalling speed and the corresponding wing loading is limited by:

$$\frac{W}{S} \leqslant \frac{1}{2}\rho V_S^2\, C_{L_{max}} \qquad (5\text{-}71)$$

For example, in the BCAR requirement Ch. D2-11, an upper limit applies to the stalling speed:
70 mph (112.5 kmh) - group C
60 mph (96.5 kmh) - group D
These values can be substituted into eq. 5-71 to find a limiting wing loading for these classes of aircraft. The reader should also refer to FAR 23.49 (b) for aircraft with a takeoff weight of 6,000 lb (2720 kg) or less.

5.4.5. Takeoff

The following cases are distinguished:

a. All-engine takeoff distance requirements

b. one-engine-out takeoff distance requirements

c. Accelerate-stop distance requirements

For aircraft to be certificated under the FAR Part 23 regulations, no explicit requirements need to be met with regard to the event of engine failure during takeoff. The all-engine takeoff distance applies to airplanes with a takeoff weight of 6,000 lb (2720 kg) or more and is generally defined as the distance required to pass the screen height of 50 ft (15.3 m) at a speed of 1.3 V_S. For a particular class of aircraft operating under the FAR Part 135 operational rules, the FAR Part 23 performance standards are considered to be inadequate. The performance standards laid down in SFAR 23 and NRPM 68-37 are intended as intermediate steps towards improving safety in the operation of small passenger airplanes and air taxis capable of carrying more than 10 persons (Ref. 5-22). Accelerate-stop distances are introduced in these requirements. Under SFAR 23 this is the distance required to accelerate to the critical engine-failure speed V_1 and then to decelerate to 35 knots, while NRPM 68-37 covers the distance needed to come to a full stop. These regulations contain no specifications for a one-engine inoperative takeoff distance.

The takeoff performance of transport category airplanes is a fairly complicated matter, which is dealt with in more detail in Appendix K. It is generally found that the most critical item of performance is determined by the case of a one-engine-out takeoff. The condition that the airplane must be brought to standstill after engine failure at the critical engine failure speed leads to the concept of the Balanced Field Length (BFL), which is usually considered as the most important design criterion as far as field performance of transport category aircraft is concerned. The simplified methods presented in this section are intended to serve as a first approximation of field length for the purpose of sizing the engine thrust or power and wing design. They can be refined if more detailed information is available to the designer; an example is given in Appendix K.

a. All-engines takeoff.

Since the takeoff consists of a takeoff run and an airborne phase, we may write:

$$S_{to} = S_{run} + S_{air} \tag{5-72}$$

The expression for S_{run} is

$$S_{run} = \frac{1}{2g} \int_0^{V_{LOF}^2} \frac{dV^2}{a/g} \tag{5-73}$$

where the momentary acceleration,

$$a/g = \frac{T}{W_{to}} - \mu - (C_D - \mu C_L) \frac{\frac{1}{2}\rho V^2 S}{W} \tag{5-74}$$

can be approximated by

$$S_{run} = \frac{V_{LOF}^2/2g}{\bar{T}/W_{to} - \mu'} \tag{5-75}$$

where $\bar{T}$ is a mean value of the thrust during the takeoff run. Assuming the lift-off speed to be approximately 1.2 V_S and the lift coefficient during the takeoff run to be equal to twice the value for minimum $(C_D - \mu C_L)$, it is found that

$$\left.\begin{aligned} \bar{T} &= \text{thrust at } V_{LOF}/\sqrt{2} \\ C_L &= \mu \, \pi A e \\ \mu' &= \mu + .72 \, C_{D_o}/C_{L_{max}} \end{aligned}\right| \tag{5-76}$$

Assuming an air maneuver after lift-off with $C_L = C_{L\text{-}LOF}$ = constant and T-D = constant, the following result can be derived from the AGARD Flight Test Manual, Vol. 1:

$$S_{air} = \frac{V_{LOF}^2}{g\sqrt{2}} + \frac{h_{to}}{\gamma_{LOF}} \tag{5-77}$$

and

$$\frac{V_3}{V_{LOF}} = \sqrt{1 + \gamma_{LOF}\sqrt{2}} \qquad (5\text{-}78)$$

where γ_{LOF} = (T-D)/W at liftoff and V_3 = velocity at the takeoff height (30 or 50 ft).

The liftoff speed corresponding to a given V_3 is:

$$V_{LOF} = V_3 \left(\frac{1}{1 + \gamma_{LOF}\sqrt{2}}\right)^{\frac{1}{2}} \qquad (5\text{-}79)$$

From eqs. 5-75, 5-76 and 5-79 the takeoff distance is now:

$$\frac{S_{to}}{f_{to}h_{to}} = \left(\frac{V_3}{V_S}\right)^2 \frac{W_{to}/S\,\{(\bar{T}/W_{to}-\mu')^{-1}+\sqrt{2}\}}{h_{to}\rho g\, C_{L_{max}}\,(1+\gamma_{LOF}\sqrt{2})} + \frac{1}{\gamma_{LOF}} \qquad (5\text{-}80)$$

Regulations	V_3/V_S	f_{to}	h_{to}
(S) FAR 23	1.3	1.0	50 ft (15.3 m)
FAR 25	1.25 to 1.30 (no requirement)	1.15	35 ft (10.7 m)

Table 5-2. Characteristic values for the all-engines takeoff according to FAR 23 and 25.

In the absence of better information, the following assumptions and approximations may be made in applying eq. 5-80.

1. In calculating μ' according to eq. 5-76, it is reasonable to assume: $.72\ C_{D_o}/C_{L_{max}} = .010\ C_{L_{max}}$, $\mu = .02$ for concrete and $\mu = .04 - .05$ for short grass.

2. $\gamma_{LOF} = .9\,\frac{\bar{T}}{W_{to}} - \frac{.3}{\sqrt{A}}$

3. The mean thrust/weight ratio at mean velocity $V_{LOF}/\sqrt{2}$, allowing for slipstream effects and power offtakes, is as follows:

jet aircraft:

$$\bar{T} = .75\,\frac{5+\lambda}{4+\lambda}\,T_{to} \qquad (5\text{-}81)$$

aircraft with constant speed propellers:

$$\bar{T} = k_p\, P_{to}\left(\frac{\sigma N_e D_p^2}{P_{to}}\right)^{1/3} \qquad (5\text{-}82)$$

where $P_{to}/(N_e D_p^2)$ is the propeller disc loading; see Fig. 6-9, for example.

k_p = 5.75 when $\bar{T}$ is in lb, P_{to} in hp, D_p in ft

k_p = .321 when $\bar{T}$ is in kg, P_{to} in kgm/s, D_p in m.

For fixed-pitch propellers the mean thrust is roughly 15-20% below the value given by eq. 5-82.

From eq. 5-77 it follows that for a specified takeoff distance, the wing loading is limited to:

$$\frac{W_{to}}{S} \leq \left\{\frac{S_{to}}{f_{to}} - \frac{h_{to}}{\gamma_{LOF}}\right\} \frac{\rho\, g\, C_{L_{max}}\,(1+\gamma_{LOF}\sqrt{2})}{(V_3/V_S)^2\,\{(\bar{T}/W_{to}-\mu')^{-1}+\sqrt{2}\}} \qquad (5\text{-}83)$$

b. Takeoff with engine failure and accelerate-stop distance.

A critical decision speed V_1 is defined so that, with a single engine failure, the total accelerate-stop distance required becomes identical with the total takeoff distance to reach screen height safely. A simple analytical method for determining the BFL in the preliminary design stage, devised by the author and presented in Ref. 5-27, will be summarized below.

As opposed to the usual subdivision (takeoff run to liftoff, transition and climb distance), the continued takeoff is split up into 2 phases (Fig. 5-16):

- Phase 0-1: acceleration from standstill to engine failure speed V_x,
- Phase 1-2: the motion after engine failure, up to the moment of attaining the screen height at takeoff safety speed V_2.

The distance travelled during phase 0-1 is:

$$S_{0-1} = \frac{V_x^2}{2\,\bar{a}_{0-1}} \qquad (5\text{-}84)$$

where $\bar{a}_{0-1}$ is calculated in the same way as for the all-engines takeoff.

The energy equation is applied to phase 1-2 (Fig. 5-16), resulting in

$$S_{1-2} = \frac{1}{\bar{\gamma}}\left(\frac{V_2^2 - V_x^2}{2\,g} + h_{to}\right) \qquad (5\text{-}85)$$

where the equivalent climb gradient $\bar{\gamma}$ is defined as follows:

$$\bar{\gamma} = \frac{\int_1^2 (T-D_{air}-D_g)ds}{W_{to}\, S_{1-2}} \qquad (5\text{-}86)$$

The distance required to come to a standstill after engine failure can be represented by:

$$S_{stop} \doteq \frac{V_x^2}{2\,\bar{a}_{stop}} + V_x\,\Delta t \qquad (5\text{-}87)$$

where Δt is referred to as an equivalent inertia time, affected in principle by the thrust/weight ratio at V_x (Fig. K-6). The condition for balancing the field length is $S_{1-2} = S_{stop}$ and Ref. 5-27 gives the following expression for the critical engine failure speed V_1:

$$\frac{V_1}{V_2} = \left\{\frac{1+2g\,h_{to}/V_2^{\,2}}{1+\bar{\gamma}/(\bar{a}/g)_{stop}}\right\}^{\frac{1}{2}} - \frac{\bar{\gamma}\,g(\Delta t-1)}{V_2} \qquad (5\text{-}88)$$

The condition that $V_1 < V_R$ must be satisfied. To check this, a more detailed analysis of the rotation and flare maneuver is necessary (Appendix K). In the case that $V_1 = V_R$, the field length is generally no longer balanced.

Combination of eqs. 5-84 through 5-88 results in the expression

$$BFL = \frac{V_2^{\,2}}{2g\left\{1+\bar{\gamma}/(\bar{a}/g)_{stop}\right\}}\left\{\frac{1}{(\bar{a}/g)_{0-1}} + \frac{1}{(\bar{a}/g)_{stop}}\right\} \times \left(1+\frac{2\,g\,h_{to}}{V_2^{\,2}}\right) + \frac{\Delta S_{to}}{\sqrt{\sigma}} \qquad (5\text{-}89)$$

In this expression the inertia distance ΔS_{to} may be assumed equal to 655 ft (200 m) for $\Delta t = 4\frac{1}{2}$ seconds, a value derived for typical combinations of wing and thrust (power) loadings. This result is valid for both propeller and jet aircraft.

To make eq. 5-89 readily applicable for preliminary design, some further simplifications can be introduced.

1. On the basis of several realistic assumptions regarding undercarriage drag, ground effect, etc., it was found (cf. Ref. 5-27) that the following approximation can be made:

$$\bar{\gamma} = .06 + \Delta\gamma_2 \qquad (5\text{-}90)$$

where $\Delta\gamma_2$ is the difference between the second segment climb gradient γ_2 and the minimum value of γ_2 permitted by the airworthiness regulations.

2. An average value of $\bar{a}_{stop} = .37g$ has been found from application of the method to 15 jet transports, although with optimum brake pressure control, lift dumpers and nosewheel braking, decelerations as high as .45g to .55g can be achieved on dry concrete. For very high decelerations the balancing condition may not be satisfied.

Using these simplifications, we find the following expression:

$$BFL = \frac{.863}{1+2.3\,\Delta\gamma_2}\left(\frac{W_{to}/S}{\rho\,g\,C_{L_2}} + h_{to}\right)\left\{\frac{1}{\bar{T}/W_{to}-\mu'} + 2.7\right\} + \frac{\Delta S_{to}}{\sqrt{\sigma}} \qquad (5\text{-}91)$$

where

h_{to} = 35 ft (10.7 m) and ΔS_{to} = 655 ft (200 m)

μ' = $.010\, C_{L_{max}} + .02$ for flaps in takeoff position

$C_{L_2} = C_L$ at V_2; normally $V_2 = 1.2\, V_S$, hence $C_{L_2} = .694 \times C_{L_{max}}$

$\bar{T}$ = mean thrust for the takeoff run, given by eqs. 5-81 and 5-82

$\Delta\gamma_2 = \gamma_2 - \gamma_{2_{min}}$

γ_2 is the second segment climb gradient calculated from eq. 5-51 at airfield altitude, one engine out (cf. example 3 of Section 5.4.3)

$\gamma_{2_{min}}$ = .024, .027 or .030 for N_e = 2,3 or 4 respectively.

For project design, the case of $\Delta\gamma_2 = 0$ presents most interest, as the corresponding weight is limited by the second segment climb requirement and the BFL is a maximum for the particular flap setting, disregarding the case of overspeed (Appendix K). Obviously, when $\Delta\gamma_2 = 0$ is substituted into eq. 5-91, the thrust/weight ratio must be chosen accordingly so that the thrust is sufficient to obtain $\gamma_2 = \gamma_{2_{min}}$.

For a given BFL, eq. 5-91 may be used to find the following limitation to the wing loading:

$$\frac{W_{to}}{S} \leq \rho\, g\, C_{L_2}\left\{\frac{1.159\,(BFL-\Delta S_{to}/\sqrt{\sigma})\,(1+2.3\,\Delta\gamma_2)}{(\bar{T}/W_{to}-\mu')^{-1}+2.7} - h_{to}\right\} \qquad (5\text{-}92)$$