CHAPTER 4: WING THEORY

The purpose of this chapter is to familiarize the reader with several key aspects of wing (or planform) theory and applications.

After a discussion of basic planform geometric parameters in Section 4.1, a discussion of circulation, downwash, lift and induced drag is given in Section 4.2.

From an airplane design viewpoint, span efficiency, aerodynamic center location, stall behavior, high speed characteristics and flaps are of prime importance. Sections 4.3 through 4.7 provide introductions to these subjects.

The material presented in this chapter is aimed at wings (or planforms). The reader will recognize that tail surfaces, canard surfaces as well as many other types of surfaces found in airplanes are also planforms. Therefore, the material in this chapter can be applied directly to such other surfaces.

4.1 DEFINITION OF WING PROPERTIES

Figure 4.1 shows a typical straight, tapered wing planform. The reader is encouraged to memorize the geometric properties shown in this figure.

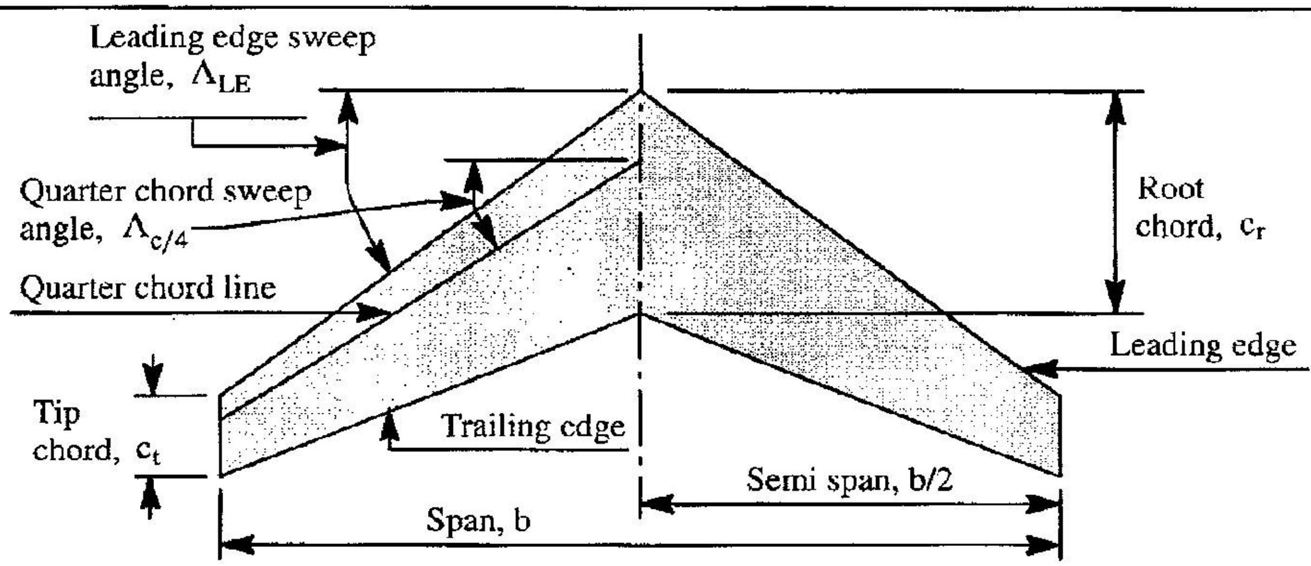

Figure 4.1 Example of a Straight, Tapered Wing Planform

The wing area, S is defined as the shaded area in Figure 4.1. In general, S is defined as the area of the wing planform, projected onto a plane of reference which is usually the wing root chord plane.

It is seen from Figure 4.1 that S may be determined from:

$$S = \frac{b}{2}(c_r + c_t) \tag{4.1}$$

In addition to wing area, other important parameters are the so–called wing aspect ratio, A and the taper ratio, λ, which are defined as:

$$A = \frac{b^2}{S} \tag{4.2}$$

and

$$\lambda = \frac{c_t}{c_r} \tag{4.3}$$

The wing sweep angle, Λ, is also of major importance. The sweep angle is normally measured either relative to the leading edge (Λ_{LE}) or relative to the quarter chord line ($\Lambda_{c/4}$).

To define lift and drag coefficients, the wing area, S, is required. To define a pitching moment coefficient it is necessary to use S in combination with a characteristic length. Normally, the so–called mean geometric chord (m.g.c.) of the wing is used for this characteristic length. The mean geometric chord of a wing is defined as:

$$\text{m.g.c.} = \bar{c} = \frac{2}{S}\int_0^{b/2} c^2 dy \tag{4.4}$$

The reader is asked to show, that for straight tapered wings the m.g.c. becomes:

$$\text{m.g.c.} = \frac{2}{3}c_r\left(\frac{\lambda^2 + \lambda + 1}{\lambda + 1}\right) \tag{4.5}$$

The simple geometric construction shown in Figure 4.2 can be used to quickly locate the m.g.c. for a straight, tapered wing.

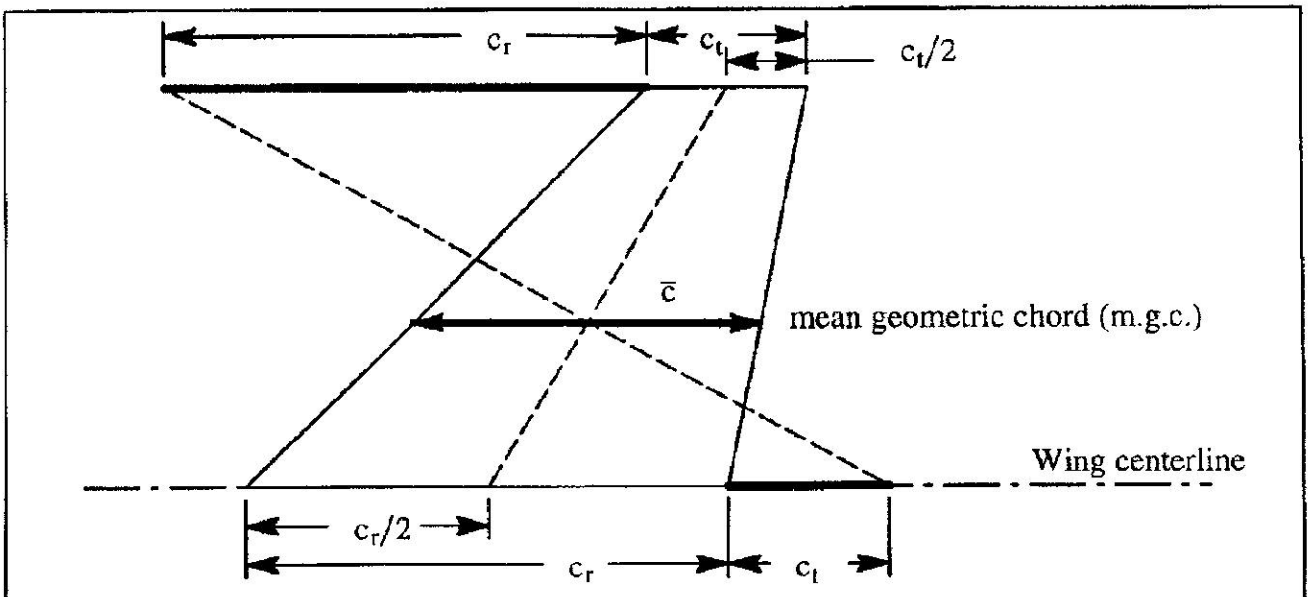

Figure 4.2 Example of a Geometric Construction of the Mean Geometric Chord (m.g.c.)

At supersonic speeds, C_{D_0} will increase with Mach Number because of the addition of wave drag. This sharp drag rise is also shown in Figure 4.9.

Ideally, if C_D of Eqn (4.35) is plotted versus C_L^2, a straight line should be obtained, with a slope of $1/\pi Ae$. This has been shown to be the case only for Reynolds numbers higher than about 5x106. Figure 4.10 backs this up with some test data.

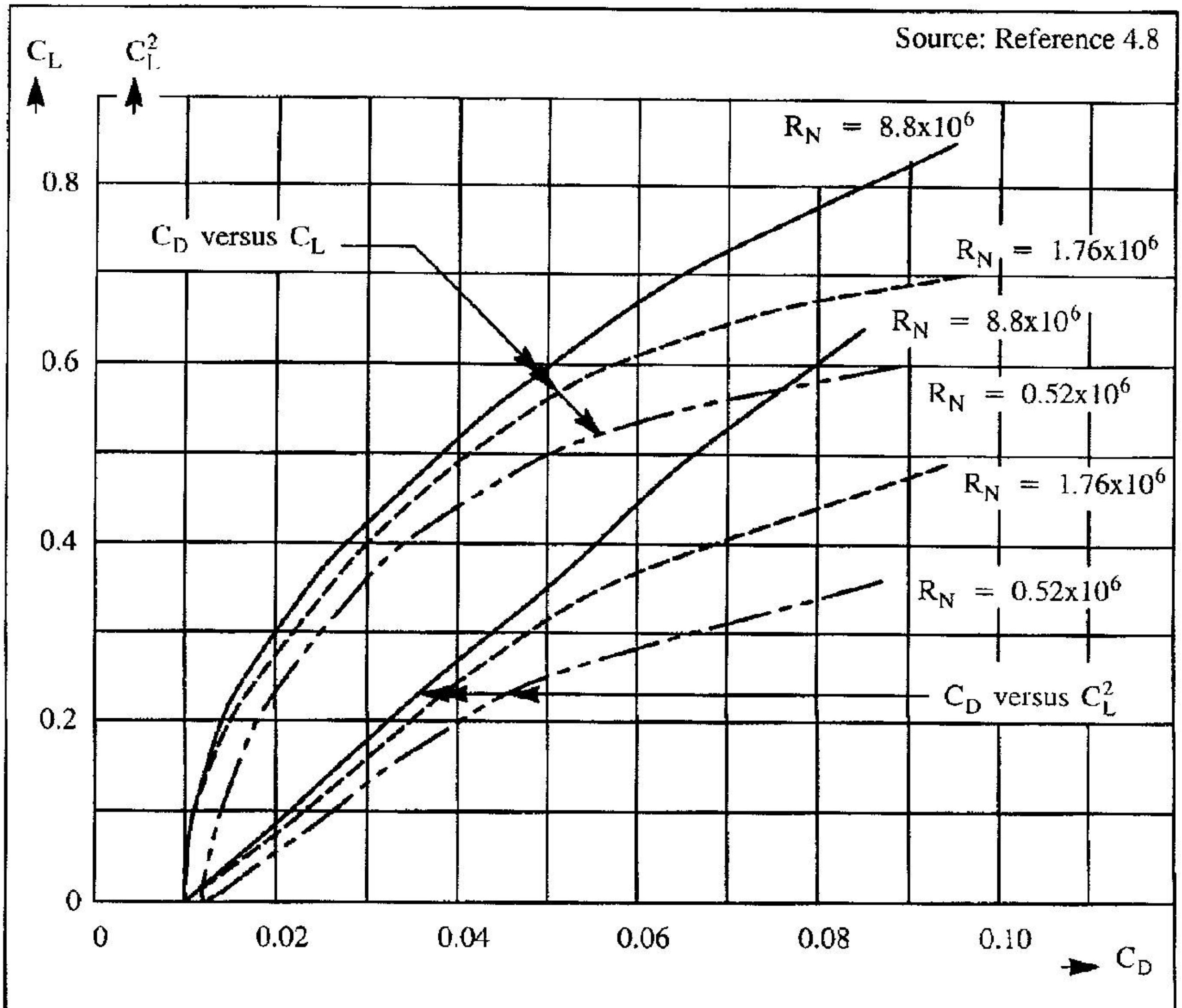

Figure 4.10 Example of the dependence of dC_D/dC_L^2 on Reynolds Number

Table 4.1 lists typical in flight Reynolds numbers associated with the wing, horizontal tail, vertical tail and fuselages of a number of airplanes.

Table 4.1 Example of Typical Flight Reynolds Numbers of Airplanes

		Reynolds Number			
		All Dimensions in Feet			
Airplane Type	Flight Condition	Wing	Hor. Tail	Vert. Tail	Fuselage
Cessna Stationair 7		$\bar{c}_w = 5.2$	$\bar{c}_h = 3.7$	$\bar{c}_v = 3.3$	$l_{fus} = 28.3$
	Max. Cruise Speed at 10,000 ft: $V_{CR} = 170$ mph	6.4×10^6	4.6×10^6	4.1×10^6	34.8×10^6
	Stall Speed (flaps down) at Sealevel: $V_S = 67$ mph	3.2×10^6	2.3×10^6	2.1×10^6	17.4×10^6
Learjet Model 36		$\bar{c}_w = 7.0$	$\bar{c}_h = 4.1$	$\bar{c}_v = 7.9$	$l_{fus} = 45.9$
	Max. Cruise Speed at 35,000 ft: $V_{CR} = 534$ mph	13.5×10^6	7.9×10^6	15.2×10^6	88.5×10^6
	Stall Speed (flaps down) at Sealevel: $V_S = 106$ mph	6.9×10^6	4.0×10^6	7.8×10^6	45.2×10^6
Boeing 727–200		$\bar{c}_w = 16.8$	$\bar{c}_h = 12.8$	$\bar{c}_v = 22.2$	$l_{fus} = 130.4$
	Max. Cruise Speed at 25,000 ft: $V_{CR} = 599$ mph	49.0×10^6	37.3×10^6	64.8×10^6	378.1×10^6
	Stall Speed (flaps down) at Sealevel: $V_S = 122$ mph	19.1×10^6	14.6×10^6	25.2×10^6	148.0×10^6
Boeing 747–200B		$\bar{c}_w = 30.8$	$\bar{c}_h = 24.6$	$\bar{c}_v = 27.0$	$l_{fus} = 229.0$
	Max. Cruise Speed at 30,000 ft: $V_{CR} = 608$ mph	77.8×10^6	62.1×10^6	68.3×10^6	578.0×10^6
	Stall Speed (flaps down) at Sealevel: $V_S = 116$ mph	32.8×10^6	26.2×10^6	28.8×10^6	244.0×10^6

4.4 AERODYNAMIC CENTER

The aerodynamic center (a.c.) of a wing is defined in the same way as that of an airfoil section (see Chapter 3). The aerodynamic center of a wing is defined as that point about which the variation of pitching moment coefficient, C_m is invariant with angle of attack, α. This definition is contrasted with that of the center of pressure (c.p.). The center of pressure is that point at which the pitching moment coefficient is zero. It follows, that the location of the c.p. varies with angle of attack.

To determine the a.c. from experimental data, assume that the moment center for the data is at a distance, x from the leading edge of the mean geometric chord of the wing: see Figure 4.11. Taking moments about the aerodynamic center it follows that:

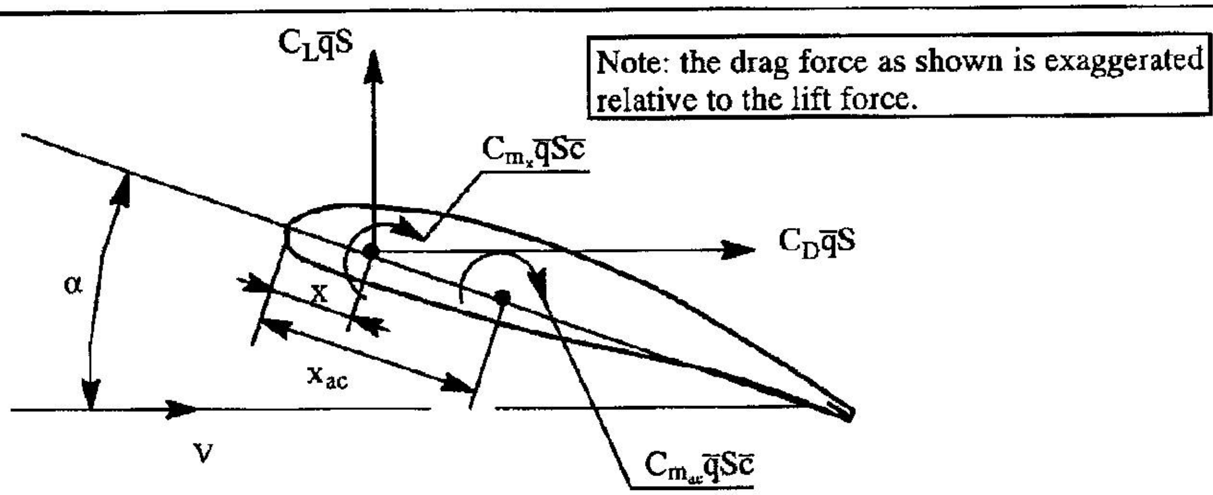

Figure 4.11 Geometry Used with Equations (4.39) and (4.40)

$$C_{m_{ac}}\bar{q}S\bar{c} = C_{m_x}\bar{q}S\bar{c} + C_L\bar{q}S(x_{ac} - x)\cos\alpha + C_D\bar{q}S(x_{ac} - x)\sin\alpha \tag{4.37}$$

Solving for x_{ac}, it is found that:

$$\frac{x_{ac}}{\bar{c}} = \frac{x}{\bar{c}} - \frac{C_{m_x} - C_{m_{ac}}}{C_L\cos\alpha + C_D\sin\alpha} \tag{4.38}$$

As long as the angle of attack is small, this can be written as:

$$\frac{x_{ac}}{\bar{c}} \approx \frac{x}{\bar{c}} - \frac{C_{m_x} - C_{m_{ac}}}{C_L} \tag{4.39}$$

At zero lift, the pressure distribution on the wing appears as a pure moment. Since a moment may be transferred to any location without changing its magnitude, and since $C_{m_{ac}}$ is independent of angle of attack, the zero lift pitching moment coefficient, $\overline{C}_{m_0}$ must be equal to the pitching moment coefficient about the aerodynamic center, $C_{m_{ac}}$. Therefore:

$$C_{m_{ac}} = \overline{C}_{m_0} \tag{4.40}$$

Figure 4.12 presents some experimental data showing typical a.c. locations for several wings. Empirical methods for determining a.c. locations and pitching moments of arbitrary wings may be found in References 4.9 and 4.10.

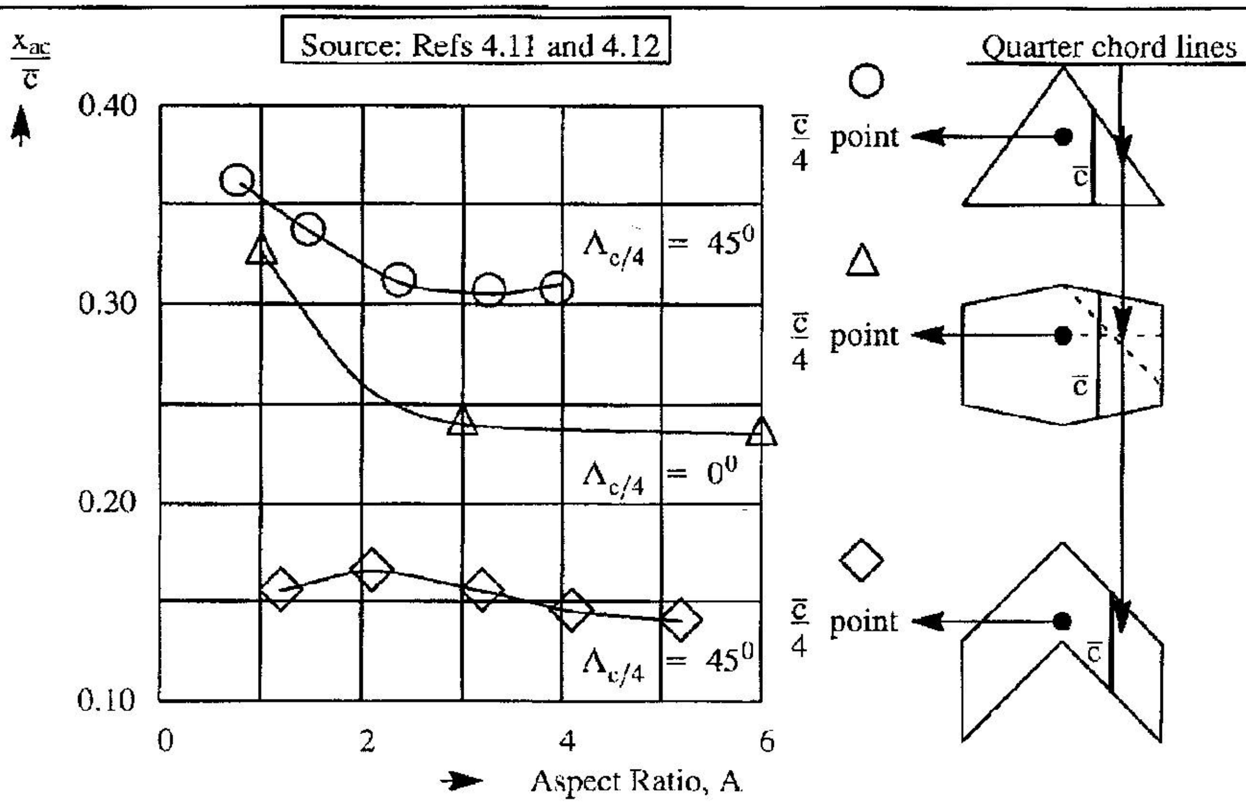

Figure 4.12 Examples of the Effect of Aspect Ratio and Sweep Angle on the Aerodynamic Center Location of Several Wings

4.5 WING STALL

Wing stall is caused by flow separation. How flow separation progresses in a chordwise and spanwise manner depends on the following items:

4.5.1 Airfoil stall characteristics

4.5.2 Planform geometry and twist

4.5.3 Stall control devices

The stall behavior of wings (or more general of lifting surfaces) is important for the following reasons:

1. In FAR* 23 airplanes, the stall speed at maximum weight may not be more than 61 knots (for W<6,000 lbs unless certain crash safety provisions have been incorporated into the design). Also, the bank angle must not exceed 15 degrees between the onset of the stall with the wings level and the completion of the recovery.

2. In FAR 25 airplanes, an airplane must not reach a bank angle of more than 20 degrees between the onset of the stall with the wings level and the completion of the recovery.

3. It is generally of great interest for either the performance of an airplane or for reasons of stability and control, to achieve the highest possible value for the maximum lift coefficient on a wing. This is true within certain constraints involving mission requirements and cost considerations.

For these three reasons, the stall characteristics of wings are of great interest to designers.

4.5.1 AIRFOIL STALL CHARACTERISTICS

Airfoil stall behavior and the factors which affect it were discussed in Chapter 3. The reader should realize, that sudden airfoil stall behavior does not necessarily imply sudden wing stall behavior. The effects of planform design can significantly modify any airfoil tendency to rapid stall. These planform effects are discussed in Sub–section 4.5.2.

4.5.2 EFFECT OF PLANFORM AND TWIST

The following planform effects are important in affecting the stall behavior of a wing (or lifting surface):

4.5.2.1	Taper ratio	4.5.2.3	Sweep angle
4.5.2.2	Aspect Ratio	4.5.2.4	Twist and camber

4.5.2.1 Taper Ratio

A wing with a rectangular planform (taper ratio of 1.0) has a larger downwash angle at the tip than at the root. Therefore, the effective angle of attack at the tip is reduced compared to that at the root. Therefore, the tip will tend to stall later than the root. However, as shown in Section 4.2 a rectangular wing planform is also aerodynamically inefficient. This is because the spanwise load distribution is far from elliptical, which is needed to minimize induced drag. To reduce induced drag a planform is tapered, to approximate the ideal, elliptical span load distribution. The result of taper is a smaller tip chord. That in turn results in a lower tip Reynolds number as well as a lower tip induced downwash angle. Both effects lower the angle of attack at which stall occurs and therefore the tip may stall before the root. This is undesirable from a viewpoint of lateral stability and lateral

* FAR: Federal Aviation Regulations

controllability as the stall is approached. To counteract these tendencies, twist is applied to many wings. Figure 4.13 illustrates the effect of wing taper on the spanwise load distribution. It is seen that decreasing the taper ratio will increase the loading at the tip, which in turn promotes tip stall. This problem can be solved with twist as shown in Sub–sub section 4.5.2.4.

Another problem with a rectangular wing is that it is also structurally inefficient: there is a lot of area outboard, which supports very little lift. Taper helps solve this problem as well.

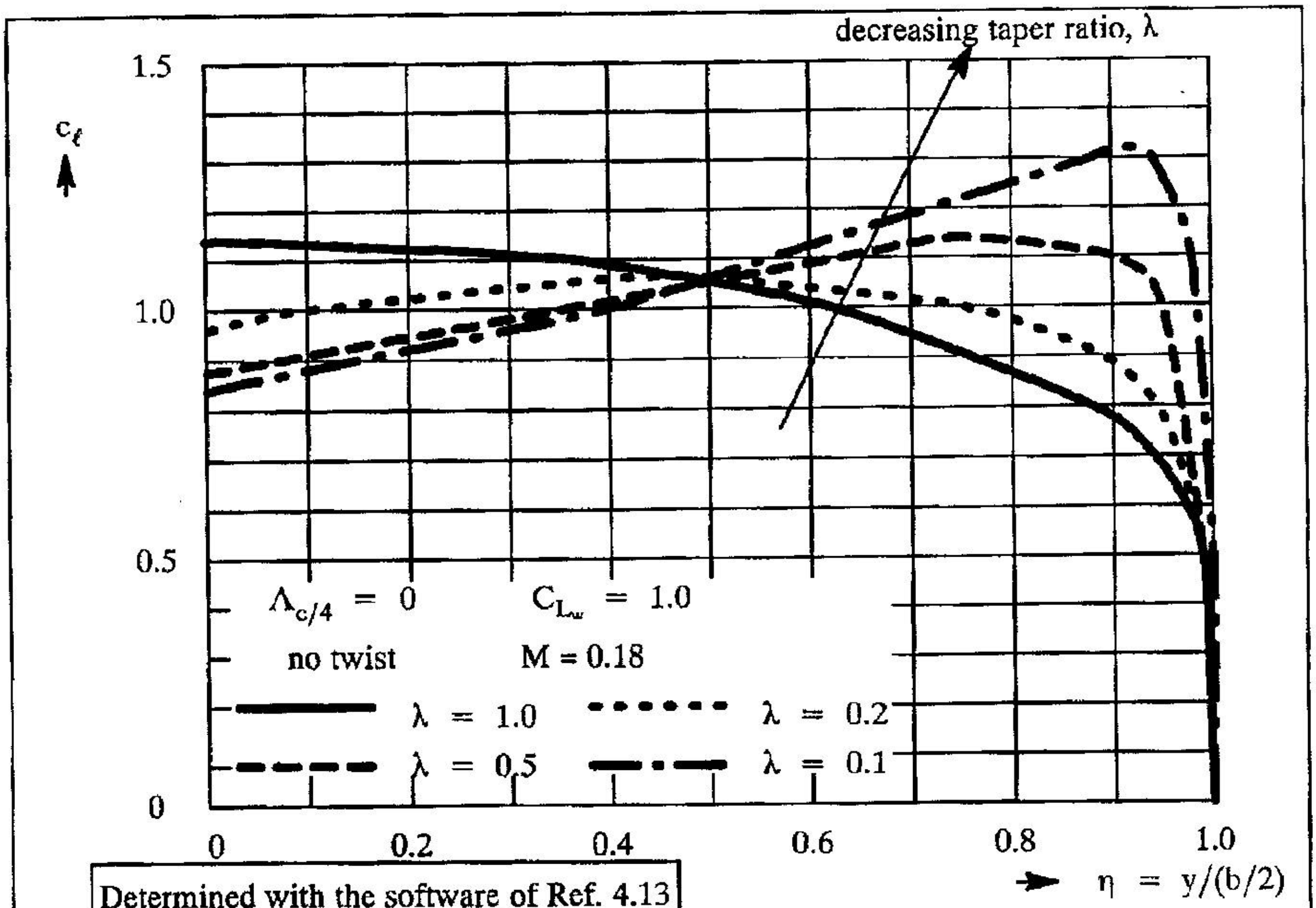

Figure 4.13 Lift Distribution for C_L = 1.0 for Unswept, Straight Tapered Wings of Varying Taper Ratio

4.5.2.2 Aspect Ratio

The following discussion applies to wings with very low sweep angles. As the wing aspect ratio increases, the wing behaves more and more like an airfoil. That is, its flow characteristics are more and more 2–dimensional. An exception is always the region at the wing tip. Therefore, it can be expected that the maximum lift coefficient, $C_{L_{max}}$ will increase with increasing aspect ratio, up to a number corresponding to section maximum lift coefficient, $c_{l_{max}}$. The increase is slight as can be seen from the experimental data in Figure 4.14 (Ref. 4.14, p. 16–3) for sweep angles around zero: see the shaded box.

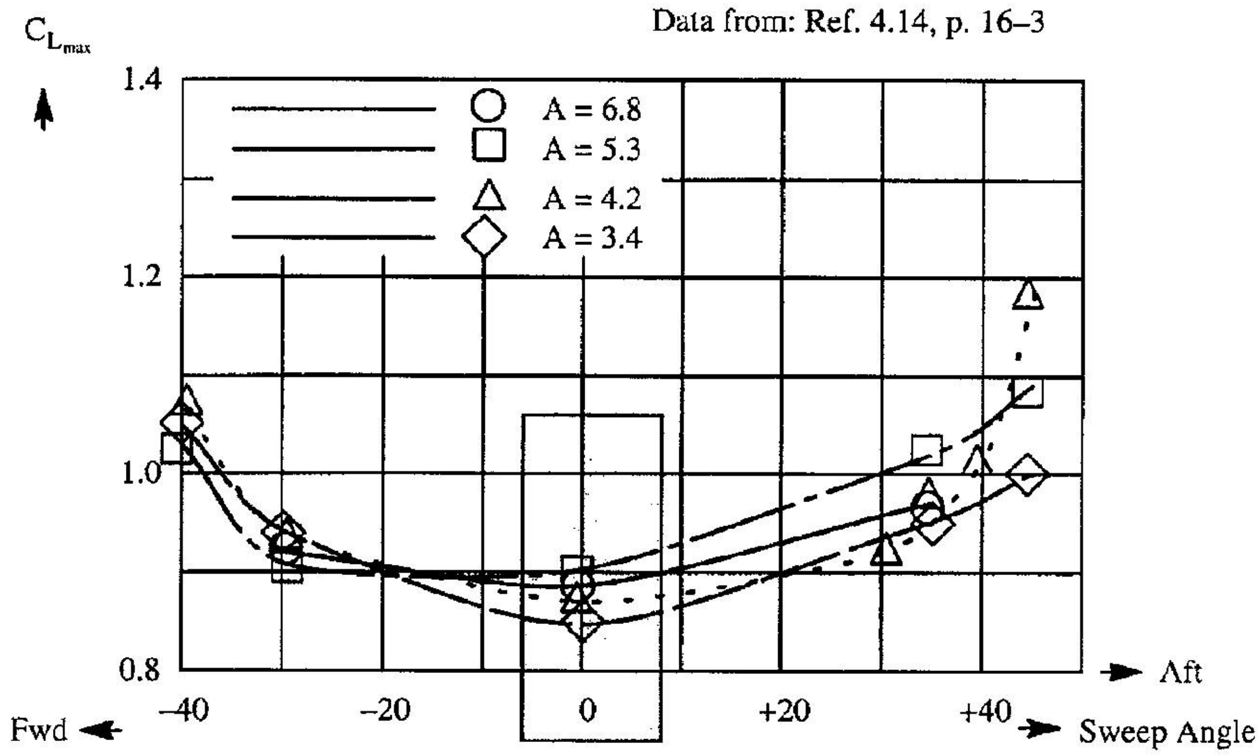

Figure 4.14 Effect of Sweep Angle on Maximum Lift Coefficient

4.5.2.3 Sweep Angle

On most aft swept wing airplanes the wing tips are located behind the center of gravity. Therefore, any loss of lift at the wing tips causes the center of pressure to move forward. This in turn will cause the airplane nose to come up. This pitch–up tendency can cause the angle of attack of the airplane to increase even further. That can result in a loss of control. The reader is asked to visualize why a forward swept wing airplane would exhibit a pitch–down tendency in a similar situation.

In addition, an aft swept wing will tend to have tip stall because of the tendency toward outboard, spanwise flow, causing the boundary layer to thicken as it approaches the tips. A swept forward wing, for the same reason, would tend toward root stall.

The maximum lift coefficient (defined as the maximum value reached by the lift coefficient as angle of attack is increased) can actually increase with increasing sweep angle. This is shown in Figure 4.14. However, the accompanying variation of pitching moment with angle of attack can lead to serious pitch control difficulties because of the tendency toward pitch–up as shown in the insert of Figure 4.15. Whether or not pitch–up occurs depends not only on the combination of aspect ratio and sweep angle, but also on airfoil type, twist and taper ratio. Figure 4.15 shows a boundary between stable and unstable pitching moment behavior as aspect ratio and sweep angle are varied. For these reasons a range of useful lift coefficients is defined as that range within which control problems are "manageable". Under this rather loose definition it can be shown that the maximum, useful lift coefficient actually decreases with increasing sweep angle. An example is shown in Figure 4.16 which is based on Reference 4.14, page 16–6.

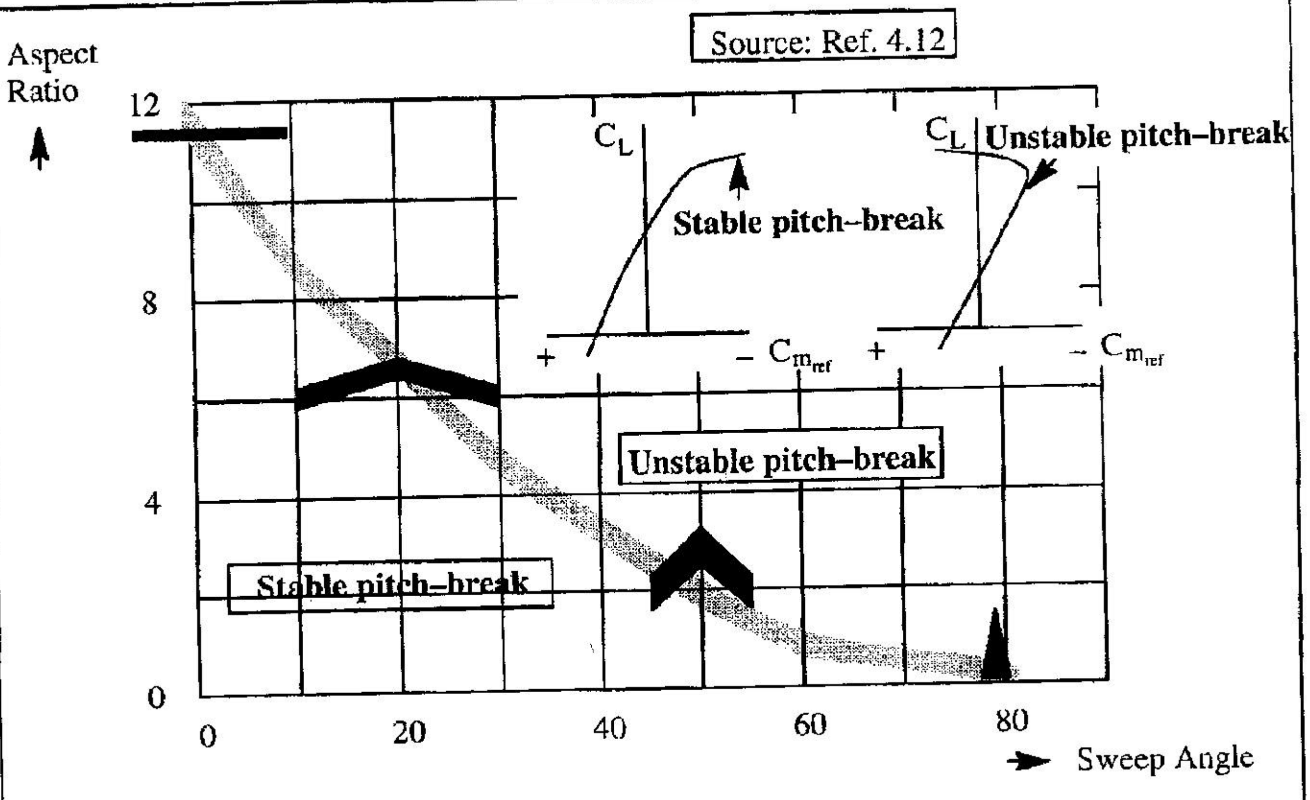

Figure 4.15 Effect of Sweep Angle and Aspect Ratio on Stable and Unstable Pitch Breaks

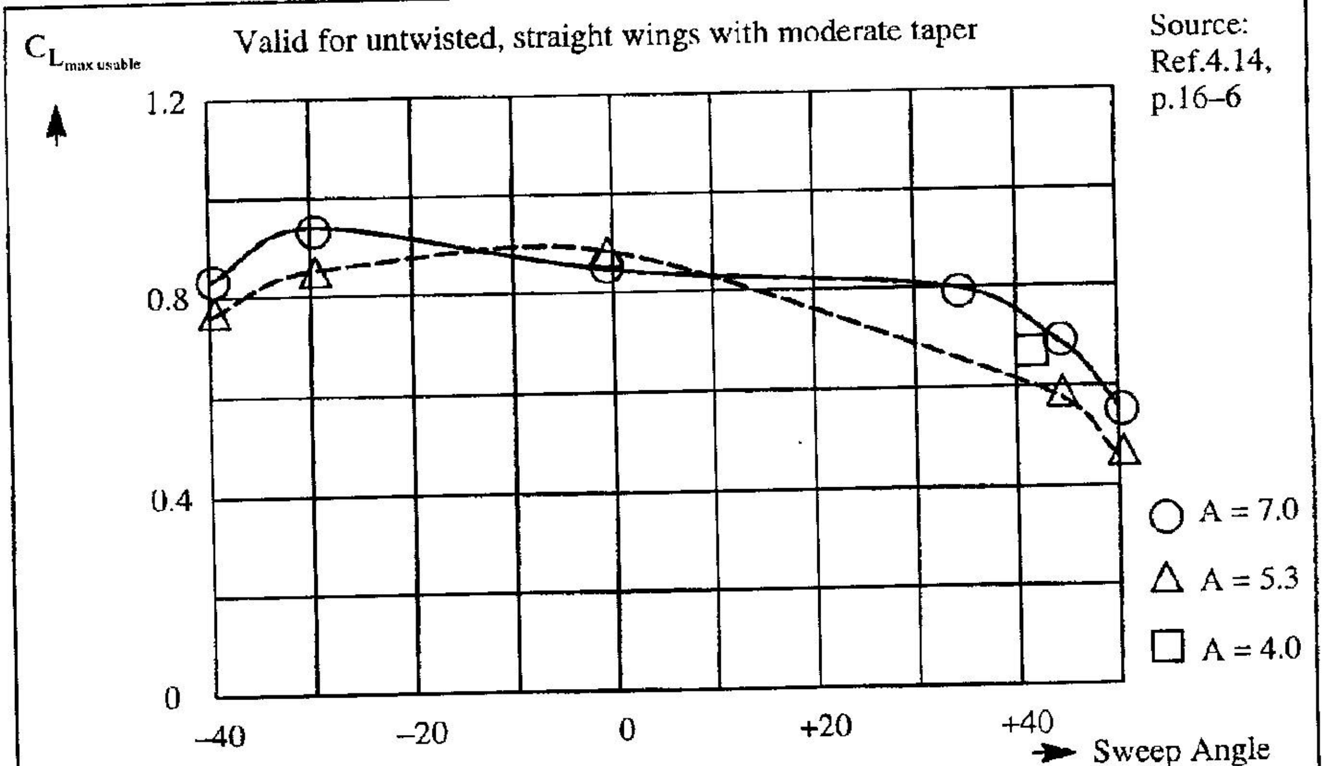

Figure 4.16 Effect of Sweep Angle on Maximum Lift Coefficient

The trend is for the useful wing $C_{L_{max}}$ to decrease with increasing aft sweep in the moderate sweep angle range of +/−25 degrees. Initially, this decrease follows the cosine–rule of Ref. 4.15, page 339):

$$C_{L_{max}}(\Lambda) = \left\{C_{L_{max}}(\Lambda = 0)\right\}\cos\Lambda \tag{4.41}$$

For higher sweep angles the maximum lift coefficient falls off rapidly with increasing sweep angles (fore and aft!).

4.5.2.4 Twist

If the angles of attack of spanwise sections of a wing are not equal, the wing is said to have twist. If the angle of attack at the tip is less than that at the root the wing is said to have wash–out or negative twist. With wash–out the wing tip will be at a lower angle of attack than the root thus delaying tip stall. Figure 4.17 illustrates how twist influences the spanwise load distribution. Note that the load is concentrated further inboard with wash–out (negative twist).

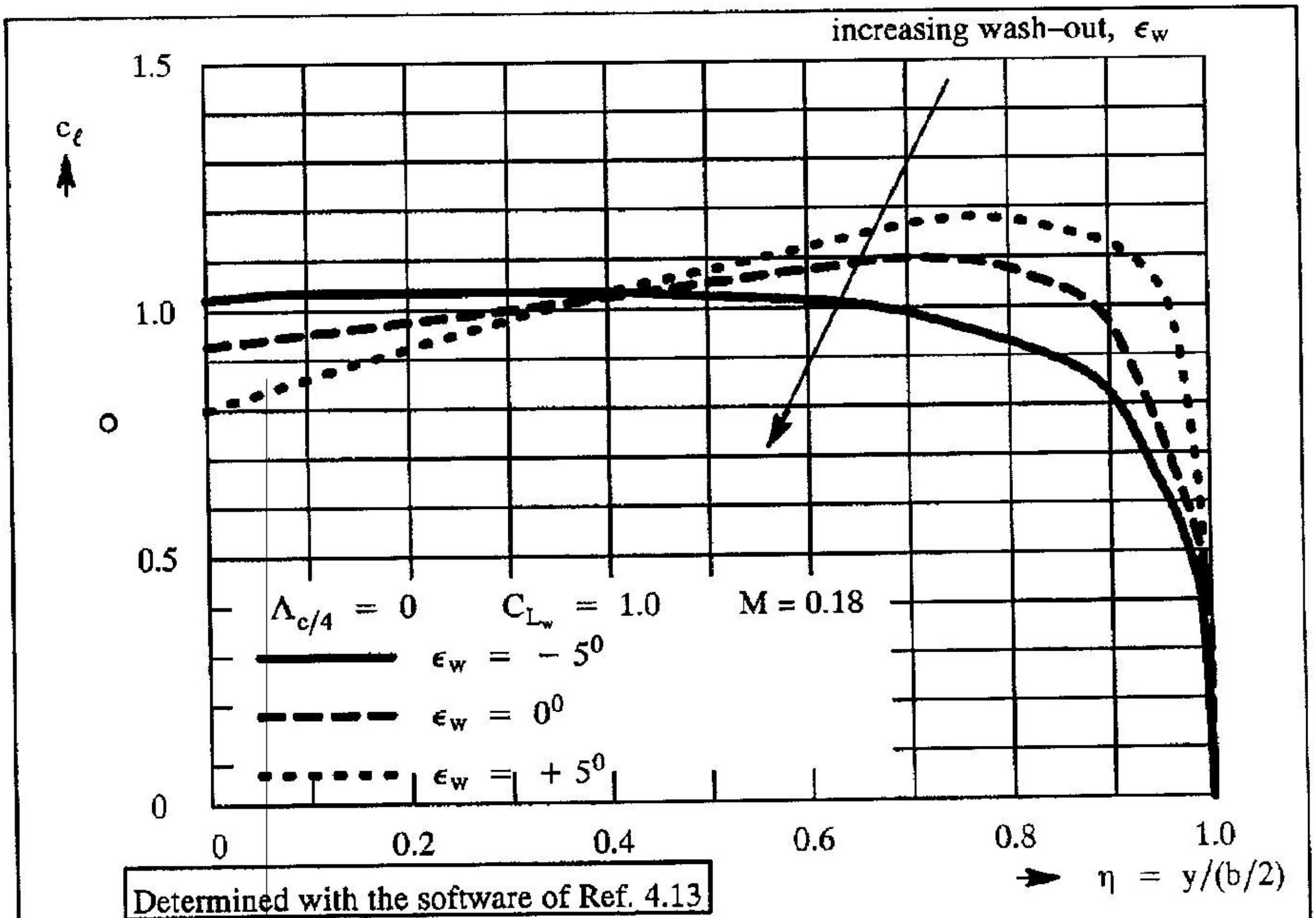

Figure 4.17 Lift Distribution for C_L = 1.0 for Unswept, Straight Tapered Wings with Three Twist Angles

4.5.3 STALL CONTROL DEVICES

In the following a number of devices for delaying tip stall are enumerated.

4.5.3.1 Twist or Wash–out

The effectiveness of washout in reducing tip stall was discussed in Sub–sub–section 4.5.2.4. Examples of the numerical magnitude of twist (or wash–out) on several airplanes are provided in Table 4.2.

Table 4.2 Examples of Washout in Several Airplanes

Airplane Type	Wing Incidence Angle in Degrees At Root	At Tip	Twist or Wash–out in degrees
Cessna Stationair 6	+ 1.5	– 1.5	3.0
Cessna 310	+ 2.5	– 0.5	3.0
Cessna Titan	+ 2.0	– 1.0	3.0
Cessna Citation I	+ 2.5	– 0.5	3.0
Beechcraft T–34C	+4.0	+1.0	3.0
Beechcraft 55 Baron	+4.0	0.0	4.0
Beechcraft Queenair	+3.9	0.0	3.9
Beechcraft Kingair	+4.8	0.0	4.8
Beechcraft T–1A Jayhawk	+3.0	–3.3	6.6
Gulfstream IV	+3.5	–2.0	5.5
Northrop–Grumman E–2C Hawkeye	+4.0	+1.0	3.0
Piper PA–28–161 Warrior	+2.0	–1.0	3.0
Piper Cheyenne	+1.5	–1.0	2.5
Piper Tomahawk	+2.0	0.0	2.0
Fokker F–50	+3.5	+1.5	2.0

4.5.3.2 Variations in Section Shape

Many airplanes have wings with spanwise varying airfoil sections. A frequently used feature which accomplishes the same as twist is to change camber in the spanwise direction. This is sometimes referred to as aerodynamic twist.