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USING INFORMATION THEORY CONCEPTS TO COMPARE ALTERNATIVE INTENSITY MEASURES FOR REPRESENTING GROUND MOTION UNCERTAINTY

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ABSTRACT

The seismic risk assessment of a structure in performance-based design may be significantly affected by the representation of ground motion uncertainty. The uncertainty in the ground motion is commonly represented by adopting a parameter or a vector of parameters known as the intensity measure (IM). In this work, a new measure, called a *sufficiency measure,* is derived based on information theory concepts, to quantify the suitability of one IM relative to another in representing ground motion uncertainty. Based on this measure, alternative IM's can be compared in terms of the expected difference in information they provide about a designated structural response parameter. Several scalar IM's are compared in terms of the amount of information they provide about the seismic response of an existing reinforced-concrete frame structure.

Introduction

Ground motion representation is a major source of uncertainty in performance-based assessments. A rigorous method for representing ground motion uncertainty consists of building a probabilistic model for the entire ground motion time-history. However, it is common to represent the uncertainty in the ground motion with a probabilistic model for a parameter or a vector of parameters related to the ground motion and known as the intensity measure (IM). One then faces the question of how suitable the adopted IM is for representing ground motion uncertainty. Since performance assessment is the main objective for ground motion modeling, it is logical that the criteria proposed for measuring the suitability of the IM be expressed in terms of the response quantities involved in the performance objectives.

Luco and Cornell (2005) have proposed *sufficiency* as one of the criteria for measuring

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the suitability of an IM in representing the dominant features of ground shaking. A *sufficient* IM has been defined as one that renders the structural response conditional on this IM to be independent of other influential ground motion parameters. Establishing sufficiency in an absolute sense is highly non-trivial since it involves independence, conditional on IM, of a designated structural response parameter from influential ground motion parameters for all possible values of the IM and the other ground motion parameters. It seems more feasible to establish sufficiency in a relative sense; i.e., to investigate whether one IM is more sufficient than other candidate IM's.

Information theory concepts can be employed in order to measure the suitability of one IM relative to another in representing ground motion uncertainty. The *entropy* of an uncertain variable is a measure of the amount of uncertainty in the value of the variable (Cover and Thomas, 1991); more specifically, it is measure of the missing information that is required (on average) to specify the value of the uncertain variable. In this work, the application of entropy and the related concept of *relative entropy*, gives a derivation of a simple analytical measure, called here the *sufficiency measure,* for comparing the suitability of alternative IM's. This measure states (on average) how much more information about the designated structural response parameter one IM gives relative to another IM.

This paper presents a preliminary case-study using the sufficiency measure to compare the suitability of alternative IM's for predicting the maximum inter-story drift ratio response in an existing reinforced-concrete moment-resisting frame located in Los Angeles. The candidate IM's include PGA (peak ground acceleration); $S_a(T_1)$ (spectral acceleration at the small-amplitude fundamental period); $IM_{2E,1I}$ (a structure-specific intensity measure proposed by Luco and Cornell (2005) which takes into account the effect of the first two modes of vibration and the effect of inelastic response); and S_a^{*} (a two-parameter intensity measure proposed by Cordova et al. (2000) which combines spectral response at the fundamental period and another (usually larger) period in order to take into account the inelastic behavior). The comparison results based on calculating the sufficiency measures agree well with the previous conclusions of others based on Luco and Cornell's sufficiency criterion.

Methodology

This section outlines the theoretical basis for comparing the candidate IM's in terms of their prediction of the designated structural response parameter, $\theta_{\max}$, the maximum inter-story drift ratio. Detailed discussion of the methodology and derivations will be presented elsewhere by the authors.

Sufficiency of an Intensity Measure in an Absolute Sense

Luco and Cornell (2005) have introduced *sufficiency* as one of the criteria for measuring the suitability of an intensity measure. Here, a variation of their original definition is given: For a designated structural response parameter, such as $\theta_{\max}$, the intensity measure IM is *absolutely sufficient* if and only if:

$$p(\theta_{\max} | IM, \underline{\ddot{x}}_g) = p(\theta_{\max} | IM) \tag{1}$$

for all ground motion (acceleration) time-histories, $\underline{\ddot{x}}_g$, expected to happen at the site. Sufficiency in this absolute sense is an extremely strong condition for an intensity measure; it is unlikely that any scalar IM satisfies this condition. Qualitatively speaking, it means that even knowing the entire ground motion time-history $\underline{\ddot{x}}_g$ does not provide any more relevant information about $\theta_{\max}$ than is already provided by the sufficient ground-motion IM. If the structural modeling uncertainty is assumed to be negligible, an absolutely sufficient IM would fully determine the structural response parameter $\theta_{\max}$.

Relative Entropy

The *relative entropy*, $D(X | I | J)$ for a continuous uncertain variable X for given information J relative to given information I, is defined as (Cover and Thomas 1991):

$$D(X | I | J) = \int p(x | I, J) \cdot \log \frac{p(x | I, J)}{p(x | I)} \cdot dx \tag{2}$$

It is also known as the *Kullback-Leibler distance* between probability density functions $p(x | I)$ and $p(x | I, J)$. Qualitatively speaking, relative entropy is a measure of the information gained about an uncertain variable X on average by gaining information J in addition to information I. It can be shown that $D(X | I | J)$ is non-negative and it is zero if and only if $p(x | I, J) = p(x | I)$, i.e. J provides no additional information relative to I (Cover and Thomas 1991).

The concept of relative entropy in Equation 2 can be applied to measure the average information gained about maximum inter-story drift $\theta_{\max}$ when the available information about the ground motion is increased from knowing only the intensity measure IM to knowing the entire ground motion time-history $\underline{\ddot{x}}_g$:

$$D(\theta_{\max} | IM | \underline{\ddot{x}}_g) = \int p(\theta_{\max} | IM, \underline{\ddot{x}}_g) \cdot \log \frac{p(\theta_{\max} | IM, \underline{\ddot{x}}_g)}{p(\theta_{\max} | IM)} \cdot d\theta_{\max} \tag{3}$$

Since the relative entropy $D(\theta_{\max} | IM | \underline{\ddot{x}}_g)$ is zero if and only if $p(\theta_{\max} | IM, \underline{\ddot{x}}_g) = p(\theta_{\max} | IM)$, the adopted IM is *absolutely sufficient* if and only if the relative entropy $D(\theta_{\max} | IM | \underline{\ddot{x}}_g)$ is zero. It should also be noted that the relative entropy concept provides a quantified measure of the sufficiency of an IM but in the sense that the farther away it is from zero, the less sufficient is the IM.

Sufficiency Measure

The relative sufficiency of alternative IM's can be measured by comparing the difference between their corresponding relative entropies as given in Equation 3. First, note that the difference between relative entropies corresponding to IM_1 and IM_2 may be expressed as:

$$\begin{aligned} &D(\theta_{\max} \mid IM_1 \mid \underline{\ddot{x}}_g) - D(\theta_{\max} \mid IM_2 \mid \underline{\ddot{x}}_g) \\ &= \int p(\theta_{\max} \mid \underline{\ddot{x}}_g) \log \frac{p(\theta_{\max} \mid IM_2)}{p(\theta_{\max} \mid IM_1)} d\theta_{\max} \end{aligned} \tag{4}$$

The difference between relative entropies is a functional of the ground-motion time history $\underline{\ddot{x}}_g$. Its expected value over all the ground motions that could happen at the site is referred to here as the *sufficiency measure* of $\theta_{\max}$ for IM_2 relative to IM_1:

$$I(\theta_{\max} \mid IM_2 \mid IM_1) = \int \log \frac{p(\theta_{\max}(\underline{\ddot{x}}_g) \mid IM_2(\underline{\ddot{x}}_g))}{p(\theta_{\max}(\underline{\ddot{x}}_g) \mid IM_1(\underline{\ddot{x}}_g))} \cdot p(\underline{\ddot{x}}_g) \cdot d\underline{\ddot{x}}_g \tag{5}$$

where $p(\underline{\ddot{x}}_g)$ is the PDF for the ground-motion time history. In deriving Equation 5, structural modeling uncertainty is ignored so that given $\underline{\ddot{x}}_g$, $\theta_{\max}$ is known. The sufficiency measure $I(\theta_{\max} \mid IM_2 \mid IM_1)$ can be interpreted as a measure of how much information on average is gained about the uncertain structural response parameter $\theta_{\max}$ by knowing IM_2 instead of IM_1. If the logarithm is calculated in base two, the sufficiency measure is expressed in terms of *bits* of information.

If the sufficiency measure is zero, this means that on average the two IM's provide the same amount of information about $\theta_{\max}$. In other words, they are "equally sufficient". If the sufficiency measure is positive, this means that on average IM_2 provides more information than IM_1 about $\theta_{\max}$, so IM_2 is "more sufficient" than IM_1. Similarly, if the sufficiency measure is negative, IM_2 provides on average less information than IM_1 and so IM_2 is "less sufficient" than IM_1.

Calculation of the sufficiency measure

In order to calculate the sufficiency measure, $p(\theta_{\max} \mid IM)$ is needed, so one has to choose probability models for the structural response given each candidate IM. Strictly speaking, then, the sufficiency measure is conditional on these probability models, in addition to being conditional on the chosen structural model.

In this study, the probability model is selected by first choosing a set of real ground motion records. The structural response for each of these ground motion records is obtained by performing non-linear dynamic analyses. Taking $p(\theta_{\max} | IM)$ as a Log Normal probability density function, the two parameters (mean and standard deviation) of each distribution can be estimated using simple linear regression of structural response versus the corresponding IM (Luco and Cornell 1998, Jalayer and Cornell 2005).

The second step in evaluating the sufficiency measure is to calculate the expectation in Equation 5 over the possible ground motions at the site. A simple approximate way to do this is to replace the expectation by an average over a selected set of ground motion records. However, the resulting average may not be a good estimate of the expected value in Equation 5, which strictly should take into account all the ground motions possible at the site, weighted by how likely each one is. It is shown in the example that this can be done using de-aggregation of the seismic hazard at the site, along with a stochastic ground motion model.

Numerical Example

The methodology described in the previous section is applied to an existing reinforced-concrete frame in order to compare the suitability of candidate intensity measures by calculating their pair-wise sufficiency measures.

Model Structure: Transverse Frame of an Existing Building

One of the transverse frames in a seven-story hotel structure located in Los Angeles is selected as the structural model. This building is an older reinforced–concrete structure that has suffered shear failures in its columns during the 1994 Northridge Earthquake. The three-bay frame is modeled using DRAIN2D-UW, which is a modified version of DRAIN2D produced at the University of Wisconsin (Pincheira et al. 1999). The structural model takes into account stiffness and strength degrading behavior in the non-linear range for both flexure and shear (Jalayer 2003). The small-amplitude natural frequencies of the first two modes are computed to be 1.25 Hz and 3.66 Hz, respectively. Mass-proportional damping is assumed and it is equal to 2% of critical damping in the fundamental mode of vibration.

The Intensity Measures

Four alternative scalar intensity measures are compared in this study. One of the most commonly used IM's is the peak of the ground motion acceleration time-history. PGA is generally perceived to be insufficient as an IM for predicting the structural response of mid- to high-rise moment-resisting frames.

Another widely used IM is the spectral acceleration at the small-amplitude fundamental period T_1 of the structure, often denoted by $S_a(T_1)$, but more briefly referred to as the spectral acceleration S_a. Unlike the PGA which is only a characteristic of the ground motion, $S_a(T_1)$ also takes into account the ground-motion frequency content around the structure's first-mode period. Shome and Cornell (1998) have demonstrated that $S_a(T_1)$ is effectively sufficient for predicting the structural response for moment-resisting frames of low to moderate fundamental period. Currently, this is the most widespread IM in use in seismic risk analyses.

Luco and Cornell (2005) have proposed a structure-specific intensity measure denoted by $IM_{1I,2E}$ that takes into account not only the ground-motion frequency content around the first two modal periods but also inelastic structural behavior to some extent. $IM_{1I,2E}$ can be calculated as:

$$IM_{1I,2E} = \frac{S_d^I(T_1,\xi_1,d_y)}{S_d(T_1,\xi_1)} \cdot \sqrt{[PF_1 \cdot S_d(T_1,\xi_1)]^2 + [PF_2 \cdot S_d(T_2,\xi_2)]^2} \tag{6}$$

where PF_1 and PF_2 are modal participation factors for the first two modes of vibration, $S_d(T_1,\xi_1)$ and $S_d(T_2,\xi_2)$ are the spectral displacements with periods T_1 and T_2 and damping ratios ξ_1 and ξ_2 corresponding to the first two modes, and $S_d^I(T_1,\xi_1,d_y)$ is the spectral displacement of an elastic-perfectly plastic oscillator with period T_1, damping ratio ξ_1 and yield displacement d_y. Luco and Cornell (2005) have demonstrated that $IM_{1I,2E}$ is relatively more sufficient in predicting the structural response of moment-resisting frames than $S_a(T_1)$.

The final IM to be considered is S_a^*, which is a two-parameter IM proposed by Cordova et al. (2000) which takes into account spectral shape information:

$$S_a{*}(T_1,T_f) = S_a(T_1) \cdot \left(\frac{S_a(T_f)}{S_a(T_1)} \right)^{\alpha} \tag{7}$$

where T_f is another period at which spectral response is calculated; it is usually a longer period in order to take into account the inelastic behavior in the structure. The optimal values (those which minimize variability in the structural response) for T_f and α are $2T_1$ and 0.5 (Cordova et al. 2000) and so these values are adopted here. It is expected that S_a^* is relatively more sufficient than $S_a(T_1)$ for predicting the inelastic structural response.

Set of Ground Motion Records

The non-linear dynamic analyzes are performed on a suite of 30 real ground-motion records that are selected from a ground motion database (see PEER 2005). The records are on stiff soil from a magnitude range of $6.5 \le M \le 7$ and source-to-site distances of $15 \le r \le 30$ km (Table 1). For each ground motion, the structural response $\theta_{\max}$, as well as the values of the four candidate IM's are also shown in Table 1.

Probability Model Parameters for $\theta_{\max}$ given the IM's

A non-linear dynamic procedure referred to as the *Cloud Method* by Jalayer and Cornell (2005) has been employed in order to calculate the parameters of the lognormal PDF, $p(\theta_{\max} | IM)$. The cloud method consists of first applying a suite of ground motion records to the structure and calculating the structural response $\theta_{\max}$. The parameters for the lognormal distribution can then be estimated by performing a simple linear regression (in a logarithmic scale) on $\theta_{\max}$ versus the candidate IM. More specifically, the expected value of $\theta_{\max}$ given IM is estimated by the regression prediction $a \cdot IM^{b}$ and the standard deviation of $\theta_{\max}$ given IM is estimated by the standard error s of the regression. The estimated parameters a, b and s, calculated in this way for each IM, are tabulated in Table 2.

Calculation of the Sufficiency Measures

The reference IM is taken to be S_a and the sufficiency measure for the other three IMs relative to S_a is first estimated by simply replacing the expectation in Equation 5 by an average over the set of ground motion records listed in Table 1. The sufficiency measures estimated in this way, $I(\theta_{\max} | PGA | S_a)$, $I(\theta_{\max} | IM_{1I,2E} | S_a)$, $I(\theta_{\max} | S_a^* | S_a)$, are presented in the second column of Table 3. The results can be interpreted as follows: a) PGA gives (on average) 0.73 bits of information less about the structural response $\theta_{\max}$ than S_a; b) $IM_{1I,2E}$ gives (on average) 0.06 bits of information more about $\theta_{\max}$ than S_a: and c) S_a^* gives (on average) 0.17 bits of information more about $\theta_{\max}$ than S_a. This ranks PGA as the least sufficient and S_a^* as the most sufficient of the IM's. However, since the suite of ground motion records in Table 1 is not a random sample drawn from an appropriate PDF for the ground motion at the site, these estimates of the sufficiency measures may be too crude.

Refined Calculation of the Sufficiency Measures

The expectation in the definition of the sufficiency measure in Equation 5 should be calculated over the range of all possible ground motions at the site. This can be achieved by expanding the right-hand side of Equation 5 with respect to source-to-site distance r and moment-magnitude M using the total probability theorem (Benjamin and Cornell 1970):

$$I(\theta_{\max} | IM_2 | IM_1) = \int \log \frac{p(\theta_{\max}(\underline{\ddot{x}}_g) | IM_2(\underline{\ddot{x}}_g))}{p(\theta_{\max}(\underline{\ddot{x}}_g) | IM_1(\underline{\ddot{x}}_g))} \cdot p(\underline{\ddot{x}}_g | M, r) \cdot p(M, r) \cdot dM dr d\underline{\ddot{x}}_g \qquad (8)$$

The integration in Equation 8 can be carried out using a standard Monte Carlo simulation scheme. This paper employs the de-aggregation of seismic hazard (McGuire 1995, Bazzurro and Cornell 1998) at different levels of ground motion intensity in order to obtain a joint probability distribution $p(M, r)$ for magnitude and distance (Jalayer and Beck 2005). The stochastic ground motion model proposed by Atkinson and Silva (2000) is used to obtain the PDF $p(\underline{\ddot{x}}_g | M, r)$ for the ground motion time history given M and r.

The simulation has been carried out using 2000 analyses and the resulting values for the sufficiency measures are presented in the third column of Table 3. These calculated values again rank PGA as the least sufficient but now rank $IM_{1I,2E}$ as the most sufficient, which is different from the conclusion drawn before by taking the average over the suite of ground motion records. However, the estimates here are more defensible than those calculated previously using the simple average over the set of recorded ground motions. It is reasonable that $IM_{1I,2E}$ is more sufficient than S_a^* because it not only takes into account frequency content relevant to the first two modes of vibration but also inelastic response to some degree; whereas S_a^* only takes into account ground motion frequency content at T_1 and at $2T_1$. It is surprising, though, that S_a^* shows no gain in information (on average) about $\theta_{\max}$ relative to S_a.

Conclusions

In the context of performance-based earthquake engineering, the uncertainty in the ground motion is commonly represented by parameters or vector of parameters known as the intensity measure. A measure of the relative suitability of alternative intensity measures for representing ground motion uncertainty can be derived based on information theory concepts. This measure is referred to here as the *sufficiency measure* and it reflects the amount of information gained about a designated structural response parameter by adopting one intensity measure instead of another.

The amount of information that four alternative scalar IM's provide about maximum inter-story drift ratio are compared using a case-study structure. Non-linear dynamic analyses on a suite of ground motion records are employed to estimate the sufficiency measures of three candidate intensity measures, PGA, $IM_{1I,2E}$ and S_a^*, relative to S_a. Refined estimates are made using de-aggregation of the seismic hazard and a stochastic ground motion model and then estimating the expectation involved in the definition of the sufficiency measure through Monte Carlo simulation. Comparison of the refined estimates of the sufficiency measures for the candidate IM's agree with previous studies of these IM's, that is, PGA is inferior to S_a in predicting $\theta_{\max}$, whereas $IM_{1I,2E}$ and S_a^* are roughly comparable to S_a, with $IM_{1I,2E}$ being slightly better; however, the computational effort in computing $IM_{1I,2E}$ is significantly more than for S_a and this may negate its slight advantage with respect to sufficiency.

In future work, the sufficiency measures will be enhanced by obtaining more complete probabilistic models for structural response given IM's. Moreover, the presented methodology will be applied for measuring and comparing the amount of information that alternative vector IM's can provide about the structural response.

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Table 1. The suite of ground-motion records

ID	Earthquake	Station	Comp	M	R	PGA	Sa(T1=0.80)	Sa*	$IM_{1I,2e}$	θ_{max}
1	Loma Prieta 10/18/89	Agnews State Hospital	90	6.9	28.2	0.159	0.23	0.17944358	0.004	0.0039
2	Northridge 01/17/94	LA - Baldwin Hill	90	6.7	31.3	0.239	0.25	0.17320508	0.0046	0.0027
3	Imperial Valley 10/15/79	Compuertas	285	6.5	32.6	0.147	0.081	0.05620498	0.0019	0.0017
4	Imperial Valley 10/15/79	Plaster City	135	6.5	31.7	0.057	0.06	0.04959839	0.0013	0.0017
5	Loma Prieta 10/18/89	Hollister Diff Array	255	6.9	25.8	0.27	0.67	0.37509999	0.014	0.0089
6	San Fernando 02/09/71	LA - Hollywood Stor Lot	180	6.6	21.2	0.174	0.14	0.083666	0.0023	0.0031
7	Loma Prieta 10/18/89	Anderson Dam (Downst)	270	6.9	21.4	0.244	0.29	0.23099784	0.0053	0.005
8	Loma Prieta 10/18/89	Coyote Lake Dam (Dowr	285	6.9	22.3	0.179	0.29	0.22268812	0.0042	0.0044
9	Imperial Valley 10/15/79	El Centro Array #12	140	6.5	18.2	0.143	0.18	0.14635573	0.003	0.0035
10	Imperial Valley 10/15/79	Cucapah	85	6.5	23.6	0.309	0.4	0.28913665	0.0082	0.0093
11	Northridge 01/17/94	LA, Hollywood Stor FF	360	6.7	25.5	0.358	0.61	0.28376046	0.01	0.0104
12	Loma Prieta 10/18/89	Sunnyvale, Colton Ave	270	6.9	28.8	0.207	0.36	0.25171412	0.0062	0.0054
13	Loma Prieta 10/18/89	Anderson Dam (Downst)	360	6.9	21.4	0.24	0.31	0.18466185	0.0047	0.0036
14	Imperial Valley 10/15/79	Chihuahua	12	6.5	28.7	0.27	0.51	0.27519993	0.0058	0.006
15	Imperial Valley 10/15/79	El Centro Array #13	140	6.5	21.9	0.117	0.13	0.10936178	0.0025	0.0029
16	Imperial Valley 10/15/79	Westmorland Fire Statio	90	6.5	15.1	0.074	0.1	0.08602325	0.0017	0.0017
17	Loma Prieta 10/18/89	Hollister South and Pine	0	6.9	28.8	0.371	1.02	0.69238717	0.0164	0.0263
18	Loma Prieta 10/18/89	Sunnyvale, Colton Ave.	360	6.9	28.8	0.209	0.25	0.20856654	0.0042	0.0067
19	Superstition Hills(B) 11/24/87	Wildlife Liquefaction Arra	90	6.7	24.4	0.181	0.26	0.20396078	0.0044	0.0054
20	Imperial Valley 10/15/79	Chihuahua	282	6.5	28.7	0.254	0.63	0.34323461	0.0071	0.0088
21	Imperial Valley 10/15/79	El Centro Array #13	230	6.5	21.9	0.139	0.11	0.10302912	0.0023	0.0021
22	Imperial Valley 10/15/79	Westmorland Fire Statio	180	6.5	15.1	0.11	0.13	0.10387492	0.0026	0.0024
23	Loma Prieta 10/18/89	Halls Valley	90	6.9	31.6	0.103	0.22	0.14071247	0.0035	0.003
24	Loma Prieta 10/18/89	Waho	0	6.9	16.9	0.37	0.8	0.30172835	0.0068	0.0078
25	Superstition Hills 11/24/87	Wildlife Liquefaction Arra	360	6.7	24.4	0.207	0.53	0.38790076	0.0078	0.016
26	Imperial Valley 10/15/79	Compuertas	15	6.5	32.6	0.186	0.16	0.08944272	0.0031	0.0026
27	Imperial Valley 10/15/79	Plaster City	45	6.5	31.7	0.042	0.03	0.0244949	0.0007	0.0007
28	Loma Prieta 10/18/89	Hollister Diff Array	165	6.9	25.8	0.269	0.67	0.45205088	0.0095	0.0115
29	San Fernando 02/09/71	LA - Hollywood Stor Lot	90	6.6	21.2	0.21	0.3	0.21213203	0.0047	0.0046
30	Loma Prieta 10/18/89	Waho	90	6.9	16.9	0.638	0.72	0.38884444	0.0095	0.0105

Table 2. Regression parameters for the adopted IM's

IM	a	b	s
PGA	0.03	1.117	0.45
$S_a(T_1)$	0.015	0.9	0.27
$IM_{1I,2E}$	1.28	1.036	0.26
S_a^*	0.026	1.022	0.24

Table 3. Sufficiency measures for alternative IM's relative to S_a

Sufficiency Measure	Approximate	Refined
$I(\theta_{max} \mid PGA \mid S_a)$	-0.73	-1.83
$I(\theta_{max} \mid IM_{1I,2E} \mid S_a)$	0.06	0.17
$I(\theta_{max} \mid S_a^* \mid S_a)$	0.186	-0.06