



Tenth U.S. National Conference on Earthquake Engineering
Frontiers of Earthquake Engineering
July 21-25, 2014
Anchorage, Alaska

CLOUD ANALYSIS REVISITED: EFFICIENT FRAGILITY CALCULATION AND UNCERTAINTY PROPAGATION USING SIMPLE LINEAR REGRESSION

F. Jalayer¹, L. Elefante², R. De Risi², G. Manfredi³

ABSTRACT

One of the most efficient non-linear dynamic analysis procedures for analytic fragility evaluation, known also as the *Cloud Method*, is based on simple regression of structural response versus seismic intensity for a set of registered records. This work presents a Bayesian take on the cloud method for efficient fragility assessment, considering both record-to-record variability and other sources of uncertainty related to structural modeling. The starting point of this method lies in the assumption of a prescribed probability distribution such as the Lognormal probability distribution. In the first stage, the structural response to a limited set of ground motions (applied to different realizations of the structural model generated through uncertainty modeling) is obtained. This is going to be used as *data* in order to update (through Bayesian updating) the joint probability distribution function for the Log Normal distribution parameters (i.e., two regression parameters and a conditional standard deviation). In the second stage, large-sample MC simulation based on the *posterior* joint probability distribution (calculated in the first stage) is used to generate a set of plausible fragility curves and their percentiles (e.g., 50th, 84th, etc.). This provides a confidence interval that takes into account also the effect of limited number of structural analyses. The application of the above-mentioned procedure for efficient fragility assessment by using the cloud method is demonstrated for a shear-critical existing RC frame designed only for gravity-loading.

¹Assistant Professor, Dept. of Structures for Engineering and Architecture, University of Naples Federico II, Naples, NA 80125, Italy.

²Postdoctoral Researcher, Dept. of Structures for Engineering and Architecture, University of Naples Federico II, Naples, NA 80125, Italy.

³Professor, Dept. of Structures for Engineering and Architecture, University of Naples Federico II, Naples, NA 80125, Italy.

Cloud Analysis revisited: Efficient fragility calculation and uncertainty propagation using simple linear regression

F. Jalayer¹, L. Elefante², R. De Risi², and G. Manfredi³

ABSTRACT

One of the most efficient non-linear dynamic analysis procedures for analytic fragility evaluation, known also as the Cloud Method, is based on simple regression of structural response versus seismic intensity for a set of registered records. This work presents a Bayesian take on the cloud method for efficient fragility assessment, considering both record-to-record variability and other sources of uncertainty related to structural modeling. The starting point of this method lies in the assumption of a prescribed probability distribution such as the Lognormal probability distribution. In the first stage, the structural response to a limited set of ground motions (applied to different realizations of the structural model generated through uncertainty modeling) is obtained. This is going to be used as data in order to update (through Bayesian updating) the joint probability distribution function for the Log Normal distribution parameters (i.e., two regression parameters and a conditional standard deviation). In the second stage, large-sample MC simulation based on the posterior joint probability distribution (calculated in the first stage) is used to generate a set of plausible fragility curves and their percentiles (e.g., 50th, 84th, etc.). This provides a confidence interval that takes into account also the effect of limited number of structural analyses. The application of the above-mentioned procedure for efficient fragility assessment by using the cloud method is demonstrated for a shear-critical existing RC frame designed only for gravity-loading.

Introduction

Analytic structural fragility assessment is arguably one of the fundamental steps in the modern performance-based engineering [1]. The structural fragility can be defined as the conditional probability of exceeding a prescribed limit state given the intensity measure (IM). If the structural limit state is defined in terms of one or more engineering demand parameters (EDPs), the fragility is going to depend significantly on the EDP-IM relationship. There are alternative non-linear dynamic analysis procedures available in the literature for characterizing the relationship between EDP and IM based on recorded ground motions, such as, the Incremental Dynamic Analysis (IDA) [2], Multiple-Stripe Analysis (MSA) [3], [4] and the Cloud Method [5],[6],[7] just to name a few. The nonlinear dynamic methods such as IDA and MSA are suitable for evaluating the relationship between the EDP and the IM for a wide range of IM

¹Professor, Dept. of Struct. for Eng. and Architecture, Univ. of Naples, NA 80125, Italy. fatemeh.jalayer@unina.it

²Postdoctoral Researcher, Dept. of Struct. for Eng. and Architecture, Univ. of Naples, Naples, NA 80125, Italy.

³Professor, Dept. of Struct. for Eng. and Architecture, Univ. of Naples Federico II, Naples, NA 80125, Italy.

values. The application of MSA and IDA can sometimes be quite time-consuming as the non-linear dynamic analyses are going to be repeated (usually for scaled ground motion time-histories) for increasing levels of ground motion intensity. The Cloud Method is particularly efficient since it involves the non-linear analysis of the structure subjected to a set of un-scaled ground motion time-histories. The simplicity of its underlying formulation makes it a quick and efficient analysis procedure for fragility assessment or safety-checking in the context of the SAC-FEMA formulation [8]. The Cloud Method has been used, not only to model the record-to-record variability in ground motion, but also to propagate structural modeling uncertainties such as uncertainty in component capacity [9] and the uncertainties in mechanical material properties and construction details [10]. Furthermore, a modified version of the Cloud Method has been proposed in [11] that implements a weighting scheme for taking into account magnitude and shape-factor dependence conditioned on the adopted IM. Finally, an information-based relative measure for the sufficiency [12] of the adopted IM has been derived based on Cloud Method's underlying probabilistic model in [13].

Well-established results in Bayesian parameter estimation [14] are going to be used herein in order to construct an analytic closed-form (*posterior*) joint probability distribution for parameters of a regression-based Log Normal fragility model. In doing so, the results of the Cloud Method expressed in terms of a (limited) set of EDP values, obtained by applying a suite of records to various realizations of the structural model taking into account structural modeling uncertainties, are going to be used as *data*. A *robust* [15], [16] estimate of the structural fragility can be obtained by integrating the Log Normal structural fragility model and the joint probability distribution for the analytic fragility parameters. Solving the integral using Monte Carlo simulation, leads to a set of plausible fragility curves and their various counted percentiles (eg., 16th, 50th, 84th). This is going to provide an equivalent of a (logarithmic) plus/minus one standard deviation confidence interval for the resulting robust fragility curve. The results are going to be compared with those obtained by employing the more accurate incremental dynamic analysis, for a shear-critical existing school building designed for gravity loading only (in its pre-retrofit state).

The small-amplitude first-mode spectral acceleration is adopted as the IM. The EDP herein is taken to be the critical demand to capacity ratio denoted as Y_{LS} and defined as the demand to capacity ratio for the component that brings the system closer to the onset of limit state LS. The formulation is based on the cut-set concept, which is suitable for cases where various potential failure mechanisms can be defined a priori:

$$Y_{LS} = \max_l^{N_{mech}} \min_j^{N_l} \frac{D_{jl}}{C_{jl}(LS)} \quad (1)$$

where N_{mech} is the number of considered potential failure mechanisms and N_l the number of components taking part in the l^{th} mechanism. D_{jl} is the demand evaluated for the j^{th} component of the l^{th} mechanism and $C_{jl}(LS)$ is the limit state capacity for the j^{th} component of the l^{th} mechanism. In the context of system reliability, a cut set is defined as any set of components whose joint failure $Y(l) = \min D_{jl}/C_{jl} > 1$, implies failure of the system, $Y = \max Y(l) > 1$.

A Brief Overview of the Cloud Method

The Cloud Method implements the non-linear dynamic analysis results in a (linear) regression-based probabilistic model. Let $\mathbf{Y}=\{Y_i, i=1:N\}$ be the critical demand to capacity ratio calculated through non-linear time-history analysis performed for a suite of N recorded ground motions with $\mathbf{S}_a=\{S_{a,i}, i=1:N\}$. The probabilistic model can be described as following:

$$E[\log Y | S_a] = \log \eta_{Y|S_a} = \log a + b \log S_a$$

$$\sigma_{\log Y|S_a} = \sqrt{\frac{\sum_{i=1}^n (\log Y_i - \log \eta_{Y|S_{a,i}})^2}{n-2}} \quad (1)$$

where $\eta_{Y|S_a}$ is the median for \mathbf{Y} given \mathbf{S}_a and $\sigma_{Y|S_a}$ is the logarithmic standard deviation for \mathbf{Y} given \mathbf{S}_a . The structural fragility obtained based on the Cloud Method can be expressed as:

$$P(Y > 1 | S_a) = P(\log Y > 0 | S_a) = 1 - \Phi\left(\frac{-\log \eta_{Y|S_a}}{\sigma_{\log Y|S_a}}\right) \quad (2)$$

Note that this is a three-parameter fragility model whose model parameters can be denoted as $\boldsymbol{\chi}=[\log a, b, \sigma_{Y|S_a}]$.

The robust fragility

Robust fragility is defined as the expected value for a prescribed fragility model taking into account the joint probability distribution for the (fragility) model parameters [15],[10],[16]. For the fragility model described in Eq. 2, the robust fragility can be written as:

$$F(LS | \mathbf{Y}) = \int_{\Omega(\boldsymbol{\chi})} \left[1 - \Phi\left(\frac{-\log \eta_{Y|S_a}}{\sigma_{Y|S_a}}\right) \right] \cdot p(\boldsymbol{\chi} | \mathbf{Y}) \cdot d\boldsymbol{\chi} \quad (3)$$

where $\boldsymbol{\chi}=[\log a, b, \sigma_{Y|S_a}]$ and $p(\boldsymbol{\chi}|\mathbf{Y})$ is the posterior joint probability distribution for fragility model parameters given the vector of EDP values \mathbf{Y} .

The posterior joint probability distribution $p(\boldsymbol{\chi}|\mathbf{Y})$ for $\boldsymbol{\chi}=[\log a, b, \sigma_{\log Y|S_a}]$

The joint probability distribution for $\boldsymbol{\chi}$ can be written as:

$$p(\boldsymbol{\chi} | \mathbf{Y}) = p(\log a, b | \sigma_{\log Y|S_a}, \mathbf{Y}) p(\sigma_{\log Y|S_a} | \mathbf{Y}) \quad (4)$$

In the following, we describe how the probability distributions $p(\sigma_{\log Y|S_a}|\mathbf{Y})$ and $p(\log a, b|\sigma_{Y|S_a}, \mathbf{Y})$ are derived [14]. The posterior probability distribution for $\sigma_{\log Y|S_a}$ can be calculated as a derived distribution based on a chi-squared distribution with $\nu=N-2$ degrees of freedom:

$$p(\sigma_{\log Y|S_a} | \mathbf{Y}) = \left[\frac{1}{2} \cdot \Gamma\left(\frac{\nu}{2}\right) \right]^{-1} \cdot \left(\frac{\nu \cdot s^2}{2} \right)^{\frac{\nu}{2}} \cdot \sigma_{\log Y|S_a}^{-(\nu+1)} \cdot e^{\left(\frac{-\nu \cdot s^2}{2 \cdot \sigma_{\log Y|S_a}^2} \right)} \quad (5)$$

where s^2 can be calculated from Eq. 1 as linear regression's best-estimate for $\sigma_{\log Y|S_a}$. The posterior joint distribution $p(\log a, b | \sigma_{\log Y|S_a}, \mathbf{Y})$ can then be expressed as a Normal bi-variate distribution:

$$\left\{ \begin{aligned} f(\log a, b | \sigma_{\log Y|S_a}, \mathbf{Y}) &= \frac{|X' \cdot X|^{\frac{1}{2}}}{(2 \cdot \pi \cdot \sigma_{\log Y|S_a}^2)^{\frac{N}{2}}} e^{\left(\frac{-(\omega - \varpi)' \cdot X' \cdot X \cdot (\omega - \varpi)}{2 \cdot \sigma_{\log Y|S_a}^2} \right)} \\ X' &= \begin{bmatrix} 1 & 1 & \dots & 1 \\ \ln(S_{a,1}) & \ln(S_{a,2}) & \dots & \ln(S_{a,N}) \end{bmatrix} \end{aligned} \right. \quad (6)$$

where ϖ is the vector of $[\log a, b]$ calculated as the linear least squares best-estimate from regression.

Deriving the robust fragility curve(s) by Monte Carlo simulation

The integral in Eq. 3 can be solved numerically, in a Monte Carlo simulation scheme, by first sampling $\sigma_{\log Y|S_a}$ based on the PDF in Eq. 5. In the next step, conditioning on the value of sampled $\sigma_{\log Y|S_a}$, $[\log a, b]$ can be sampled based on the PDF in Eq. 6. Conditioning on the sampled vector $\chi = [\log a, b, \sigma_{\log Y|S_a}]$, the Log Normal analytical fragility curve can be calculated from Eq. 2. This leads to a *plausible* fragility curve based on the underlying probabilistic regression model. Counted percentiles for the plausible fragility curves can be finally obtained by taking into account a large number of fragility curves.

The numerical example

As numerical example, the application of the Bayesian inference in the assessment of robust fragility based on the Cloud Method is demonstrated for a shear-critical existing RC school building.

Description of the structural model

An existing school structure located in Avellino, Italy is considered for the application of the robust fragility method. The structure is situated in seismic zone II according to the Italian seismic guidelines (see [10] for more information).

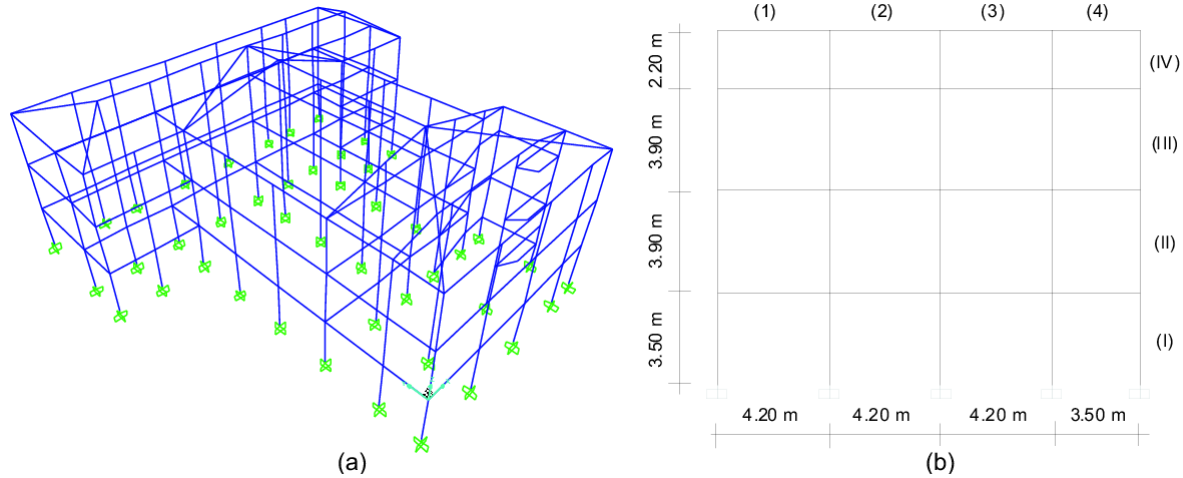


Figure 1. (a) The tri-dimensional view of the scholastic building. (b) The central frame of the case-study building.

The structure is consisted of three stories and a semi-embedded story and its foundation lies on soil type B. For the structure in question, the original design notes and graphics have been gathered. The building is constructed in the 1960's and it is designed for gravity loads only, as it is frequently encountered in the post second world war construction. In Fig. 1a, the tri-dimensional view of the structure is illustrated; it can be observed that the building is highly irregular both in plane and elevation. In order to reduce the computational effort, the main central frame in the structure is extracted and used as the structural model (Fig. 1b). The columns have rectangular section with the following dimensions: first storey: $40 \times 55 \text{ cm}^2$, second storey $40 \times 45 \text{ cm}^2$, third storey: $40 \times 40 \text{ cm}^2$, and forth storey: $30 \times 40 \text{ cm}^2$. The beams, also with rectangular section, have the following dimensions: $40 \times 70 \text{ cm}^2$ at first and second storey, and $30 \times 50 \text{ cm}^2$ for the ultimate two floors. The finite element model of the frame is constructed, using the OpenSees software, assuming that the non-linear behavior in the structure is concentrated in plastic hinges.

Characterization of uncertainties

Typically, the uncertainties present in a seismic vulnerability assessment problem can be classified in different groups; namely, the uncertainties in the representation of the ground motion (GM), the modeling uncertainties associated with the component capacity models, and the uncertainties in the structural modeling parameters. In order to take into account the uncertainty in the representation of the GM, a set of 20 GM records [11] based on Mediterranean events are chosen from European Ground motion database (18 recordings) and the database of the Next Generation Attenuation of Ground Motions (NGA) Project (2 recordings). They are all main-shock recordings and include only one of the horizontal components of the same registration. The soil category on which the GMs are recorded is stiff soil ($400 \text{ m/s} < V_{s30} < 700 \text{ m/s}$) which is consistent with the soil-type B (the soil-type for the site of the case-study presented in this work). The earthquake events have moment magnitude between 5.3 and 7.2, and closest distances ranging between 7 km and 87 km (see the spectra in Fig. 2a).

Component capacities are modeled as the product of semi-empirical formulas and unit-median Log Normal variables accounting for the uncertainty in component capacity [9], according to the general format:

$$C_i = \hat{C}_i \cdot \varepsilon_{Ci} \quad (7)$$

Table 3 illustrates the values of the logarithmic standard deviation β_{Ci} for unit-median Lognormal variables ε_{Ci} . These variables represent the uncertainty in the yield chord rotation capacity, the ultimate chord rotation capacity, and the ultimate shear capacity.

Table 1. Logarithmic standard deviation values for component capacity models (see [9] for source references).

Unit-Log Normal variable	β_{Ci}
ε_{Cyield}	36%
ε_{Cult}	47%
ε_{Cshear}	40%

The parameters identifying the probability distributions for the material mechanical properties (concrete strength and steel yielding force) have been based on the values typical of the post world-war II construction in Italy (see [10] for more detail). It is assumed that the material properties are homogeneous across each floor. Table 2 illustrates the parameters that are used to define the lognormal probability distributions for the material properties; f_{ci} denotes the compressional strength for concrete for storey i and f_y denotes the yield strength of steel.

Table 2. The uncertainties in the material mechanical properties.

Material	Type	Median	COV
f_{c1}	LN	165	0.15
f_{c2}	LN	165	0.15
f_{c3}	LN	165	0.15
f_y	LN	3200	0.08

Having a shear-critical model, the spacing of the shear rebar is expected to affect significantly the structural fragility. It is assumed that the information about the shear rebar is limited to knowledge of the intervals in which it is going to vary. Hence, a uniform distribution is assumed in that interval (Table 3).

Table 3. The uncertainties in the spacing of shear rebar.

Defect	Type	Min	Max
spacing of shear rebar	Uniform (beams)	15cm	30cm
spacing of shear rebar	Uniform (column)	20cm	35cm

The results of the Cloud Method

The results obtained using the Cloud Method for first-mode spectral acceleration $S_a(T_1)$ and critical demand to capacity ratio Y data pairs are shown in Fig. 2. In particular, Fig. 2b illustrates the results obtained considering only the uncertainties related to the representation of the GM (case 1); Fig. 2c illustrates the results considering also the uncertainties in component capacity models (case 2); Fig. 2d, illustrates the results obtained considering also the epistemic uncertainties in the structural modeling (material properties plus construction details, case 3).

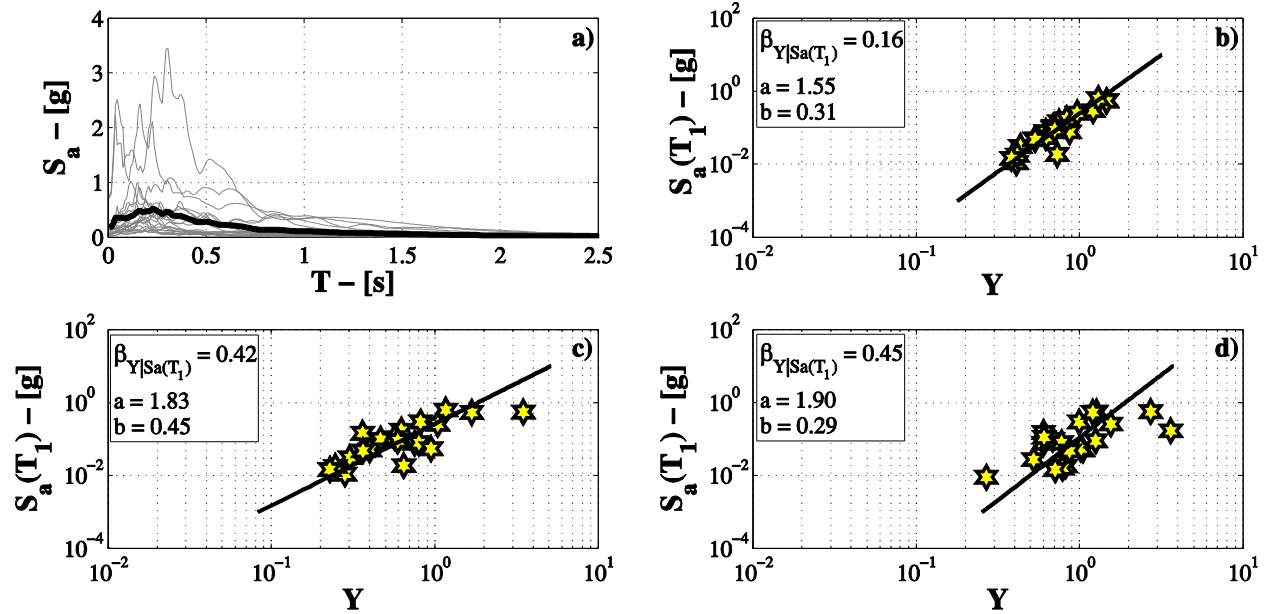


Figure 2. Cloud method results (a) considering only record to record variability (case 1); (b) accounting also for uncertainties in component capacity models (case 2); and (c) accounting also for structural modeling uncertainties (case 3).

The robust fragility curves

Figure 3 illustrates the fragility percentiles (16th, 50th and 84th) calculated through the robust fragility procedure outlined herein (Eq. 2) for the three cases described above. The Cloud Method results Y_i are used to update the joint probability distribution for the Log Normal fragility parameters. In the next step, large sample MC simulation is used in order to solve the integral in Eq. 2 numerically, by first sampling $\sigma_{\log Y|S_a}$ based on the PDF in Eq. 5. Conditioning on the value of sampled $\sigma_{\log Y|S_a}$, $[\log a, b]$ can be sampled based on the PDF in Eq. 6. Conditioning on the sampled vector $\chi = [\log a, b, \sigma_{\log Y|S_a}]$, the Log Normal analytical fragility curve can be calculated from Eq. 2. It can be observed that the consideration of component capacity uncertainty leads to an increase in the dispersion of the fragility curve but not affecting significantly the median. On the other hand, considering also the structural modeling uncertainties leads to both a significant shift in the median value and also an increase in the dispersion.

The comparison with robust fragility curves based on IDA results

Figure 4(a) and (b) illustrate the empirical distributions (plotted in red) for spectral acceleration corresponding to $Y=1$ obtained by employing the IDA method, for cases 1 and 3, respectively. The Log Normal distributions fitted to these empirical distributions are also plotted in red color. It can be observed that for both cases, the Log Normal fit to the empirical data (obtained by employing IDA) is well within the robust fragility plus/minus one standard deviation interval predicted by the Cloud Method.

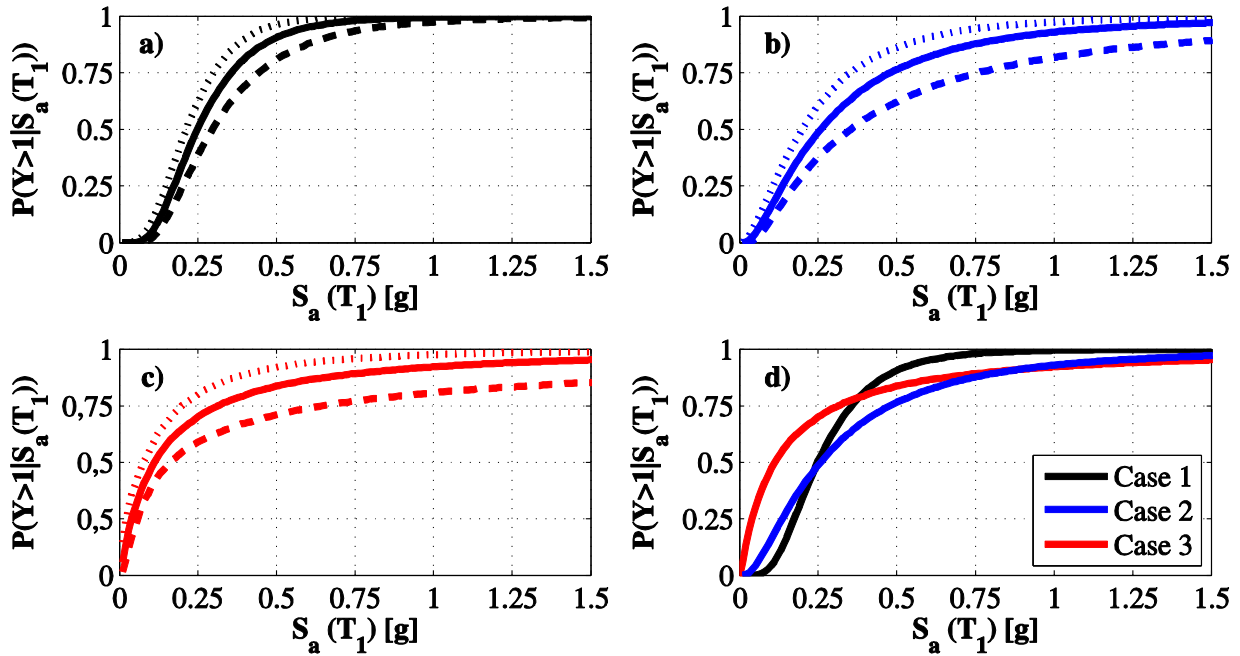


Figure 3. The robust fragility curves (a) considering only record to record variability (case 1); (b) accounting also for uncertainties in component capacity models (case 2); and (c) accounting also for structural modelling uncertainties (case 3).

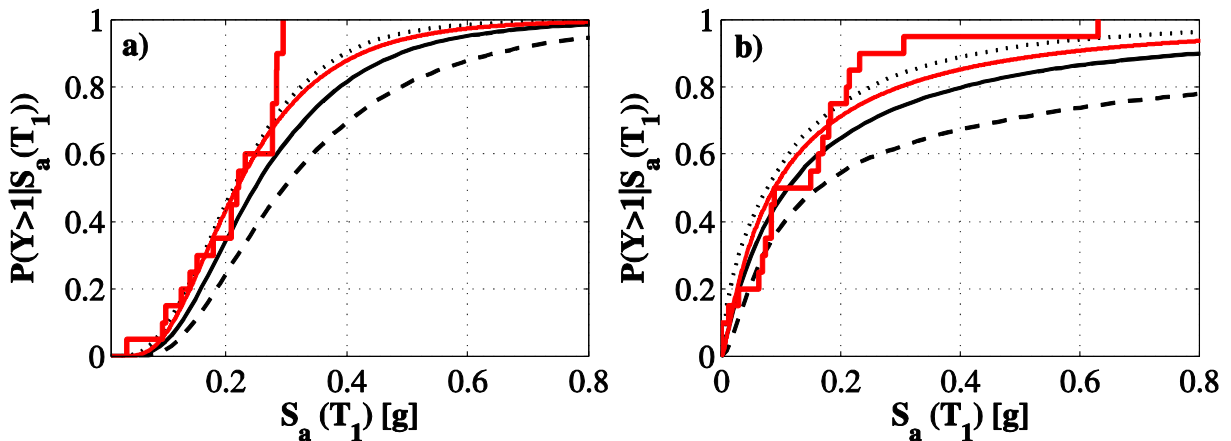


Figure 4. The robust fragility curves (a) case 1 together with the IDA results; (b) case 3 together with the IDA results.

The confidence band as a function number of records

A confidence interval is constructed herein by the 16th and 84th percentile curves around the 50th percentile curve. The error term β_H is defined as half of the logarithmic distance between the 16th and 84th percentile curves Fig. 5(a). The procedure for the calculation of the robust fragility is repeated for a number of records varying from $n=4$ to 20. For each n , several combinations of n records from a total of twenty are considered. The median β_H value corresponding to each n is plotted versus n in Fig. 5(b) for case 1 (the green squares) and case 3 (the yellow circles).

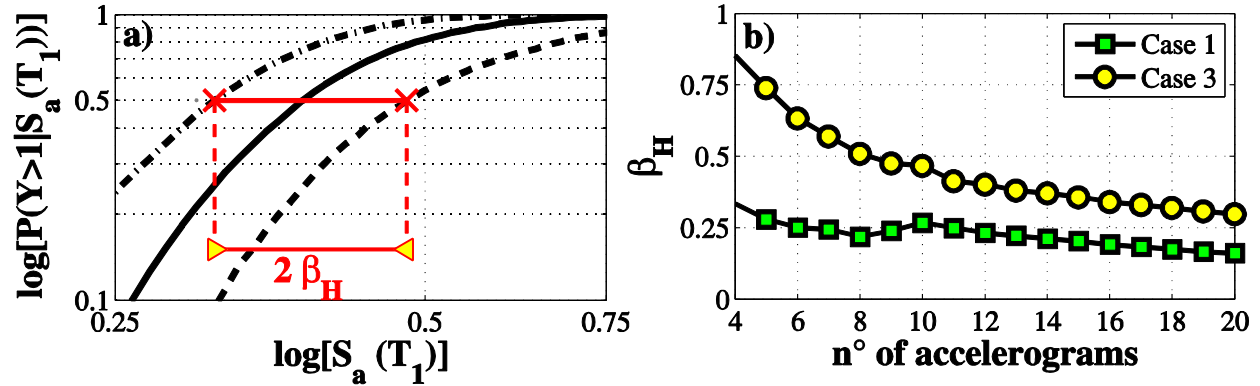


Figure 5. (a) The logarithmic error term β_H ; (b) β_H varying as the number of records.

It can be observed that in both cases, β_H tends to decrease as the number of records increases. Moreover, the error term corresponding to case 3 is larger than that of case 1 and shows a more pronounced decrease as a function of the number of records.

Conclusions

Cloud Method is based on linear least squares regression of the structural response EDP versus the intensity measure IM, in the logarithmic scale. This procedure has many advantages such as simplicity, use of original records, and relatively small computational effort. Not to mention draw-backs such as but not limited to modeling of record-to-record variability with constant standard deviation. This work looks more closely to the probabilistic model underlying the linear regression. Employing the results of Cloud Method in a Bayesian framework, leads to closed-form and analytic joint probability distribution for the parameters of the (Log Normal) Cloud fragility. A robust fragility curve is then calculated as the expected value of the Log Normal Cloud fragility, taking into account the joint probability distribution for the fragility parameters. The robust fragility curves can also be represented as curves constructed by the 16th, 50th and 84th percentiles of the set of plausible fragility curves.

The robust fragilities are obtained for a shear-critical existing RC frame designed for gravity loading only, taking into account record-to-record variability, component capacity uncertainties, and structural modeling uncertainties (material properties and construction details). It can be seen that the consideration of component capacities leads to an increase in the dispersion of the fragility curve without significantly influencing the median. On the other hand, considering the structural modeling uncertainties leads also to a significant shift in median. The confidence bands constructed by the 16th and 84th percentile curves manage (in cases 1 and 3) to contain the Log Normal Fragility curves built based on IDA results. As a measure of error in the

robust fragility curve, (half of) the logarithmic distance between the 16th and 84th percentile curves is used. It can be observed that this error measure tends to decrease as the number of records increases. In particular, the error is larger for cases with more sources of uncertainty and demonstrates a more pronounced decrease as the number of records increases.

Acknowledgments

This work was supported in part by the executive project ReLUIIS 2010/2013 – Dipartimento della Protezione Civile. This support is gratefully acknowledged.

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