APPROXIMATE METHODS FOR PROBABILITY-BASED ASSESSMENT
of Jacket-type Offshore Platforms by Using Incremental Wave Analysis

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INTRODUCTION

The main concepts of the proposed probability-based assessment method called *Probabilistic Incremental Wave Analysis* (PIWA) are investigated in the companion paper by the authors [1]. This probability-based approach can be used for assessment of jacket-type offshore platforms against extreme sea states. In this methodology, the performance objective can be stated in terms of the mean annual frequency (MAF) of exceeding a given level of structural Demand Parameter (*DP*) or a limit state, e.g., the collapse prevention (CP) limit state. The CWH-based approach is suggested as a wave height-based scheme in order to estimate the CP limit state frequency [1].
However, there are some complexities in numerical calculations associated with the PIWA framework, which can affect the practical implications of this probabilistic approach. As a result, to simplify the PIWA procedure, approximate closed-form analytical expressions are suggested herein in order to estimate the MAF of exceeding the CP limit state. The two alternative solutions are comprised of the ultimate capacity-based approach, and the Collapse Wave Height (CWH)-based approach. Hence, the proposed approximate methodologies make the PIWA framework generally suitable for being used in practical assessment of jacket-type platforms, and being implemented in guidelines and codes. Furthermore, the closed-form analytical expression regarding the CWH-based criterion provides an alternative interpretation for the design or assessment against extreme waves. It provides a simple and consistent method for probabilistic evaluation of the jacket structures. A case study jacket-type platform located in South Pars Gas Field in the Persian Gulf region, the same used in [1], is used for application of the aforementioned issues.

1 NECESSARY ELEMENTS

The main elements including source of uncertainties, sampling procedure, the case-study offshore platform, and finally the *Incremental Wave Analysis* (IWA) concept and the associated *Multiple-Stripe Analysis* (MSA) are described in the companion paper (see [1]); the same specifications are implemented in this study. Moreover, the exact estimation of the limit state frequency by the CWH-based methodology is used as a benchmark for comparison with the proposed closed-from approximations provided herein.

2 CLOSED-FORM EVALUATION OF THE CP LIMIT STATE FREQUENCY

2.1 General

This section provides analytical approximate expressions (closed-form solutions) for computing the MAF of exceeding the CP limit state, λ_{LS}, by following two different approaches. These closed-form expressions are generally based on the previous work in [2-4], which has been proposed for probabilistic seismic demand analysis of building structures. An application of this technique is utilized in this study. The analytical approximate expression can be considered to deliver reliable results if one is willing to accept a set of assumptions as follows:

- The wave hazard curve is locally approximated by a power-law function of the form:

$$\lambda_{H_{\max}}\left(h\right) = k_o \cdot h^{-k} \tag{1}$$

where k_o and k are parameters defining the shape of the wave height hazard curve.

- The conditional median of the *DP* for a given level of wave height, *h*, is approximated by a power-law function as follows:

$$\eta_{DP|H_{\max}}\left(h\right) = a \cdot h^{b} \tag{2}$$

- The conditional standard deviation of *DP* for a given level of wave height is assumed to have a constant value over the entire interval of the wave heights:

$$\sigma_{\ln DP|H_{\max}}\left(h\right) = \beta_{DP|H_{\max}} \tag{3}$$

Under the above assumptions, the first step for estimating the closed form expressions associated with λ_{LS}, for the case-study SPD2 platform, can be summarized as:

(1) Performing multiple IWA or MSA (shown previously in *Fig. 3* of [1])

(2) Calculating the conditional median and dispersion of *DP* at each wave height intensity according to the MSA

(3) Approximating the wave height hazard curve according to *Eq. (1)* and the conditional median of the demand parameter by means of a power-law curve, i.e. *Eq. (2)*.

Fig. 1 illustrates aforementioned steps 2 through 3 for the case of static and dynamic IWA (see [1], [5-6] for more description of static and dynamic IWA denoted as SIWA and DIWA). *Fig. 1*a and *Fig. 1*b show the 16th, 50th, and 84th percentiles of the distribution of the base shear given wave height intensity for the case study SPD2 platform based on SIWA and DIWA (step 2). In step 3, the wave height hazard curve of the desired site in the Persian Gulf region (see [1] and [5]) is approximated by a power-law curve shown in *Eq. (1)*, as shown in *Fig. 1c*. Based on this figure, a line with slope *k* and intercept k_o is fitted to the hazard curve in the logarithmic scales. The initial point of the approximate curve indicates the wave height corresponding to 100-year return period.
Subsequently, the 50th percentile of the conditional *DP* obtained in step 1 is approximated based on *Eq. (2)* as shown in *Fig. 1*d and *Fig. 1*e; hence, the linear regression in logarithmic scale is used to fit a power-law curve to the 50th percentile of the distribution of $DP|H_{\max}$ for the SPD2 jacket. Similarly, the initial wave height associated with the approximate median curve is the same as that shown in *Fig. 1*c.

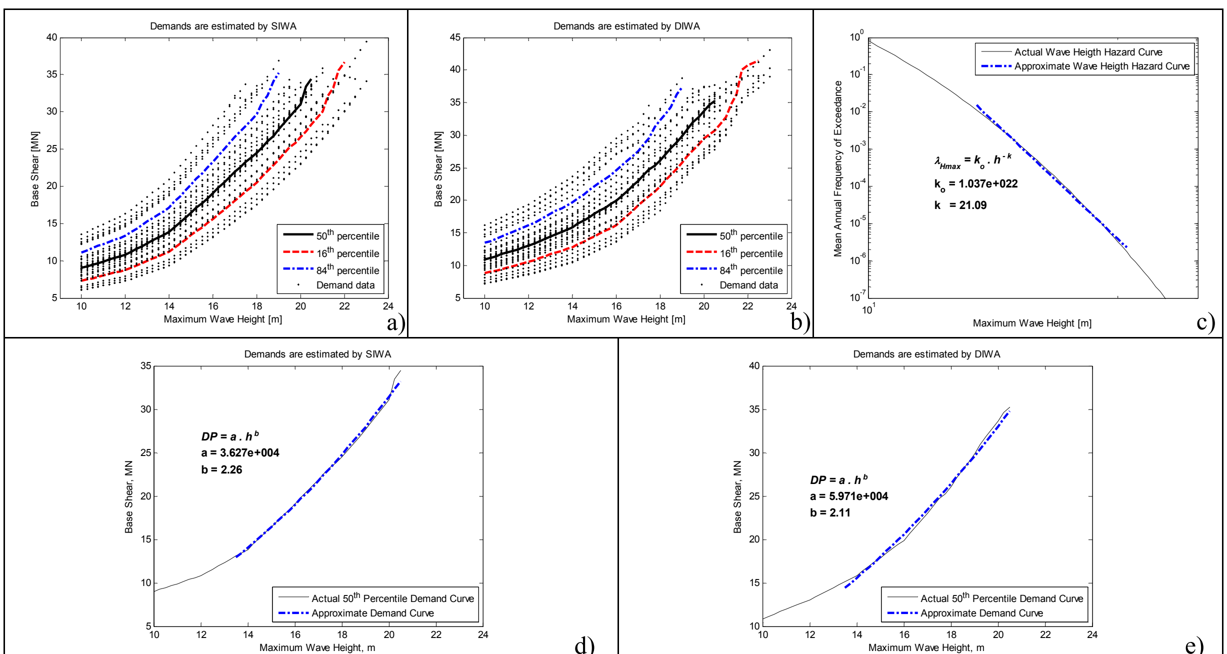

Figure 1. a, b) The 16th, 50th, and 84th percentiles of the distribution of base shear given wave height intensity by employing SIWA and DIWA; c) the site-specific wave hazard curve and its associated approximation in the region of interest; d, e) the 50th percentile of the conditional distribution of base shear and the corresponding approximate curve by SIWA and DIWA

2.2 Closed-form expression using the ultimate capacity-based approach

The MAF of exceeding the CP limit state (denoted as limit state frequency, λ_{LS}) is composed of all possible combination of ($DP = x$) and ($C < x$), in which C denotes the ultimate capacity random variable. Utilizing the total probability theorem, the expression for the limit state frequency by using the ultimate capacity-based approach (see also [5]) can be defined as follows:

$$\lambda_{LS} = \nu \sum_{all\ x} P\left[\left(DP = x\right) \bullet \left(C < x\right)\right] = \sum_{all\ x} P\left[C < DP \middle| DP = x\right] \cdot \left(\nu P\left[DP = x\right]\right)$$
$$= \int_x F_C\left(x\right) \left| \mathrm{d}\lambda_{DP}\left(x\right) \right| \tag{4}$$

where $(\bullet)$ is the logical "and"; ν represents the annual rate of occurrence of the events (storms) or the number of 3-hour sea states in one year (see [5], [7]); F_C is the conditional Cumulative Density Function (CDF) of the ultimate capacity for the CP limit state known as the fragility function; $\lambda_{DP}(x)$ represents the MAF of exceeding a specified level of response known as the DP hazard (see [5] for more details), and $\mathrm{d}\lambda_{DP}$ is the differential of the DP hazard. The expression in *Eq. (4)* illustrates the exact solution based on numerical integration. The aim, herein, is to provide an approximate closed-form solution for this estimate. This closed-form solution is based on the previous work in [3] and [4] which has been utilized for estimating the annual frequency of exceeding a limit state in the probabilistic seismic demand analysis of building structures. Hence, *Eq. (4)* can be approximated as:

$$\lambda_{LS} \cong k_o \left(\frac{\eta_C}{a}\right)^{-\frac{k}{b}} \cdot \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^2_{DP|H_{\max}}\right) \cdot \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^2_C\right)$$
$$= \lambda_{H_{\max}}\left(h^{DP=\eta_C}\right) \cdot \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^2_{DP|H_{\max}}\right) \cdot \exp\left(\frac{1}{2}\frac{k^2}{b^2}\beta^2_C\right) \tag{5}$$

This closed-form expression is equal to the wave hazard evaluated at the wave height associated with the median of demand capacity, i.e. $\lambda_{H\max}(h^{DP=\eta_C})$, times two coefficients which accounts for the uncertainties in the demand for a given wave height and in the capacity itself. The parameters k_o, k, a and b are considered to have the values shown in *Fig. 1*. The median, η_C, and dispersion, β_C, are estimated from the ultimate capacity fragility function, F_C. *Fig. 2* shows the fragility curve associated with the ultimate capacity from SIWA and DIWA by applying both empirical and lognormal distributions. It is estimated based on the scattered ultimate capacity data illustrated in (see *Fig. 3* in [1]) for the SPD2 jacket. The median, η_C, and the coefficient of variation (COV) of the ultimate capacity estimated by the lognormal distribution are indicated on *Fig. 2*. The resulted COV in the current study is close to the lower limit recommended by [8] for the base shear capacity of jacket structures to be 0.05-0.10.

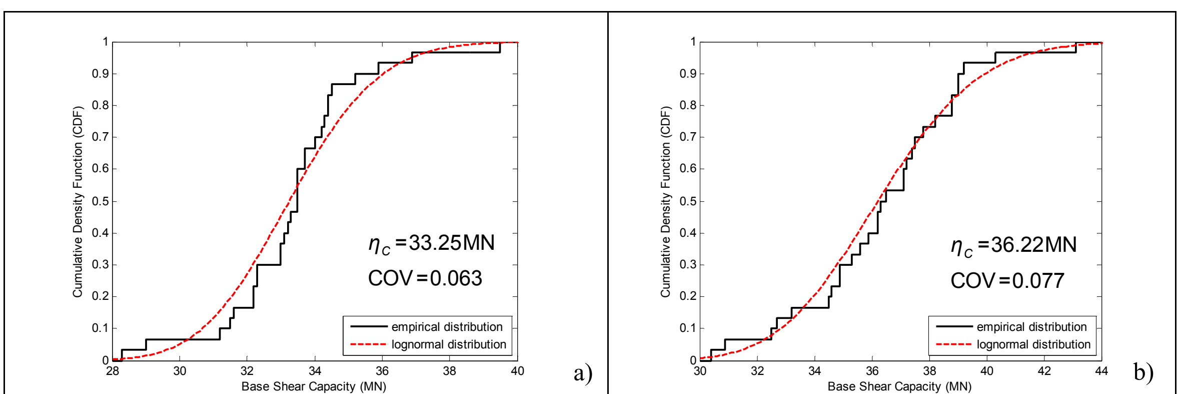

Figure 2. The ultimate-capacity fragility curves based on a) SIWA, and b) DIWA

Based on *Eq. (5)*, the approximate closed-form solutions for the CP limit state frequency is demonstrated in *Table 1* by employing the SIWA and DIWA.

2.3 Using the CWH-based approach

The MAF of exceeding the CP limit state can also be estimated by following a CWH-based approach, i.e. collapse occurs when the wave height intensity exceeds the collapse wave height variable, *CWH*. Therefore, the CP limit state frequency can be written as (see [1]):

$$\lambda_{LS} = \int_h F_{CWH}\left(h\right) \cdot \left|\mathrm{d}\lambda_{H_{\max}}\left(h\right)\right| \tag{6}$$

where F_{CWH} is the conditional CDF of the collapse wave height known as the CWH fragility function, and $\mathrm{d}\lambda_{H\max}$ represents the differential of the wave height hazard curve [1]. The approximate closed-form formulation for the MAF of exceeding the CP limit state, by following the CWH-based approach, can be expressed as (see [3], [4], and [7]):

$$\lambda_{LS} \cong \left(k_o\, \eta_{CWH}^{-k}\right) \cdot \exp\left(\frac{1}{2} k^2 \beta_{CWH}^2\right) = \lambda_{H_{\max}}\left(\eta_{CWH}\right) \cdot \exp\left(\frac{1}{2} k^2 \beta_{CWH}^2\right) \tag{7}$$

This approximate formulation requires only four parameters comprising k_o, k, η_{CWH} , and β_{CWH}, for which the first two parameters are shown in *Fig. 1*c; moreover, the median, η_{CWH}, and dispersion, $\sigma_{\ln CWH}=\beta_{CWH}$, are obtained from the CWH fragility function, F_{CWH}, illustrated in (see [1], *Fig. 4*a, and *Fig. 4*b).

However, the closed-form expression for the CP limit state by the ultimate capacity-based method, i.e. *Eq. (5)*, requires seven parameters; Hence, it can be inferred that the formulation of the CWH-based approach not only requires lesser computational efforts, but also provides more accurate estimates. The latter can be concluded as the degree of approximation is much lower compared to the ultimate capacity-based formulation (evidently, there is no approximation for $DP|H_{\max}$ by the CWH-based approach). Moreover, η_{CWH} for the case study jacket platform indicates less sensitivity to the static and dynamic wave analyses in comparison with η_C (see [1]).

Table 1 compares the results of the closed-form solution with those obtained from the exact method [1]. It can be concluded that the CWH-based closed-form solution leads to perfectly acceptable results compared to the exact solution. Furthermore, the similarity between the exact and the approximate limit state frequencies in this case are more apparent compared to those obtained from the ultimate capacity-based approach.

As a general result, it is recommended to use the closed-form solution of the CP limit state by utilizing the CWH-based approach, since it provides a simple, practical and reliable approach for estimating the limit state frequency. It is noteworthy that the limit state frequencies by the ultimate capacity-based approach provides conservative results; hence, it implies that this approximate method can appropriately be utilized as an upper-bound solution for estimating the CP limit state frequency of jacket structures.

Table 1. Different estimates of the CP limit state frequency

Type of Solution	Distribution of Fragility Function	SIWA	DIWA
Exact	Empirical	5.8575×10^{-6}	7.2374×10^{-6}
	lognormal	7.2117×10^{-6}	7.7720×10^{-6}
Closed-form (CWH-based approach)	lognormal	8.1833×10^{-6}	8.7085×10^{-6}
Closed-form (ultimate capacity-based approach)	lognormal	1.2764×10^{-5}	1.1651×10^{-5}

3 FACTORED COLLAPSE WAVE HEIGHT

In the following, an alternative interpretation for the design or assessment of offshore platforms, considering a CWH-based design criterion, is presented. This is mainly based on the concept of "*factored capacity*" for performance-oriented design procedure proposed in the SAC/FEMA Steel Project for seismic actions (see [2-4] for more details). Accordingly, a certain design criteria herein is to check whether the MAF of exceeding the CP limit state (CP limit state frequency) is less than or equal to an allowable annual frequency λ_o, i.e. $\lambda_{LS} \le \lambda_o$. Thus, utilizing the closed-form expression of λ_{LS} from *Eq. (7)*, and with the objective of allocating one side to the parameters corresponding to the *CWH* capacity, the aforementioned inequality will take the following form:

$$k_o\, \eta_{CWH}^{-k} \cdot \exp\left(\frac{1}{2} k^2 \beta_{CWH}^2\right) \le \lambda_o \tag{8}$$

$$\eta_{CWH} \cdot \exp\left(-\frac{1}{2} k \beta_{CWH}^2\right) \ge \left(\frac{\lambda_o}{k_o}\right)^{-\frac{1}{k}} \tag{9}$$

$$\eta_{CWH} \cdot \exp\left(-\frac{1}{2} k \beta_{CWH}^2\right) \ge h^{\lambda_o} \tag{10}$$

where h^{λ_o} is equal to the wave height associated with the annual exceedance frequency, λ_o (see wave hazard curve in [1] *Fig. 4*c). Subsequently, the "*Factored Collapse Wave Height*" (FCWH) can be expressed by:

$$\text{FCWH} = \eta_{CWH} \cdot \exp\left(-\frac{1}{2} k \beta_{CWH}^2\right) \tag{11}$$

Therefore, the design or assessment criterion can be presented as FCWH $\ge h^{\lambda_o}$. As a result, in order to have a safe design, the wave height with a frequency of exceedance equal to the allowable rate λ_o, i.e. h^{λ_o}, extracted from the wave height hazard curve, should be lower than the FCWH. The exponential expression in *Eq. (11)* is a reduction factor which takes into account the uncertainties in the CWH. Since this exponential term rises to a non-positive power, the FCWH is always less than or equal to η_{CWH}. *Fig. 3* shows the sensitivity of the FCWH to different values of COV (associated with the CWH) in the reasonable interval of [0.05, 0.1]. According to this Figure, it can be concluded that the FCWH is not very sensitive to the variation in β_{CWH}.

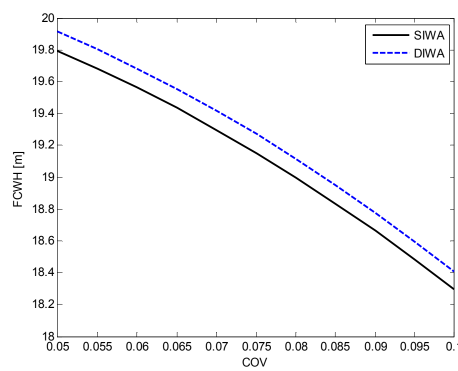

Figure 3. The variation in the FCWH by considering both SIWA and DIWA

In addition to the aforementioned application of the FCWH, the mean annual frequency of exceeding a limit state, λ_{LS}, can be approximately obtained from the wave hazard by finding the exceedance rate associated with the FCWH, i.e $\lambda_{H\max}$(FCWH), as shown in *Table 2*. By comparing the limit state frequencies associated with the FCWH with those summarized in *Table 1*, it can be concluded that the exceedance rate $\lambda_{H\max}$(FCWH) can be used as the CP limit state frequency.

Table 2. The FCWH and the associated exceedance rate

	FCWH (m)	$\lambda_{H\max}$(FCWH)
SIWA	19.29	7.8321×10^{-6}
DIWA	19.24	8.3775×10^{-6}

4 SUMMARY

Approximate closed-form expressions are proposed for estimating the Mean Annual Frequency (MAF) of exceeding the Collapse Prevention (CP) limit state for a jacket offshore platform against extreme sea states. It can simplify mainly the PIWA framework proposed in [5]. The main advantage of these approximations is that they can not only be utilized for practical assessment of jacket-type offshore platforms, but also be definitely implemented in the offshore design or assessment guidelines and codes. These closed-form analytical solutions are introduced step-by-step through a case-study jacket platform located in the South Pars Gas Field in the Persian Gulf region. However, the results obtained for the case-study jacket are only valid for platforms in this specific site, since these structures encompass the same design specifications and general configuration.

Two approximate closed-form analytical solutions are provided for estimating the CP limit state by means of the ultimate capacity- and CWH-based approaches. Comparison between these two closed-form expressions with the exact solution [1] demonstrates that both closed-form solutions are able to properly estimate the limit state frequency; however, the CWH-based approach provides not only a simple and practical expression, but also more accurate estimates since the degree of approximation is much lower compared to the ultimate capacity-based formulation.

Furthermore, the closed-form analytical expression, by the CWH-based criterion, provides an alternative interpretation for the design or assessment against extreme waves. This new indicator, called herein the Factored Collapse Wave Height (FCWH), is introduced. The FCWH is a novel probabilistic design criterion, which is stated in terms of the wave height, in order to check whether the MAF of exceeding the CP limit state is less than or equal to an allowable frequency. It is further revealed that the exceedance rate associated with the FCWH can be considered as a particularly suitable upper-bound solution for the CWH-based limit state frequency.

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KEYWORDS: Assessment, Jacket-type offshore platform, Incremental wave analysis, Extreme wave, Mean annual frequency of exceedance.

ABSTRACT

Two approximate closed-form solutions are presented for calculating the mean annual frequency (MAF) of exceeding a specified limit state. They can be used for assessment of existing jacket-type offshore platforms under extreme waves, which are mainly based on the Probabilistic Incremental Wave Analysis (PIWA) framework (see [1] and [2]). The PIWA is a robust procedure for probabilistic evaluation of jacket offshore platforms, which can properly estimate the Mean Annual Frequency (MAF) of exceeding a limit state as well as calculating the demand hazard. Nevertheless, the simplified proposed closed-form analytical expressions aim to calculate the limit state frequency more convenient for practical assessment of jacket structures. Strictly speaking, instead of conducting complicated computations, approximate analytical expressions are proposed which enable practical engineers to simply follow the PIWA framework. Moreover, they can be definitely implemented in the offshore design or assessment guidelines and codes. The closed-form methodology is comprised of ultimate capacity-based and Collapse Wave Height-based (CWH-based) approaches for estimating the limit state frequency.

Furthermore, the closed-form analytical expression, by the CWH-based criterion provides an alternative interpretation for the design or assessment against extreme waves. It provides a simple and consistent method for probabilistic evaluation of the jacket structures. A case study jacket-type platform located in South Pars Gas Field in the Persian Gulf region, the same used in [1] and [2], is used in this study for application of the aforementioned issues.

CONCLUSIONS

Comparison between these two closed-form expressions with the exact solution [1] demonstrates that both closed-form solutions are able to properly estimate the limit state frequency; however, the CWH-based approach provides not only a simple and practical expression, but also more accurate estimates since the degree of approximation is much lower compared to the ultimate capacity-based formulation.

Furthermore, the closed-form analytical expression, by the CWH-based criterion, provides an alternative interpretation for the design or assessment against extreme waves. This new indicator, which is called herein the Factored Collapse Wave Height (FCWH), is derived. By using the FCWH stated in terms of the wave height, one can check whether the MAF of exceeding the CP limit state is less than or equal to an allowable frequency. It is further shown that the exceedance rate associated with the FCWH can be considered as a particularly suitable upper-bound solution for the CWH-based limit state frequency.

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