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Seismic hazard analysis for alternative measures of ground motion intensity employing stochastic simulation methods

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ABSTRACT: The wide application of spectral acceleration as a ground motion intensity measure (IM) is in part due to the fact that various empirical (attenuation) relationships, that relate spectral acceleration to ground motion source and path parameters, are already available and tabulated. In the recent years, various scalar and vector IM's have been proposed and shown to be more suitable than spectral acceleration; however, their wider use is hindered by limited availability of site-specific empirical attenuation relations for these variables. Subset Simulation is an advanced simulation method which is particularly efficient for calculating small failure probabilities. The (target) failure region in the Subset Simulation is regarded as the last in a sequence of nested intermediate failure regions in which, a Markov Chain Monte Carlo algorithm is used to sample from the original probability distribution conditioned on the previous intermediate failure region in the sequence. This study demonstrates that Subset Simulation, based on a stochastic model for ground motion, can be effectively employed in order to develop hazard curves for alternative scalar and vector IM's. Two example applications are illustrated in which hazard curves are derived for a scalar structure-specific IM that includes the effect of both higher modes and inelastic response and a vector IM consisting of spectral acceleration and spectral shape for a California site. The algorithm is efficient and relatively straight-forward, however, some care should be taken in defining the sequence of nested failure regions which guide the simulation procedure into regions with small exceedance probabilities.

1 INTRODUCTION

The state of the practice in probability-based seismic performance assessment of structures is to adopt an intensity measure (IM) in order to represent the uncertainty in the future ground motion. This involves a probabilistic seismic hazard analysis (PSHA, McGuire, 1995) that relies on empirical attenuation relations in order to obtain the (mean and standard deviation of) IM as a function of parameters such as magnitude and distance.

Spectral acceleration at the first-mode period, denoted by $S_a(T_1)$, is a common choice for a measure of earthquake ground motion intensity. Its wide-spread use is in part facilitated by availability of tabulated empirical attenuation relations and regional hazard curves. However, studies (Shome and Cornell 1999, Luco and Cornell 2006, Baker and Cornell, 2006) demonstrate that first-mode spectral acceleration is not always *sufficient* as a single parameter for relaying the ground-motion source characteristics.

For example, for high-rise long-period structures, the effect of higher modes becomes important. Another case involves near-source ground motions, where studies (Alavi and Krawinkler 2000) have demonstrated that, even for an SDOF structure, $S_a(T_1)$ may not be sufficient and the effect of inelastic response needs to be taken into account.

Considerable research effort has been focused on finding ways to employ more suitable intensity measures in the scalar or vector form (Luco and Cornell 2006, Baker and Cornell 2005 and Cordova et al., 2002); however, these efforts have been restrained by the practical need to base the results on the PSHA for spectral acceleration, for which site-specific empirical attenuation relations already exist.

A vector IM which consist of $S_a(T_1)$ and $S_a(T)$ at a period T other than the first-mode one could represent a better, more informative IM compared to $S_a(T_1)$ alone. The ratio $S_a(T)/S_a(T_1)$, which is known as the spectral shape, is found to be a parameter that directly affects structural response (Shome and Cornell 1999, Baker and Cornell 2006). Depending on the structure, the period T could be either shorter than T_1 (e.g., second mode period) to reflect the higher mode effect, or longer, to reflect the softening effect in the regime of significant non-linear behavior. The application of this vector IM, however, would require calculation of the joint probability distribution of $S_a(T_1)$ and $S_a(T)$ or at the very least would ask for estimation of the correlation between the two spectral values.

Subset Simulation (Au and Beck 2001) is an advanced simulation procedure which is efficient for calculating the small probabilities corresponding to strong earthquake occurrence. Moreover, it is robust with respect to the number of uncertain parameters in the problem and it is applicable to any type of structural model and loading.

This work shows how the subset simulation scheme can be efficiently employed, together with a stochastic ground motion model (Atkinson and Silva 2000) and the results of site-specific PSHA for $S_a(T_1)$, in order to generate ground motions and to calculate the hazard for alternative intensity measure, such as the vector $[S_a(T_1),\ S_a(T)]$. Marginal distributions for each of the components are computed from the obtained joint probability distribution and compared to the respective PSHA results. Moreover, the procedure is used to calculate the hazard curve for a scalar structure-specific intensity measure $IM_{1I,2E}$ that is proposed by Luco and Cornell (2006) which takes into account the effect of the first two modes of vibration and the effect of inelastic response.

PROPOSAL

A standard probabilistic seismic hazard analysis involves calculation of the probability of exceeding a given level of the adopted IM in a given time interval or calculation of the mean annual rate of exceeding such level due to all possible seismic events capable of producing a ground motion of intensity greater than the specified level at the site. In order to achieve this result, PSHA requires the integration of the contributions from all influential seismogenetic zones (Cornell, 1968 and McGuire, 1995). A PSHA procedure relies on attenuation relations in order to obtain the (mean and standard deviation of) IM as a function of parameters such as magnitude and distance, whereas, magnitude and distance are sampled based on the geometry of the surrounding seismic zone, the magnitude scaling laws, and the marginal probability distribution for moment magnitude.

Alternatively, the above integration can be carried out by means of simulation. When using simulation, instead of using an attenuation relation, the ground motion time-history can be directly generated based on semi-empirical stochastic ground motion models which provide Fourier amplitude spectrum for a given magnitude and distance. This in turn can be used as the transfer function for a linear SDOF system in the frequency domain which takes Gaussian white noise as the input and produces the ground motion time-history as the output. The magnitude and distance could be simulated similar to a standard PSHA. Given the very low probability levels of interest, use of an efficient simulation scheme is mandatory in order to make such an approach feasible. This work employs the Subset Simulation (Au and beck, 2001, 2003) coupled with a stochastic ground motion model (Atkinson and Silva, 2000) in order to calculate the joint complementary CDF and the corresponding hazard surface for both the vector IM consisting of $S_a(T_1)$ and $S_a(T)$ and the scalar intensity measure $IM_{1I,2E}$.

1.1 *The Subset Simulation Procedure*

Given a n-dimensional vector $\boldsymbol{\theta}$ of uncertain parameters, with joint density $f(\boldsymbol{\theta})$, and a failure domain $F \subset R^n$ in the space of $\boldsymbol{\theta}$, the probability of failure can be written as:

$$P(F) = \int_F f(\boldsymbol{\theta}) d\boldsymbol{\theta} = \int I_F(\boldsymbol{\theta}) f(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{1}$$

The right-most term in the above expression shows that $P(F)$ can be evaluated as the expectation of the failure indicator function $I_F(\boldsymbol{\theta}) = 1$ if $\boldsymbol{\theta} \in F$ and zero otherwise. This is the starting point of Monte Carlo simulation methods.

Subset simulation method achieves its great efficiency in computing estimates of very low $P(F)$'s by breaking up the problem into a sequence of smaller ones. If the failure domain F can be decomposed in a ordered sequence of nested failure regions $F_1 \supset F_2 \supset \ldots \supset F_m = F$ such that $F_k = \bigcap_{i=1}^{k} F_i$, $P(F)$ can be correspondingly expressed as the product of a sequence of (much larger) conditional probabilities according to:

$$P(F) = P(F_m) = P\left(\bigcap_{i=1}^{m} F_i\right) = P(F_1) \prod_{i=1}^{m-1} P(F_{i+1} | F_i) \tag{2}$$

Central to this simulation method are two aspects: 1) the availability of an algorithm to simulate samples based (in an asymptotic manner) on the original probability distribution conditioned on being in an intermediate failure region (the Metropolis-Hastings algorithm) 2) the choice of the intermediate failure regions. As it regards the latter, consideration of the shape of the failure domain helps in the choice of the nested sequence.

The failure region for a system modeled as n_s serially connected sub-systems, with the set of component indices of the j-th subsystem denoted by I_j, can be written as union and the intersection of component failure regions:

$$F = \bigcup_{j=1}^{n_s} \bigcap_{i \in I_j} F_i(\boldsymbol{\theta}) = \bigcup_{j=1}^{n_s} \bigcap_{i \in I_j} \{\boldsymbol{\theta} : D_i(\boldsymbol{\theta}) > C_i(\boldsymbol{\theta})\} \quad (3)$$

where $F_i(\boldsymbol{\theta})$ is the failure domain for the *ith* component with its demand and capacity denoted by $D_i(\boldsymbol{\theta})$ and capacities $C_i(\boldsymbol{\theta})$, respectively.

It is possible to parameterize F with a scalar parameter such that the sequence of failure regions can be generated by varying a single parameter. For a failure region that can be expressed according to the general format in (3) such a parameter is given by:

$$Y = \max_{j=1,n_s} \left(\min_{i \in I_j} \frac{D_i(\boldsymbol{\theta})}{C_i(\boldsymbol{\theta})} \right) \quad (4)$$

It is immediate to show that $F = \{\boldsymbol{\theta} : Y(\boldsymbol{\theta}) > 1\}$ from which it follows that the sequence of failure regions can be generated as: $F_k = \{\boldsymbol{\theta} : Y(\boldsymbol{\theta}) > y_k\}$ with $0 < y_1 < \ldots < y_m = 1$. The choice of the intermediate y_k-values usually results from a compromise between the number of nested domains (*levels*) and the number of simulations per level. A convenient choice is to choose these thresholds *adaptively* by keeping the magnitude of the conditional probabilities in (2) constant (e.g. equal to $P(F_1) = p_0$).

1.2 *The stochastic ground motion model*

In this work, a stochastic ground motion model proposed by Atkinson and Silva (2000) is used to obtain the (mean) Fourier amplitude spectrum denoted by $A(f; M, r)$ for a given magnitude M and source-to-site distance r. This mean amplitude spectrum is used as the transfer function for a linear SDOF oscillator in the frequency domain that takes as input a windowed time-series of zero-mean Gaussian uncertain variables and produces ground motion time-history as output. The Fourier amplitude can be written as:

$$A(f; M, r) = A_{AS2000}(f; M, r) \cdot \varepsilon_{\text{model}} \quad (5)$$

where $A_{AS2000}(f; M, r)$ is the Fourier amplitude proposed in (Atkinson and Silva 2000) and $\varepsilon_{\text{model}}$ is assumed to be a unit-median Lognormal uncertain variable which takes into account — in the absence of more specific information — the overall effect of uncertainty in ground motion parameters on the spectra predicted by the stochastic model.

1.3 *Application of subset simulation to the determination of hazard for a vector IM*

Determination of the hazard and/or complementary CDF for an alternative scalar IM (one for which there is no attenuation available) is a straightforward application of subset simulation as described in Section 1.1. One can regard the problem as that of a system with one component of deterministic unit-capacity. In this case equation (4) reduces to $Y = D(\boldsymbol{\theta}) = IM(\boldsymbol{\theta})$. In general, the algorithm can be used in order to calculate the joint probability distribution for a vector-valued IM, denoted by $\mathbf{S}(\boldsymbol{\theta}) = (S_1(\boldsymbol{\theta}), \ldots, S_n(\boldsymbol{\theta}))$.

For a vector-valued IM, the sequence of nested failure regions can be represented by a scalar variable similar to the one stated in Equation 4:

$$Y(\boldsymbol{\theta}) = \min_{i=1,n} \frac{S_i(\boldsymbol{\theta})}{s_i} \quad (6)$$

which parameterizes the nested failure boundary sequence $F_k = \{k : 1, \ldots, m\}$ with (monotonically) increasing scalar sequence $y_k = \{k : 1, \ldots, m\}$:

$$F_k = \bigcap_{i=1}^{n} \{\boldsymbol{\theta} : S_i(\boldsymbol{\theta}) > s_i . y_k\} = \{\boldsymbol{\theta} : Y(\boldsymbol{\theta}) > y_k\} \quad (7)$$

where $P(F_k) = P(Y > y_k)$ can be calculated by performing the Subset Simulation procedure for the scalar variable Y and the s_i values are *shape factors* that control the aspect ratio of the failure boundary. In this particular case, the failure surface has the shape of an n-dimensional box. Finally, the results of the Subset Simulation need to be post-processed in order to render the values of $F_S(\mathbf{s})$ for a desired mesh of $\mathbf{s} = (s_1, \ldots, s_n)$ values.

Based on the sequence of failure regions, a set of mutually exclusive and collectively exhaustive (MECE) regions can be defined as (Figure 1):

$$\begin{aligned} &A_i = F_i - F_{i+1} = \{\boldsymbol{\theta} : y_i < Y(\boldsymbol{\theta}) \le y_{i+1}, i = 1, \ldots, m-1\} \\ &A_0 = \overline{F_1} = \{\boldsymbol{\theta} : Y(\boldsymbol{\theta}) \le y_1\} \quad A_m = F_m \end{aligned} \quad (8)$$

Identification of a set of MECE regions defined as in (8) allows the determination of CCDF (G_S) values by use of the total probability theorem:

$$F_S(\mathbf{s}) = \sum_{i=0}^{m} F_S(\mathbf{s} | A_i) P(A_i) \quad (9)$$

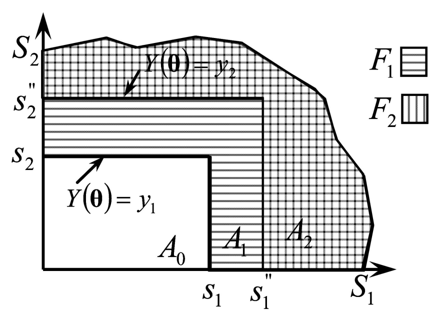

Figure 1. Decomposition of the failure domain into mutually exclusive and collectively exhaustive regions.

The probability terms $F_S(s|A_i)$ and $P(A_i)$ in (9) are easily evaluated: $F_S(s|A_i)$ can be calculated based on simple statistics on the $\boldsymbol{\theta}$-samples that belong to domain A_i, (i.e. those for which $y_i < Y(\boldsymbol{\theta}) \leq y_{i+1}$); $P(A_i)$ is given by:

$$P(A_i) = p(F_i)P(\overline{F}_{i+1} \mid F_i) = P(F_i)(1 - P(F_{i+1}))$$
$$= p_0^i(1 - p_0) \quad \mathrm{i = 0,.....,m - 1} \tag{10}$$
$$P(A_m) = p_0^{m-1}$$

2 NUMERICAL EXAMPLES

2.1 *Example 1:* $\mathrm{S}_a(\mathrm{T}_1)$ *and* $\mathrm{S_a}(\mathrm{T}_2)$

As the first example, the hazard curve for a vector IM consisting of the spectral acceleration at periods T_1 and T_2 is calculated using Subset Simulation. This is equivalent to an IM consisting of first-mode spectral acceleration and the spectral shape factor at another period. In order to determine the modeling error for the stochastic ground motion model, the marginal distributions for $S_\mathrm{a}(T_1)$ and $S_\mathrm{a}(T_2)$ calculated using Subset Simulation are (roughly) fit to the results of PSHA.

2.1.1 *Fine-tuning of stochastic-model variability term*

In the absence of error estimations specific to the stochastic ground motion model, a single parameter denoted by $\varepsilon_\mathrm{model}$ is adopted to model the overall effect of uncertainties in the ground motion source and path parameters. $\varepsilon_\mathrm{model}$ is assumed to be a unit-median lognormal variable whose variance is determined by matching the spectral acceleration hazard estimations provided by subset simulation with those provided by a standard PSHA. Figures 2-a and 2-b show the spectral acceleration hazard curves $\lambda_{S_a(T)}(x)$ obtained for $T_1 = 0.8$s and $T_2 \approx 2T_1 = 1.5$s and the corresponding value of ε.

It can be observed that $\sigma_{\ln \varepsilon_{\mathrm{mod}el}} = 0.50$ and $\sigma_{\ln \varepsilon_{\mathrm{mod}el}} = 0.45$ achieve a good match with PSHA results at $T = 0.80$s and $T = 1.50$, respectively.

2.1.2 *Calculating the joint complementary CDF for* $\mathrm{S_a}(\mathrm{T}_1)$ *and* $\mathrm{S_a}(\mathrm{T}_2)$

The Subset Simulation procedure is employed to calculate the joint complementary CDF for $S_a(T_1)$ and $S_a(T_2)$, where the failure boundary sequence is parameterized by :

$$Y = \min\left(S_a(T_1), \frac{S_a(T_2)}{0.40}\right) \tag{11}$$

The Subset Simulation is performed at 6 successively increasing level, each with the same conditional probability of failure of 10%. $P(Y > y)$ at the highest

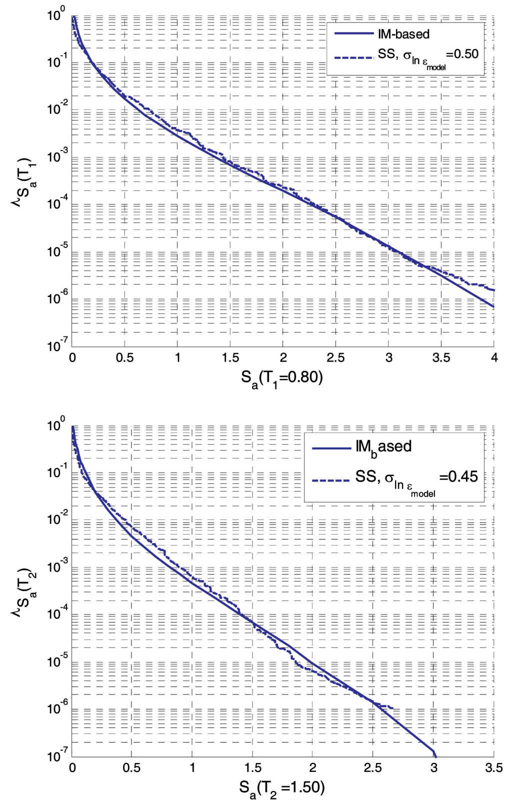

Figure 2. Spectral acceleration seismic hazards from PSHA (IM-based) and Subset Simulation (probabilistic representation of ground motion) for a) $\mathrm{T}_1 = 0.80$s and b) $\mathrm{T}_2 = 1.50$s.

level, which is equal to the product of the conditional failure probabilities at all levels, is equal to 10^{-6}. At each level 500 simulations are performed. In order to calculate exceedance probabilities as small as 10^{-6} using standard Monte Carlo simulation, one needs to perform on the order of 4 million (4×10^6) simulations to get a coefficient of variation in the estimate for the failure probability equal to 50%; while Subset Simulation has been carried out by performing $500 + (500 - 50) \times 5 = 2750$ analyses, giving a coefficient of variation for the lowest level of 13% that increases with each intermediate level to approximately 66% at the highest level. This demonstrates the efficiency of Subset Simulation for calculating very small failure probabilities. It is observed, however, that this CoV refers to the estimate of the distribution of the scalar parameter Y and the estimate of the joint distribution of the vector IM could be characterized by a different CoV.

Figure 3 illustrates the joint complementary CDF surface for $S_a(T_1)$ and $S_a(T_2)$ that is calculated using$\varepsilon_{\mathrm{mod}el} = 0.45$ following the procedure outlined

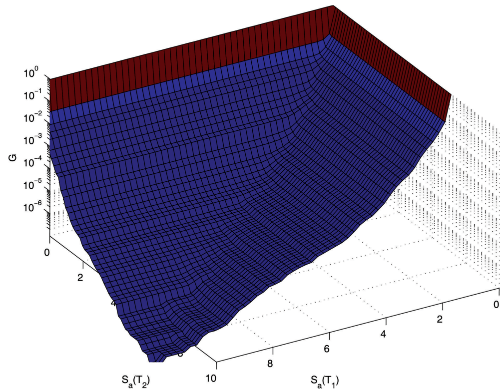

Figure 3. Joint complementary distribution function of $S_a(T_1)$ and $S_a(T_2)$.

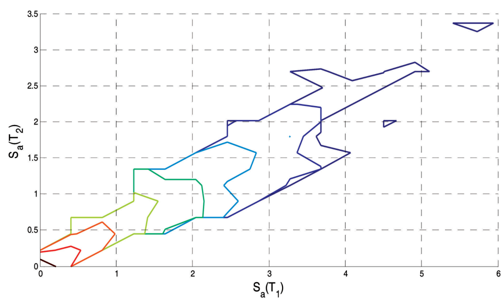

Figure 4. Joint PDF of $S_a(T_1)$ and $S_a(T_2)$ (the contour values are in logarithmic scale).

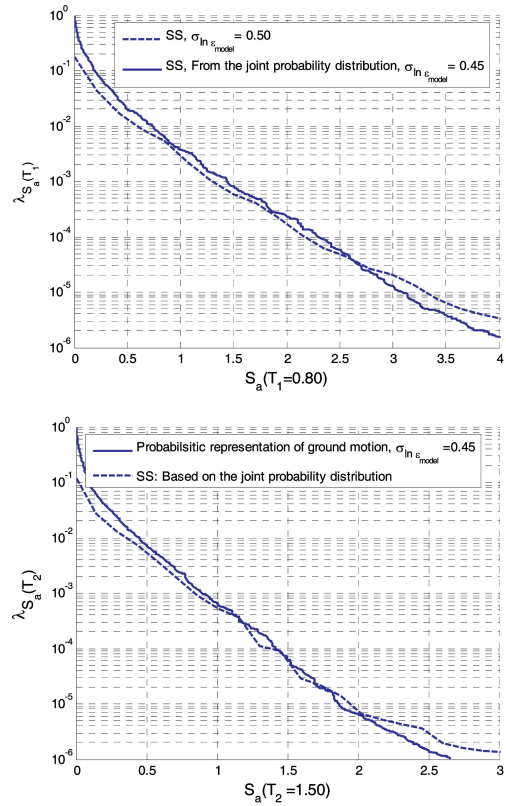

Figure 5. Marginal hazards from: (a) marginal subset simulation (b) integration of joint distribution.

in Section 1.3 with . This procedure also provides the joint PDF whose contours are plotted in Figure 4.

The significant correlation between the two spectral values can be observed in the contour lines of Figure 4. For the sake of presentation, the contour values are shown in the logarithmic scale.

2.1.3 *Comparing the marginal distributions*

In order to benchmark the above procedure for calculating the joint distribution for the two variables, the marginal distributions obtained by integration of the joint distribution are compared against those obtained by performing separate "scalar" Subset Simulation for each variables. Figures 5-a and 5-b demonstrate the results for the two periods. The marginal distributions are plotted in dashed lines and the solid lines represent the results of the Subset Simulation carried out for each of the variables individually. The results seem to have a good agreement. However, it should be noted that the choice of the failure region parameter Y is fundamental in achieving a good agreement. More specifically, the ratio of the factors s_1 and s_2 (i.e., the aspect ratio of the failure boundary) in Equation 6 should be consistent with the correlation observed between the two variables. This is partly because the algorithm performs the simulations and moves forward conditioned on the sequence of failure regions. The results of PSHA for each variable are not shown in the figures; nevertheless, as observed in Figures 2-a and 2-b, the hazard curves for each variable were reasonably matched with the corresponding PSHA results.

2.2 *Example 2: Luco's IM*

As a second example, the subset simulation method is used to simulate stochastic ground motion in order to calculate the hazard curve for a scalar IM which is a functional of both elastic and inelastic spectral values. Luco and Cornell (2006) have proposed a scalar structure-specific intensity measure denoted by $IM_{1I,2E}$ that takes into account not only the ground-motion frequency content around the first two modal

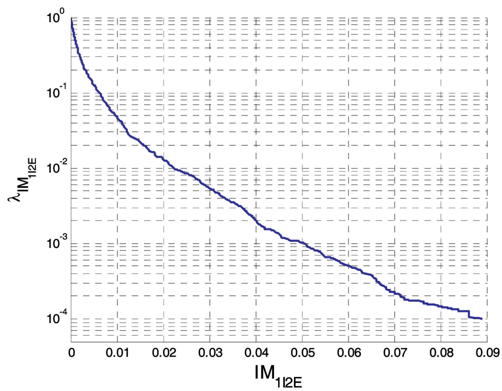

Figure 6. The mean annual frequency of exceeding $IM_{1I,2E}$

periods but also, to some extent, the inelastic structural behavior. $IM_{1I,2E}$ can be calculated as:

$$IM_{1I,2E} = \frac{S_d^I(T_1,\xi_1,d_y)}{S_d(T_1,\xi_1)} \cdot \sqrt{[PF_1 \cdot S_d(T_1,\xi_1)]^2 + [PF_2 \cdot S_d(T_2,\xi_2)]^2} \tag{7}$$

where PF_1 and PF_2 are the modal participation factors for maximum inter-story drift corresponding to the first two modes of vibration, $S_d(T_1, \xi_1)$ and $S_d(T_2, \xi_2)$ are the spectral displacements with periods T_1 and T_2 and damping ratios ξ_1 and ξ_2 corresponding to the first two modes, and $S_d^I(T_1, \xi_1, d_y)$ is the spectral displacement of an elastic-perfectly plastic oscillator with period T_1, damping ratio ξ_1 and yield displacement d_y. $IM_{1I,2E}$ is decidedly more sufficient than $S_a(T_1)$ in predicting the maximum inter-story drift ratio response of moment-resisting frames.

The Subset Simulation is performed at 3 successively increasing level, each with the same conditional probability of failure of 10%. At each level 500 simulations are performed. Figure 6 shows the corresponding hazard curve.

3 CONCLUSIONS

An advanced simulation method known as Subset Simulation is applied to generate ground motions from a stochastic ground motion model and to calculate the hazard values and exceedance probabilities for alternative scalar and vector intensity measures (IM). This is while the state of the practice is to perform a probabilistic seismic hazard analysis (PSHA) in order to calculate the hazard curve for an IM. However, PSHA relies on the availability of empirical (attenuation) relations between the considered IM and ground motion parameters such as magnitude and distance.

This work demonstrates the efficiency of the Subset Simulation in calculating the hazard for those IM's for which the PSHA results cannot be readily obtained for lack of the corresponding attenuation relationships. Magnitude and distance are simulated from a joint distribution that is obtained by post-processing (deaggregation) the results of a PSHA for spectral acceleration.

As a first example, the joint hazard surface for a vector IM consisting of spectral acceleration at two different periods is calculated. Since the spectral shape is found to be a parameter that directly affects structural response, it is important to have the hazard information for this type of IM. The resulting marginal hazard curves for the two spectral acceleration values demonstrated a good match with the available hazard curves for each of the spectral acceleration values. However, the final results and their accuracy is sensitive to the (pre-determined) shape of the nested failure region sequence used in the algorithm. In comparison to the computation effort needed for a standard Monte Carlo, Subset Simulation proves to be an efficient means for calculating the hazard information (based on the PSHA results available for spectral acceleration) for a vector-valued IM.

As the second example, the proposed procedure is used to calculate the mean annual frequencies of exceedance for a scalar intensity measure $IM_{1I,2E}$ in the form of generalized maximum inter-story drift ratio taking into account the participation of the first two modes and the inelastic behavior in the structure to some extent. $IM_{1I,2E}$ is observed to be more sufficient than spectral acceleration in representing ground motion intensity; thus, the availability of hazard information for it would facilitate it use as an intensity measure.

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