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A case-study on scenario-based probabilistic seismic loss assessment for a portfolio of bridges

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ABSTRACT The majority of bridge infrastructure in Italy has been built in the 60's and 70's without specific seismic provisions. Therefore, it is expected that they reveal high seismic vulnerability if subjected to a significant seismic event. Given this background, it is natural that rapid and accurate assessment of economic losses incurred to bridge infrastructure can play a crucial role in emergency management in the immediate aftermath of an earthquake. Focusing on the infrastructure system of highway bridges in the Campania region, it is shown how state of the art methodologies in portfolio loss assessment and available data can be implemented in order to assess the probability distribution of the repair costs incurred to the portfolio in question due the Irpinia 1980 earthquake. Formulating the probabilistic loss assessment for the portfolio of bridges as a standard Monte Carlo Simulation problem helps in resolving the spatial risk integral efficiently. One of the specific features of this case-study is the use of statistical methods for updating, a) ground motion prediction; b) vulnerability/fragility; and c) exposure/cost models based on available data. It has been observed that alternative hypotheses with regard to the ground motion correlation structure can significantly affect the distribution of the direct economic loss. Furthermore, updating the ground motion prediction based on available recordings may significantly reduce the dispersion in the estimation of the direct economic losses.

In the immediate aftermath of a strong earthquake, the road networks play a crucial role in rescue and recovery operations. Since their loss of functionality may undermine the performance of the entire network, the bridge infrastructure can be considered as the so-called weak links within a road network affected by an earthquake. A large part of the Italian bridge infrastructure portfolio, dating back to the first half/beginning of the second half of the last century, is not designed based on earthquake resistant criteria. Therefore, the Italian highway bridges are potentially vulnerable to seismic actions, considering that the Italian territory is classified by medium to high seismicity. The total direct losses incurred to the bridge infrastructure in a road network can be used as a scalar proxy for the performance of the entire network right after the seismic event. This quantity encompasses various parameters, such as seismic hazard, the seismic vulnerability of the bridges and the repair costs. A prompt assessment of incurred losses to a network is rendered more complicated by the spatial correlation in the ground motion and the possible inter-relations between the vulnerability and repair costs of various infrastructures. It is logical to exploit the available registrations related to a given seismic event in order to improve and update the seismic hazard and vulnerability evaluations, given the advances made in measurements, sensors and data transfer techniques. This work strives to employ various established statistical techniques in order to obtain updated and estimates of total direct losses for a portfolio of bridges for a given earthquake scenario, based on the available data.

Treating the GMPE's as a regression probability model, provides many flexibilities; such as, consideration of spatial correlation in the GMPE regression residuals (e.g., Jayaram and Baker (2009)) and model updating using the available registrations. In particular Park, Bazzurro et al. (2007) have demonstrated the effect of spatial correlation in ground motion intensity for a given scenario on portfolio loss estimation. Moreover, treating the GMPE as probabilistic regression model defining a joint Log Normal distribution between various intensity values, they have shown how (the statistics of such model can be updated based on available registrations). The suggested spatial correlation structure and model updating based on available registrations has been later implemented by Crowley, Stafford et al. (2008) for loss assessment. In fact, many recent research efforts have been concentrated on evaluating the effect of spatial correlation in ground motion on loss estimation (e.g., Yoshikawa and Goda (2013)). With specific reference to bridges, Sokolov and Wenzel (2011) have carried out an assessment of the effects of the spatial correlation on economic losses, for extended targets (hypothetical portfolios) and critical elements (bridges), for a specific seismic event. Apart from the correlation in ground motion intensities, also the incurred damages and the costs of repair can be correlated for spatially distributed infrastructure (e.g., Bazzurro and Luco (2005)). In fact, Lee and Kiremidjian (2007) have presented a framework for treating both the ground motion and structure to structure correlations in risk analysis, with a practical example for a transportation network in the San Francisco Bay region.

In this work, a spatially-distributed portfolio of bridges, was considered as a case study for scenario-based loss assessment. Available data such as, fragility curves and repair costs found in literature and ground motion recordings were used in order to update GMPE, vulnerability and repair cost estimates for the earthquake of Irpinia, 1980. More specifically, available recordings of the Irpinia earthquake were used in order to update the Italian GMPE by Bindi, Pacor et al. (2011) and the corresponding spatial correlation structure between its residuals (Esposito and Iervolino 2012). The fragility curve for the considered class (herein, also the portfolio) of bridges was updated in a Bayesian updating scheme based on the bridge-specific fragility curves available in the literature for a few bridges in the highway road network of the case-study. The direct costs incurred as a result of past earthquakes is used in a Bayesian updating scheme in order to obtain updated repair cost distributions for two distinct limit states of damage and collapse, assuming perfect correlation between repair costs for individual bridge infrastructures. Finally, the effect of various alternative hypotheses with regard to the spatial correlation structure in the GMPE residuals on the estimation of the total repair costs is investigated.

1. THE MONTE CARLO SIMULATION FRAMEWORK

The total repair costs L for the entire network can be calculated as the sum of the total repair costs for each individual bridge (assuming that only the bridge infrastructure contributed to the incurred costs):

$$L = \sum_{i=1}^{N_{bridge}} RC_i \quad (1)$$

where N_{bridge} is the total number of bridges within the portfolio and RC_i is the total repair costs for bridge *i*. The probability that the total repair cost exceeds a specific value *l* can be calculated as:

$$P(L>l)=\int_{\Omega_{\boldsymbol{\theta}}} I(L>l\,|\,\boldsymbol{\theta})\,p(\boldsymbol{\theta})d\boldsymbol{\theta} \approx \frac{\sum_{i=1}^{N_{sim}} I(L_i>l\,|\,\boldsymbol{\theta}_i)}{N_{sim}} \qquad (2)$$

where $\boldsymbol{\theta}$ is the vector of all the uncertain parameters that affect L; $p(\boldsymbol{\theta})$ is the joint density function for the components of $\boldsymbol{\theta}$; $\mathbf{I}(L>l|\boldsymbol{\theta})$ is an indicator function which is equal to one only when L>l and otherwise equal to zero. In the above equation, the vector integral in Eq. 2 can be approximated using the Monte Carlo simulation with N_{sim} simulations. Let vector $\boldsymbol{\theta}$ be consisted of $\boldsymbol{\theta}=[\mathbf{IM},\mathbf{DS},\mathbf{RC}]$, where **IM** is the vector of intensity measure values at the site of each bridge (neglecting the dimension of the bridge); **DS** be the vector of the damage states for each bridge; and **RC** be the vector of repair costs evaluated for each bridge. The expression in Eq. (2) can be expanded as follows:

$$P(L>l)=\int_{\Omega_{\boldsymbol{\theta}}} I(L>l\,|\,\boldsymbol{\theta})\,f(\mathbf{RC}\,|\,\mathbf{DS})P(\mathbf{DS}\,|\,\mathbf{IM})g(\mathbf{IM})d\mathbf{RC}d\mathbf{IM} \qquad (3)$$

where g(**IM**) is the joint probability density function for the vector of IM values **IM**; P(**DS**|**IM**) is the joint probability mass function for the damage states of the all the bridges within the network given **IM**; f(**RC**|**DS**) is the joint probability density function for the vector of repair costs **RC** given **DS**. Based on some simplified assumptions, the dimension of the vector of uncertain parameters $\boldsymbol{\theta}$ can be reduced. Herein: we have assumed that: a)the repair costs for various bridges in the highway network are perfectly correlated (the repair cost for each damage state is defined by the same probability distribution over the entire network); b)the damage states for the various bridges given the value of IM at each corresponding location are perfectly correlated (i.e., portfolio is represented by a single fragility curve); c)the distribution for the repair costs given the damage state is independent of the IM. In the following, we illustrate the case-study probabilistic loss estimation for the Campania highway bridges by explaining how each of the terms within Eq.(3) are estimated.

2. GENERATION OF GROUND MOTION FIELDS FOR A SPECIFIC SEISMIC SCENARIO

The joint probability density function g(**IM**) for the vector of IM values at the location of each bridge of interest for a given earthquake scenario can be evaluated by employing a GMPE. Herein, the model proposed by Bindi, Pacor et al. (2011) for the peak ground acceleration (PGA) as the intensity measure is used. It has been assumed that the vector $\log_{10}\mathbf{IM}$ is described by a joint Normal probability distribution with the mean of the logarithm (base 10) denoted as vector **M** and covariance (of the logarithm) equal to matrix **Σ**. The components of the vector M for a given earthquake scenario and site conditions are calculated according to Bindi, Pacor et al. (2011). The covariance matrix, Σ, is defined as the sum of inter-event and intra-event components:

$$\boldsymbol{\Sigma}=\sigma^2_{INTER}\cdot\mathbf{e}+\sigma^2_{INTRA}\cdot\mathbf{R} \qquad (4)$$

where σ_{intra} represents the intra-event variability and σ_{inter} representst the inter-event variability (both parameters are tabulated in Bindi, Pacor et al. (2011)); **e** is the all ones matrix and **R** is the matrix of correlation coefficients (with diagonal terms equal to unity and off-diagonals equal to ρ). The covariance matrix is calculated according to the following alternatives:

1. ρ are calculated as (Esposito and Iervolino (2012)):
   $$\rho=\exp(-3\cdot h/b(T)) \qquad (5)$$
   where h represents the distance between sites i and j and b(T) is a coefficient equal to 10.8km;
2. ρ equal to zero (spatial correlation only due to inter-event term);
   $$\boldsymbol{\Sigma}=\sigma^2_{INTER}\cdot\mathbf{e}+\sigma^2_{INTRA}\cdot\mathbf{I} \qquad (6)$$

where **e** is the all-ones matrix and **I** is the identity matrix;

3. zero correlation between IM values:

$$\mathbf{\Sigma} = (\sigma_{INTER}^2 + \sigma_{INTRA}^2)\mathbf{I} \quad (7)$$

2.1. Updating the joint probability distribution for the IM values based on registrations

One of the specific characteristics of a joint Normal distribution for a vector of variables is that any given partition of the vector conditioned on the remaining components of the vector is still going to be a joint Normal distribution. With specific reference to the case of the vector of $\log_{10}\mathbf{IM}$ values, let the vector of mean values M and the covariance matrix $\mathbf{\Sigma}$ be partitioned as follows (Park et al. 2007):

$$M = \begin{Bmatrix} M_1 \\ M_2 \end{Bmatrix}; \Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \quad (8)$$

where:

- M_1 is the mean vector of $\log_{10}$PGA for the bridges of interest (according to the GMPE);
- M_2 is the mean vector of calculated $\log_{10}$PGA for the the stations within the area of interest (according to the GMPE);
- Σ_{11} is the covariance matrix for the calculated $\log_{10}$PGA for the bridges of interest;
- Σ_{12}=Σ_{21} is the cross-covariance matrix for the $\log_{10}$PGA values calculated at the location of the bridges and those calculated at the location of the stations;
- Σ_{22} is the covariance matrix for the $\log_{10}$PGA values calculated at the stations.

As described briefly above, the conditional distribution of the calculated $\log_{10}$PGA values given the registered $\log_{10}$PGA values at the stations is a joint Normal distribution with mean vector $M_{1|2}$ and covariance matrix $\Sigma_{11|22}$:

$$M_{1|2} = M_1 + \Sigma_{12}\Sigma_{22}^{-1}(x_2 - M_2) \quad (9)$$

$$\Sigma_{11|22} = \Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21} \quad (10)$$

where x_2 is the vector of the registered $\log_{10}$PGA values for the stations.

It should be noted that the updating scheme described herein can be applied only to the first two alternatives outlined before. This is because, assuming independence between PGA values, the cross-covariance matrices Σ_{12} and Σ_{21} are going to be equal to zero. Later on, the sensitivity of the loss estimations to five alternative cases are going to be investigated:

1. case 1 with updating (both inter- and intra-type correlations);
2. case 2 with updating (only inter- type correlation);
3. case 1 without updating (both inter- and intra-type correlations);
4. case 2 without updating (only inter- type correlation);
5. uncorrelated PGA values.

Figure 1 illustrates a PGA field generated for the bridge portfolio in question for case 1. The vector of PGA values is simulated based on the conditional joint Normal distribution outlined in Eqs. (9) and (10).

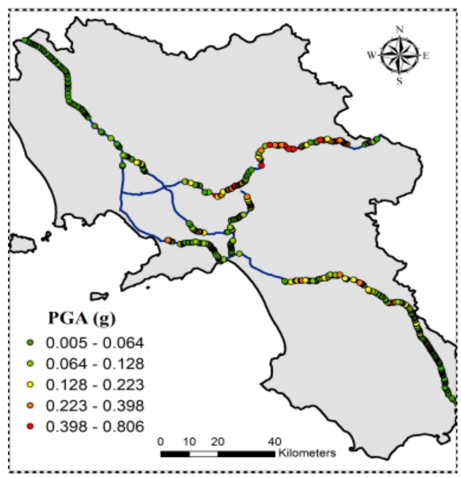

Figure 1 Example of simulation of a field of PGA for the portfolio of bridges with reference to case 1.

3. VULNERABILITY MODEL

This section describes how the probability mass function P(**DS**|**IM**) in Equation 2 is going to be evaluated. As mentioned before, it is assumed that the damage states for the various bridges given the value of IM at each corresponding location are perfectly correlated. In other words, the portfolio of bridges is represented by a single fragility curve for each damage state DS. The fragility curve for the portfolio can be represented in the scalar notation as P(DS>ds|im) or alternatively by the probability mass function

P(ds|im). The portfolio studied herein has been characterized by a single class of reinforced concrete girder bridges (a mono-class portfolio). In this work, two distinct damage states of ds=*collapse* and ds=*damage* are considered, These limit states are consistent with those defined by Noto and Franchin (2012) and with the restrictions imposed by Eucocode 8 Part 3 (1998). The probability mass function P(ds|im) can be written as:

$$P(ds\,|\,im)=\begin{cases} ds=\text{no damage} & P(ds\le damage\,|\,im)\\ ds=\text{damage} & P(ds>damage\,|\,im)\text{ - }P(ds>collapse\,|\,im)\\ ds=\text{collpase} & P(ds>collapse\,|\,im)\end{cases} \quad (11)$$

It can be argued that the median value of the PGA value marking the onset of the damage state DS denoted as $\lambda = PGA_{DS}$, under the assumption of a homogeneous mono-class portfolio, represents the onset of the damage state for the entire portfolio. Hence, the portfolio fragility curve for damage state DS can also be interpreted as the cumulative probability distribution of λ. The Bayesian updating is employed herein in order to update the *prior* portfolio fragility curves for the damage states *collapse* and *damage* based on bridge-specific fragility curves available in the literature. Let us assume that the prior probability distribution for λ is Log Normal (i.e., Log Normal prior portfolio fragility) and that the likelihood function is expressed by a Log Normal probability density. It can be shown (e.g., Singhal and Kiremidjian (1998)) that the posterior distribution of the parameter λ, f(λ), is also Log Normal with mean value $\mu''_{\log\lambda}$ and standard deviation $\sigma''_{\log\lambda}$:

$$\mu''_{\log\lambda}=\frac{\mu'_{\log\lambda}\left(\dfrac{\zeta_p^{\,2}}{n_p}\right)+\mu'_p\,\sigma'^{\,2}_{\log\lambda}}{\left(\dfrac{\zeta_p^{\,2}}{n_p}\right)+\sigma'^{\,2}_{\log\lambda}} \quad (12)$$

$$\sigma''^{\,2}_{\log\lambda}=\frac{\left(\dfrac{\zeta_p^{\,2}}{n_p}\right)\sigma'^{\,2}_{\log\lambda}}{\left(\dfrac{\zeta_p^{\,2}}{n_p}\right)+\sigma'^{\,2}_{\log\lambda}} \quad (13)$$

where:

$\mu'_{\log\lambda}$ is the prior mean of logλ;

μ'_p is the mean of the natural logarithm of the $\lambda = PGA_{DS}$ values obtained as data;

$\sigma'^{\,2}_{\log\lambda}$ is the prior variance of logλ;

ζ^2_p is the variance of the natural logarithm of the of the $\lambda = PGA_{DS}$ values obtained as data;

n_p is number of $\lambda = PGA_{DS}$ values obtained as data.

3.1. *Updating the portfolio fragility curves based on bridge-specific fragility data*

The Bayesian updating scheme outlined above has been employed in order to update the portfolio analytical fragility curves for the Italian roadway girder bridges derived in Borzi, Ceresa et al. (2014), in press, and Noto and Franchin (2012). The parameters $\mu'_{\log\lambda}$ and $\sigma'^{\,2}_{\log\lambda}$ have been estimated as the logarithmic mean and variance of the median fragility curve reported in Borzi, Ceresa et al. (2014), in press. The database, used for the updating of the fragility curves, consists of eight curves for the limit state of collapse and six curves for the limit state of damage. These fragility curves all correspond to RC girder highway bridges belonging to the portfolio of interest. This updating scheme should be viewed as a statistical exercise in portfolio fragility updating and needs to be further validated in terms of the consistency of the damage states.

Herein, the parameters μ_p and ζ^2_p have been estimated as the mean and variance of the logarithm of the median PGA_{DS} values obtained from the fragility curves used as data. In other words, from each fragility curve, only the median PGA value for each DS has been used as data.

Figures 2 and 3 show, for both limit states, the prior fragility curve, the updated fragility and the bridge-specific fragility curves. Table 1 shows the statistical parameters of the prior and of the updated fragility curves.

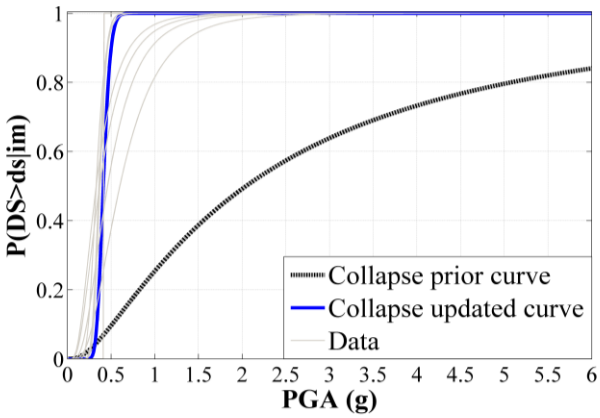

Figure 2 Collapse fragility curves.

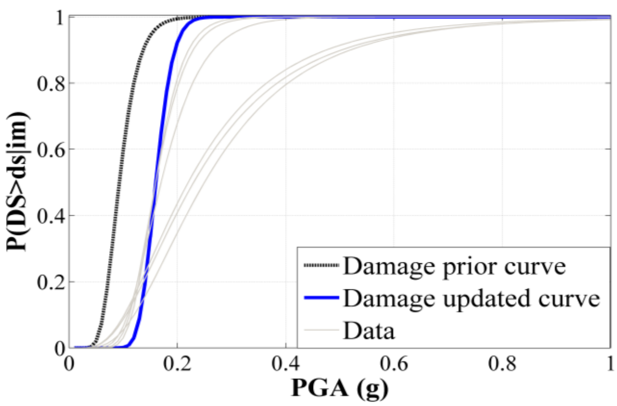

Figure 3 Damage fragility curves.

Table 1 Fragility curves parameters prior and posterior to updating.

Collapse	η (g)	β
Prior curve	2.044	1.081
Updated curve	0.411	0.155
Damage	η (g)	β
Prior curve	0.093	0.302
Updated curve	0.160	0.155

4. THE REPAIR COST MODEL

The repair cost model in Eq. (3) is expressed as the joint probability density function f(**RC**|**DS**). Assuming that the repair costs for the various bridges in the portfolio given their damage state are fully correlated, it would suffice to estimate the scalar probability density functions f(RC|DS=ds) for ds=*collapse* and ds=*damage*. Given the scarcity of documentation on direct repair costs incurred to the bridge infrastructure in the aftermath of Irpinia earthquake, we carried on the probabilistic characterization of repair costs based on available literature. The Bayesian updating framework was adopted in order to update general repair cost distributions, based on available earthquake-specific data. Assuming Log Normal prior distributions for the average costs of repair, the information provided by the South Carolina Department Of Transportation (SCDOT) has been used in order to estimate the logarithmic mean and standard deviation of the prior distributions for the damage states of *collapse* and *damage*. The repair cost data used for updating is gathered with reference to the costs of repair for the damage states of collapse and damage for bridges damaged due various past earthquakes. The updating scheme employed for the damage state *damage* is based on the closed-form reported in Eqs. (12) and (13) assuming a Log Normal likelihood function. The updating scheme for the damage state of collapse is based on an exponential likelihood function. In this case, the posterior distribution for the median repair cost $rc = (RC_{DS})$ was obtained through the following formula:

$$f(rc \mid Data, ds) = c^{-1} \cdot f(Data \mid rc, ds) \cdot f(rc \mid ds) \qquad (14)$$

where f(rc|Data,ds) is the posterior density of the median cost rc;
c^{-1} is the bayesian factor;
f(Data|rc,ds) is the likelihood density function (exponential);
f(rc) is the prior density of rc (Log Normal)

Table 2 reports the parameters of the prior, the posterior and the likelihood density functions for both damage states.

Table 2 Cost distributions parameters.

Collapse	Mean (euro/m^2)	Median (euro/m^2)	Std (euro/m^2)	Std log	COV (%)
Prior distribution	851	841	132	0.15	15.5
Likelihood distribution	2340	2020	1380	0.54	58.7
Posterior distribution	1110	1100	143	0.13	12.9
Damage	Mean (euro/m^2)	Median (euro/m^2)	Std (euro/m^2)	Std log	COV (%)
Prior distribution	68.1	67.3	10.6	0.15	15.5
Likelihood distribution	121	64.2	194	1.13	160
Posterior distribution	68.8	68.3	8.44	0.12	12.3

5. PORTFOLIO LOSS ASSESSMENT

After describing the steps for characterizing the ground shaking map, the vulnerability and the repair costs for the case-study portfolio, in this section the probability distribution for the total repair costs are calculated from Eq. (3) by using the Monte Carlo Simulation (N_{sim}=3000). For each simulation {i:i=1:N_{sim}}, a vector of IM values (or the PGA field) is generated based on the joint Log Normal distribution both before and after updating based on the available Irpinia PGA recordings in the five alternative cases described beforehand in Section 3. For each IM value generated, the probability mass function p(ds|IM) is calculated according to Eq.(11) as a function of the updated posterior fragility curves derived based on bridge-specific fragility data for the portfolio in question, as described in Section 4. This probability mass function is used in order to simulate the damage state for each bridge. For each simulation of the PGA field, the repair costs (per square meter) for the damage states *collapse* and *damage* are also simulated from the updated probability distributions reported in Table 2. Given, the damage state of each bridge in the portfolio, the corresponding repair cost is calculated as the product of the bridge area by the simulated repair cost per square meter. The total portfolio repair cost for the simulation *i*, is calculated as the sum of the repair costs for each bridge. Finally, the probability of exceeding a specific loss value *l* is calculated as the ratio of the number of bridges with L>*l* over N_{sim}. Table 3 reports the parameters of the loss probability density function for the five cases described in Section3.

Table 3 PDF parameters for the analyzed five cases.

	Mean (euro/m^2)	C.O.V. (%)
Case 1: with updating (both inter- and intra- type correlations)	2.93·10^7	63.66
Case 2: with updating (only inter- type correlation)	3.42·10^7	48.22
Case 3: without updating (both inter- and intra- type correlations)	4.26·10^7	103.8
Case 4: without updating (only inter- type correlation)	4.35·10^7	96.73
Case 5: Uncorrelated PGA values	4.22·10^7	35.92

Figure 4 below illustrates the probability of exceeding a given loss value for the 5 cases reported in Table 3.

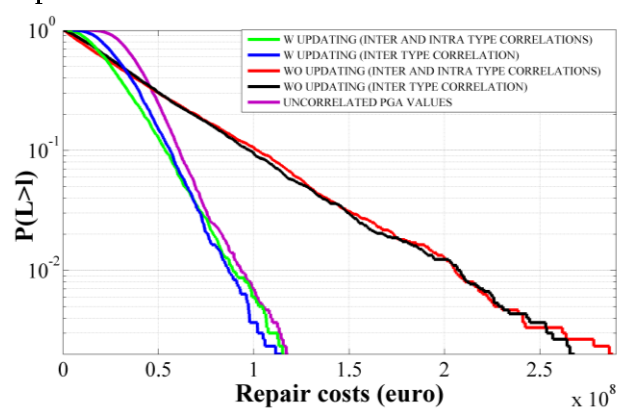

Fig. 4 The probability of exceeding a given loss value

It can be observed that in case 5 (uncorrelated PGA values) the loss dispersion is underestimated with respect to cases 3 and 4 where spatial correlation is considered (before updating). In cases 1 and 2 where updating is done (based on available PGA recordings) a slight reduction in the estimated expected repair costs and a significant reduction in the dispersion is evident. In comparison, taking into account the spatial correlation in the intra-event residuals of the GMPE seems to be not very significant in the final loss estimations for the portfolio considered. This is to be expected since most of the bridges are much more than b(T)=10.8km apart (the intra-event correlation term in Eq. 6 becomes very small for distances larger than b(T)).

6. CONCLUSIONS

In this preliminary work, a scenario-based probabilistic assessment of direct losses (repair costs) for the highway bridges in Campania was performed. This was achieved by using the available ground motion recordings for the Irpinia 1980 earthquake, and literature in order to update the ground motion prediction, fragility and repair cost models. The advantage of the updating schemes adopted herein is that they can be reproduced promptly for any seismic event that may occur in the future.

It can be observed that the spatial correlation assumptions are going to affect in a significant manner the dispersion in the portfolio repair cost distribution. Moreover, it can be seen that the updating of the GMPE based on available recordings leads to a significant reduction in the dispersion of the loss distribution. The findings of the paper are based on the assumption of a homogenous mono-class portfolio of RC girder bridges and also the assumption that the repair costs for the considered damage states are fully correlated for the bridges within the portfolio. The uniformity in the definition of the damage states through various sources of data is expected to be an important issue.

7. ACKNOWLEDGEMENTS

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