Towards quantifying the effect of aftershocks in seismic risk assessment

Fatemeh Jalayer  
*Associate Professor, Dep. of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy*

Hossein Ebrahimian  
*Post-doctoral Researcher, Dep. of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy*

Gaetano Manfredi  
*Professor, Dep. of Structures for Engineering and Architecture, University of Naples Federico II, Naples, Italy*

**ABSTRACT:** The state of art in seismic design and assessment of structures implicitly relies on structural redundancy in order to deal with the effect of the triggered sequence of aftershock events on a building. Calculating the time-dependent limit state exceedance probability for a structure considering both the main event and the triggered sequence of aftershocks is complicated both by the time-dependent rate of aftershock occurrence and also the cumulative damage caused by the sequence of events. Taking advantage of a methodology developed in previous works by the authors for post-mainshock risk assessment, the limit state probability due to a sequence of mainshock and the triggered aftershocks is calculated herein. Moreover, a closed- and analytic-form approximation to the post-mainshock limit state probability given the mainshock magnitude is derived. This closed- and analytic formulation facilitates the estimation of the limit state probability by employing the standard tools in risk assessment such as the fragility curve for the intact structure. Applying the proposed methodology for a RC concrete moment-resisting frame with infills subjected to the main event and the triggered sequence emphasizes the importance of taking into account the cumulative damage caused by the triggered sequence for the case-study model. Moreover, it is demonstrated that the proposed analytical approximation based on the fragility curve for the intact structure leads to a surprisingly close agreement with the best-estimate results obtained by considering the time- and event-dependent degradation in the structure.

1. **INTRODUCTION**

Considerable efforts have been made in the past few years in order to take into account additional damage potential due to aftershocks in seismic vulnerability assessment. This issue is addressed by subjecting the structure to various mainshock-aftershock (MS-AS) sequences. These sequences are constructed by combining a MS record with another one or by repeating it to account for aftershock record (see e.g., Luco et al. 2004, Yeo and Cornell 2009, Goda and Taylor 2012, Raghunandan et al. 2014), or by using real MS-AS duos (e.g., Goda and Taylor 2012, Goda and Salami 2014). These procedures provide a sound estimation of post-earthquake functionality.

On a different note, Jalayer et al. (2011a, b) proposed a methodology for time-dependent risk assessment in the post-mainshock environment by calculating the probability of exceeding prescribed limit states. Conditioned on the occurrence of a given MS event, this method aimed to consider the progressive damage induced by the occurrence of a sequence of aftershocks (represented by a sequence of strong-
motion events). Ebrahimian et al. (2014b) refined and implemented this methodology within a performance-based and adaptive framework in order to perform operational aftershock risk forecasting in the post-mainshock environment.

This study strives to quantify and to compare the limit state exceedance probability due to a MS and the triggered sequence of aftershocks (MS+AS sequence) with respect to the risk calculated by only considering the strong motion (MS). The procedure outlined herein for calculating the limit state probabilities considers explicitly both the time-dependent rate of occurrence for aftershocks and the cumulative damage caused by the seismic sequence (MS+AS sequence).

This non-trivial task is facilitated by deriving a simple closed-form and analytical approximation to the post-mainshock limit state exceedance probability. Based on the derived analytical formulation, a simple and approximate solution is presented later on by employing only the fragility curve for the intact structure.

As a numerical example, time-dependent risk related to (MS+AS sequence) is calculated through both the outlined “best-estimate” and “approximate” procedures. Risk is evaluated in terms of the probability of exceeding the Near Collapse (NC) limit state for a typical RC frame building with infill panels located in L’Aquila, central Italy.

2. METHODOLOGY

In this section, the “best-estimate” procedure for calculating the limit state exceedance probability due to a MS and the triggered aftershock sequence (MS+AS sequence) is presented. Risk is calculated herein in terms of the first-excursion probability of a desired (discrete) limit state. Since the triggered aftershocks following a given MS usually take place in a span in order of weeks or months, it is assumed that no additional main event will take place in the considered time interval.

2.1. Time-dependent Risk Formulation

Let \( P = P(\text{LS}|I_1) \) be the probability of the first-excursion of a desired limit state \( \text{LS} \) in time interval \([0,t]\) given information \(I_1\).

\( I_1 \): The structure may be subjected to a MS event (neglecting the probability of having more than one MS event of interest) with magnitude \( M_{ms} \) so that \( M_L \leq M_{ms} \leq M_u \) (\( M_L \) and \( M_u \) are the site-specific lower- and upper-bound magnitudes) followed by a sequence of triggered aftershocks (AS sequence) with magnitudes \( M_{as} \) in the range of \( M_{l,as} \leq M_{as} \leq M_{u} \) (\( M_{l,as} \) is the lower bound magnitude for aftershocks).

The dependence on \( I_1 \) has been dropped hereafter for the sake of brevity and readability. Nevertheless, unless otherwise specified, the probability terms derived in this work are all conditioned on \( I_1 \).

The probability \( P \) can be further broken down as follows: \( \text{LS} \) is exceeded after the MS (\( P_{ms} \)), or it is exceeded during the AS sequence that is triggered by the MS given that it is not exceeded due to the MS (\( P_{as} \)):

\[
P = P_{ms} + P_{as} (1 - P_{ms})
\]

The probability term \( P_{ms} \) can be interpreted as the limit state probability associated with the MS event. Adopting the first-mode spectral acceleration \( Sa(T) \) as the ground motion intensity measure and assuming that the limit state exceedance can be described by a homogenous Poisson Process with rate \( \lambda_{LS} \),

\[
\lambda_{LS} = \int_{s_{ms}}^{s_{u}} \pi_0(s_{ms}) \frac{d\lambda_{ms}(s_{ms})}{ds_{ms}} ds_{ms}
\]

where \( s_{ms} \) is the \( Sa(T) \) associated with the MS event; \( \pi_0 = P(\text{LS}|s_{ms}) \) is the structural fragility for the MS event (also known as the fragility of the intact structure); and \( \lambda_{ms} \) is the site-specific hazard defined as the mean rate of exceeding \( Sa(T) \) (note that the rates \( \lambda_{LS}, \lambda_{ms} \) and time \( t \) should have consistent units; i.e., the product of the rate and time is dimensionless). Accordingly, the probability of exceeding the limit state \( P_{ms} \) in time interval \([0,t]\) can be calculated as:
The probability \( P_{as} \) is the limit state first-excursion probability associated with the AS sequence following the MS event given that the MS has not caused the limit state excursion. This term can be expanded based on Total Probability Theorem (Benjamin and Cornell 1970) over all possible MS wave forms \( x_g \) and magnitudes \( m \):

\[
P_{as} = \int P(LS_{as} \mid m, x_g) p(x_g \mid m) p(m) dx_g dm
\]

\[
\approx 1/N_{gm} \sum_{i=1}^{N_{as}} P(LS_{as} \mid m_i, x_{ig})
\]

(4)

where \( P(LS_{as} \mid m, x_g) \) is the probability of \( LS \) first-excursion for an arbitrary AS event belonging to the AS sequence, given that the MS has a wave-form denoted by \( x_g \) and magnitude \( m \); \( p(x_g \mid m) \) is the probability of MS wave-form given magnitude \( m \); and \( p(m) \) is the probability of having MS with a magnitude \( m \). Based on the working assumption (see e.g., Jalayer et al. 2012) that different wave-forms are equally likely to occur, Eq. (4) can be approximated by the average of \( P(LS_{as} \mid m, x_g) \) values over a set of \( N_{gm} \) ground motion records (which have not caused \( LS \) excursion for the intact structure).

To estimate \( P(LS_{as} \mid m, x_g) \) in Eq. (4), the aftershock risk assessment procedure derived by Jalayer et al. (2011a) and later refined in Ebrahimian et al. (2014b) is used herein. This procedure leads to the estimation of time-dependent limit state first-excursion probabilities given that the MS magnitude and its wave-form is known, taking into account both the time-varying aftershock occurrence rate and also the cumulative damage caused by the sequence of aftershocks.

Let \( N_{as} \) be the maximum number of AS events that may take place in the time interval \([0, t]\). The limit state probability \( P(LS_{as} \mid m, x_{ig}) \) can be expanded over all possible AS events that can take place in the time-interval \([0, t]\) based on Total probability Theorem (Benjamin and Cornell, 1970):

\[
P_{as} = 1 - \exp(-\lambda_{as,t})
\]

where \( P(LS_{as} \mid m, x_{ig}) \) is the probability of exceeding the limit state \( LS \) for the first time given that exactly \( n_{as} \) AS events take place given \( m \) and \( x_{ig} \). The term \( P(n_{as} \mid m, x_{ig}) \) is the conditional probability that exactly \( n_{as} \) AS events take place given MS magnitude and its wave-form. This probability is defined herein by a non-homogenous Poisson process with parameter \( \nu_{as} \), the probability \( P(n_{as} \mid m, x_{ig}) \) can be calculated as (note that we have dropped the dependence on MS wave-form):

\[
P(n_{as} \mid m) = \left( \nu_{as} \right)^{n_{as}} e^{-\nu_{as}}
\]

(6)

Note that the parameter \( \nu_{as} \) is both time- and MS magnitude-dependent (see Ebrahimian et al. 2014a for more details). The parameters of the MO model can be obtained from a generic territorial model.

The probability \( P(LS_{as} \mid m, x_{ig}, n_{as}) \) in Eq. (5) can be calculated by taking into account the set of mutually exclusive and collectively exhaustive (MECE) events that \( LS \) first-excursion happens at one and just one of the \( n_{as} \) AS events (Ebrahimian et al. 2014b):

\[
P(LS_{as} \mid m, x_{ig}, n_{as}) = \sum_{k=1}^{n_{as}} \left( \Pi_{k} \prod_{i=1}^{k-1} (1-\Pi_{i}) \right)
\]

(7)

where \( \Pi_{k} \) denotes the probability of \( LS \) first-excursion due to the occurrence of the \( k \)th AS event in the sequence given that the limit state has not exceeded in the previous \((k-1)\) events. The
probability term $\Pi_k$ can be expanded with respect to the first-mode spectral acceleration $Sa(T)$ as follows (Ebrahimian et al. 2014b):

$$\Pi_k = \frac{1}{\nu_{as}} \int \pi_k (sa_{as}) \frac{d\lambda_{as} (sa_{as})}{ds_{as}} ds_{as} \quad (8)$$

where $sa_{as}$ is $Sa(T)$ associated with the $k$th aftershock event; $\pi_k = P(LS_{as} | sa_{as})$ is an event-dependent fragility for the $k$th aftershock event (defined as the probability of exceeding the limit state $LS$ due to the $k$th event given that it has not been exceeded due to the previous $k-1$ events; and $\lambda_{as}$ is the mean (in unit of time $t$) rate of exceeding $Sa(T)$ equal to $sa_{as}$ (see Ebrahimian et al. 2014a for the description of aftershock hazard calculation).

2.2. Closed-form approximation of the time-dependent limit state probabilities

In this section, the term $P(LS_{as} | m_i, x_{ig})$ in Eq. (5) is approximated with a closed-form analytic expression. A preliminary version of this closed-form expression was proposed by Jalayer et al. (2014a). Assume that the set of probability terms \{\Pi_k | k=1:n_{as}\} are identical and equal to the time- and event-invariant function $\Pi$. Thus, $P(LS_{as} | m_i, x_{ig}, n_{as})$ in Eq. (7) is calculated as the sum of a geometric series:

$$P(LS_{as} | m_i, x_{ig}, n_{as}) = \Pi \left( 1 - \sum_{k=1}^{n_{as}} (1-\Pi)^{k-1} \right) = 1 - (1-\Pi)^{n_{as}} \quad (9)$$

Substituting $P(LS_{as} | m_i, x_{ig}, n_{as})$ from Eq. (9) and $P(n_{as} | m_i)$ from Eq. (6) into Eq. (5), and performing some simple algebraic manipulations (considering that the sum of the Poisson probability mass function terms is asymptotically equal to unity for large $N_{as}$ values):

$$P(LS_{as} | m_i, x_{ig}) = 1 - \sum_{n_{as}=1}^{N_{as}} \left( 1 - \Pi \right)^{n_{as}} = 1 - \sum_{n_{as}=1}^{N_{as}} \left( 1 - \Pi \right)^{n_{as}} \quad (10)$$

The second term in Eq. (10) is a filtered Poisson probability mass function with a binary filter having probability $\Pi$. Thus, Eq. (10) can be simplified as follows:

$$P(LS_{as} | m_i, x_{ig}) = 1 - e^{-\Pi \nu_{as} \sum_{n_{as}=1}^{N_{as}} \left( (1-\Pi) \nu_{as} \right)^{n_{as}} e^{-((1-\Pi)\nu_{as})}} \quad (11)$$

Finally, the closed-form approximation to the time-dependent limit state probability $P(LS_{as} | m_i, x_{ig})$ is derived as:

$$P(LS_{as} | m_i, x_{ig}) = 1 - \exp(-\Pi \nu_{as}) \quad (12)$$

The closed-form expression in Eq. (12) provides a simple analytic equation for calculating the time-dependent $LS$ first-exursion probability due a sequence of aftershocks triggered by a MS of magnitude $m_i$ and wave-form $x_{ig}$. An extremely convenient proposal for $\Pi$ can be $\Pi_0=\lambda_{L, S}/\nu_{ms}$, which is calculated by dividing the limit state exceedance rate $\lambda_{L, S}$ due to the MS from Eq. (2) by the seismicity rate of MS events $M_{L, S} \leq M_{ms} \leq M_{as}$, denoted by $\nu_{ms}$, taking care that $\lambda_{L, S}$ and $\nu_{ms}$ should have the same temporal unit.

3. NUMERICAL APPLICATION

The methodology described in Section 2 is applied herein in order to perform risk assessment for (MS+AS sequence) for a typical RC building with infill panels located in L’Aquila, central Italy.

3.1. Case Study Definitions

The case study structure is a shear building model of a 3-story 2-bay RC two-dimensional frame having one-bay infill panel. The first-mode period of the building is equal to 0.27sec. A comprehensive representation of the model with the attributed nonlinear behavior of columns and infill panel can be found in Ebrahimian et al. (2014b). Since the damage is mainly accumulated in the first story, the displacement of the first story is taken to be the engineering demand parameter (EDP) in this study. The nonlinear dynamic analyses are performed using OpenSees (http://opensees.berkeley.edu, ver. 2.4.4) by adopting properly calibrated hysteretic models.
Only one discrete limit state of Near Collapse (NC) is considered in this study, which is conservatively set to 10% drop in ultimate strength of the columns based on the pushover analysis of the structure (see Ebrahimian et al. 2014b for more details).

The latitudinal and longitudinal coordinates of the site where the reference structure is located are [42.3450, 13.4009]. This area is located within the seismic zone 923 based on the ZS9 Italian Seismogenetic Zonation (Gruppo di Lavoro, 2004), in which the key seismicity parameters are: $v_{ms}=0.14$ (/year), $M_L=4.76$, $M_d=7.06$. The aftershock zone considered herein is the one presented in Ebrahimian et al. (2014a), which is located within the prescribed latitude/longitude cells of a $0.1\times0.1$ degree grid of area $[13.15-13.65\, E, 42.10-42.70\, N]$. The occurrence of aftershocks are modeled by the MO model proposed by Jalayer et al (2011a). Based on a Bayesian updating framework, they have updated the Italian generic parameters of MO model based on the sequence of aftershocks following the L’Aquila earthquake of 6th April 2009.

3.3. Calculation of Event-dependent Fragilities

The methodology for calculating the event-dependent fragilities denoted as $\{\pi_k|k=1:n_{as}\}$ is comprehensively discussed in the previous works by the authors (Ebrahimian et al. 2014b). The structural performance variable denoted as $Y_{LS}$ is calculated as following:

$$Y_{LS}^{(k)} = \frac{D_{\max}^{(k)} - D_r^{(k-1)}}{C_{LS} - D_r^{(k-1)}} \quad \text{(13)}$$

where $D_{\max}^{(k)}$ is the maximum EDP due to the $k$th event; $D_r^{(k-1)}$ is the associated residual demand corresponding to the sequence of $(k-1)$ events; $C_{LS}$ is the limit state capacity of the (intact) structure. This history-dependent parameter, which is the ratio of maximum demand increment to a reduced limit state capacity, reveals appropriate correlation with $S_a(T)$ of the $k$th event within the sequence.

For performing the Cloud Analysis (see Elefante et al. 2010 and Jalayer et al. 2014b), a set of 50 European (especially Italian) ground motion records, including strong ground-motions as well as aftershocks, are selected from the NGA-West2 database (Ancheta et al. 2014). The set of records covers a wide range of magnitudes from 4.5 up to 7.5, and closest distance to ruptured area up to 80 km. It is to note that accurate record selection can significantly affect the results (see e.g. Goda 2014); however, this issue is not a primary focus in this study. The set of event-dependent fragility curves $\{\pi_k|k=1:13\}$ are shown in Figure 1 together with the cloud regression associated with $\pi_0$, which is the fragility of the intact structure. For simplicity of calculations while $k>13$, the event-dependent fragility $\pi_k=\pi_{13}$ is considered for calculation of $\Pi_k$ in Eq. (8).
The probability of exceeding the limit state in a reference time interval \([t, t+\Delta t]\) (say e.g. \(\Delta t=1\) day) can also be calculated as the difference of the exceedance probabilities in intervals \([0,t+\Delta t]\) and \([0,t]\) (see Jalayer et al. 2011a, b). The resulting daily limit state probabilities as a function of time \(t\) for \(\Delta t=1\) day are plotted in Figure 3. Assuming equal number of events in 1 year between two Poisson processes with daily and annual unit

3.4. Time-dependent Risk Calculation

The time-dependent limit state probability due to (MS+AS sequence) \(P=\text{P}(\text{LS}|I_1)\) is calculated according to Eq. (1). The limit state exceedance probability in time \([0,t]\) due to MS denoted by \(P_{ms}\) is calculated according to Eq. (3).

The first-excursion limit state probability due to the AS sequence, \(P_{as}\) in Eq. (1), is calculated from Eq. (4) by integrating the post-mainshock limit state probability given MS magnitude and wave-form \(P(\text{LS}_{as}|m_i, x_{ig})\) over all possible MS magnitudes \(m\) and wave-forms \(x_{ig}\). As mentioned before, this expression is approximated as the average of \(P(\text{LS}_{as}|m_i, x_{ig})\) values for a suite of strong motion records which have not led to limit state excursion of the intact structure. \(P(\text{LS}_{as}|m_i, x_{ig})\) is calculated according to Eq. (5). With reference to Eq. (5), \(P(n_{as}|m_i, x_{ig})\) is estimated by the Poisson probability distribution from Eq. (6), and the limit state probability \(P(\text{LS}_{as}|m_i, x_{ig}, n_{as})\) (due to aftershocks) given the MS magnitude \(m_i\) wave-form \(x_{ig}\) and the number of aftershock events \(n_{as}\) can be calculated from the recursive formulation in Eq. (7) as a function of \(\{\Pi_k|k=1:n_{as}\}\). Accordingly, \(\Pi_k\) is the probability of \(LS\) first-excursion due to \(k\)th aftershock event given that the limit state has not exceeded in the previous \((k-1)\) events, and is estimated from Eq. (8).

The probabilities \(P_{ms}\) (red dotted line), and the “best-estimate” for \(P\) (calculated through the procedure described in Section 2 and plotted as black solid lines) are all plotted in Figure 2. It can be observed that the consideration of the cumulative damage caused by the aftershocks for the studied case (building severely damaged by the MS), will significantly increase the estimated time-variant risk \(P\).

Now, it is going to be interesting to see how the derived analytical closed-form for \(P(\text{LS}_{as}|m_i, x_{ig})\) in Eq. (12) is going to estimate the effect of the cumulative damage caused by the aftershocks. The resulting “approximate” \(P\) is plotted in blue dashed line in Figure 2. It can be observed that, although obtained with significantly reduced analytic effort (almost the same analytical effort as the one needed for obtaining \(P_{ms}\)), the “approximate” time-dependent limit state probability \(P\) roughly manages to capture the effect of the cumulative damage caused by the AS sequence. It should be mentioned that, had we used the MS damaged fragility \(\Pi_1\) from Eq. (8) (instead of \(\Pi_0\)), the “approximate” \(P\) would be almost identical to the “best-estimate”.

The probability of exceeding the limit state in a reference time interval \([t, t+\Delta t]\) (say e.g. \(\Delta t=1\) day) can also be calculated as the difference of the exceedance probabilities in intervals \([0,t+\Delta t]\) and \([0,t]\) (see Jalayer et al. 2011a, b). The resulting daily limit state probabilities as a function of time \(t\) for \(\Delta t=1\) day are plotted in Figure 3. Assuming equal number of events in 1 year between two Poisson processes with daily and annual unit
times, we have calculated the equivalent admissible mean daily rate of exceedance equal to $5.6 \times 10^{-6}$ corresponding to a mean annual admissible rate of exceedance equal to $2 \times 10^{-3}$ (475 year return period). For the first 100 days elapsed after the MS, the daily limit state probability is decisively above the admissible threshold (in green), while it is below the green line considering only MS (red dotted line). A perfect agreement between the “best-estimate” and the “approximate” results can be observed, as well.

4. CONCLUSIONS
The findings in this work can be synthesized as follows:

1. A procedure for calculating the limit state exceedance probability due to (MS+AS sequence) is proposed.
2. This methodology explicitly takes into account the cumulative damage caused by the AS sequence for MDOF structures with degrading behavior.
3. A closed-form analytical expression is derived for calculating the post-mainshock limit state probability. The simplest application of this analytic formulation based on the fragility of intact structure leads to a much-better-than-expected agreement with the computationally extensive “best-estimate” results.
4. For the particular building studied herein, considering the triggered AS sequence significantly affects the limit state probability in the immediate aftermath of the main event. The obtained results seem to be consistent with those obtained in the previous work by the authors (Ebrahimian et al. 2014b).
5. It is expected that the results are sensitive to the amount of damage sustained by the building due to the MS and the characteristics of the AS sequence.

The proposed procedure is based on the strong working assumption that various MS events are equally likely to take place. It should also be reminded that the Cloud Analysis, employed herein for the calculation of event-dependent fragilities, is quite sensitive to the selection of records.

5. ACKNOWLEDGEMENT
This work was supported in part by project STRIT (Strumenti e Tecnologie per la gestione del Rischio delle Infrastrutture di Trasporto). This support is gratefully acknowledged.

6. REFERENCES


