# MCMC-based Updating of an Epidemiological Temporal Aftershock Forecasting Model

F. Jalayer[1] and H. Ebrahimian[2]

[1]Assisstant Professor, Department of Structures for Engineering and Architecture, University of Naples Federico II, Via Claudio 21, NA 80125, Naples, Italy; PH (0039) 081-7683671; FAX (0039) 081-7683671; e-mail: fatemeh.jalayer@unina.it
[2]Post-doctoral Researcher, Department of Structures for Engineering and Architecture, University of Naples Federico II, Via Claudio 21, NA 80125, Naples, Italy; e-mail: ebrahimian.hossein@unina.it

## ABSTRACT

The first few days elapsed after the occurrence of a strong earthquake and in the presence of an ongoing aftershock sequence are quite critical for emergency decision-making purposes. Epidemic Type Aftershock Sequence (ETAS) models are used frequently for forecasting the spatio-temporal evolution of seismicity in the short-term. In such a context, the forecasted seismicity usually makes reference to a 24-hour forecasting interval. Focusing the attention on clustering of events in time only, a robust seismicity forecast based on (a simplified version of) the ETAS model takes into account the joint probability distribution for the model parameters conditioned on the events already registered with the ongoing sequence. The advanced simulation procedures such as Markov Chain Monte Carlo simulation provide very efficient means of estimating the robust seismicity forecasts. In addition to the uncertainty in model parameters, the robust simulation-based forecasting of seismicity can also take into account the uncertainty in the sequence of events that are going to happen during the sampling interval. With regard to a specific application to the L'Aquila 2009 seismic sequence, the daily robust ETAS forecasts in the first ten days elapsed after the main event can predict the seismicity within plus/minus one standard deviation interval.

## INTRODUCTION

After the occurrence of high-magnitude earthquakes and in the presence of aftershocks, short-term seismicity forecasts (in the order of days to months) are of utmost importance for decision-making. The short-term forecasts can be made based on stochastic models that describe the spatio-temporal clustering of earthquakes in space and time (for an extensive review see Jordan *et al.* 2011). The Epidemic Type Aftershock Sequence (ETAS, Ogata, 1988; 1998) model is an epidemic stochastic point process in which every earthquake is a potential triggering event for subsequent earthquakes. The ETAS model performed quite well in operational seismic forecasting during the recent L'Aquila seismic sequence (Marzocchi and Lombardi, 2009).

The ETAS model parameters are usually calibrated a priori and based on a set of events that do not belong to the on-going seismic sequence (Marzocchi and Lombardi 2009). However, adaptive model parameter estimation, based on the events in the on-going sequence, may have several advantages such as, tuning the model to the specific sequence characteristics, and capturing possible variations in time of the model parameters. For instance, Jalayer *et al*. (2011) and Ebrahimian *et al*. (2014) estimated the parameters of the Modified Omori Model (Utsu, 1961) based on the ongoing sequence catalogue and by employing Bayesian parameter estimation.

Focusing the attention on the temporal dimension (the spatial dimension is not considered herein), simulation-based methods are employed in order to provide a *robust* estimate for the forecasted number of events in a prescribed forecasting time interval (i.e., a day) after the main event. The robust estimate takes into account the uncertainty in the model parameters expressed as the posterior joint probability distribution for the model parameters conditioned on the events that have already occurred (i.e., before the beginning of the forecasting interval) in the on-going seismic sequence. The Markov Chain Monte Carlo simulation scheme is used in order to sample directly from the posterior probability distribution for ETAS model parameters. Moreover, the sequence of events that is going to occur during the forecasting interval (and hence affect the seismicity in an epidemic type model like ETAS) is also generated through a stochastic procedure. The procedure leads to a probability distribution for the forecasted number of events and the uncertainty in estimating the probability of exceeding a certain number of events. The robust ETAS forecasts can be directly implemented in adaptive daily aftershock hazard and risk assessment procedures (Ebrahimian *et al*. 2013, 2014)

## ROBUST SEISMICITY FORECASTING

Let the aftershock occurrence be described by a non-homogenous Poisson point process over time. The process can be identified by $\lambda(t,m|\mathbf{seq})$ which represents the rate of occurrence of events with magnitude greater than or equal to $m$ at time $t$ (elapsed after the main event) in the forecasting interval $[T_{start},T_{end}]$ and given the sequence of aftershocks, **seq**. This sequence of events **seq** includes the aftershock events before the forecasting interval. The average number of events with magnitude greater than or equal to $m$ in the forecasting interval can be calculated as (see also Ebrahimian *et al*. 2014):

$$N\left(m\middle|\mathbf{seq},M_l\right)=\int_{T_{start}}^{T_{end}}\lambda\left(t,m\middle|\mathbf{seq},M_l\right)\mathrm{d}t \tag{1}$$

where $M_l$ is the lower cut-off magnitude.

Let $\mathbf{\Theta}$ denotes the vector of model parameters for $\lambda(t,m|\mathbf{seq},M_l)$. Conditioned on a particular space-time model and a specific realization of the vector of model parameters $\mathbf{\Theta}$, one can calculate a plausible value for the rate of occurrence denoted as $\lambda(t,m|\mathbf{\Theta},\mathbf{seq},M_l)$ (we have not included the conditioning on the model assumptions for the sake of brevity). A robust (Papadimitriou *et al*. 2001; Beck and Au 2002; Jalayer and Beck 2008; Jalayer *et al*. 2010) estimate of the average (expected) number of events with magnitude greater than or equal to $m$ in the forecasting interval $[T_{start}, T_{end}]$ and over the domain of the model parameters $\mathbf{\Omega_\Theta}$ can be calculated as:

$$\mathbb{E}\left[N\left(m\middle|\mathbf{seq},M_l\right)\right]=\int\limits_{\mathbf{\Omega_\Theta}}\int_{T_{start}}^{T_{end}}\lambda\left(t,m\middle|\mathbf{\Theta},\mathbf{seq},M_l\right)p(\mathbf{\Theta}\mid\mathbf{seq},M_l)\,\mathrm{d}\mathbf{\Theta}\mathrm{d}t \tag{2}$$

where $p(\mathbf{\Theta}|\mathbf{seq},M_l)$ is the joint conditional probability distribution for $\mathbf{\Theta}$ given the **seq** and the lower cut-off magnitude $M_l$.

## AN EPIDEMIOLOGICAL MODEL FOR TIME CLUSTERING OF AFTERSHOCKS

The ETAS model (Ogata 1988; Ogata 1998; Zhuang *et al*. 2002; Marzocchi and Lombardi 2009) is an epidemic stochastic point process in which every earthquake is a potential triggering event for subsequent earthquakes. Thus, the seismicity is expressed as the superposition of the triggering effect induced by previous events on the background/base seismicity. Herein, a simplified version of the ETAS model, with temporal clustering only, is considered. In this simplified version, the seismicity rate for events with magnitude greater than $m$ in time t and denoted by $\lambda_{\mathrm{ETAS}}(t,m|\mathbf{\Theta},\mathbf{seq},M_l)$ is calculated as:

$$\lambda_{\mathrm{ETAS}}\left(t,m\middle|\mathbf{\Theta},\mathbf{seq},M_l\right)=\lambda_{\mathrm{ETAS}}\left(t,M_l\middle|\mathbf{\Theta},\mathbf{seq}\right)e^{-\beta(m-M_l)} \tag{3}$$

$$\lambda_{\mathrm{ETAS}}\left(t,M_l\middle|\mathbf{\Theta},\mathbf{seq}\right)=\sum_{t_i<t}\frac{Ke^{\beta(M_i-M_l)}}{\left(t-t_i+c\right)^p} \tag{4}$$

where $\mathbf{seq}=\{(t_i,M_i),\ t_i<T_{start}\}$ refers to the catalogue of events up to time $T_{start}$; parameters $K$, $c$, and $p$ are those of the Modified Omori's Law (Utsu 1961) defining the decay in time of the short-term triggering effect, $\beta$ defines the dependence of triggering capability on the magnitude of an earthquake; $\beta=b\ \ln(10)$ for which $b$ represents the seismicity rate of the considered site (note that, in general, the parameter that defines the magnitude-dependence of the triggering capability is different from $\beta$). Therefore, the vector of ETAS model parameters is $\mathbf{\Theta}=[\beta, K, c, p]$.

The rate of events with magnitude exactly equal to $m$ can be calculated by taking the derivative of Eq. 3 with respect to magnitude $m$:

$$\begin{aligned}\mu_{\mathrm{ETAS}}\left(t,m\mid\mathbf{\Theta},\mathbf{seq},M_l\right)&=\left|\partial\lambda_{\mathrm{ETAS}}\left(t,m\mid\mathbf{\Theta},\mathbf{seq},M_l\right)/\partial m\right|=\\ &\beta\,\lambda_{\mathrm{ETAS}}\left(t,M_l\mid\mathbf{\Theta},\mathbf{seq},M_l\right)e^{-\beta(m-M_l)}\end{aligned} \tag{5}$$

## SIMULATION OF THE SEQUENCE OF EVENTS IN THE FORECASTING TIME INTERVAL

As mentioned above, the sequence of events **seq**=$\{(t_i,M_i),\ t_i<T_{start}\}$ refers to the registered aftershocks taking place before the beginning of the forecasting interval $[T_{start},T_{end}]$. Given the fact that ETAS is an epidemic type model, the sequence of aftershock events, taking place during the forecasting interval is simulated / generated herein. This generated sequence is denoted as **seqg.**

Let us assume that a possible **seqg** is defined as a set of pairs of arrival times and magnitudes defined as **seqg**=$\{(t_i,M_i),\ T_{start}\le t_i\le T_{end}\}$. A robust estimate for the number of aftershock events based on ETAS model should also take into account the uncertainty in the sequence of events **seqg** that is going to happen during the forecasting time interval:

$$\mathbb{E}\left[N\left(m\middle|\mathbf{seq},M_l\right)\right]=$$
$$\int_{\Omega_{\mathbf{seqg}}}\int_{\Omega_{\mathbf{\Theta}}}\int_{T_{start}}^{T_{end}}\lambda_{\mathrm{ETAS}}\left(t,m\middle|\mathbf{\Theta},\mathbf{seq},\mathbf{seqg},M_l\right)p\left(\mathbf{seqg}\mid\mathbf{\Theta},\mathbf{seq},M_l\right)p\left(\mathbf{\Theta}\mid\mathbf{seq},M_l\right)\mathrm{d}t\,\mathrm{d}\mathbf{\Theta}\,\mathrm{d}\mathbf{seqg} \tag{6}$$

where $p(\mathbf{seqg}|\mathbf{\Theta},\mathbf{seq},M_l)$ is the conditional probability distribution for the generated sequence **seqg** within the forecasting time interval, and consequently, $\lambda_{\mathrm{ETAS}}(t,m|\mathbf{\Theta},\mathbf{seq},\mathbf{seqg},M_l)$ is the time-dependent rate of ETAS model conditioned on both the registered and generated sequence of events.

### Generating sequences according to $p(\mathbf{seqg}|\mathbf{\Theta},\mathbf{seq},M_l)$

The probability distribution $p(\mathbf{seqg}|\mathbf{\Theta},\mathbf{seq},M_l)$ can be written as follows (based on the probability product rule, Jaynes 2003):

$$p\left(\mathbf{seqg}\mid\mathbf{\Theta},\mathbf{seq},M_l\right)=$$
$$\left[\prod_{i=1}^{n\mathrm{gen}}p\left(t_i,M_i\mid\mathbf{\Theta},\mathbf{seq},\mathbf{seqg_{i-1}},M_l\right)\right]\cdot P\left(t_{n\mathrm{gen}+1}>T_{end},M>M_l\mid\mathbf{\Theta},\mathbf{seq},\mathbf{seqg},M_l\right) \tag{7}$$

where $n$gen in the first term (within the brackets) is the number of generated events within the forecasting time interval $[T_{start},T_{end}]$, which is unknown at the time of generation. The second term defines the Cumulative Density Function (CDF) of inter-arrival time for event $n$gen+1 with magnitude $M>M_l$ for the time interval between the last arrival time $t_{n\mathrm{gen}}$ and the end of the forecasting interval $T_{end}$. It is noteworthy that for the first generated event ($i$=1), the condition $t_1>T_{start}$ should be satisfied. The probability distribution $p(t_i,M_i|\mathbf{\Theta},\mathbf{seq},M_l)$ can be further expanded (again using the probability product rule):

$$p\left(t_i,M_i\mid\mathbf{\Theta},\mathbf{seq},M_l\right)=p\left(t_i\mid\mathbf{\Theta},\mathbf{seq},M_l\right)p\left(M_i\mid t_i,\mathbf{\Theta},\mathbf{seq},M_l\right) \tag{8}$$

where $p(t_i|\mathbf{\Theta},\mathbf{seq},M_l)$ is the marginal probability distribution for the arrival time given the information (i.e., vector of parameters $\mathbf{\Theta}$, the sequence **seq**, and the lower cut-off magnitude $M_l$). Moreover, $p(M_i|t_i,\mathbf{\Theta},\mathbf{seq},M_l)$ is the marginal probability distribution for $M_i$ given that the value of arrival time is equal to $t_i$, and the above-mentioned prior information.

In the context of the robust estimation method outlined in Equation (6), the sequence of events **seqg** can be generated in an adaptive manner with reference to Eqs. 7 and 8. Hence, the break-down into the product of several marginal probability distributions (as shown in Eq. 8) is necessary during the sequence generation process. This means that the $i$th event within the **seqg**, which is distinguished by the pair $(t_i,M_i)$, is simulated by conditioning on all the previous events within the **seq**. This event is generated by first simulating the arrival time $t_i$ from $p(t_i|\mathbf{\Theta},\mathbf{seq},M_l)$. Subsequently, the $M_i$ is simulated from $p(M_i|t_i,\mathbf{\Theta},\mathbf{seq},M_l)$. The simulation procedure is started with $t_1>T_{start}$ and continued while $t_i<T_{end}$. This latter condition will be equivalent of sampling from the probability of having no events with magnitude greater than $M_l$ after the $n$gen simulated events in the forecasting interval (see Eq. 7).

***Sampling arrival times from $p(t_i|\mathbf{\Theta},\mathbf{seq},M_l)$ by employing the thinning method***

Considering the probability distribution $p(t_i|\mathbf{\Theta},\mathbf{seq},M_l)$ as a non-homogeneous Poisson process with rate $\lambda_{\mathrm{ETAS}}(t_i,M_l|\mathbf{\Theta},\mathbf{seq})$ according to Eq. 4, the arrival times can be simulated by employing the very efficient thinning algorithm (Lewis and Shedler 1978; Ogata 1981).

***Sampling magnitudes from $p(Mi|ti,\mathbf{\Theta},\mathbf{seq},M_l)$***

The conditional marginal probability distribution for magnitude $p(M_i|t_i,\mathbf{\Theta},\mathbf{seq},M_l)$ is calculated herein by assuming that the occurrence of events with $M=M_i$ is independent of the its arrival time, $t_i$. Assuming a Gutenberg-Richter magnitude recurrence model, it can be shown that:

$$p\left(M_i \mid \mathbf{\Theta},\mathbf{seq},M_l\right)=\beta e^{-\beta(M_i-M_l)} \tag{9}$$

Thus, $M_i$ is sampled (independently from the arrival time) from a truncated exponential distribution with the rate $\beta$.

**Using Markov Chain Monte Carlo (MCMC) Simulation for Calculation of $\mathbb{E}[N(m|\mathbf{seq},M_l)]$**

In the robust simulation-based procedure outlined in Eq. 6 for forecasting the number of events with magnitude more than $m$, the MCMC simulation procedure can be used for taking into account the uncertainty in ETAS model parameters $\mathbf{\Theta}$. As mentioned before, the uncertainty in $\mathbf{\Theta}$ is represented by sampling directly from the posterior probability distribution $p(\mathbf{\Theta}|\mathbf{seq},M_l)$. This probability distribution can be calculated as (using Bayesian parameter estimation):

$$p\left(\mathbf{\Theta} \mid \mathbf{seq},M_l\right)=c^{-1}p\left(\mathbf{seq} \mid \mathbf{\Theta},M_l\right)p\left(\mathbf{\Theta} \mid M_l\right) \tag{10}$$

where $p(\mathbf{seq}|\mathbf{\Theta},M_l)$ denotes the likelihood of the observed sequence given the vector of model parameters $\mathbf{\Theta}$ and cut-off magnitude $M_l$; $p(\mathbf{\Theta}|M_l)$ is the prior distribution for the model parameters $\mathbf{\Theta}$, and $c^{-1}$ is a normalizing constant. The MCMC simulation routine is particularly useful for cases where the sampling needs to be done from a probability distribution that is known up to a constant value (Beck and Au 2002). This method employs the Metropolis-Hastings (MH) algorithm (Metropolis *et al.* 1953, Hastings 1970) in order to generate samples as a Markov Chain sequence which is used later to estimate the robust reliability (Eq. 6).

***The likelihood of the observed sequence***

The likelihood for the observed events within the **seq** can be calculated as:

$$p\left(\mathbf{seq}\,|\,\mathbf{\Theta},M_l\right)=\left[\prod_{i=1}^{nseq}p\left(t_i,M_i\,|\mathbf{\Theta},\mathbf{seq}_{i-1},M_l\right)\right]P\left(t_{nseq+1}>T_{start},M>M_l\,|\,\mathbf{\Theta},\mathbf{seq},\mathbf{seqg},M_l\right)=$$
$$\prod_{i=1}^{n\mathrm{seq}}\left[\mu_{\mathrm{ETAS}}\left(t_i,M_i\,|\,\mathbf{\Theta},\mathbf{seq}_{i-1},M_l\right)\cdot e^{-\int_{t_{i-1}}^{t_i}\mu_{\mathrm{ETAS}}\left(t,M_l|\mathbf{\Theta},\mathbf{seq}_{i-1},M_l\right)\mathrm{d}t}\right]\cdot e^{-\int_{t_{n\mathrm{seq}}}^{T_{start}}\lambda_{\mathrm{ETAS}}\left(t,M_l|\mathbf{\Theta},\mathbf{seq}\right)\mathrm{d}t} \tag{11}$$

where $n$seq denotes the number of registered events within the **seq**. Note that the following relation holds: $\mathbf{seq}_i=\{\mathbf{seq}_{i-1},(t_i,M_i)\}$. Index $i$=0 indicates the main event; despite the fact that the calculations start from $i$=1 (the first registered aftershock).

## NUMERICAL EXAMPLE

As the numerical example, the L'Aquila 2009 (central Italy) aftershock sequence is used herein. The methodology described in the previous section is applied in order to perform robust forecasting for the distribution of number of events within the first few days elapsed after the main-shock with local magnitude equal to 5.9. The hypothetic site is located near the recording station AQK (http://itaca.mi.ingv.it/ItacaNet/). To make reference to a provisional catalog, a quasi real-time catalog used by Marzocchi and Lombardi (2009) is utilized (for more details on the aftershock zone and the considered catalog, see also Ebrahimian *et al.* 2014).

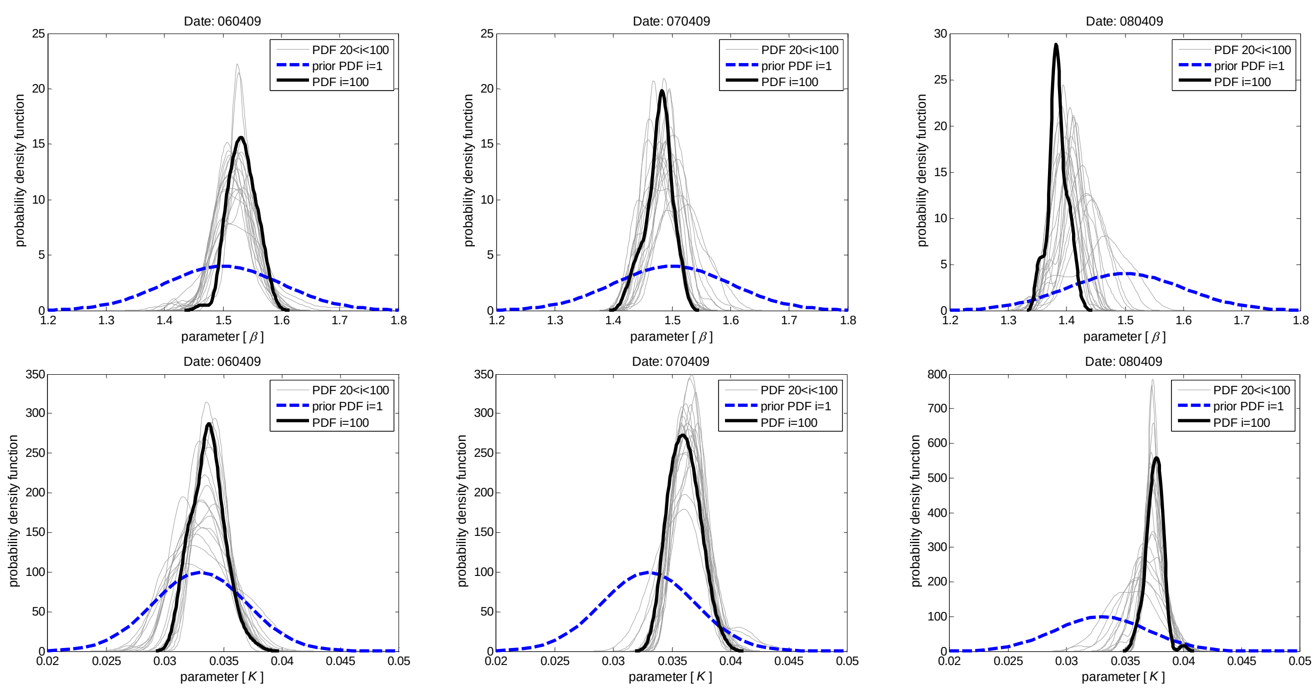
In this study, prediction time window $[T_{start},T_{end}]$ indicates a 24-hour interval associated with the desired day elapsed after the main event. Daily forecasts are provided at 6:00AM UTC every day since April 6, 2009. Each forecast uses available information at the time when the forecast is issued. This corresponds to the sequence of events, **seq** that is comprised of the events registered in the above-mentioned catalog right after the main-shock up to $T_{start}$ of the upcoming day. It has been shown that for the first time interval (i.e. after the main-shock up to 6:00AM UTC of 6 April 2009), the completeness magnitude of the catalog is equal to 3.0. The cut-off magnitude for the subsequent time intervals is parctically equal to 2.5 (Ebrahimian *et al*. 2014).

## Model parameters calibrated for L'Aquila aftershock sequence

The vector of model parameters $\boldsymbol{\Theta}$ is updated on a daily basis by applying the Bayesian updating routine illustrated in Eq. 11. As prior information, the non-evolutionary ETAS model parameters calibrated for the L'Aquila aftershock sequence (Marzocchi and Lombardi 2009) are used. However, in lieu of such prior information, one can also use uniform probability distributions which require the knowledge of the upper- and lower-bound intervals for the model parameters.

By employing the MH algorithm, samples for $\boldsymbol{\Theta}$ are generated as a Markov Chain sequence directly from the posterior (target) probability distribution $p(\mathbf{seq}|\boldsymbol{\Theta},M_l)$. The MCMC procedure is carried out by generating *nchain*=20 independent Markov chains. The 20 seeds corresponding to Markov Chains' initial state ($i$=1) are generated by taking the proposal PDF equal to the prior PDF $p(\boldsymbol{\Theta}|M_l)$. In all other states $i$>1, the candidate sample $\theta_{candidate}$ is sampled from a proposal distribution constructed as a Normal distribution centered around $\theta_{i-1}$ (for a complete discussion on adaptive sampling techniques see Au and Beck 2001). Within each chain, *ns*=100 samples are generated (the first 20 samples within each chain are later discarded; therefore the effective number of states for each chain is equal to 80). It can be shown that (Beck and Au 2002) the samples $\theta_i$ are asymptotically distributed as the posterior distribution $p(\mathbf{seq}|\boldsymbol{\Theta},M_l)$.

Figure 1 shows the evolution of the marginal PDF's corresponding to the four model parameters for first 3 days after the main-shock. Each plot contains the prior PDF, a set of PDF's corresponding to states 20<$i$<100, and the marginal PDF's for $i$=100.



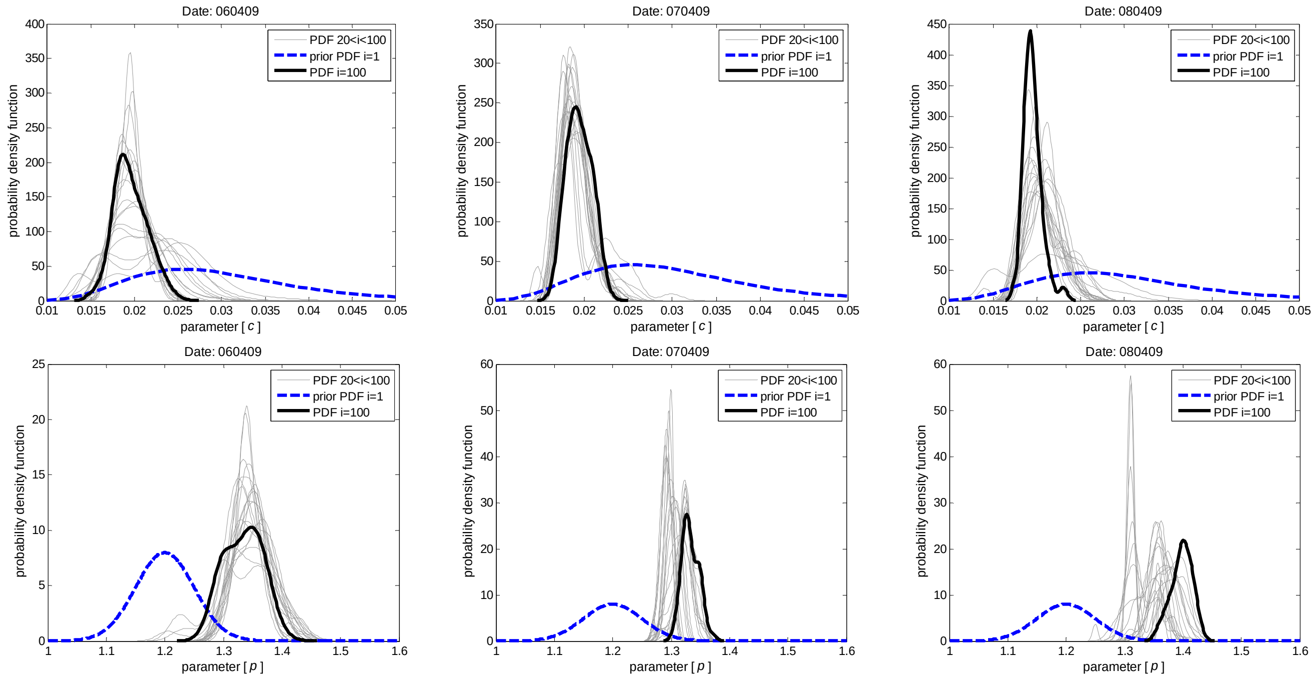



**Figure 1. The marginal PDF's for the four model parameters for selected days**

## Robust estimate for the number of aftershock events

The probability of having events greater than a specified value, i.e. $p(N>n)$, can be obtained as a direct result of the robust estimation procedure within Eq. 6. This probability can be estimated as follows:

$$p\left[N(m)>n\,|\,\mathbf{seq},M_l\right]=\frac{1}{nchain\cdot ns}\sum_{i=1}^{nchain}\sum_{j=1}^{ns}\mathrm{I}_{N\left(m\,|\,\mathbf{seq},M_l\right)>n}\left(i,j\right)=\frac{N_f}{nchain\cdot ns} \tag{12}$$

where $\mathrm{I}_{N>n}$ is an indicator function that is equal to one if $N>n$, otherwise equal to zero; $N_f$ is the number of cases with $\mathrm{I}_{N>n}=1$ ($N>n$). The coefficient of variation of the estimator for probability $p(N>n)$ can calculated through a procedure described in detail in Au and Beck (2001) and Beck and Au (2002). Figure 2(a) illustrates the complementary CDF for the number of events N, for the first five days elapsed after the main-shock and for magnitudes larger than the corresponding cut-off magnitude $M_l$. In addition, Figure 2(b) shows the histogram of the number of events for the first two days elapsed after the main-shock.

The distribution of the number of events within the forecasting time interval (i.e. Figure 2b) can be directly used for adaptive aftershock vulnerability and risk assessment (see Ebrahimian *et al.* 2013).

## Comparison between the seismicity rates of both models

The daily earthquake forecasts are provided for an event larger than the cut-off magnitude $M_l$ (i.e., $M_l$=3 for the first day elapsed after the main event and $M_l$=2.5 for all other days) based on both the robust ETAS model estimates (Eq. 6) and the original model (Marzocchi and Lombardi 2009). Figure 3(a) shows the evolution of the forecasts made according to both models, and the comparison with the number of observed data in the catalog. It can be observed that both models perform quite well in capturing the trend in the number of aftershocks. It is worth noting that the robust ETAS forecasts are obtained through a fully automated simulation procedure and without considering the background seismicity compared to the original model. In particular, the robust ETAS forecasts are provided based on only 4 parameters; whereas, the original model is comprised of 6 parameters (including a parameter related to the base seismicity) regarding the temporal seismicity evolution.

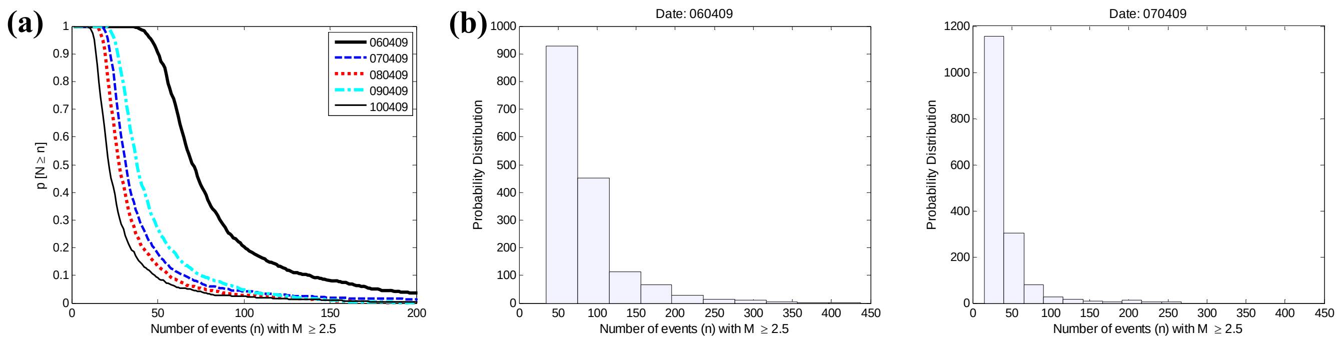



**Figure 2. (a) Probability of exceeding various levels of number of events within the first 5 days elapsed after the main-shock; (b) Histogram of the number of events for the first 2 days**

Furthermore, the mean and mean±1 standard deviation of the forecasted number of events (based on the distributions shown in Figure 2) are illustrated in Figure 3(b) for the first ten days. The observed number of events lies within the ±1 standard deviation interval of the forecasted distribution. The forecasted robust number of events provided by the ETAS model can be directly applied in adaptive daily forecasting of seismic aftershock hazard (see Ebrahimian *et al.* 2014).

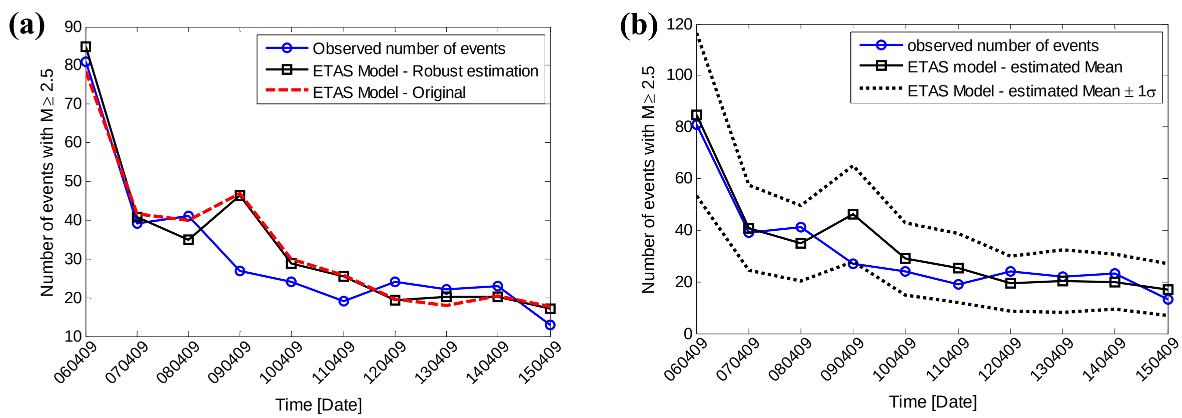



**Figure 3. (a) Daily observed and forecasted number of events; (b) The robust forecasted number of events, mean, mean±1 standard deviation**

## CONCLUSION

Robust daily forecasts of seismicity (number of events with magnitude larger than a prescribed threshold), in the presence of an ongoing aftershock sequence, are obtained through a simulation-based procedure. These forecasts are based on a simplified version of the ETAS model with temporal clustering only, where the model parameters are updated adaptively based on the aftershock events already occurred in the ongoing sequence. The model parameters are simulated using MCMC algorithm directly from the posterior joint probability distribution for the model parameters conditioned on the events already registered in the ongoing sequence. Given the epidemic nature of ETAS model, the robust forecasts also take into account uncertainty in the sequence of aftershock events that is going to take place during the forecasting interval.

It is observed that the robust forecasts, in the first ten days elapsed after the L'Aquila 2009 main-shock, are quite close to the original ETAS model. It should be noted that the original ETAS model has been quite successful in forecasting the seismicity for this particular seismic sequence. Moreover, the observed number of events occurred during the first ten days elapsed after the main event fall within ±1 standard deviation of the robust ETAS forecasts. The robust daily seismicity forecasts provided through this procedure are directly applicable to adaptive daily hazard, vulnerability and risk assessments. The procedure should be extended to take into account the spatio-temporal clustering of aftershock events as in the original ETAS model.

## ACKNOWLEDGEMENTs

This work was supported in part by the project TEMASAV CUP B25B09000090009 funded by the ESF POR Campania 2007-2013. This support is gratefully acknowledged.

## REFERENCES

Au, S. K., and Beck, J. L. (2001). “Estimation of small failure probabilities in high dimensions by subset simulation.” *Probab. Eng. Mech.*, 16(4), 263-277.

Beck, J. L., and Au, S. K. (2002). “Bayesian updating of structural models and reliability using Markov Chain Monte Carlo simulation.” *J. Eng. Mech.*, 128(4), 380-391.

Ebrahimian, H., Jalayer, F., Asprone, D., Lombardi, A. M., Marzocchi, W., Prota, A., Manfredi, G. (2013). “An outlook into time-dependent aftershock vulnerability assessment.” In: *M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.), 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering* (*COMPDYN2013*), Kos Island, Greece, 12-14 June 2013; Paper No. 1386.

Ebrahimian, H., Jalayer, F., Asprone, D., Lombardi, A. M., Marzocchi, W., Prota, A., Manfredi, G. (2014). “Adaptive daily forecasting of seismic aftershock hazard.” *Bull. Seismol. Soc. Am.*, DOI:10.1785/0120130040.

Hastings, W. K. (1970). “Monte-Carlo sampling methods using Markov chains and their applications.” *Biometrika*, 57(1), 97-109.

Jalayer, F., and Beck, J. L. (2008) “Effects of two alternative representations of ground-motion uncertainty on probabilistic seismic demand assessment of structures.” *Earthq Eng Struct Dyn*, 37(1), 61-79.

Jalayer, F., Iervolino I., Manfredi G. (2010). “Structural modeling uncertainties and their influence on seismic assessment of existing RC structures.” *Struct. Safety*, 32(3), 220-228.

Jalayer, F., Asprone, D., Prota, A., Manfredi, G. (2011). “A decision support system for post-earthquake reliability assessment of structures subjected to aftershocks: an application to L'Aquila earthquake, 2009.” *Bull. Earthq. Eng*, 9(4), 997-1014.

Jaynes, E.T. (2003) *Probability theory: The logic of science*, Cambridge University Press, London.

Jordan, T. H., Chen, Y. T., Gasparini, P., Madariaga, R., Main I., Marzocchi, W., Papadopoulos G., Sobolev, G., Yamaoka K., Zschau, J. (2011). “Operational earthquake forecasting: State of knowledge and guidelines for implementation.” *Ann. Geophys*., 54(4), 315-391, DOI: 10.4401/ag-5350.

Lewis, P. A. W., and Shedler, G. S. (1978). Simulation methods for Poisson processes in nonstationary systems. In: *proceedings of the 10$^{th}$ conference on winter simulation*, IEEE Press, 155-163.

Marzocchi, W., and Lombardi, A. M. (2009). “Real-time forecasting following a damaging earthquake.” *Geophys. Res. Lett.* 36, L21302, doi:10.1029 / 2009GL040233.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., Teller, E. (1953). “Equations of state calculations by fast computing machines.” *J. Chem. Phys.*, 21, 1087-1092.

Ogata, Y. (1981). On Lewis’ simulation method for point processes. *IEEE Trans Inform Theory*, 27(1), 23-31.

Ogata, Y. (1988). “Statistical models for earthquake occurrences and residual analysis for point processes.” *J. Am. Stat. Assoc.*, 83, 9-27.

Ogata, Y. (1998). “Space-time point-process models for earthquake occurrences.” *Ann. Inst. Statist. Math.*, 50(2), 379-402.

Papadimitriou, C., and Beck, J.L., Katafygiotis, L.S. (2001). “Updating robust reliability using structural test data.” *Probab. Eng. Mech.*, 16(2), 103-113.

Utsu, T. (1961). “A statistical study of the occurrence of aftershocks.” *Geophys. Mag.*, 30, 521-605.

Zhuang, J., Ogata, Y., Vere-Jones, D. (2002). “Stochastic declustering of space-time earthquake occurrences.” *J. Am. Stat. Assoc.*, 97, 369- 380.