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Seismic Reliability of Steel Frames

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Abstract

This paper presents an introduction to the probabilistic basis for a new set of seismic design and assessment procedures that consider explicitly nonlinear dynamic displacements. Two new seismic design and assessment guidelines make use of these developments. The new 2000 SAC steel moment-resisting seismic design and assessment guidelines prepared for FEMA and a new draft of the ISO offshore seismic guidelines use two different formats derived from this same probabilistic model.

Basic Approach:

The three primary random elements begin with the ground motion intensity, as characterized here by the level of the spectral acceleration, S_a (at some period roughly equal to the first natural period of the structure, and 5% or higher damping), Shome et al., (1998). The other two elements are the (maximum interstory) drift demand, D, and the drift capacity, C. Future intense ground motions at the site are represented in the standard way by the hazard function, $H(s_a)$, which gives the annual probability that the (random) intensity S_a at the site will equal or exceed level s_a. This is provided by earth scientists, e.g., in the US on the USGS web site. The prediction of the drift demand given any particular level of ground motion and the estimation of the capacities of various ‘failure modes’ are the purview of the structural engineer. These developments focus on these second two elements including their probabilistic representations. Finally it must be recognized that all such predictions and representations are estimates; quantification and analysis of these uncertainties will be addressed subsequently.

First we couple S_a hazard and drift demand (versus S_a) to produce a (structure-specific) drift hazard curve, $H_D(d)$, i.e., the annual probability (or strictly the mean annual frequency) that the drift demand D exceeds value d. The second step combines this with the drift capacity representation to produce P_{PL}, the (annual) probability of the performance level not being met (e.g., of collapse).

$$H_D(d) = \int P[D \geq d \mid S_a = x] \, |dH(x)| \tag{1}$$

and

$$P_{PL} = \int P[C \leq d] \, |dH_D(d)| \tag{2}$$

In order to reduce the conclusions to simple demand and capacity factors, these integrals must be tractable; this is achieved by three analytical approximations of the representations. First, assume that the site hazard curve can be approximated in the region around $s_a^{P_{PL}}$, i.e., in the range of spectral accelerations in the region of probabilities around the limit state probability P_{PL}, by the form:

$$H(s_a) = P[S_a \geq s_a] = k_o s_a^{-k} \tag{3}$$

Typical values of the important (log) slope k are 1.5 to 3**.**

Next looking more closely at the structural elements, assume that, given the level of S_a, the predicted (median) drift demand, $\hat{D}$, can be represented (again in the region around $s_a^{P_{PL}}$, at least) by

$$\hat{D} = a(S_a)^b \tag{4}$$

Experience to date suggests that $b = 1$ may be an effective default value, which is consistent, for example, with the "equal displacement rule" for moderate period structures. The coefficient a can be estimated by various approximate methods or by nonlinear time history analyses; in the latter case, for accuracy in what follows it is necessary only that the leading coefficient a be estimated from records with S_a levels near $s_a^{P_{PL}}$. In order to complete their probabilistic representation, assume that drift demands are distributed lognormally about the median with "dispersion" (formally, the standard deviation of the natural log), $\beta_{D|S_a}$. This notation $\beta_{D|S_a}$ emphasizes that this is the (record-to-record) dispersion for D at a given S_a level, but the simpler notation β_D will be used normally. There are several practical ways to estimate the three parameters, a, b, and $\beta_{D|S_a}$ (or β_D). The most direct, in principle, is to conduct a number of nonlinear analyses and then conduct a regression analysis of $\ln D$ on $\ln S_a$. One may also use Incremental Dynamic Analysis (Luco and Cornell, 1998), as will be shown below, or with an *a priori* estimate of the slope b, it is sufficient to conduct nonlinear (or even simpler displacement prediction analyses) at a <u>single</u> ground motion intensity level, as will be clear below.

With these assumptions it follows that the first factor in Eq. 1 is:

$$P[D \geq d \mid S_a = x] = 1 - \Phi(\ln[d / ax^b] / \beta_{D|S_a}) \tag{5}$$

The drift capacity, C, is assumed to have a median value, $\hat{C}$, and to be lognormally distributed with dispersion, β_C. Therefore the first factor in Eq. 2 is

$$\Phi(\ln[d / \hat{C}] / \beta_C) \tag{6}$$

Substituting and carrying out the integrations one finds this *primary result* (Jalayer and Cornell, 2000):

$$P_{PL} = H\left(s_a^{\hat{C}}\right)\exp\left[\frac{1}{2}\frac{k^2}{b^2}\left(\beta_{D|S_a}^2 + \beta_C^2\right)\right] \tag{7}$$

in which for simplicity we have introduced $s_a^{\hat{C}}$ as the spectral acceleration "corresponding to" the median drift capacity:

$$s_a^{\hat{C}} = (\hat{C}/a)^{1/b} \tag{8}$$

To transform this result into a convenient, more conventional LRFD checking format, one sets the P_{PL} equal to the performance objective, P_o, e.g., 1/2500 per year (or 2% in 50 years), and rearranges (making use of the Eq. 3), yielding:

$$\{\exp[-\frac{1}{2}\frac{k}{b}\beta_C^2]\}\,\hat{C} \;\geq \{\exp[\frac{1}{2}\frac{k}{b}\beta_{D|S_a}^2]\}\,\hat{D}^{s_a^{P_o}}$$

or

$$\phi\hat{C} \geq \lambda\hat{D}^{P_o} \tag{9}$$

in which $\hat{D}^{s_a^{P_o}}$ (or $\hat{D}^{P_o}$ for future simplicity of notation) is the median drift demand under a given ground motion of a particular intensity $s_a^{P_o}$, which in turn is defined as the S_a level with annual probability P_o of being exceeded, i.e., $\hat{D}^{P_o} = a(s_a^{P_o})^b$.

A preliminary practical conclusion: Eq. 9 implies that to confirm whether an existing building or a design of new building meets the performance objective, P_o, one (1) finds from the hazard curve the ground motion with the corresponding intensity, $s_a^{P_o}$, (2) determines the (median) drift demand, $\hat{D}$, for this S_a, and (3) compares the factored (median) capacity, $\hat{C}$, versus the factored $\hat{D}$.

Examples

Figure 1 shows the results of 30 Incremental Dynamic Analysis (IDA) curves of a steel braced frame (an offshore jacket). Each is a spline fit to several nonlinear dynamic analyses of the structure under increasing levels of the same record. The maximum story drift is plotted versus the spectral acceleration of the record (at 1.8 seconds, the natural period, and 5% damping). At a given S_a there are thus 30 values of displacement. The variety and dispersion of these curves is remarkable. The conditional median $\hat{D}$ is plotted in Figure 2 versus S_a together with the conventional (maximum story) *static* pushover. The latter has a negative slope in the displacement regime where diagonal braces are buckling, while the "dynamic pushover" softens there and then re-stiffens beyond 0.01 drift. This median and the 16th and 84th fractiles of the conditional displacement distribution are plotted in log scale in Figure 3; the regions of proportional dependence ($b = 1$), such as the elastic region and the region above 0.01 drift, appear as straight lines with unit slope. The slope, b, is about 2.3 in the intermediate (brace-buckling) regime. The definition of global system dynamic capacity is an open question; for many of the curves (records) numerical non-convergence occurs at higher S_a levels. The flattening of any one such curve is effectively analogous to a static instability: large displacement increases for small "load" increases. In any case the capacity varies from record to record.

Following the SAC format we adopt here the definition that the displacement capacity is that value at which the local slope of the IDA curve is 20% of the initial (elastic) slope, or 0.02 drift (whichever is smaller). These points are plotted in Figure 1 where the median and dispersion of the displacement capacity are reported (0.018 and 0.42). A representative (Santa Barbara Channel) seismic hazard curve is shown in Figure 4 for the same S_a definition.

With these results we can consider examples. Consider the onset-of-buckling limit state; its median capacity appears to be about 0.003 with a dispersion (arbitrarily selected to be) 0.2. The S_a corresponding to the median capacity is 0.25g and the "slope" of the (log) median drift curve in this region is $b = 2.3$. The dispersion of drift given S_a is about 0.2 in this regime. The value of the hazard at 0.25g is 4×10^{-3} with a local (log) slope of about $k = 2.56$. Substituting into Eq. 7 we find for the annual probability of brace buckling

$$4 \times 10^{3} \cdot \exp\left[1/2 \cdot \left(2.56^2 / 2.3^2\right) \cdot \left(0.2^2 + 0.2^2\right)\right] = 4 \times 10^{-3} \cdot (1.05) = 4.2 \times 10^{-3}.$$

Next, consider the use of Eq. 9 to confirm whether the probability of collapse is less than the allowable value of 1/2500. The value of S_a at this hazard level is 0.65g, and the local slope of the (log) hazard curve about 3.3. The median displacement at this S_a level is about 0.013 in a regime where $b = 1$ and the dispersion is 0.35. As discussed above the global collapse limit state has a median capacity of 0.016 with dispersion 0.44. Substituting for the LRFD factors we obtain $\phi = 0.72$ and $\lambda = 1.22$, and the checking result (Eq. 9) is 0.72(0.016) >(?) 1.22(0.013) or 0.115 < 0.0159, implying that the allowable failure probability has been exceeded (even though the median drift given the 1/2500 ground motion intensity is less than the median capacity). Appendix A shows IDA results for two models of a steel building frame.

Uncertainty and its Treatment

The (epistemic) uncertainty in the hazard curve is commonly represented in probabilistic hazard analysis practice, including the 50% confidence level (or *median estimate*, $\hat{H}(s_a)$) and other levels (e.g., the 84%, the 95%, etc.) from which one can deduce a dispersion, β_H, and a *mean estimate*, $\overline{H}(s_a)$. As above, here it is sufficient to assume that β_H is constant in the region around $s_a^{P_o}$. It is assumed too that the lognormal distribution is an adequate representation of this uncertainty.

To represent the uncertainty in the drift demand estimation, it is assumed that a in Eq. 6 is a (lognormally distributed) uncertain quantity with median estimate $\hat{a}$ and dispersion β_a. The implication is that (always given $S_a = s_a$) the median drift $\hat{D}$ is uncertain with median $\hat{\hat{D}} = \hat{a}(s_a)^b$ and (uncertainty) dispersion $\beta_{\hat{D}} = \beta_a$. (For future notational simplicity we shall use simply $\hat{D}$ for $\hat{\hat{D}}$.) For further notational simplicity we shall use β_{DU} for this *uncertainty* in (median) drift demand, and β_{DR} for, in contrast, the (record-to-record) *randomness* in drift associated with $\beta_{D|S_a}$ (or β_D) defined in the section above.

Finally to represent the uncertainty in drift capacity it is assumed that the median drift capacity, $\hat{C}$, is (lognormally) uncertain with 'best estimate' (median) $\hat{\hat{C}}$ (or simply again just $\hat{C}$) and dispersion β_{CU}; the latter uncertainty dispersion is in contrast to randomness in drift capacity measured by β_{CR} (which we now use in place of the notation β_C used in the section above).

Next it must be recognized that P_{PL} is now itself an uncertain quantity because it is a function (Eq. 7) of the uncertain quantities $H(s_a)$, $\hat{D}$, and $\hat{C}$ just described. It is straightforward to deduce that, because of this uncertainty, the probability P_{PL} is lognormally distributed with parameters below. For cost-benefit-risk assessments and other purposes it is useful to know the *mean* value (or mean estimate) of P_{PL}:

$$\begin{aligned}\overline{P}_{PL} &= \hat{H}(s_a^{\hat{C}})\exp\left[\frac{1}{2}\beta_h^2\right]\exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2)\right]\\ &= \overline{H}(s_a^{\hat{C}})\exp\left[\frac{1}{2}\frac{k^2}{b^2}(\beta_{DR}^2+\beta_{DU}^2+\beta_{CR}^2+\beta_{CU}^2)\right]\end{aligned} \tag{10}$$

One can see that second version of Eq. 10 looks much like Eq. 7 except it is now specified that it is the *mean* hazard curve into which one must substitute $s_a^{\hat{C}}$, and now all four dispersion contributions appear. Some say that the effect of the uncertainty in the hazard curve is "captured" by using this mean (rather than the median) estimate. The dispersion (standard deviation of the natural log) of P_{PL} is

$$\beta_{P_{PL}} = \sqrt{\beta_H^2 + \frac{k^2}{b^2}(\beta_{DU}^2+\beta_{CU}^2)} \tag{11}$$

The median estimate of P_{PL} is just the mean times $\exp\left[-1/2\beta_{P_{PL}}^2\right]$. From these results one can produce, for example, an upper confidence interval estimate:

$$P_{PL}^x = \hat{P}_{PL}\exp[K_x\beta_{P_{PL}}] \tag{12}$$

in which K_x is the appropriate value from a gaussian table for specified confidence x.

Finally, the following are examples of safety or performance checking schemes that can be developed from the information above. The simplest form results from using the mean probability as the objective: substitute the performance objective P_o in the second form of Eq. 10, and rearrange as done in the section above (when uncertainty was not recognized), obtaining now:

$$\left\{\exp\left[-\frac{1}{2}\frac{k}{b}(\beta_{CR}^2+\beta_{CU}^2)\right]\right\}\cdot\hat{C} \geq \left\{\exp\left[\frac{1}{2}\frac{k}{b}(\beta_{DR}^2+\beta_{DU}^2)\right]\right\}\cdot\hat{D}^{s_a^{P_o}}$$

or

$$\phi\hat{C} \geq \lambda\hat{D}^{P_o} \tag{13}$$

in which the capacity and demand factors are defined by the obvious two exponential terms. The application of this format would parallel that of the example above except that the mean (estimate of the) hazard curve should be used and the two factors need to be based on the total aleatory and epistemic uncertainty in capacity and in dynamic response (given ground motion intensity). A structure or design satisfying the condition above (Eq. 13) can be said to have a *mean* P_{PL} less than or equal to the performance objective P_o. Somewhat analogous reasoning has been used for the DOE 1020 seismic criteria, which is, however, explicitly neither displacement nor nonlinear-analysis based, and which is couched in terms of different factors.

The current draft of the ISO offshore guidelines (Younan et al., 2001) follow a scheme that has the engineer compare the median capacity versus the median displacement estimated for a ground motion level that has been appropriately enhanced, by a factor $C_c = \exp[(k/2b^2)\cdot(\beta^2_{TD|S_a} + \beta^2_{TC})]$, such that the check is equivalent to Eq. 13. (The notation such as β^2_{TC} is shorthand for the total of random and uncertain - aleatory and epistemic - squared dispersions.)

In the development of the SAC structural checking procedure (Hamburger, et al., 2000) a decision was made to focus on the uncertainty in the two structural elements of the problem, i.e., drift demand (given ground motion intensity) and capacity. The uncertainty in the hazard is, in effect, presumed to have been dealt with as per the second version of Eq. 10 above, i.e., by using the mean hazard curve, which reflects β_H. Then the uncertainty in P_{PL} (given the mean hazard curve) is considered explicitly, i.e., the uncertainty due to the two structural elements. Recall that satisfying Eq. 13 implies that the mean P_{PL} is less than the target value. If, as SAC has chosen, one wants to *set* the criterion that there must be a confidence of at least 95%, say, that the actual probability of the limit state is less than the objective P_o, then the checking procedure or format becomes: insure that the ratio of factored capacity to factored demand, $\gamma_{con} = \phi\hat{C}/\lambda\hat{D}^{P_o}$, is greater than a certain critical value:

$$\gamma_{con} = \exp\left[K_x \beta_{UT} - \frac{1}{2}\frac{k}{b}\beta^2_{UT}\right] \tag{14}$$

in which $\beta^2_{UT} = \beta^2_{CU} + \beta^2_{DU}$ is the "total" epistemic uncertainty (again given the mean hazard curve). This result can be derived from the mean and dispersion of the uncertain P_{PL} which look like the results above (Eq. 10 and 11) without the hazard curve dispersion.

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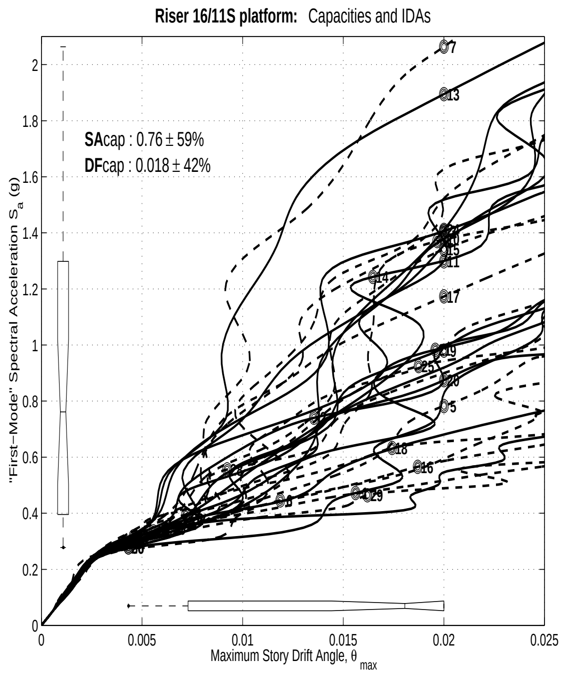

Figure 1: Braced Frame: Multiple IDA Curves.

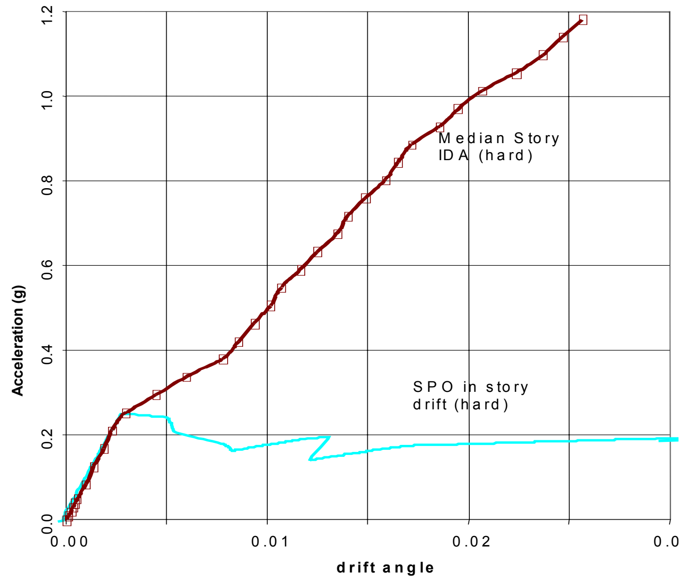

Figure 2: Median Drift vs. Static Pushover

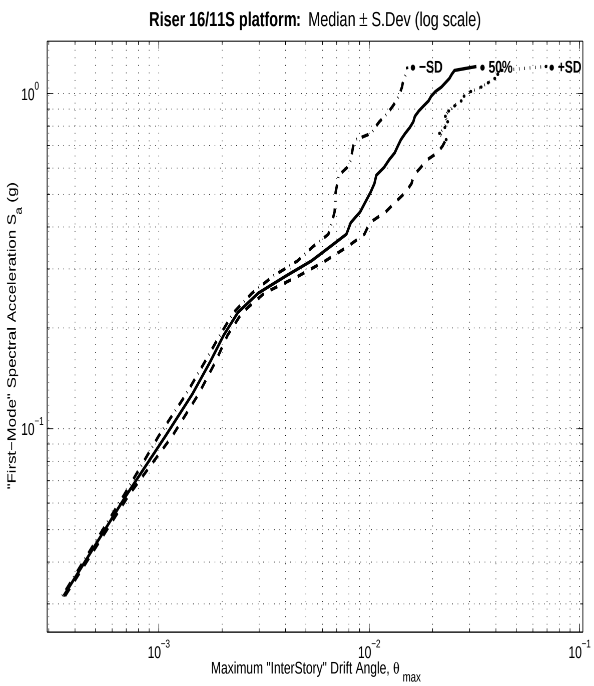

Figure 3: Median, 16^{th}, and 84^{th} Fractiles

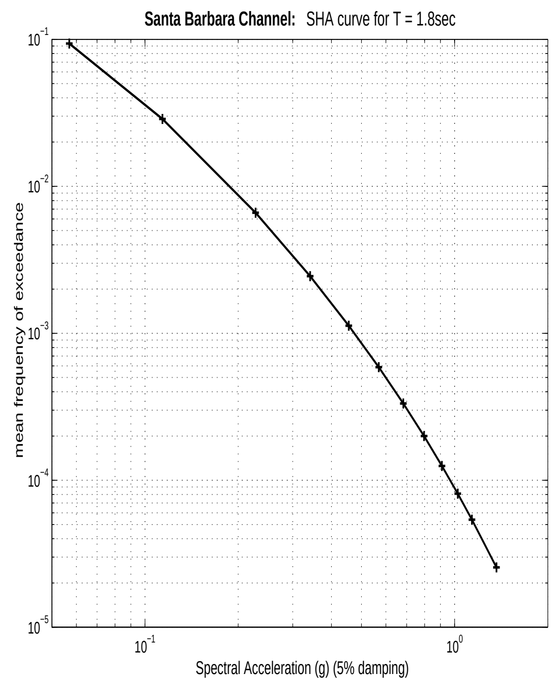

Figure 4: Seismic Hazard Curve

Appendix A. Building Frame Examples

Figures A1 and A2 show summary plots of IDA analyses of two models of a 9-story steel moment-resisting building frame. The first is a model with ductile connections and hence represents the anticipated behavior of such frames prior to the 1994 Northridge earthquake when many connections were found to have fractured. The second model contains connection representations that mimic the fracturing behavior observed. The summary plots report the median and 16th and 84th fractiles, as well as the two most robust and two most flexible curves (records) in the data sets considered. Capacities defined as above are also shown. Note that the fracturing model shows larger median displacements especially at larger spectral accelerations and typically smaller displacement capacities.

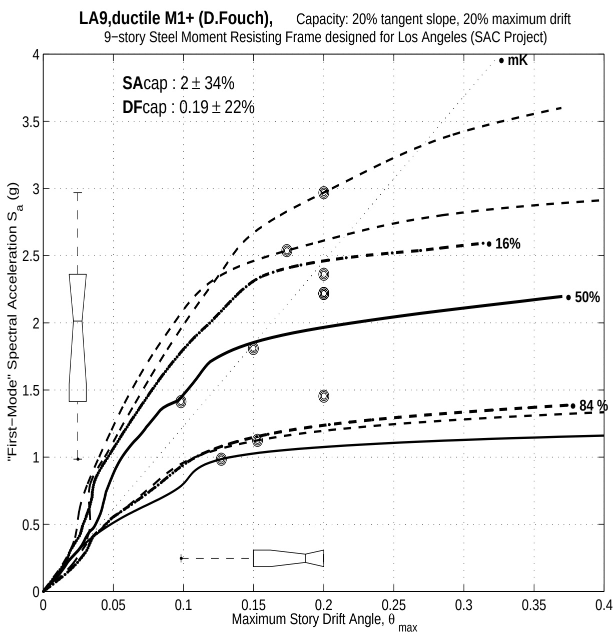

Figure A1: IDA Summary: Ductile 9-Story Building Moment Frame

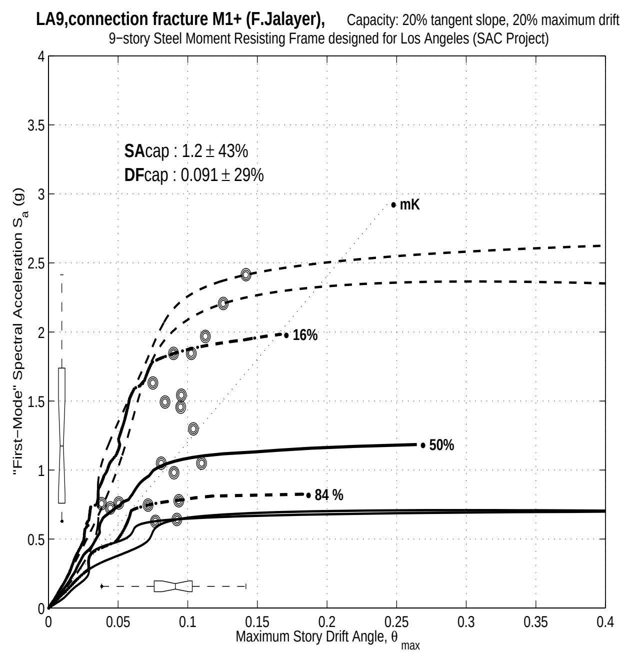

Figure A2: IDA Summary: Fracturing Connections Model of 9-Story Frame