CHAPTER 3

Probability-based Demand and Capacity Factor Design (DCFD) Seismic Formats

F. Jalayer and C. A. Cornell (2003). RMS Report No. 43- Part II Seismic Design Format

3.1 Introduction – Format development

Chapter 2 was dedicated to developing an analytical foundation for the probability-based seismic assessment of structures. The final product of this foundation development was the mean annual frequency of exceeding a limit state or the "limit state frequency" in short. Limit state frequency H_{LS} was calculated taking into account the uncertainty in various elements involved in the seismic assessment and design of the structural system. An analytical framework for calculating the limit state probability is helpful for seismic assessment of the structures, e.g., calculating the limit state probability for an existing structure and checking to see if its design falls within the acceptable region. However, in a design problem, the actual structural members and connections are not known beforehand. They are , rather, the end product of the design process. Conversely, the performance objective for the design is usually set beforehand and can be expressed in terms of the limit state probability. From a designer's point of view, the structure can be designed to satisfy a specified performance objective.

This part of the report addresses problems similar to the following: "how to assess a proposed (or existing) design for a structure for a known collapse limit state frequency of, say, 0.04 percent per annum?" or "how to address the uncertainty (due to limited knowledge) involved in evaluation of the design parameters?". This uncertainty is usually stated through questions such as, "how to design a structure for a known mean annual collapse limit state frequency of 0.04% <u>with a confidence interval level of 95%</u>?".

We shall discuss various alternative design formats that stem from the probability-based foundation developed in the first part of this document. These formats are in general suitable for guidelines and code implementation. A major class of these formats are analogous in form to (linear, static, force-based) Load and Resistance Factored Design (LRFD) procedures (AISC LRFD code). However, these formats are based on generic, random (usually) displacement-based, non-linear dynamic response variables: "demand" and "capacity", and they are therefore referred

to as Demand and Capacity Factored Design (DCFD). The DCFD format can also be formulated in terms of spectral acceleration-based (referred to as IM-based in Chapter 2) generic demand and capacity variables. Fragility-hazard format is another IM-based format discussed in this chapter, which is useful for designing/assessing the global behavior of a structure or a class of structures.

Unlike the foundation, which is unique, the formats are numerous. They are just various representations of a common foundation. In other words, the choice among these alternative formats is subjective. It is to be made on grounds such as familiarity, practicality, etc.

3.2 Randomness: The only source of uncertainty

Similar to the foundation development, the alternative design formats discussed in this chapter are also presented in two parts. The probability-based foundations developed in this section are based on the assumption that randomness is the only source of uncertainty, and hence, they are based on the expression for limit state frequency derived in the first part of Chapter 2 (Equations 2-38 and 2-48).

Recalling from Equation 2-38 in Chapter 2, the limit state probability, expressed in displacement-based terms, was derived as:

$$H_{LS} = \nu \cdot P[D > C] = H_D(\eta_C) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta_C^2} = H_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \qquad (3\text{-}1)$$

where $H(\cdot)$ denotes the mean rate of exceedance in general[1]. We are going to re-arrange the above equation into alternative forms, also known as "DCFD design formats". The purpose for this re-arrangement is to present this probability-based formulation in a way that is easy to be implemented in the design practice. The basic components of Demand and Capacity Factored Design Format (DCFD) are outlined in the following equation,

$$\eta_{D|^{P_0}s_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_{D|S_a}^2} = \eta_C \cdot e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2} \qquad (3\text{-}2)$$

where $\eta_{D|^{P_0}s_a}$ is the median drift *demand* for a given spectral acceleration, ${}^{P_0}s_a$, corresponding to hazard levels in the proximity of an acceptable limit state probability, P_0. η_C is the median drift

[1] Later in this thesis we shall use the notation $\lambda(.)$ to represent mean annual rate of exceedance.

capacity, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ is the *Demand Factor*, and $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_C)$ is the *Capacity Factor*. Equation 3-2 offers an alternative presentation of the formal foundation equation (Equation 3-1), and is obtained by re-arranging Equation 3-1. We shall go through the re-arrangement step-by-step below.

The Fragility-Hazard Format is another format discussed in this chapter. This format is derived by re-arranging the $S_a - based$ presentation of the foundation equation in Chapter 2 (Equation 2-48):

$$^{P_0}s_a \cdot e^{\frac{1}{2}\cdot\frac{k}{b^2}\cdot\beta^2_{D|S_a}} = s_a^{\eta_C} \cdot e^{-\frac{1}{2}\cdot\frac{k}{b^2}\cdot\beta^2_C} \qquad (3\text{-}3)$$

where $^{P_0}s_a$ is the spectral acceleration with a *Hazard* level equal to an acceptable limit state probability, P_0, and $s_a^{\eta_C}$ is the spectral acceleration with a *Fragility* of 50%. Each format will be developed and discussed in detail in the corresponding section. Before proceeding to the details of the derivations, we are going to define or overview a few parameters (variables) that are going to be very helpful in our future format presentations.

3.2.1 Spectral acceleration s_a^d corresponding to a displacement-based demand equal to *d*

s_a^d or the spectral acceleration corresponding to displacement-based demand value, d, is defined as the spectral acceleration corresponding to the value, *d*, from the median displacement-based curve as a function of the spectral acceleration, i.e., as the inverse of that function:

$$s_a^d = \eta^{-1}_{D|S_a}(d)$$

Recalling from the last chapter, the median displacement-based demand was approximated as a power-law function of the spectral acceleration, $\eta_{D|S_a}(x) = a \cdot x^b$. Based on this approximation, s_a^d or the spectral acceleration corresponding to the displacement-based response *d* can be expressed as:

$$s_a^d = \eta^{-1}_{D|S_a}(d) = \left(\frac{d}{a}\right)^{\frac{1}{b}} \qquad (3\text{ - }4)$$

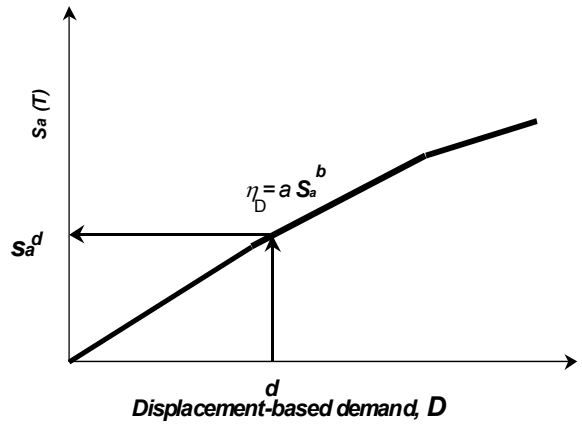

Figure 3-1 - Spectral acceleration corresponding to a displacement-based demand equal to *d*.

s_a^d is illustrated graphically in Figure 3-1. In simple words, s_a^d represents the spectral acceleration value corresponding to a given demand value *d* from a median curve approximated as, $a \cdot S_a^{\,b}$.

3.2.2. Spectral acceleration ${}^{P_0}s_a$ for a hazard level equal to P_0

${}^{P_0}s_a$ is the spectral acceleration with a mean annual frequency of being exceeded (hazard, defined in Chapter 2) equal to P_0:

$$ {}^{P_0}s_a = \lambda_{S_a}^{-1}(P_0) \cong \left(\frac{P_0}{k_0}\right)^{\frac{-1}{k}} \qquad (3\text{-}5) $$

In which we make sue of the fact that (Chapter 2) the mean annual frequency of exceeding a given spectral acceleration value (also known as the spectral acceleration hazard curve) can be approximated (at least locally) by the power-law function, $\lambda_{S_a}(x) = k_0 \cdot x^{-k}$. Figure 3-2 illustrates the graphical presentation of ${}^{P_0}s_a$. The spectral acceleration s_a^d corresponding to a drift demand equal to *d* is also plotted on the same figure.

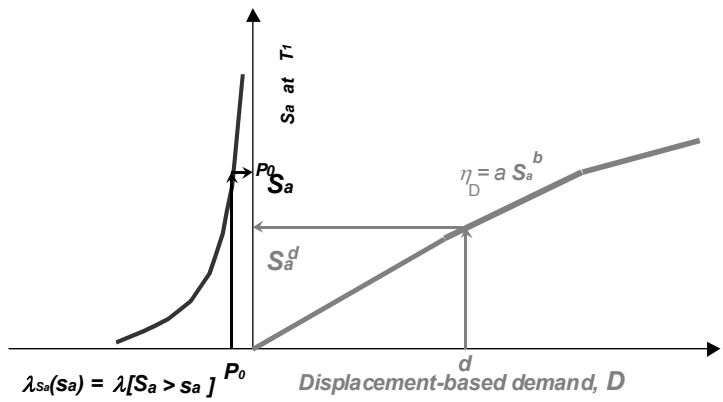

Figure 3-2 - Spectral acceleration for a hazard level equal to P_0

3.2.3 DCFD Format

DCFD format is analogous in form to the Load and Resistant Factored Design (LRFD) procedures (see AISC design procedures, 1994). As the name suggests, this format is constituted of demand and capacity multiplied by their respective factors. The same as LRFD procedures, the DCFD format can be used to design a building against a certain factored demand by finding a factored capacity. The probabilistic demand and capacity factors for the DCFD format are very similar in concept to the partial safety factors applied to the load and resistance in LRFD design procedures.

This format stems directly from the expression for limit state probability (Equation 3-1) after some re-arrangements. It should be noted that the same simplifying assumptions that led to the derivation of the closed-form foundation equation in the previous chapter are implicit here in the derivation of the DCFD format. In order to develop a design format, we first need to set a design criteria. One certain criteria is to design a structure so that the mean annual frequency of exceeding a certain limit state (limit state frequency in short) is less than or equal to the allowable annual probability P_0 [2]:

[2] Note the mixing of the usage of the terms "mean annual frequency" and "annual probability". Although the more precise term to be used in this derivations is "mean annual frequency", for the type of rare events that we are interested, the corresponding values are virtually numerically identical.

$$H_{LS} \le P_0 \qquad (3\text{-}6)$$

where the equality holds at the onset of the limit state. Recalling from the previous chapter, the limit state frequency can be expressed through a closed-form relationship (Equation 2-38 or 3-1). This closed-form expression can be substituted for H_{LS} in Equation 3-6:

$$H_{S_a}(s_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \le P_0 \qquad (3\text{-}7)$$

where $H_{S_a}(s_a^{\eta_C})$ is the hazard value (mean annual frequency of exceedance) for a spectral acceleration equal to $s_a^{\eta_C}$ (i.e., the spectral acceleration corresponding to the median capacity η_C), and it can be derived from Equation 2-36 as:

$$H_{S_a}(s_a^{\eta_C}) = k_0\left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \qquad (3\text{-}8)$$

If $H_{S_a}(s_a^{\eta_C})$ from the Equation 3-8 is replaced in Equation 3-7:

$$k_0\left(\frac{\eta_C}{a}\right)^{\frac{-k}{b}} \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot\beta_C^2} \le \mathrm{P}_0 \qquad (3\text{-}9)$$

After re-arranging the above equation in order to solve for median capacity, η_C, we get the following expression for the median capacity required so that the limit state frequency H_{LS} is less than or equal to the allowable probability P_0:

$$\eta_C \le a \cdot \left(\left(\frac{P_0}{k_0}\right)^{-\frac{1}{k}}\right)^b \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k}{b}\cdot\beta_C^2} \qquad (3\text{-}10)$$

The expression in the parenthesis $(P_0/k_0)^{-1/k}$ is nothing but the spectral acceleration ${}^{P_0}s_a$, having a hazard value equal to the allowable probability P_0 as given in Equation (3-5). Substituting $(P_0/k_0)^{-1/k}$ with ${}^{P_0}s_a$ in Equation 3-10 will make it look more simple:

$$\eta_C \le a \cdot \left({}^{P_0}s_a\right)^b \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\frac{k}{b}\cdot\beta_C^2} \qquad (3\text{-}11)$$

where $a\left({}^{P_0}s_a\right)^b$ is in turn equal to the median drift demand $\eta_{D|^{P_0}S_a}$ for a given spectral acceleration of ${}^{P_0}s_a$ (Equation 2-4). Thus, Equation 3-11 can be further simplified by replacing $a\left({}^{P_0}s_a\right)^b$ with $\eta_{D|^{P_0}S_a}$:

$$\eta_{D|^{P_0}S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2} \leq \eta_C$$

Finally we transfer the capacity-related exponential term, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_C)$, to the other side of the equation changing the sign of the exponent:

$$\eta_{D|^{P_0}S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} \leq \eta_C \cdot e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2} \qquad (3\text{-}12)$$

Equation 3-12 represents the DCFD format in its final form. The right-hand side or the "capacity side" of the equation is called the "factored capacity", parallel to LRFD's factored resistance. In a similar manner, the left-hand side of the equation or the "demand side" is called the "factored demand for the allowable probability P_0", parallel to LRFD's factored load. It should be noted that the factored demand (the equivalent to factored load) is a function of the allowable probability level, P_0, whereas the factored capacity does not depend on P_0. In contrast to the two factors in the AISC LRFD, where neither the demand nor capacity factor depends on P_0. The DCFD format offers an alternative and equivalent statement for the design criterion, according to which the factored demand for the allowable probability P_0 should be less than or equal to the factored capacity. This implies that at the onset of the limit state, the factored demand for the allowable probability P_0 is equal to the factored capacity. One of the main advantages of the DCFD design format is that the probabilistic design criteria can be stated in terms of familiar displacement-based response parameters. This makes the DCFD format compatible with existing (deterministic) design procedures.

The following sections are going to discuss in more detail the components of the DCFD format (Equation 3-12).

3.2.3.1 Displacement-based demand, $\eta_{D|^{P_0}S_a}$

$\eta_{D|^{P_0}S_a}$ is the median displacement-based demand for a spectral acceleration equal to spectral acceleration, ${}^{P_0}s_a$, (i.e., spectral acceleration with a frequency of exceedance equal to the allowable probability, P_0). We may also refer to it as the median demand for a given ground motion intensity ${}^{P_0}s_a$ in short. Adopting the analytical development above this median demand was shown to be equal:

$$\eta_{D|^{P_0}s_a} = a\left({}^{P_0}s_a\right)^b = a\left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \qquad (3\text{-}13)$$

But Figure 3-3 illustrates a graphical presentation of $\eta_{D|^{P_0}s_a}$ that demonstrates its more general applicability. Looking at the figure, we can see that $\eta_{D|^{P_0}s_a}$ can be calculated in two simple steps. Step 1 is to find the spectral acceleration ${}^{P_0}s_a$ that has a frequency of exceedance (i.e., hazard) equal to the allowable probability, P_0, from the hazard curve for the spectral acceleration. Step 2 is to find the displacement-based demand $\eta_{D|^{P_0}s_a}$ that corresponds to a spectral acceleration equal to ${}^{P_0}s_a$ from the median demand curve. Note that in application neither the hazard curve nor the median demand curve need to be in analytical form to evaluate ${}^{P_0}s_a$ and $\eta_{D|^{P_0}s_a}$. This fact will be exploited in applications that follow.

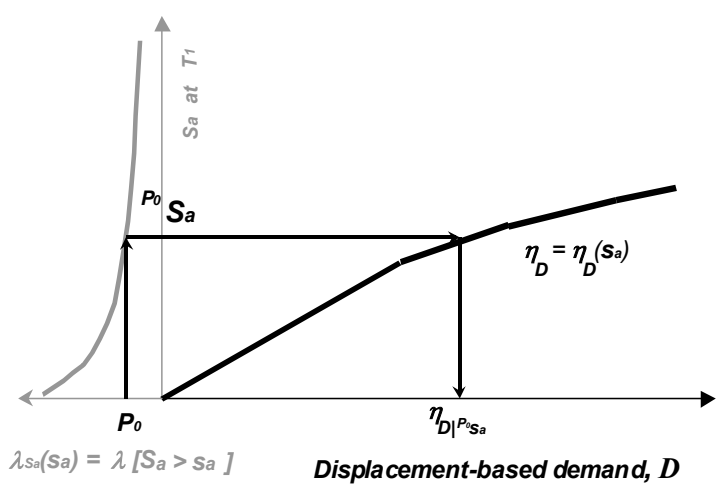

Figure 3-3 - Graphical presentation of median demand $\eta_{D|^{P_0}s_a}$ for a spectral acceleration equal to ${}^{P_0}s_a$

3.2.3.2 Displacement-based capacity, η_C

η_C is the median displacement-based capacity for the structure. Figure 3-4 illustrates the median drift demand $\eta_{D|^{P_0}S_a}$ and capacity η_C on the same graph.

3.2.3.3 Demand factor $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$

$\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ or the displacement-based demand factor is a magnifying factor that takes into account the randomness in the displacement-based demand. The randomness represented by the factored demand is usually due to record-to-record variability. $\beta_{D|S_a}$, a dispersion measure for the displacement-based demand, is equal to the standard deviation of the (natural) logarithm of displacement-based demand for a given spectral acceleration. Typical values for $\beta_{D|S_a}$, in the non-linear range, are about 0.30 to 0.60. In the special case (e.g., a linear SDOF oscillator) where there is no dispersion in demand (given S_a), the demand factor will be equal to unity. k/b can be interpreted as the sensitivity of probability to a the change in the displacement-based demand; which means that a factor of x change on the displacement scale will cause a factor of $x^{-k/b}$ change on the probability scale.

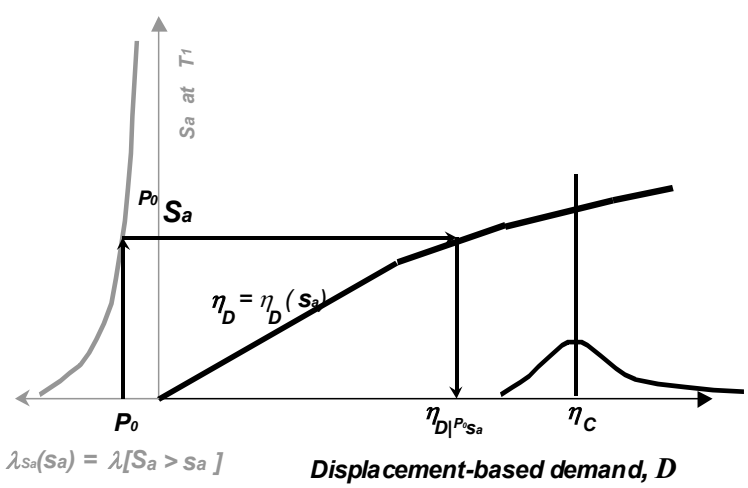

Figure 3-4 - Graphical presentation of median drift capacity η_C

Similar to the LRFD design procedures, the demand factor $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ is also denoted by γ. Clearly, γ is always greater than or equal unity (an exponential raised to a nonnegative power). Thus, the "design" displacement-based demand is always greater than or equal to the median demand due to the randomness-type of uncertainty in displacement-based demand.

3.2.3.4 Capacity factor $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$

$\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ or the factored capacity is a reduction factor that takes into account the randomness type of uncertainty in the displacement-based capacity. It is an exponential term raised to a non-positive power and hence is always smaller than one. Therefore, the "design" capacity is always less than or equal to the median capacity due to the randomness-type of uncertainty. The dispersion term in the exponential power, β_C, is the standard deviation of the (natural) logarithm of the displacement-based capacity. Also k/b is a factor reflecting the sensitivity of the probability to a change in displacement-based capacity. Similar to the LRFD design procedures, the capacity factor $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ is also denoted by ϕ.

3.2.3.5 Factored demand and demand hazard

In the following, we are to present an alternative interpretation of factored demand. This alternative derivation relates the factored demand to the demand hazard (mean frequency of exceedance).In DCFD format, the factored demand (FD) was derived as:

$$FD = \eta_{D|^{P_0}{}_{S_a}} \cdot \exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$$

Replacing back the analytic expression for $\eta_{D|^{P_0}{}_{S_a}}$ from Equation 3-13:

$$FD = a\left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \cdot \exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$$

We can solve the above equation for P_0:

$$P_0 = k_0 \left(\frac{FD}{a} \right)^{-\frac{k}{b}} \cdot \exp(\frac{1}{2} \cdot \frac{k^2}{b^2} \cdot \beta^2{}_{D|S_a}) \qquad (3\text{-}14)$$

Realizing that (according to Equation 3-8) the term $k_0 \cdot (FD/a)^{-k/b}$ is equal to the hazard value for a spectral acceleration corresponding to a (median) demand value equal to FD:

$$H_{S_a}(s_a^{FD}) = k_0 \left(\frac{FD}{a} \right)^{-\frac{k}{b}} \qquad (3\text{-}15)$$

Replacing the term $k_0 \cdot (FD/a)^{-k/b}$ in Equation 3-14 with its equivalent from Equation 3-15:

$$P_0 = H_{S_a}(s_a^{FD}) \cdot \exp(\frac{1}{2} \cdot \frac{k^2}{b^2} \cdot \beta^2{}_{D|S_a}) \qquad (3\text{-}16)$$

We can observe that right side of the above equality is equal to (demand) hazard for a demand value equal to FD (Equation 2-25):

$$P_0 = H_D(FD) \qquad (3\text{-}17)$$

Conversely, the factored demand can be written as the inverse of the hazard function at value P_0:

$$FD = H_D^{-1}(P_0) \qquad (3\text{-}18)$$

The above equation states that the factored demand for an allowable probability P_0 is equal to the (displacement-based) demand with a mean annual frequency of exceedance (hazard) equal to P_0. This alternative interpretation for the factored demand is going to be helpful later in Chapters 4 and 5 where we need to evaluate the factored demand for more general cases (i.e., the analytic assumptions underlying the derivation of DCFD have not been employed).

3.2.3.6 General Form for the DCFD design format:

We have already discussed the derivation of the closed-from for DCFD format (Equation 3-12), which resulted from re-arranging the expression for limit state probability in Equation 3-1:

$$\eta_{D|^{\lambda_{LS}} S_a} \cdot e^{\frac{1}{2} \cdot \frac{k}{b} \cdot \beta^2{}_{D|S_a}} \leq \eta_C \cdot e^{-\frac{1}{2} \frac{k}{b} \cdot \beta_C^2}$$

However, it should be kept in mind that this format is based on the same simplifying assumptions that we made for the foundation derivation in Chapter 2. The general form for the DCFD design can be introduced based on the format we derived in Equation 3-12, but replacing $\eta_{D|^{P_{LS}}S_a}$ with[3] D, η_C with C, $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$ with γ, and $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ with ϕ:

$$D\cdot\gamma \le C\ \cdot\phi \qquad (3\text{-}19)$$

where D and C refer to a demand and capacity displacement-based parameters and γ and ϕ are their corresponding factors. It can be noted that the DCFD format presented in its general form as in Equation 3-19 looks similar to the LRFD format presentation. Another alternative general way to present the DCFD format is by simply comparing the factored demand to factored capacity:

$$F.D. \le F.C. \qquad (3\text{-}20)$$

The benefit of this alternative representation is that factored demand and factored capacity can be defined in a different manner from the DCFD format. A generalized definition for factored demand is already discussed in the previous section. According to this definition the factored demand is the demand value has a mean annual frequency of exceedance equal to the allowable probability P_0. Generalized definitions for factored demand and factored capacity are going to be discussed in Chapters 4 and 5.

3.2.3.7 Numerical Example: (Performance Evaluation for an Existing Building)

Returning to the numerical example presented in Chapter 2, now we can assess the performance of an existing 3-story frame for the collapse limit state for an allowable probability of $P_0 = 4\times10^{-4}$ (2% in 50 years). Based on the DCFD design format, we are going evaluate and compare the factored demand for the allowable probability $P_0 = 4\times10^{-4}$ and the factored capacity for the collapse limit state.

[3] Despite their capital letter designation, D and C do not represent stochastic variables in this DCFD context. (They do typically represent stochastic variables in Chapters 2and 3). Here, they are just referring to some generic displacement-based demand and capacity parameter.

Factored Demand $D \cdot \gamma$

Evaluation of the factored demand consists of two parts, a) calculating the median drift demand $\eta_{D|^{0.0004}S_a}$ for a spectral acceleration with a hazard equal to 4×10^{-4}, and b) calculating the demand factor. The median demand $\eta_{D|^{0.0004}S_a}$ itself can be calculated in two steps. The first step is to calculate the spectral acceleration ${}^{0.0004}s_a$ with a hazard equal to 4×10^{-4}. This can be done either by using Equation 3-5 or more generally by simply finding the spectral acceleration corresponding to $P_0 = 4 \times 10^{-4}$ from the hazard curve. The advantage of the second approach is that the hazard curve does not necessarily need to be a power-law. This approach will be used extensively in Chapters 4 and 5. Here, we are going find ${}^{0.0004}s_a$ both analytically and graphically. ${}^{0.0004}s_a$ can be calculated from Equation 3-5 for $P_0 = 4 \times 10^{-4}$:

$$ {}^{0.0004}s_a = \left(\frac{0.0004}{k_0}\right)^{\frac{-1}{k}} $$

Recalling from the first part of the numerical example in the previous chapter, the parameter estimates for k_0 and k were equal to:

$$ k_0 = 0.00124 $$
$$ k = 3.03 $$

Finally, ${}^{0.0004}s_a$ can be calculated as:

$$ {}^{0.0004}s_a = \left(\frac{0.0004}{0.00124}\right)^{\frac{-1}{3.03}} = 1.458 \text{ [g]} $$

Graphically, ${}^{0.0004}s_a$ is estimated by the spectral acceleration corresponding to the hazard value equal to 0.0004 from the spectral acceleration hazard curve. The hazard curve with parameters k_0 and k (listed above) is plotted in Figure 3-5. It can be observed that a hazard value 0.0004 corresponds to ${}^{0.0004}s_a$ equal to 1.45. After the ${}^{0.0004}s_a$ is calculated (estimated), the next step is to find the median displacement-based demand that corresponds to this spectral acceleration. Again, the median demand can be either calculated from the power-law approximation, $\eta_{D|S_a}(x) = a \cdot x^b$, or estimated graphically from the median displacement-based demand curve that is plotted versus spectral acceleration.

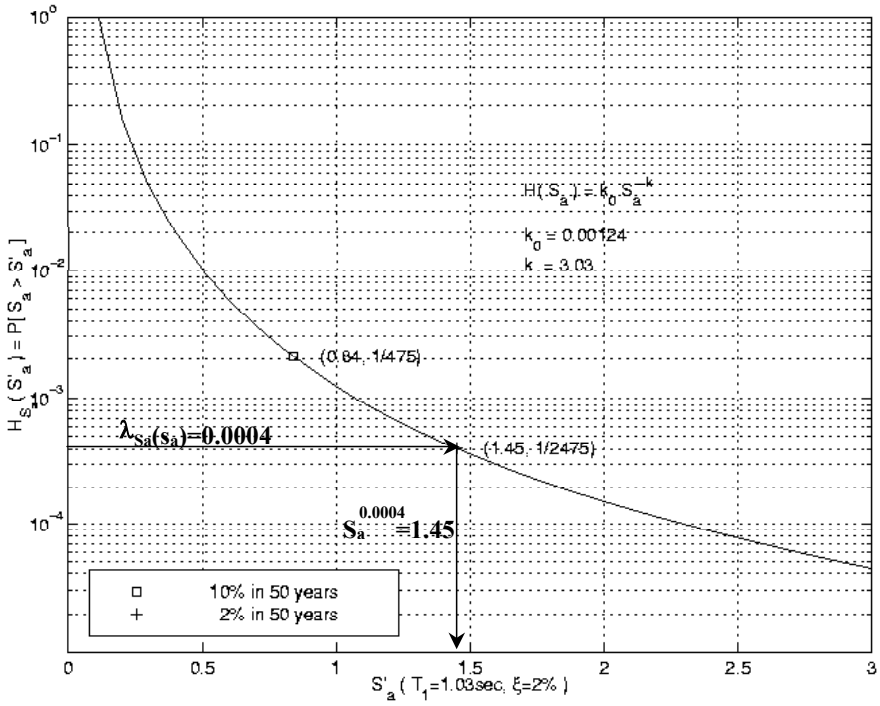

Figure 3-5 - The hazard curve for $S_a(T=1, \xi=2\%)$

The median demand corresponding to a spectral acceleration equal to ${}^{0.0004}s_a = 1.458[g]$ can be calculated from the power-law relation as:

$$\eta_{D|S_a}({}^{0.0004}s_a) = a \cdot \left({}^{0.0004}s_a\right)^b$$

Recalling from the previous chapter, the parameter estimates for *a* and *b* were equal to:

$$a = 0.0325$$
$$b = 1.002$$

Finally, $\eta_{D|S_a}({}^{0.0004}s_a)$ can be calculated as:

$$\eta_{D|S_a}({}^{0.0004}s_a) = 0.0325 \cdot \left({}^{0.0004}s_a\right)^{1.002} = 0.0325 \cdot (1.458)^{1.002} = 0.047$$

We can also estimate $\eta_D({}^{0.0004}s_a)$ graphically by finding the median demand value corresponding to a given spectral acceleration of ${}^{0.0004}s_a$ or 1.45 [g] from the median demand curve. In this example we have chosen the maximum inter-story drift angle (the "absolute" maximum of the response time-history over all the stories in the structure) as the displacement-based demand parameter. The maximum inter-story drift angle is plotted versus spectral acceleration in Figure 3-6 below. In Chapter 4, we shall explore other ways to evaluate this

median drift, e.g., by simply making a small set of non-linear dynamic runs with all input records scaled to the single level, ${}^{0.0004}s_a = 1.45g$.

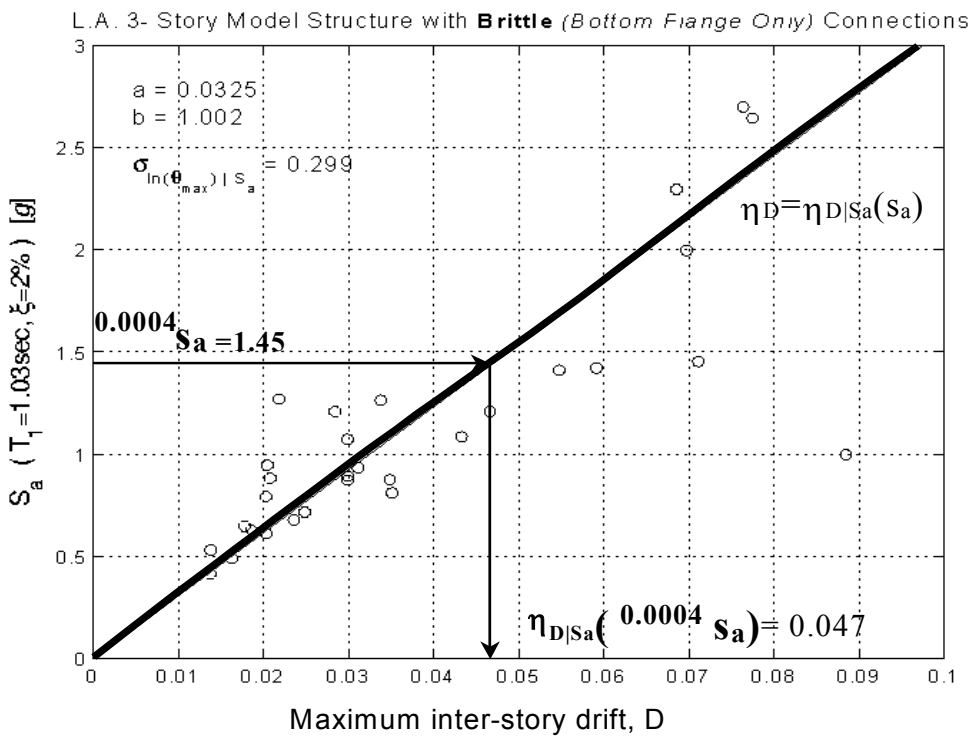

Figure 3-6 - Spectral acceleration plotted versus maximum inter-story drift angle, and fitted power-law relation (a line on the two-way logarithmic paper)

The next step is to calculate the demand factor or $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$. As mentioned in Section 3.2.3.5, $\beta_{D|S_a}$ is equal to the standard deviation in the (natural) logarithm of the demand given spectral acceleration denoted by $\sigma_{\ln D|S_a}$. This can be approximated by the reported $\sigma_{\ln\theta_{\max}|S_a}$ on the graph in Figure 3-6 (it should be noted that the maximum inter-story drift angle, $\theta_{\max}$, is in fact the demand parameter used in this example) that is approximately equal to 0.3. The hazard slope parameter *k* is reported on Figure 3-5 as 3.0. Also the median demand-spectral acceleration slope factor *b* is equal to 1.0 (Figure 3-6). Now that are the parameter estimates are obtained, we are ready to calculate the demand factor or $\exp(\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a})$:

$$\gamma = e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{D|S_a}} = \mathrm{e}^{(\frac{1}{2})(\frac{3}{1})(0.3^2)} = 1.144$$

Finally, the factored demand is calculated by multiplying the median demand $\eta_{D|S_a}(^{0.0004}s_a) = 0.047$ and the demand factor $e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2_{D|S_a}} = 1.144$:

$$D \cdot \gamma = \eta_{D|S_a}(^{0.0004}s_a) \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2_{D|S_a}} = (0.047)(1.144) = 0.0538 \qquad (3\text{-}21)$$

Factored Capacity $C \cdot \phi$

As it was mentioned before, this numerical example demonstrates the assessment of structural performance for the limit state of collapse, using maximum inter-story drift angle as the displacement-based parameter. Therefore, the displacement-based capacity is represented by maximum inter-story drift angle at the onset of the collapse limit state. Similar to factored demand estimation, the estimation of factored capacity consists of two parts, a) calculating the median capacity η_C for the collapse limit state, and b) calculating the capacity factor. Recalling from the previous parts of this numerical example in Chapter 2, the median (drift) capacity η_C for the collapse limit state and its dispersion parameter β_C (i.e., the standard deviation of the natural logarithm of maximum inter-story drift angle capacity values) were estimated to be equal to:

$$\eta_C \cong 0.07$$
$$\beta_C = 0.20$$

Hence, the capacity factor or $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2)$ can be calculated as:

$$\phi = e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2_C} = \mathrm{e}^{-(\frac{1}{2})(\frac{3}{1})(0.2^2)} = 0.94$$

Finally, the factored capacity is calculated by multiplying the median capacity $\eta_C = 0.07$ by the capacity factor $\exp(-\frac{1}{2}\cdot\frac{k}{b}\cdot\beta_C^2) = 0.94$:

$$C \cdot \phi = \eta_C \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2_C} = (0.07)(0.94) = 0.0658 \qquad (3\text{-}22)$$

Comparing the factored demand from Equation 3-21 and factored capacity from Equation 3-22, we can observe that:

$$D \cdot \gamma = 0.0538 \le C \cdot \phi = 0.0658$$

As we can see, the structure satisfies the design criteria in Equation 3-19 for an allowable probability equal to 0.0004 (i.e., 10% in 50 years) corresponding to the collapse limit state. This conclusion implies that the actual structure's limit state frequency (probability of failure) is less than 0.0004 per annum.

3.2.4 Fragility/Hazard Format – An IM-based probabilistic format

In the previous sections, we outlined in detail the main components of DCFD format, which is a displacement-based probabilistic design format. Here, we are going to discuss fragility/hazard format, a design format that stems from the IM-based framework equation in Chapter 2. One main advantage of an IM-based design format is that design and/or assessments are performed in the level of spectral acceleration and do not involve explicitly the displacement-based response.

The fragility/hazard format (see e.g., DOE 1020, 1994, and Kennedy and Short, 1994) is a graphical design format in which the design criterion involves comparing "fragility" curves to "hazard" curves. The hazard curves represent the probabilistic ground motion intensity or in general terms the "loading" characteristics, whereas the "fragility" curves represent the probabilistic structural capacity or in general terms the structural "resistance.

The same as the DCFD design format, the first step in developing a format is to set the design criteria. In the case of the fragility/hazard format the IM-based design criterion can also be stated as in Equation 3-6 for a given allowable annual probability, P_0: $H_{LS,S_a-based} \leq P_0$

Where the limit state frequency is calculated from the IM-based expression for the limit state probability in Chapter 2 (Equation 2-48):

$$H_{LS,S_a_based} = H_{S_a}(\eta_{S_{a,C}}) \cdot e^{\frac{1}{2} \cdot k^2 \cdot \beta^2_{S_{a,C}}} \qquad (3\text{-}23)$$

The expression for the IM-based limit state frequency can be substituted in the design criterion (Equation 3-6):

$$H_{S_a}(\eta_{S_{a,C}}) \cdot e^{\frac{1}{2} \cdot k^2 \cdot \beta^2_{S_{a,C}}} \leq P_0 \qquad (3\text{-}24)$$

where $S_{a,C}$ is the IM-based random variable representing the limit state capacity (or spectral acceleration capacity in short) and *k* is the parameter reflecting the steepness of the hazard curve for spectral acceleration. It will be demonstrated in the next chapter how to estimate the statistical properties (i.e., median, $\eta_{S_{a,C}}$, and standard deviation of the natural logarithm, $\beta_{S_{a,C}}$) of this random variable using non-linear dynamic analysis procedures such as Incremental Dynamic Analysis (see Vamvatsikos and Cornell, 2001). Recalling from Chapter 2, the spectral acceleration hazard can be approximated (at least locally) by a power-law relationship:

$$\lambda_{S_a}(x) = k_0 \cdot x^{-k} \qquad (3\text{-}25)$$

Therefore, $\lambda_{S_a}(\eta_{S_{a,C}})$, or the mean annual frequency of exceeding the median spectral acceleration capacity can be calculated from the above expression and then substituted in Equation 3-24:

$$k_0 \cdot \eta_{S_{a,C}}{}^{-k} \cdot e^{\frac{1}{2}\cdot k^2 \cdot \beta^2{}_{S_{a,C}}} \le P_0 \qquad (3\text{-}26)$$

After some simple re-arrangements with the objective of separating the "load" and "resistance" sides, Equation 3-26 would take the following form:

$$\eta_{S_{a,C}} \cdot e^{-\frac{1}{2}\cdot k \cdot \beta^2{}_{S_{a,C}}} \le \left(\frac{P_0}{k_0}\right)^{-\frac{1}{k}} \qquad (3\text{-}27)$$

Recalling from Equation 3-5, the right-hand side of the equation is in fact the indicator spectral acceleration ${}^{P_0}s_a$ for a hazard level equal to the allowable probability P_0 (or indicator spectral acceleration for allowable probability in short):

$${}^{P_0}s_a \le \eta_{S_{a,C}} \cdot e^{-\frac{1}{2}\cdot k \cdot \beta^2{}_{S_{a,C}}} \qquad (3\text{-}28)$$

This is the expression for the fragility/hazard format. A similar expression, with the exponential term (with a positive sign) applied to ${}^{P_0}s_a$ has been used in the current draft of ISO seismic criteria for offshore structures (Banon et al. 2001). It will be shown below how this expression

relates to the fragility and hazard curves. Similar to the DCFD format, the left-hand side of the expression represents the "factored demand for the allowable annual probability P_0", and the right-hand side represents the "factored capacity". However, if compared to the expression for the DCFD format in Equation 3-12, one can observe that the demand factor representing the dispersion in displacement-based demand is missing in the demand side of the expression. Nonetheless, the factored capacity looks similar to that of the DCFD format except for the fact that the *b* value is missing from the capacity factor. This is to be expected since the *b* value represents the (log) slope of the displacement-based demand parameter versus spectral acceleration; and the fragility/hazard format does not explicitly involve the displacement-based demand. Therefore, the design criterion based on the fragility/hazard format can be stated in terms of the IM-based factored capacity being less than or equal to the IM-based factored demand for a given allowable annual probability, P_0. The following sections are going to discuss fragility and hazard curves and how they can be employed in order to make parameter estimates for the fragility/hazard format in Equation 3-28.

3.2.4.1 Hazard curves

The hazard function, $H_{S_a}(s_a)$, for a given spectral acceleration value, s_a, can be defined as the mean annual frequency of exceeding the spectral acceleration value, s_a. The hazard function $H_{S_a}(s_a)$ is discussed in more detail in chapter 2. Figure 3-7 illustrates a schematic hazard curve. As it is mentioned before, the hazard curve is approximated by a power-law relation, $H_{S_a}(s_a) = k_0 \cdot s_a^{-k}$ in the derivation of the framework equations in Chapter 2, where the parameter *k* represents the steepness of the hazard curve. It is shown only schematically in Figure 3-7. In fact it is strictly the slope of the power-law hazard curve on a two-way logarithmic scale graph. It will be discussed in Chapter 4 that the slope parameter *k* can be estimated as the local slope of the hazard curve (Figure 3-7) in the region of hazard/spectral acceleration values that are of interest.

In the context of the fragility/hazard format, the hazard curve represents the probabilistic characteristics of "load" or demand. It is demonstrated in Figure 3-7 how the "factored demand for the allowable probability P_0" in the fragility/hazard format, ${}^{P_0}s_a$, can be derived from the

hazard curve. As it is shown in the figure, ${}^{P_0}s_a$ is the spectral acceleration with a mean annual frequency of exceedance (hazard) equal to P_0.

3.2.4.2 Fragility curves

The structural fragility for a limit state is defined as the conditional probability of exceeding the limit state capacity for a given level of ground motion intensity (conditional probability of failure in short). If the ground motion intensity is represented in terms of the spectral acceleration, the fragility can be expressed as:

$$F_{LS}(s_a) = P[S_a \geq S_{a,C} \mid S_a = s_a] = P[S_{a,C} \leq s_a] \qquad (3-29)$$

where $F_{LS}(s_a)$ is the structural fragility at spectral acceleration s_a for the limit state *LS*. It can be observed from the above equation that the fragility is expressed as the probability that the random variable $S_{a,C}$ is less than or equal the value s_a. Therefore, the fragility can also be stated as the cumulative distribution function of the random capacity, $S_{a,C}$. If it is assumed that the probability distribution of the spectral acceleration capacity, $S_{a,C}$, is lognormal with median, $\eta_{S_{a,C}}$, and standard deviation of the natural logarithm, $\beta_{S_{a,C}}$, the fragility can be expressed in terms of the widely tabulated "standardized" Gaussian distribution function:

$$F_{LS}(s_a) = P[S_{a,C} \leq s_a] = \Phi(\ln(\frac{s_a}{\eta_{S_{a,C}}}) \Big/ \beta_{S_{a,C}}) \qquad (3\text{-}30)$$

It can be observed from the above equation that the structural fragility for a given limit state can be plotted as a function of spectral acceleration. For a certain limit state, a monotonically increasing "fragility curve" can be plotted. A schematic fragility curve is shown in Figure 3-7. The median spectral acceleration capacity $S_{a,C}$ is marked as the spectral acceleration that corresponds to a fragility of 50%, since according to Equation 3-30:

$$F_{LS}(\eta_{S_{a,C}}) = \Phi(\ln(\frac{\eta_{S_{a,C}}}{\eta_{S_{a,C}}}) \Big/ \beta_{S_{a,C}}) = \Phi(0) = 0.50$$

Also the standard deviation of the (natural) logarithm of $S_{a,C}$ is marked as the difference between the spectral accelerations (on the logarithmic paper) corresponding to fragility values 16% and 50%, since again according to Equation 3-30:

$$F_{LS}(\eta_{S_{a,C}} \cdot e^{-\beta_{S_{a,C}}}) = \Phi(\ln(\frac{\eta_{S_{a,C}} \cdot e^{-\beta_{S_{a,C}}}}{\eta_{S_{a,C}}}) \Big/ \beta_{S_{a,C}}) = \Phi(-1) = 0.16$$

$$\beta_{S_{a,C}} = -\ln \frac{\eta_{S_{a,C}} \cdot e^{-\beta_{S_{a,C}}}}{\eta_{S_{a,C}}} = \ln F_{LS}^{-1}(0.50) - \ln F_{LS}^{-1}(0.16)$$

The fragility curve for a certain limit state represents the probabilistic characteristics of structural resistance or capacity for that limit state. Once the fragility curve is available for a limit state, the "factored capacity" according to the fragility/hazard format, $\eta_{S_{a,C}} \cdot \exp(-\frac{1}{2} \cdot k \cdot \beta^2{}_{S_{a,C}})$, can be calculated based on the parameter estimates for, k, $\eta_{S_{a,C}}$, and $\beta_{S_{a,C}}$ from the hazard and fragility curves (Figure 3-7).

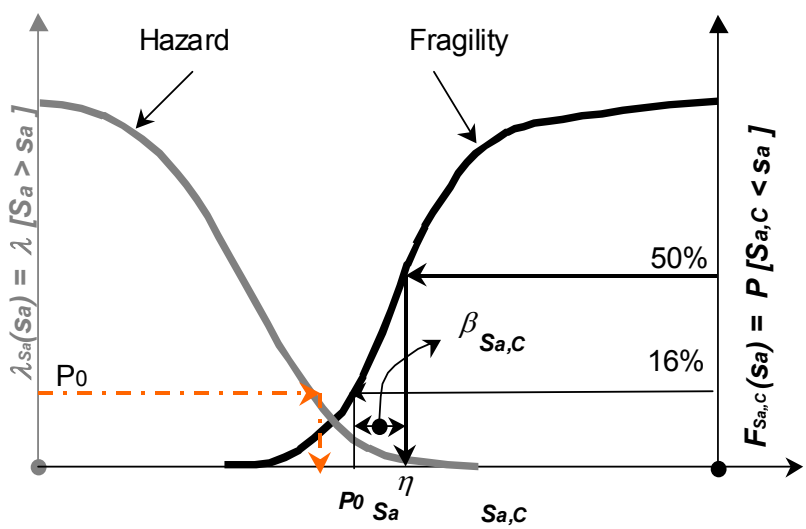

Figure 3-7 – A schematic plot of the hazard and fragility curves. The main parameters of hazard/fragility format are also shown on the figure.

3.2.4.3 The IM-based limit state frequency in terms of the fragility and hazard functions

It was demonstrated in the previous sections that fragility and hazard curves are very helpful graphic tools for estimating the IM-based factored demand and capacity. Moreover, it will be shown in this sections that the fragility and hazard curves can also be used in order to calculate

the limit state frequency. The IM-based limit state frequency is derived from the following expression (Equation 2-39):

$$H_{LS,S_a-based} = \int P[x \geq S_{a,C}] \cdot \left| dH_{S_a}(x) \right| \qquad (3-31)$$

where the first term in the integrand is nothing but the fragility $F_{LS}(s_a)$ at a spectral acceleration equal to s_a from Equation 3-29. Therefore, the limit state frequency in Equation 3-31 can also be written as:

$$H_{LS,S_a-based} = \int F_{LS}(x) \cdot \left| dH_{S_a}(x) \right| \qquad (3-32)$$

Where the IM-based limit state frequency is derived in terms of fragility and hazard. This equation states that the limit state frequency can be calculated as the area under the product of the structural fragility curve for the limit state multiplied by the (absolute value of) the increment in the hazard.

3.2.4.4 Numerical Example: (Performance Evaluation for an Existing Building)

In the numerical example presented earlier for the DCFD format, we assessed the performance of an existing 3-story frame for the collapse limit state for an allowable probability of $P_0 = 4 \times 10^{-4}$ (2% in 50 years). Here, we are going to use the same example in order to demonstrate probabilistic assessments based on the fragility/hazard format. Based on the fragility/hazard design format, we are going evaluate and compare the IM-based factored demand for the allowable probability $P_0 = 4 \times 10^{-4}$ and factored capacity for the collapse limit state.

Factored demand: The IM-based factored demand for a given probability $P_0 = 4 \times 10^{-4}$ is equal to ${}^{P_0=0.0004}s_a$ (Equation 3-28). Back in Section 3.2.3.7, the spectral acceleration with $P_0 = 4 \times 10^{-4}$ frequency of exceedance was found to be equal to ${}^{P_0=0.0004}s_a = 1.45[g]$. Therefore the factored demand for an allowable probability $P_0 = 4 \times 10^{-4}$ is equal to:

$$F.D.(P_0) = {}^{P_0=0.0004}s_a = 1.45\,[g]$$

Factored capacity: In order to calculate the factored capacity for fragility/hazard format, we assume that the (random variable) spectral acceleration capacity has median and (fractional) standard deviation equal to:

$$\eta_{S_{a,C}} \cong 2.15[g]$$

$$\beta_{S_{a,C}} \cong 0.20$$

The slope parameter *k* for the spectral acceleration hazard curve is reported to be equal to 3 on Figure 3-5. Now that we have the parameter estimates for estimates for, k, $\eta_{S_{a,C}}$, and $\beta_{S_{a,C}}$, we can calculate the factored capacity:

$$F.C. = \eta_{S_{a,C}} \cdot \exp(-\frac{1}{2} \cdot k \cdot \beta^2{}_{S_{a,C}}) = 2.15.\exp(-\frac{1}{2} \times 3 \times 0.2^2) = 2.15x0.94 = 2.0[g]$$

Comparing the factored demand for the allowable probability $P_0 = 4 \times 10^{-4}$ that was calculated above with the factored capacity, we can observe that:

$$F.D.(P_0) = 1.45[g] \leq F.C. = 2\ [g]$$

Therefore, we can conclude that the collapse limit state fragility/hazard design criterion is satisfied for an allowable annual probability of $P_0 = 4 \times 10^{-4}$ (i.e., 10% in 50 years). However, it should be noted that the parameter estimates used in this section for the spectral acceleration capacity are only for the sake of demonstration. In practical applications, non-linear dynamic analysis procedures (see Chapter 4 or Vamvatsikos and Cornell, 2001) can be implemented in order to build the structural fragility curve(s). Then the factored capacity can be calculated using the structural fragility curve as it is shown in Section 3.2.4.2.

3.3 Randomness and uncertainty, the sources of uncertainty

The design/assessment formats introduced so far only considered the randomness (or aleatory) type of uncertainty in the estimations for demand and capacity. This type of uncertainty is normally caused by record-to-record variability in the demand and capacity. However, it is of interest to include the uncertainty due to incomplete knowledge (epistemic uncertainty) in the estimations for parameters related to hazard, demand, and capacity. As seen in Chapter 2, consideration of the uncertainty due to incomplete knowledge affects the mean estimate of the

limit state frequency and/or the confidence statements that can be made about bounds on estimates of that probability. confidence in the probabilistic assessments. Therefore, it is desirable to measure the epistemic uncertainty involved in the estimation of the parameters, and also to represent such uncertainty in the design or the assessments. One way to do this is to simply replace H_{LS} in the previous section (3.2) everywhere by its mean estimate, $\overline{H}_{LS}$. As per Equation 2-86 in Chapter 2:

$$\overline{H}_{LS} = \nu \cdot \overline{P}[D > C] = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\cdot\frac{k^2}{b^2}\cdot(\beta^2{}_{RD}+\beta^2{}_{UD})} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\cdot(\beta^2_{RC}+\beta^2_{UC})} \qquad (3\text{-}33)$$

In which $\overline{H}_{S_a}$ is the mean estimate of the hazard curve, $\beta_{RC}{}^2 + \beta_{UC}{}^2$ and $\beta_{RD}{}^2 + \beta_{RC}{}^2$ are the total aleatory and epistemic uncertainty variances in demand and capacity , respectively. Then it is clear comparing this to Equation 3-1, that both of the DCFD formats introduced in Section 3.2 can be "up-graded" to include epistemic uncertainty by simply replacing H_{LS} by its mean estimate $\overline{H}_{LS}$ and the aleatory uncertainty variances, β^2 for demand ($D \mid S_a$) and capacity by their total β^2's, i.e., the total aleatory and epistemic variances. For example, Equation 3-2 becomes:

$$\eta_{D|^{P_0}s_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot(\beta^2_{RD|S_a}+\beta^2_{UD|S_a})} = \eta_C \cdot e^{-\frac{1}{2}\cdot\frac{k}{b}\cdot(\beta^2_{RC}+\beta^2_{UC})} \qquad (3\text{-}34)$$

where it is understood that the spectral acceleration ${}^{P_0}s_a$ is obtained from the mean estimate of the hazard curve at P_0. It is implied that the tolerable limits state frequency is specified in terms of its mean estimate. While not in this same format, DOE 1020 is based in using such a mean estimate approach with combined or total (aleatory plus epistemic) variances used for the demand and capacity. In this section we chose to outline a hybrid scheme. The DCFD format is extended to account for the epistemic uncertainty in the demand and capacity parameter estimations by associating a level of confidence with the frequency of exceeding a certain limit state. This format has recently been implemented for performance evaluation of existing steel moment-resisting structures in FEMA 351.

The DCFD format presented in this section mainly features consideration of epistemic uncertainty in the structural demand and capacity parameter estimations. However, it does implicitly take into account the epistemic uncertainty in the seismic hazard estimations by incorporating the "mean" estimate for the hazard instead of the "median" estimate.

3.3.1 A confidence-based DCFD format

In Chapter 2, the frequency of exceeding a limit state was derived by taking into account the uncertainty due to both aleatory and epistemic uncertainty. In such derivations, the limit state frequency was not a single quantity but a range of possible values represented by a central value (median) and a dispersion measure (standard deviation of the natural logarithm):

$$\hat{H}_{LS} = \hat{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RC}}$$

$$\beta_{H_{LS}} = \sqrt{\beta_H{}^2 + \frac{k^2}{b^2}\beta_{UD}{}^2 + \frac{k^2}{b^2}\beta_{UC}{}^2} \qquad (3\text{-}35)$$

Where $\hat{H}_{LS}$ is the median estimate for the limit state frequency and $\hat{H}_{S_a}(.)$ is the median estimate for the spectral acceleration hazard. $\beta_{\lambda_{LS}}$ is the dispersion measure (standard deviation of the natural logarithm) for the limit state frequency; and it contains the epistemic uncertainty-related dispersion terms for hazard, demand and capacity. We note that these could be used to develop one or more DCFD formats that treat epistemic uncertainty in hazard, demand, and capacity uniformly. Here, however, we chose to develop the hybrid scheme introduced above. Suppose we assume that there is no epistemic uncertainty in the estimation of the median spectral acceleration hazard (i.e., $\beta_H = 0$), the dispersion term $\beta_{\lambda_{LS}}$ in Equation 3-35 would be simplified to:

$$\beta_{H_{LS}} = \frac{k}{b}\sqrt{\beta_{UD}{}^2 + \beta_{UC}{}^2} = \frac{k}{b}\beta_{UT} \qquad (3\text{-}36)$$

where β_{UT} is the dispersion measure representing the total epistemic uncertainty in the displacement-based demand and capacity parameters. In order to account for the epistemic uncertainty in the estimation of hazard, we substitute the "median" estimate of the spectral acceleration hazard, $\hat{H}_{LS}$, in Equation 3-35 by "mean" estimate of the spectral acceleration hazard, $\hat{H}_{S_A} \cdot \exp(\frac{1}{2}\beta_H^2)$. The resulting median estimate for the limit state probability is denoted by "$\hat{H}_{LS}$" (in order to distinguish it from the median hazard $\hat{H}_{LS}$ in Equation 3-35:

$$"\hat{H}_{LS}" = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RC}} = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \qquad (3\text{-}37)$$

where the bar "–" represents the mean estimate; and (parallel to β_{UT}) β_{RT} is the dispersion measure representing the total aleatory uncertainty in the displacement-based demand and capacity parameters. It should be noted the hazard curves provided by the seismologists are usually in terms of the mean estimates of the annual frequency of exceedance, or "mean hazard" in short.

Now, we can build a confidence interval around the "median" estimate for the limit state frequency reflecting the epistemic uncertainty in the estimation of the demand (given S_a) and capacity parameters. The limit state frequency corresponding to the confidence level, *x*, denoted by, H_{LS}^{x}, can be expressed as:

$$H_{LS}^{x} = "\hat{H}_{LS}" \cdot e^{K_x \cdot \beta_{H_{LS}}} \qquad (3\text{-}38)$$

where K_x is the standard Gaussian variate associated with the probability x of <u>not</u> being exceeded. K_x can be found in standard probability tables under the Normal distribution as a function of the number of standard deviations above or below the mean. Substituting the "median" estimate for the frequency of exceeding the limit state from Equation 3-37 into Equation 3-38, one obtains the upper *x%* confidence limit H_{LS}^{x} on the limit state frequency:

$$H_{LS}^{x} = \overline{H}_{S_a}(S_a^{\eta_C}) \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2_{RT}} \cdot e^{K_x \cdot \beta_{H_{LS}}} \qquad (3\text{-}39)$$

Recalling from Chapter 2 and earlier sections in this chapter, the mean annual frequency of exceeding (hazard) the spectral acceleration corresponding to median displacement-based capacity can be estimated by power-law approximations:

$$\overline{H}_{S_a}(S_a^{\eta_C}) = k_0 \cdot (S_a^{\eta_C})^k = k_0 \cdot (\frac{\eta_C}{a})^{-\frac{k}{b}} \qquad (3\text{-}40)$$

Clearly this result represents a theoretically inconsistent treatment of the total epistemic uncertainty, as that in hazard, β_H, is incorporated in $\overline{H}$, while that in capacity and demand (given S_a) is treated via the confidence factor $e^{K_x \cdot \beta_{H_{LS}}}$. The objective is to focus on the structural epistemic uncertainties. More precisely, one should say that this represents an *x%* confidence limit on H_{LS} given the mean hazard curve.

After substituting the estimate for $\overline{H}_{S_a}(S_a^{\eta_C})$ from Equation 3-40 into Equation 3-39, H_{LS}^x or the limit state frequency corresponding to the confidence level *x* is derived as:

$$H_{LS}^x = k_0 \cdot \left(\frac{\eta_C}{a}\right)^{-\frac{k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \cdot e^{K_x \cdot \beta_{\lambda_{LS}}} \qquad (3\text{-}41)$$

Parallel to the derivation of the DCFD format in Section 3.2.3 (Equation 3-6), the design criterion can be stated by comparing the limit state frequency H_{LS}^x corresponding to a confidence level *x* to an allowable probability, P_0:

$$H_{LS}^x \le P_0 \qquad (3\text{-}42)$$

Substituting the expression for H_{LS}^x from Equation 3-41 into the design criterion in Equation 3-42 above:

$$k_0 \cdot \left(\frac{\eta_C}{a}\right)^{-\frac{k}{b}} \cdot e^{\frac{1}{2}\frac{k^2}{b^2}\beta^2{}_{RT}} \cdot e^{K_x \cdot \beta_{\lambda_{LS}}} \le \mathrm{P}_0 \qquad (3\text{-}43)$$

Similar to the derivation of the DCFD format in section 3.2.3, we next make some re-arrangements mainly in order to separate the "demand" and "capacity" sides:

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RT}} \cdot e^{-K_x \cdot \frac{b}{k}\beta_{\lambda_{LS}}} \ge a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \qquad (3\text{-}44)$$

Based on the relationship between the dispersion measure for the limit state frequency and the one measuring the total epistemic uncertainty in Equation 3-36:

$$\beta_{UT} = \frac{b}{k}\beta_{\lambda_{LS}} \qquad (3\text{-}45)$$

Therefore, Equation 3-44 is simplified to:

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RT}} \cdot e^{-K_x \cdot \beta_{UT}} \ge a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \qquad (3\text{-}46)$$

Now, we multiply both sides of the inequality in $\exp(-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT})$ (in order to make this format look similar to DCFD):

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RT}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT}} \cdot e^{-K_x \cdot \beta_{UT}} \geq a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT}} \qquad (3\text{-}47)$$

After further simplifications noting that: $\eta_{D|S_a}({}^{P_0}s_a) = a \cdot \left(\frac{P_0}{k_0}\right)^{-\frac{b}{k}}$, and also breaking up the total variance terms into the corresponding demand and capacity variances:

$$\eta_C \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{RC}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UC}} \geq \eta_{D|S_a}({}^{P_0}s_a) \cdot e^{\frac{1}{2}\frac{k}{b}\beta^2{}_{RD}} \cdot e^{\frac{1}{2}\frac{k}{b}\beta^2{}_{UD}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2{}_{UT}} e^{K_x \cdot \beta_{UT}} \qquad (3\text{-}48)$$

Equation 3-48 already resembles the DCFD format. Now, we can define the demand and capacity factors as:

$$\gamma = \gamma_R \cdot \gamma_U = e^{\frac{1}{2}\frac{k}{b}\beta^2_{RD}} \cdot e^{\frac{1}{2}\frac{k}{b}\beta^2_{UD}}$$
$$\phi = \phi_R \cdot \phi_U = e^{-\frac{1}{2}\frac{k}{b}\beta^2_{RC}} \cdot e^{-\frac{1}{2}\frac{k}{b}\beta^2_{UC}} \qquad (3\text{-}49)$$

Also the confidence factor corresponding to the confidence level, *x*, denoted by λ_x can be defined as:

$$\lambda_x = e^{-\beta_{UT}(K_x - \frac{1}{2}\frac{k}{b}\beta_{UT})} \qquad (3\text{-}50)$$

After the demand, capacity, and confidence factors are substituted from Equations 3-49 and 3-50 into Equation 3-48:

$$\eta_{D|S_a}({}^{P_0}s_a) \cdot \gamma \leq \eta_C \cdot \phi \cdot \lambda_{\mathrm{x}} \qquad (3\text{-}51)$$

or:

$$\frac{\eta_{D|S_a}({}^{P_0}s_a) \cdot \gamma}{\eta_C \cdot \phi} \leq \lambda_x \qquad (3\text{-}52)$$

This is the final form for the DCFD format taking into account both aleatory and epistemic uncertainty. This looks very similar to Equation 3-12 for DCFD format considering only the aleatory; except for the confidence factor γ_x and also the fact the demand and capacity factors in Equation 3-51 also include the effect of epistemic uncertainty. As it was mentioned before, this format is implemented in FEMA 351 for the purpose of the performance evaluation of the existing steel moment-resisting structures. If Equation 3-51 is satisfied, one can say that the probability of failure is less than P_0 with confidence *x%*.

It is also interesting to re-arrange Equation 3-51 in terms of the ratio of the factored demand to factored capacity related to aleatory uncertainty (according to Equation 3-49):

$$\frac{F.D.(P_0)}{F.C.} = \frac{\eta_{D|S_a}({}^{P_0}s_a)\cdot\gamma_R}{\eta_C\cdot\phi_R} \le e^{-K_X\cdot\beta_{UT}} \qquad (3\text{-}53)$$

or:

$$F.D.(P_0)\cdot e^{K_X\cdot\beta_{UT}} \le F.C. \qquad (3\text{-}54)$$

Which is the equivalent design criterion for the DCFD format taking into account the epistemic uncertainty. It should be noted that the factored demand and capacity in Equation 3-54 take into account only the aleatory uncertainty and are identical to those of Section 3.2.3.

The design criterion in Equation 3-53 can also be implemented in order to assess the level of confidence in an existing design for an allowable probability P_0, by following the steps outlined below:

1. Calculate the factored demand for an allowable probability P_0 and also the factored capacity from Equation 3-12 taking into account only the aleatory uncertainty.
2. Find the ratio of the calculated factored demand to factored capacity.
3. Estimate the dispersion measure $\beta_{UT} = \sqrt{\beta_{UD}{}^2 + \beta_{UC}{}^2}$ accounting for the total uncertainty in the estimation of the demand and capacity factors; examples appear in DOE 1020, FEMA 351, etc.
4. Solve the equation,

$$\frac{F.D.(P_0)}{F.C.} = e^{-K_X\cdot\beta_{UT}} \qquad (3\text{-}55)$$

   in order to find the corresponding Gaussian variate K_x. Note that Equation 3-55 is a special case of the design criteria in Equation 3-54 at the onset of the limit state.
5. Find the corresponding confidence level *x* for the existing design

3.3.1.1 Numerical Example

The procedure outlined above for finding the confidence level corresponding to an existing design can be applied to the numerical example in Section 3.2.3.7 where the factored demand for an allowable probability of $P_0 = 0.0004$ and factored capacity for the collapse limit state where calculated. The ratio of the factored demand to factored capacity is equal to:

$$\frac{F.D.(0.0004)}{F.C.} = \frac{0.0538}{0.0658} = 0.817$$

We have used the tables in FEMA 351 guidelines in order to estimate β_{UT}. For a 3-story (low-rise) structure, the tables recommend the value $\beta_{UD} = 0.15$ representing the uncertainty involved in the estimation of the displacement-based response using non-linear dynamic procedures for the collapse prevention limit state. Also the guidelines recommend the value $\beta_{UC} = 0.15$ associated with the uncertainty in the estimation of the global dynamic capacity for a low-rise structure. Therefore, β_{UT} can be estimated as:

$$\beta_{UT} = (0.15^2 + 0.15^2)^{\frac{1}{2}} = 0.212$$

The next step is to calculate the Guassian variate K_x from Equation 3-55:

$$\frac{F.D.(0.0004)}{F.C.} = 0.817 = e^{-K_X \cdot 0.212}$$

which implies that: $K_x = 0.953$

Hence, the corresponding confidence level for $K_x = 0.953$ can be found from a normal table:

$$x = \Phi(0.953) = 0.83$$

Therefore, we can conclude that the confidence that associated with the existing design of this structure is 83%. Precisely the same conclusion would be reached following the SAC-like format with definitions in Equations 3-49 and 3-50. The factored demand and capacity would differ in value as would their ratio, but the numerical confidence calculated via 3-50 would be the same.

3.4 Summary and conclusions

A probabilistic framework for the assessment of the performance of structures under seismic excitations was developed in the previous chapter. This chapter discusses several of many possible alternative design and assessment formats that stem from this probabilistic framework. The design formats discussed can all be traced back to a general probabilistic design criterion, which is satisfied when the frequency of exceeding a certain limit state is less than or equal to an allowable probability P_0. A design format usually offers equivalent displacement-based or spectral acceleration-based criteria parallel to the general design criterion. The advantage of these equivalent criteria is that they are expressed in terms of structural response parameters and hence the resulting format can be incorporated more easily into the existing design codes.

These formats can be categorized based on the types of uncertainty involved in the parameter estimations. The first category takes into account the randomness, also known as the aleatory uncertainty, in the assessment of the demand and capacity. The second category takes into account both the randomness (aleatory uncertainty) inherent in the estimation of the demand and capacity and also the uncertainty due to limited knowledge (epistemic uncertainty) in the estimation of the hazard, demand and capacity parameters.

Within the first category, the Demand and Capacity Factored Design (DCFD) format was discussed. This format is a (displacement-based) design format analogous to the LRFD procedures that stems directly from the expression for the frequency of displacement-based demand exceeding capacity for a certain limit state. The DCFD format offers a displacement-based design criterion in which the factored (displacement-based) demand (representing "load") for the allowable probability P_0 should be less than or equal to the factored (displacement-based) capacity (representing "resistance") for a certain limit state. Another format discussed under the first category is a spectral acceleration-based format known as the Fragility/Hazard format, in which the fragility curves represent the structural "resistance" and the hazard curves represent the seismic "load". This format offers a design criterion in which the spectral acceleration with a hazard value (i.e., frequency of exceedance) equal to the allowable probability P_0, is less than or equal to the factored capacity expressed in spectral acceleration terms. Each fragility curve is specific to a certain limit state and provides the necessary parameter estimates for the calculation of the factored capacity for that limit state. The fragility/hazard format has been implemented for

the design and evaluation of energy facilities (e.g., nuclear power-plants) in the DOE 1020 guidelines and for offshore structures in ISO guidelines.

Within the second category of the design formats, a more general form of the displacement-based DCFD format is discussed. This format associates a level of confidence with the estimated frequency of exceeding a limit state. This confidence level represents explicitly the epistemic uncertainties involved in the estimation of the demand and capacity parameters and implicitly (and approximately) the epistemic uncertainty in the hazard estimation. The displacement-based design criterion offered by this format is similar to that of the DCFD with aleatory uncertainties except for an additional factor that reflects the level of confidence in the estimation of the limit state frequency. The DCFD format can be used for both designing a building with a certain level of confidence and also determining the level of confidence associated to an existing design. This format is implemented in the guidelines for the performance evaluation of existing and earthquake damaged buildings in FEMA 351.

As a final note, it should be mentioned that there are numerous ways to transform the probabilistic design criterion into design criteria that are suitable for code implementation. This chapter only manages to discuss the most commonly-used of these formats. Nevertheless, the fundamentals used for deriving these formats can be applied towards developing new alternative formats.

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