A Special Application of Non-linear Dynamic Analysis Procedures in Probability-Based Seismic Assessments in the Region of Global Dynamic Instability

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Demand and Capacity Factor Design (DCFD)

Design Criterion for a given allowable probability level, P_o:

$$\text{Factored Capacity} \geq \text{Factored Demand } (P_o)$$

Assumptions Leading to a Closed-form Expression for (DCFD)

Spectral acceleration hazard is a power-law function of the spectral acceleration:

$$\lambda_{S_a}(x) = k_o \cdot x^{-k}$$

Demand (given spectral acceleration) can be described by a lognormal distribution with constant standard deviation.

Median demand is a power-law function of the spectral acceleration:

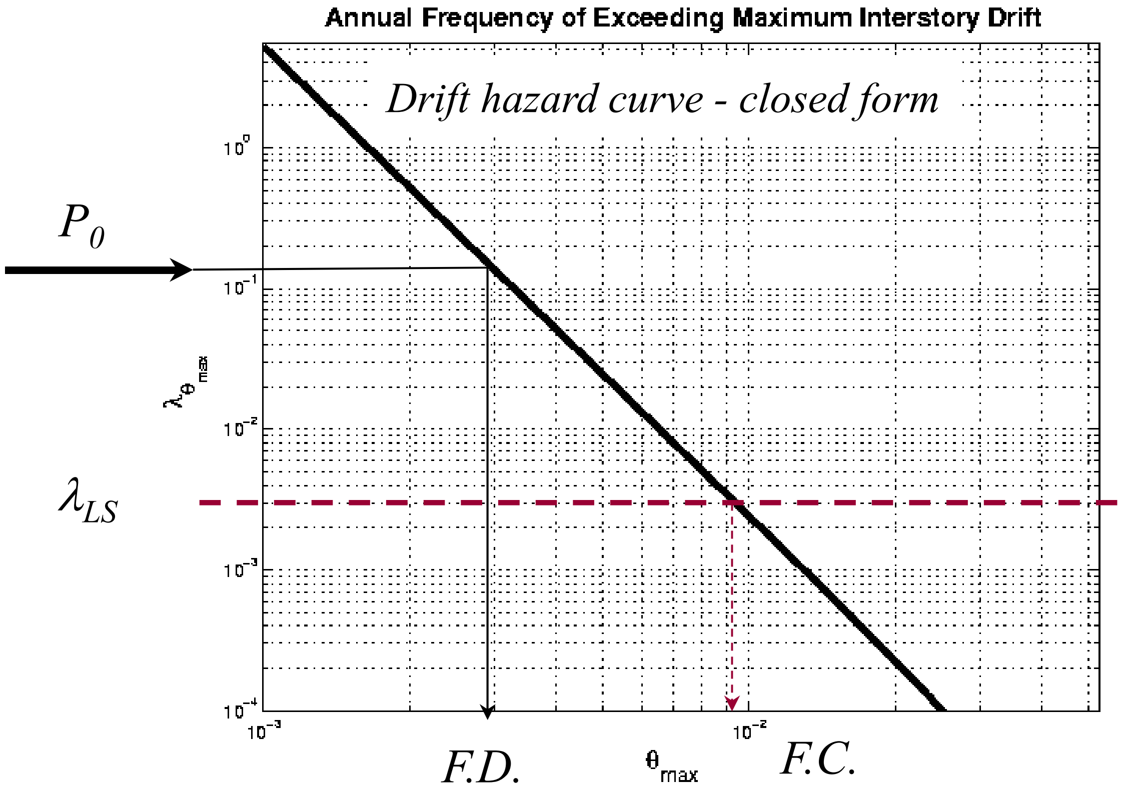
$$\eta_{\theta_{\max}|S_a=x} = a \cdot x^{b}$$

Median capacity is described by a lognormal distribution with constant median and standard deviation (with respect to spectral acceleration).

Closed-form Presentation of DCFD format

$$\eta_{\theta_{cap}} \cdot e^{-\frac{1}{2}\frac{k}{b}\cdot\beta^2_{\theta_{cap}}} \geq \eta_{\theta_{\max}|^{P_0} S_a} \cdot e^{\frac{1}{2}\cdot\frac{k}{b}\cdot\beta^2{}_{\theta_{\max}|S_a}}$$

A Graphic Presentation of DCFD format:



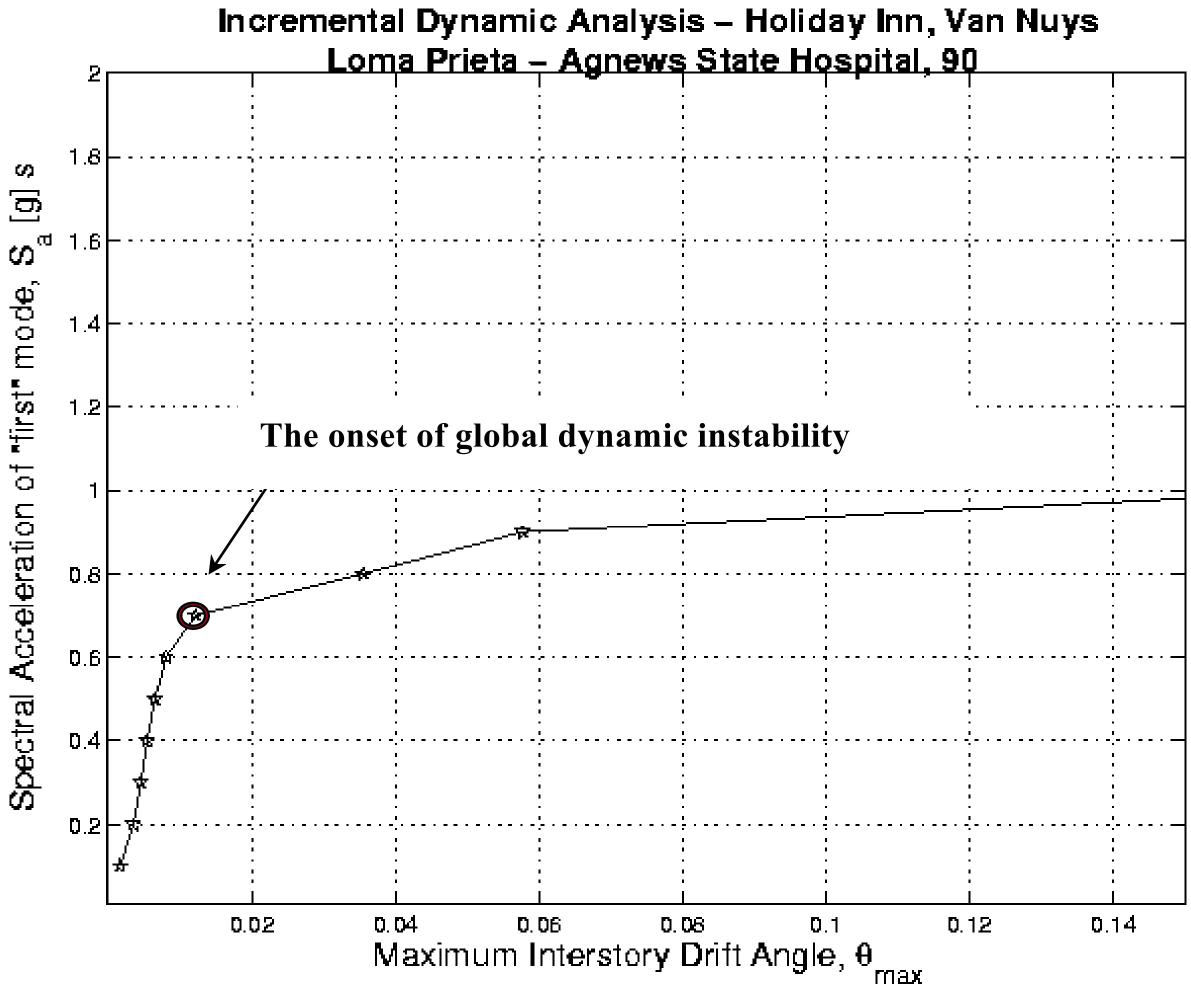
Factored Capacity $\leq$ Factored Demand

- However, in many cases, the assumptions leading to a closed-form solution for DCFD format are not valid.

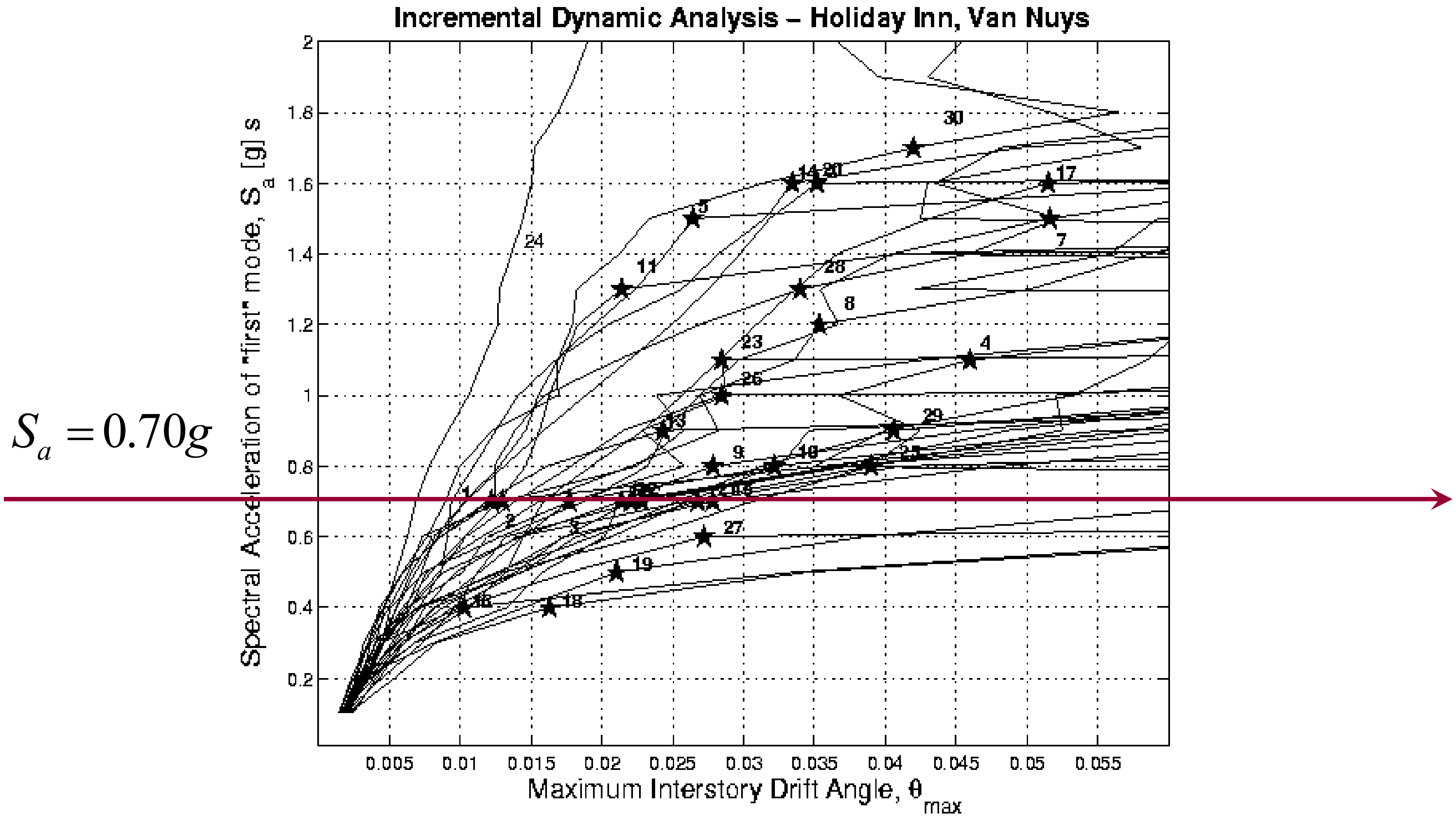
- Can we estimate the DCFD parameters locally, although it is in contrast with its basic underlying assumptions?

- A special case is when the displacement response of the structure becomes arbitrarily large for a small change in ground motion intensity (global dynamic instability)

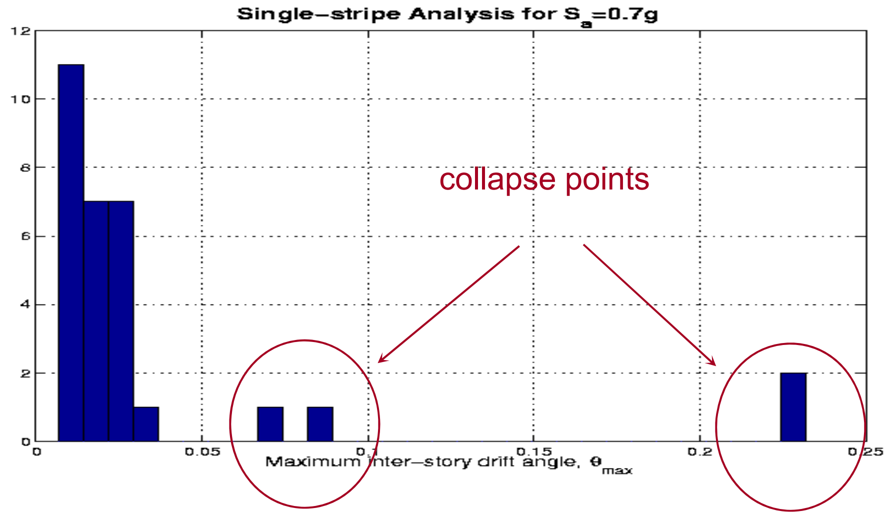
Incremental Dynamic Analysis (IDA)

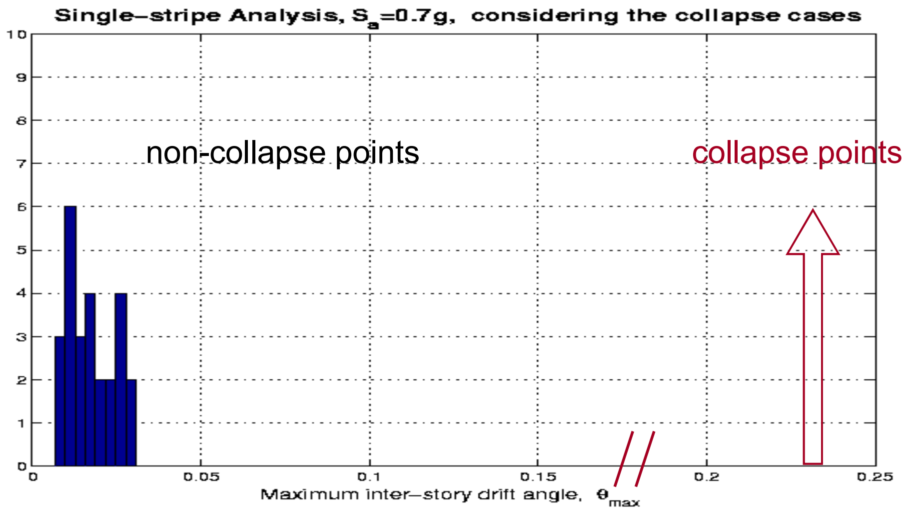


Incremental Dynamic Analysis (IDA)



Displacement Response in the Region of Global Dynamic Instability in the Structure





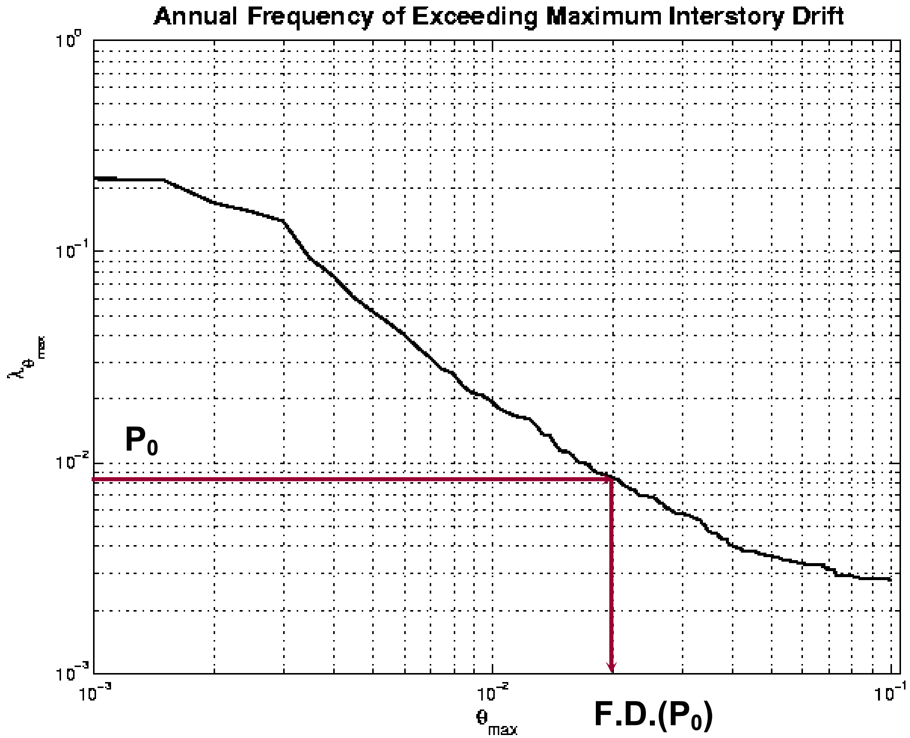
Probability Distribution of Response in the Region of Global Dynamic Instability

The probability distribution of structural response in the region of global dynamic instability can be divided into collapse and non-collapse parts.

$$G_{\theta_{\max}|S_a}(y\,|\,x) = P[\theta_{\max} > y\,|\,S_a = x] = G_{\theta_{\max}|NC,S_a}(y\,|\,x)\cdot P_{NC|S_a}(x) + (1 - P_{NC|S_a}(x))$$

Factored Demand and the Mean Annual Frequency of Exceeding a Demand Level (Demand Hazard)

$$F.D.(p_0) = \lambda_{\theta_{\max}}{}^{-1}(p_0)$$



Factored demand for the allowable probability P_0, is also the demand value that is exceeded with a mean annual frequency P_0 .

A Three-Parameter Distribution

The non-collapse complementary cumulative distribution (CCDF) can be represented by lognormal distribution:

$$G_{\theta_{\max}|S_a}(y\,|\,x) = \Phi^{C}_{\theta_{\max}|NC,S_a}\left(\frac{\ln(y) - \ln(\eta_{\theta_{\max}|NC,S_a}(x))}{\beta_{\theta_{\max}|NC,S_a}(x)}\right) \cdot P_{NC|S_a}(x) + 1 - P_{NC|S_a}(x)$$

The percentile *p* of the three-parameter distribution can be derived by solving the above equation for *y* for $G_{\theta_{\max}|S_a}(y\,|\,x) = p$:

$$y_p(x) = \eta_{\theta_{\max}|NC,S_a}(x) \cdot \exp\left(\beta_{\theta_{\max}|NC,S_a}(x) \cdot \Phi^{-1}_{\theta_{\max}|NC,S_a}\left(\frac{p}{P_{NC|S_a}(x)}\right)\right)$$

Estimating the Parameters of TPD with Power-law functions

The probability of non-collapse is presented by a power-law distribution:

$$P_{NC|S_a}(x) = \left(\frac{x}{s_{a0}}\right)^{-\beta_C} \qquad x > s_{a0}$$

$$P_{NC|S_a}(x) = 1 \qquad x \le s_{a0}$$

The displacement-based response given non-collapse is approximated by a power-law relation:

$$\theta_{\max|NC,S_a}(x) = A \cdot x^B$$

or

$$\ln\ \theta_{\max|NC,S_a}(x) = \ln A + B \cdot \ln x$$

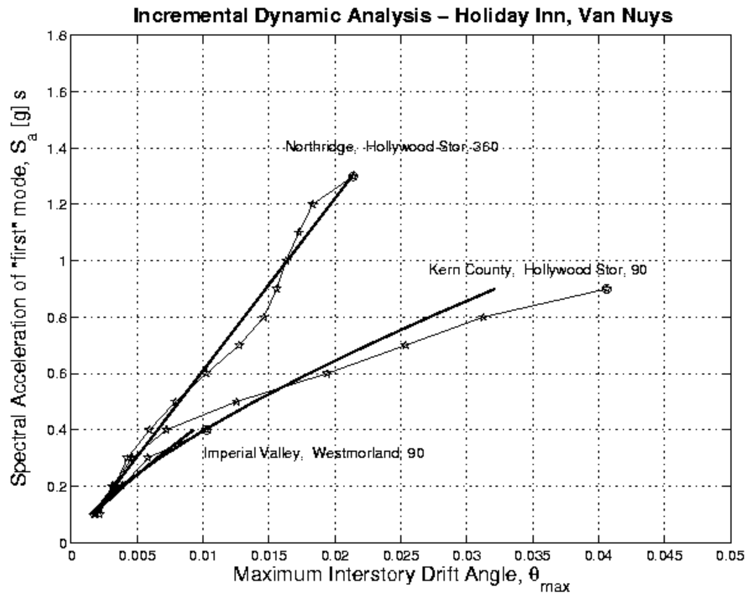
This results in the following expressions for the median and (fractional) standard deviation given non-collapse:

$$\eta_{\theta_{\max}|NC,S_a}(x) = \eta_A \cdot x^{\mu_B}$$

$$\beta^2{}_{\theta_{\max}|NC,S_a}(x) = \beta_A^2 + 2 \cdot \ln x \cdot \rho \cdot \sigma_B \cdot \beta_A + (\ln x)^2 \cdot \sigma^2{}_B$$

Using the IDA curves in order to estimate random parameters A and B

$$\theta_i(x) = a_i \cdot x^{b_i}$$

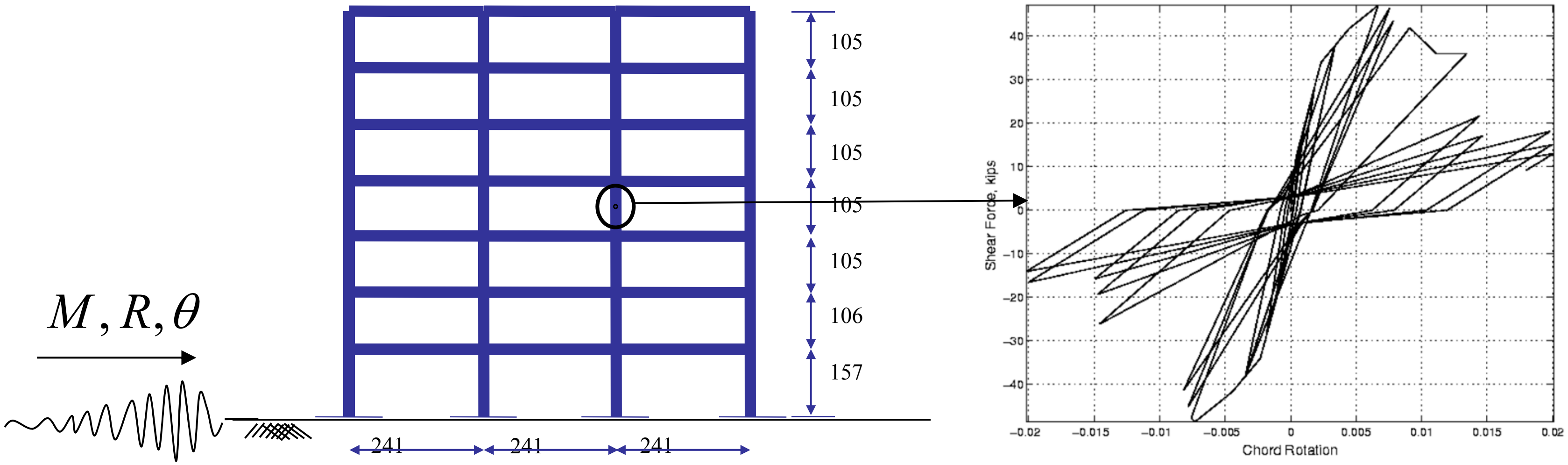


IDA curves in the non-collapse regime approximated by power-law functions.

The points marked as circles in the end are the points at which the global instability capacity was reached.

Illustrative Example

Structural Model



Beam-column model with stiffness and strength degradation in shear and flexure using DRAIN2D-UW by J. Pincheira et al.

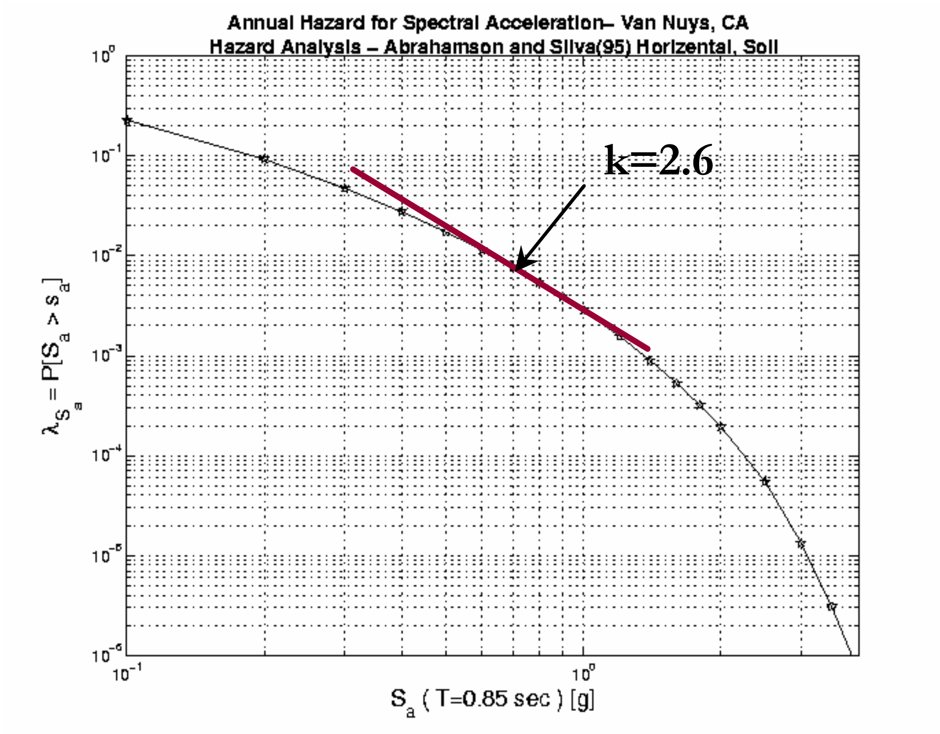
Ground Motion Records

30 ground motion records are taken from the PEER strong motion database, with:

$5.5 < M < 7.5$, $15 < R < 120$ km

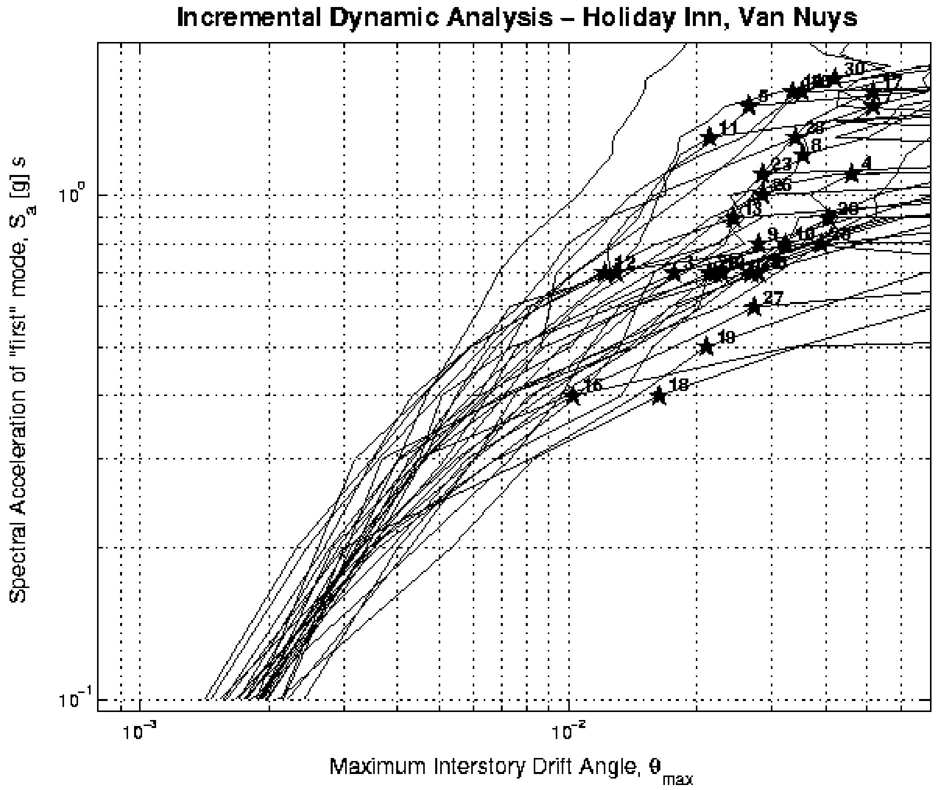
The records are all California events recorded on deep soil (Geo Matrix C and D soil types).

A site-specific hazard curve for the model structure



Site-specific hazard curve, Van Nuys CA, and a linear approximation to the hazard curve.

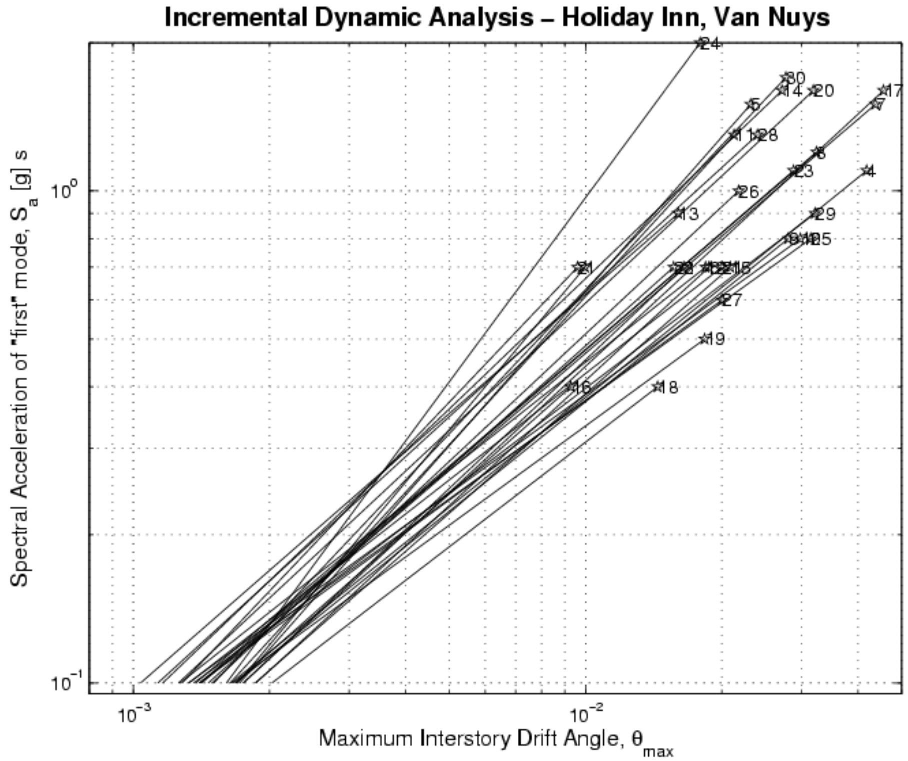
IDA curves for the structure



IDA curves for the model structure. The onset of global dynamic instability is marked on each IDA curve

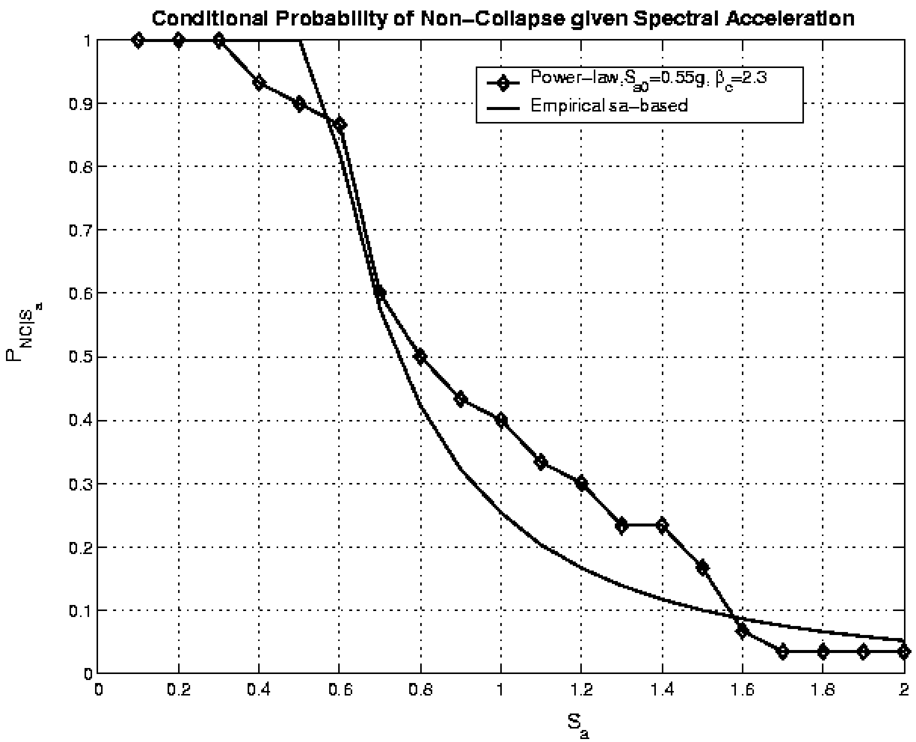
Using the resulting IDA curves to get the percentiles for the TPD

The non-collapse portion of the IDA curves (in log-log plane) is approximated with straight lines:



η_A	β_A	μ_B	σ_B	ρ
0.025	0.43	1.23	0.19	0.93

Using the resulting IDA curves to get the percentiles for the TPD (continued)



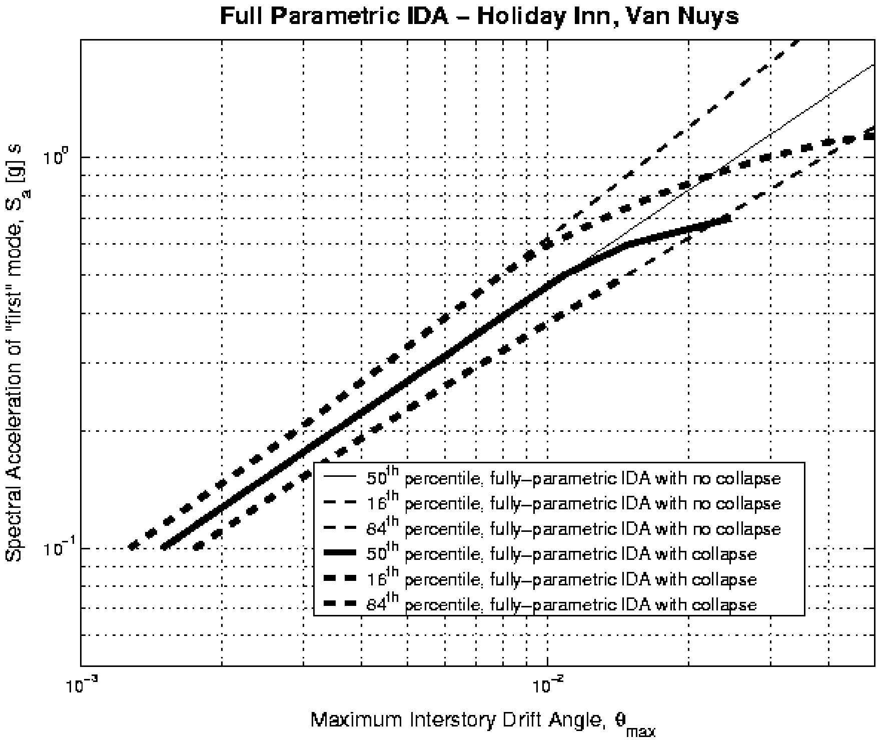
$$\hat{P}_{NC|sa}(x) = \frac{\sum_{i}^{N} I[x \geq s_{a,c_i}]}{N}$$

s_{a0}	β_c
0.55g	2.3

The probability of non-collapse is estimated by the ratio of the collapsed to non-collapsed points on the IDA curves for a given spectral acceleration.

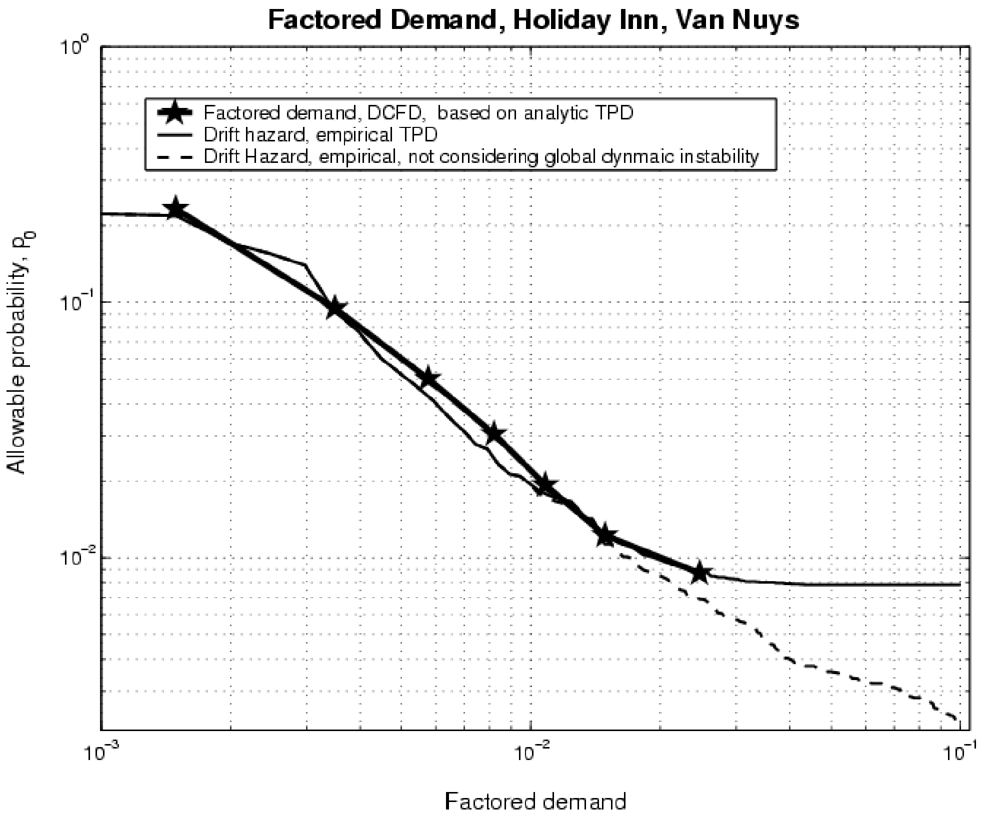
Using the resulting IDA curves to get the percentiles for the TPD (continued)

$$y_p(x) = \eta_{\theta_{\max}|NC,S_a}(x) \cdot \exp(\beta_{\theta_{\max}|NC,S_a}(x) \cdot \Phi^{-1}_{\theta_{\max}|NC,S_a}(\frac{p}{P_{NC|S_a}(x)}))$$



The 16th, 50th, and 84th percentiles for the three-parameter distribution are plotted as functions of spectral acceleration.

Estimating the factored demand for different allowable probability levels



$$F.D.(p_0) = \eta_{\theta_{\max}|^{p_0} S_a}(x) \cdot \exp\left(\frac{1}{2} \cdot \frac{k(x)}{b(x)} \cdot \beta^2_{\theta_{\max}|S_a}(x)\right)$$

Summary and Conclusions

- DCFD is an analytic probability-based design/assessment format based on a set of assumptions.
- These assumptions may not be valid in the range of global dynamic stability in the structure.
- An application of DCFD in the range of global dynamic instability is investigated, using local parameter estimates in the original formulation.
- These local parameter estimates are obtained as a function of ground motion intensity (spectral acceleration) using median and (fractional) standard deviation of a three-parameter distribution used to describe the structural response in the range of dynamic instability.
- The demand part of the DCFD format is estimated based on the local parameter estimates and compared to a case where it is calculated without assuming any particular analytic forms. The results are very close.

Summary and Conclusions (Continued)

- Incremental dynamic analysis (IDA) is used as an analysis tool in the assessments.

- In this study, individual IDA curves are approximated analytically with power-law functions given non-collapse. A three-parameter distribution is used take into account the collapse cases.

- Performing incremental dynamic analysis requires considerable analysis effort. The proposed analytical approximation using three-parameter distribution, makes it possible to obtain them with considerably less analysis effort.

- The focus of the comparisons was on the demand side of DCFD. There is also need for investigating the capacity side.

Thank You.