LIMIT ANALYSIS MODELLING OF CATENARY EFFECTS IN PROGRESSIVE COLLAPSE ANALYSIS OF RC FRAME STRUCTURES

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ABSTRACT
Progressive collapse is a phenomenon in which local failure of a structural component may lead to an overall loss of load bearing capacity. Moment-resisting frame structures can resist progressive collapse through various mechanisms like frame action and catenary action. The frame action is known to contribute as the main mechanism resisting progressive collapse. In this paper, the effect of catenary action on the resistance of reinforced concrete frame structures against progressive collapse is evaluated. This is done by modelling the catenary effect as a second order term in the formulation of the virtual work theorem. It is observed that after the rupture of the rebar, the presence of catenary effects may significantly increase the resistance of the structure to progressive collapse. The results have resaonable agreement with experimental results in the literature.

INTRODUCTION

In the past few decades, many cases of progressive collapse due to gas explosion or blast have taken place. The collapse of a 22 story building in Ronan Point, London (UK), as a result of gas explosion in 1968, first attracted the attentions of engineers to this phenomenon (McGuire 1974, Leyendecker and Ellingwood 1977). The Alfred P. Murrah Federal Building bombing in 1995 in Oklahoma City (USA) (Corley et al. 1998) and the destruction of the World Trade Center (WTC), in New York (USA) in 2001 (Bazant and Zhou 2002, Bazant and Verdure 2007, Bazant et al. 2008, Seffen 2008) are among the most devastating cases of blast-induced progressive collapse.

In the United States, the Department of Defense (DOD 2005) and General Service Administration (GSA 2003) issued design guidelines in an attempt to mitigate the potential for progressive collapse in structures. Both guidelines use the alternate load path (ALP) method for progressive collapse analysis. The ALP method is an event-independent procedure. In this method, a single column is assumed to be suddenly missing and an analysis is conducted to determine whether or not the structure can bridge across the missing column.

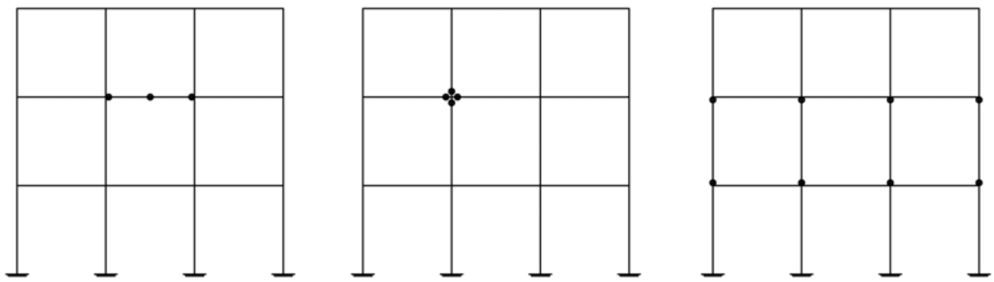
RC structures typically resist progressive collapse through frame action, catenary action, (Izzuddin 2005) and membrane action in floor slabs (Alashker and El-Tawil 2011, Dat and Hai 2011). In frame action, the applied load is resisted by the formation of plastic hinges in beams and columns. Catenary or cable action contributes to increasing the load bearing capacity through tension caused in the structural members due to large deflections. In concrete structures, the catenary action is typically activated after the rupture of bottom bars (Sasani and kropelnicki 2008). Finally, the membrane action of slabs play a key role in increasing the progressive collapse resistance of RC structures (Alashker and El-Tawil 2011).

Many experimental and analytical research efforts have been performed to study the catenary action and its influence on the overall load bearing resistance of the structure. A static experimental study on progressive collapse resistance of concrete frame structures is done by Yi et al. (2008) where a four bay and three storey model is tested. A constant vertical load was applied on the top of the middle column while step-by-step unloading was initiated by lowering the mechanical jack on the first story until the bottom steel bars near the end of the first floor beam adjacent to the middle column ruptured. It is shown that catenary mechanism led to an overall 30% of increase in the resistance with respect to teh case where only plastic mechanism is considered. Sadek et al. (2011) presented the results of full-scale testing and finite element modeling for two steel and two reinforced concrete 2D assemblies. Each assembly comprised of three columns and two beams represented portions of structural framing systems of a 10 storey building designed as intermediate moment frames (IMF) and special moment frames (SMF). It is observed that more stringent seismic design and detailing of steel SMF assembly leads to an increase in both the ultimate vertical deflection and the ultimate vertical load bearing capacity. Sasani and kropelnicki (2008) carried out an experimental program to study the behavior of a continuous perimeter beam in a reinforced concrete frame structure after a supporting column removal The 2D test was conducted utilizing displacement control at the center span. After bar fracture, catenary action provided by the top reinforcement causes the resistance of the beam to increase.

This study proposes a second-order virtual work formulation for taking into account the catenary action in the progressive collapse analysis of frame structures. The ultimate load bearing capacity of the structure is calculated employing non-linear optimization in order to minimize the ratio of internal to external virtual work. A Matlab-SAP user interface is created in order to facilitate the definition of the geometry and material properties of the structure. The validity of the formulation is demonstrated by comparison of the results with the results of the experimental work done by Sasani and kropelnicki (2008).

1. PROGRESSIVE COLLAPSE ANALYSIS USING PLASTIC LIMIT ANALYSIS

Plastic limit Analysis strives to determine the smallest load factor for which a mechanism forms. A mechanism is formed if the structural elements between plastic hinges are able to move as rigid bodies without an increase of loads. This method is based on the virtual work principle in which a structure in equilibrium under the action of a system of external forces is subjected to a virtual deformation pattern compatible with its support conditions. The theorem states that the external work done by the external forces on the displacements associated with the virtual deformation is equal to the internal work done by the internal stresses on the strains associated with this displacement. The plastic limit can be considered an application of the virtual work theorem in which a set of independent structural mechanism deformations are applied as the virtual deflection patterns.



(a) Beam mechanism (b) Joint mechanism (c) Story mechanism

Fig. 1. Independent mechanisms

The independent mechanisms, as shown in Fig. 1, are classified as: the beam mechanism in which at least three hinges in a row are formed in a given beam, the joint mechanism in which the end hinges of all elements connected to a joint are activated and the story mechanism in which the plastic hinges at both ends of columns in a given story are activated.
The collapse load factor λ_C is the ratio of internal to external virtual works corresponding to a given structural mechanism, denoted by u and e respectively:

$$\lambda_c = \frac{u}{e} \tag{1}$$

Grierson and Gradwill (1971), proposed a linear programming formulation for deriving the load factor λ_C:

$$\text{Minimize } u = \sum_{j=1}^{s} M_{Pj}\theta_j \tag{2}$$

$$\text{Subject to: } \begin{cases} \theta_j = \sum_{i=1}^{m} t_i \theta_{ij} \quad (j = 1, 2, ..., s) \\ \sum_{i=1}^{m} t_i e_i = e \end{cases} \tag{3}$$

where M_{pj} is the plastic moment capacity of critical section; θ_j is the inelastic rotation increment taking place at critical section j during formation of the mechanism i; s is the total number of critical sections; m is the number of independent mechanisms; t_i is a scale factor defining the way and the extent to which elementary mechanism i enters into the combination that forms the mechanism; θ_{ij} is the inelastic rotation increment occurring at critical section j during the formation of elementary mechanism i; e_i is the external work done by the applied service loads during the formation of elementary mechanism i.

Asprone et al. (2010), applied the linear programming in order to evaluate the collapse load factor. They have then implemented the linear programming algorithm in a simulation-based probabilistic procedure for multi-hazard risk assessment of blast-induced progressive collapse in a seismic zone.

2. CONSIDERING THE CATENARY ACTION AS A SECOND ORDER EFFECT IN THE PROGRESSIVE COLLAPSE ANALYSIS

As mentioned before, RC structures can resist progressive collapse through frame action, catenary action and membrane action in floor slabs. Catenary or cable action enables the resistance of gravity forces through tension in an event of large deflection. According to Fig. 2, catenary action is a tensile force that is composed of a vertical and horizontal component due to deflection of the member without any flexural reaction. The vertical component provides some resistance to the gravity loads.

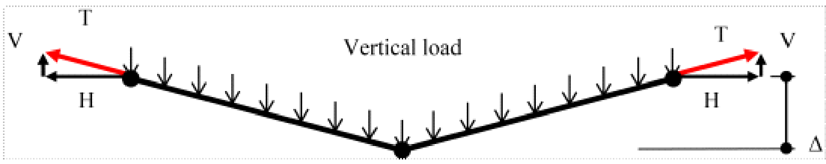


Fig. 2. Forming catenary action in a beam

The response of an RC beam to vertical load in catenary action generally depends on the geometry of the element, reinforcement ratio and its distribution along the beam, and the concrete and steel mechanical properties (Sasani and kropelnicki, 2008). The load-displacement curve looks like Fig. 3. At first, frame action is activated. Then, because of cracking in concrete, the load-.bearing capacity of the beam decreases until the rupture of the bottom bars. The catenary action is activated normally after the rupture of the bottom rebar. Similar to the frame action, the collapse load factor due to the activation of catenary action can be calculated using limit analysis based virtual work principles. The internal energy can be calculated as the product of the catenary tensile forces and the mechanism deformation:

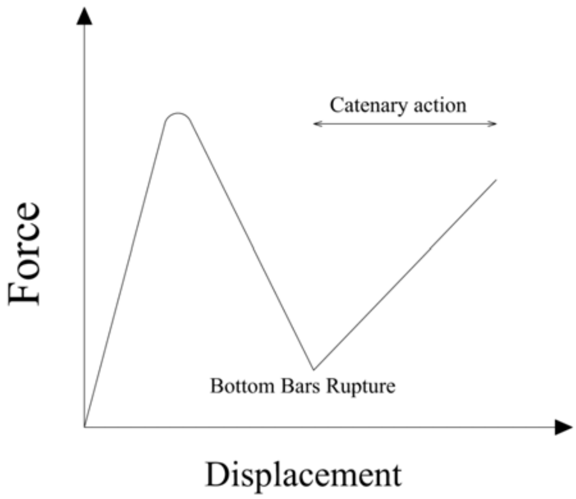


Fig. 3. Typical load-displacement curve of a RC beam

$$u_{ca} = \sum_{k=1}^{nElem} F_k \Delta_k \qquad (4)$$

where u_{ca} is the internal energy dissipated during the formation of an individual mechanism by catenary action; F_k is the tension force initiated in the member k during catenary action; Δ_k is the elongation of member k due to F_k. If an element with length L, have vertical displacement with end rotation θ, it can be shown according to Fig. 4 that:

$$\Delta = L\left(\sqrt{1+(\tan\theta)^2} - 1\right) \approx \frac{L}{2}\theta^2 \qquad (5)$$

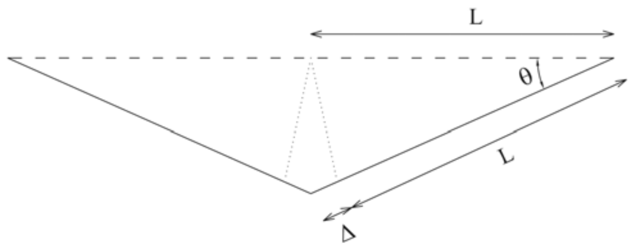


Fig. 4 Beam deformation after column removal

Substituting Eq. 5 in Eq. 4 and summing up on the total number of critical sections:

$$u_{ca} = \sum_{j=1}^{s} F_j \frac{L_j}{4} \theta_j^{\,2} \tag{6}$$

Where F_j is the tensile force in the element on which hinge j is located; and L_j is the length of the element. It is assumed that each of the hinges located in the element ends contribute to half of the deflection in the element. Not considering the effect of concrete section on tensile force, the force F_j can be calculated as:

$$F_j = f_{yj} . A_{stj} \tag{7}$$

where f_{yj} is yield tension of bars; A_{stj} is total area of top reinforcement. The problem of finding the smallest load factor corresponding to the catenary action can be formulated as:

$$\text{minimize}\left(\lambda_c\right) = \text{minimize}\left(\frac{\sum_{j=1}^{s} \left(F_j \frac{L_j}{4} \theta_j^{\,2} \right)}{\sum_{i=1}^{m} t_i e_i} \right) \tag{8}$$

$$\text{Subject to:} \quad \theta_j = \sum_{i=1}^{m} t_i \theta_{ij} \tag{9}$$

$$\text{Subject to:} \quad \sum_{i=1}^{m} t_i e_i > 0 \tag{10}$$

where the optimal λ_C value can be calculated by solving the above constrained non-linear optimization problem. It can be shown that this optimization problem is a convex problem (Boyd and Vandenberghe (2009)). It should be mentioned that the objective function in Eq. 8 may have a linear term that includes a percentage of plastic moment capacity of the section at the time that catenary action is activated. In this work it is assumed that the moment resisting capacity of the section is zero when the catenary effect is activated.

3. NUMERICAL EXAMPLES

3.1. Creating a Matlab interface with SAP

A user interface between Matlab and SAP is created which simplifies the process of defining the structural geometry and material properties in Matlab. This interface reads a SAP input file and generates the required vectors and matrices in Matlab necessary for performing limit analysis based on solving the optimization problems outlines in Eqs. 2-3 and 8-9-10. The following numerical exmaples are generated using this interface.

3.2 Example 1. A two bay RC beam with clamped ends

To demonstrate the validity of the proposed method, the frame used by Sasani and kropelnicki (2008) is analyzed. The material properties are: f_y=525MPA, E_s=2.5MPA and f_t=2.5MPA (the ultimate tensile stress). The strain corresponding with the maximum stress for compressive concrete is taken as ε_{CO}=0.0025 and ultimate strain of concrete is taken as ε_{CU}=0.008. A point gravity load equal to 20 kN is applied to the frame of Fig. 5. The detail of reinforcement of the beam is shown in Fig. 6.

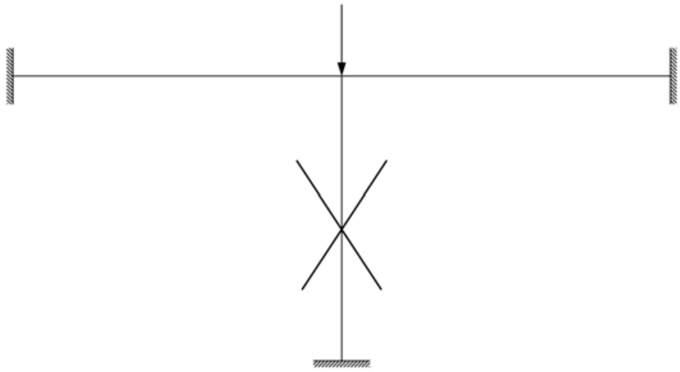


Fig. 5. Two bay beam being analyzed

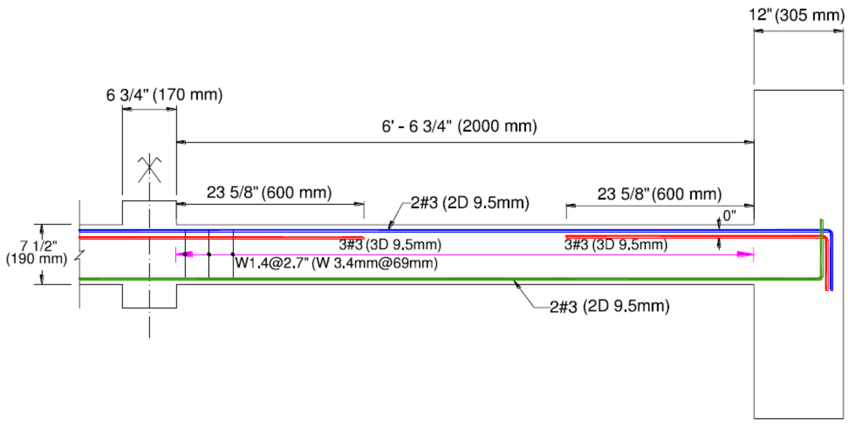


Fig. 6. Reinforcement Detailing of the two span beam (Sasani and kropelnicki (2008))

The value of λ_C is calculated for the two cases of frame action λ_{pl} and catenary action λ_{ca}. The load factor corresponding to frame action λ_{pl}, is calculated based on Eqs. 2-3 employing the simplex algorithm using Matlab's optimization toolbox. The load factor corresponding to catenary action λ_{ca} is calculated based on Eqs. 8-9-10 using matlab's constrained optimization routine and the interior point algorithm.

The values of λ_{pl} and λ_{ca} are 3.7 and 2.9 respectively. According to these values, the maximum load that the structure can sustain at the end of its frame action and catenary action is 74 and 58 kN, respectively which has good agreement with the result of the test done by Sasani and kropelnicki (2008).

3.3 Example 2: A 3D low rise RC structure

As the second example, a generic two-storey frame according to Fig. 7 has been considered. The structure is designed for dead load and live load equal to 5.76 (kN/m^2) and 2 (kN/m^2), respectively. In order to take into account the self-weight of the beams, a linear the column removal scenario depicted in Fig. 8 and the load combination of DL+0.25LL, according to GSA. The value of f_y is assumed to be 320 MPa, whilst other material properties are the same as those listed for example 1.

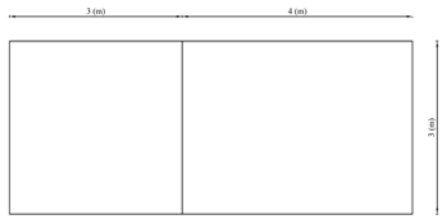


Fig. 7. Plan view of the two storey structure

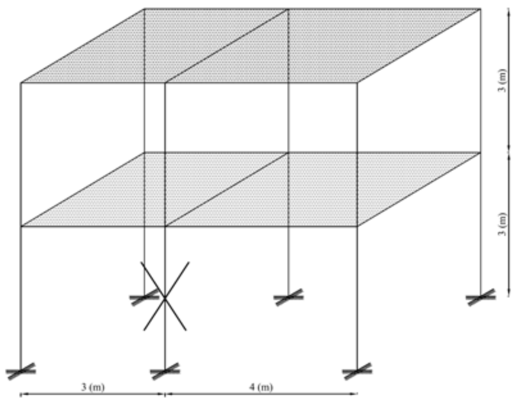


Fig. 8. The column removal scenario

The smallest load factors for both frame action and catenary action are calculated by solving the optimization problems outlined in Eqs. 2-3 and 8-9-10 using the Matlab optimization toolbox the same as Example 1. The calculated values for λ_{pl} and λ_{ca} are 1.67 and 1, respectively. It is interesting to observe that although --according to GSA recommendations-- this structure does not have equal redundancy against progressive collapse, the catenary action mechanism alone can sustain an external load equal to the applied load.

3.4 Example 3: A 3D RC structure with 3x4 bays and 4 stories

As the third example, a four-storey generic RC frame is considered (the plan view is shown in Fig. 9. The load application and material properties are the same as Example 2. The structure is analyzed for the column removal scenario depicted in Fig. 10. The calculated values for λ_{pl} and λ_{ca} are equal to 2 and 1.2, respectively.

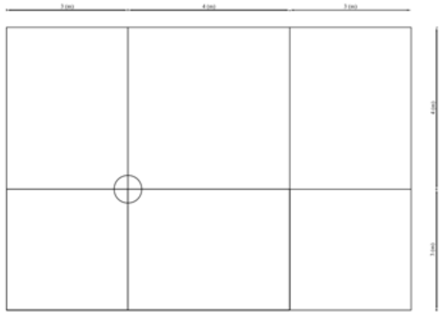


Fig. 9. Plan of the four story structure

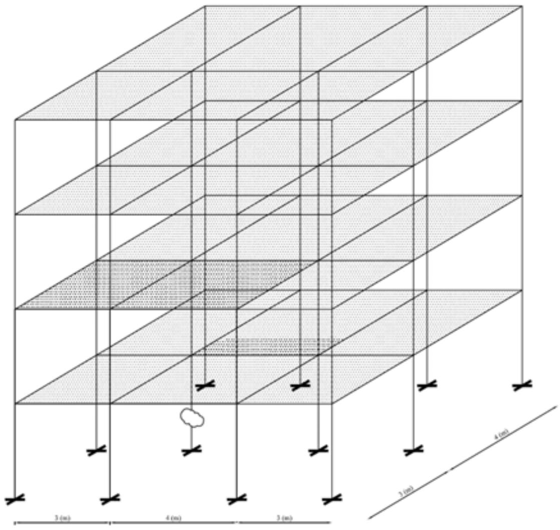


Fig. 10. Column removal scenario

4. CONCLUSION

A virtual work formulation for modelling the catenary action is proposed and the problem of deriving the corresponding smallest load factor is defined as a nonlinear convex optimization problem. The non-linear optimization problem laid out in this work can be solved using the optimization toolbox in Matlab. In order to facilitate the definition of geometry and material properties for multi-degree of freedom frame structures, a use interface linking SAP and Matlab is developed. The proposed modelling of catenary action using limit analysis is tested for three different RC frames employing the developed user interface. The first example is applied to the experimental test set-up in the work of Sasani and Kropelnicki (2008). It is observed that load factors corresponding to frame action and catenary action based on the formulation proposed in this work show reasonable agreement with the results of the above-mentioned experimental results. In order to demonstrate the applicability of the proposed formulation and the developed user interface, two 3D generic RC frames have been considered. The structure geometry and material properties for both case-study frames are generated using the SAP-Matlab user interface.

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