

ESERCIZI SULLA SOMMA, SOTTRAZIONE E PRODOTTO TRA MATRICI

ESERCIZI SUL CALCOLO DEL DETERMINANTE E DELLA MATRICE INVERSA

## USO DEI PULSANTI

Visualizza solo la soluzione dell'esercizio

Visualizza le soluzioni di tutti gli esercizi

Nasconde le soluzioni

Torna all'indice degli esercizi

Eseguire , quando possibile , le seguenti operazioni di somma , sottrazione e moltiplicazione tra matrici :

$$1. \quad A = \begin{bmatrix} -1 & +2 \\ +3 & -1 \end{bmatrix} , \quad B = \begin{bmatrix} -4 & -1 \\ +1 & -2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1 & +2 \\ +3 & -1 \end{bmatrix} + \begin{bmatrix} -4 & -1 \\ +1 & -2 \end{bmatrix} = \begin{bmatrix} -5 & +1 \\ +4 & -3 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -1 & +2 \\ +3 & -1 \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ +1 & -2 \end{bmatrix} = \begin{bmatrix} +3 & +3 \\ +2 & +1 \end{bmatrix}$$

$$A * B = \begin{bmatrix} -1 & +2 \\ +3 & -1 \end{bmatrix} * \begin{bmatrix} -4 & -1 \\ +1 & -2 \end{bmatrix} = \begin{bmatrix} 4+2 & 1-4 \\ -12-1 & -3+2 \end{bmatrix} = \begin{bmatrix} 6 & -3 \\ -13 & -1 \end{bmatrix}$$

$$2. \quad A = \begin{bmatrix} 0 & -5 \\ 2 & -1 \end{bmatrix} , \quad B = \begin{bmatrix} -2 & +2 \\ +6 & -1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -5 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} -2 & +2 \\ +6 & -1 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ +8 & -2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 0 & -5 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} -2 & +2 \\ +6 & -1 \end{bmatrix} = \begin{bmatrix} +2 & -7 \\ -4 & 0 \end{bmatrix}$$

$$A * B = \begin{bmatrix} 0 & -5 \\ 2 & -1 \end{bmatrix} * \begin{bmatrix} -2 & +2 \\ +6 & -1 \end{bmatrix} = \begin{bmatrix} 0-30 & 0+5 \\ -4-6 & 4+1 \end{bmatrix} = \begin{bmatrix} -30 & +5 \\ -10 & +5 \end{bmatrix}$$

$$3. \quad A = \begin{bmatrix} -\frac{1}{2} & -1 \\ +1 & +\frac{1}{3} \end{bmatrix}, \quad B = \begin{bmatrix} -3 & +4 \\ -1 & -2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -\frac{1}{2} & -1 \\ +1 & +\frac{1}{3} \end{bmatrix} + \begin{bmatrix} -3 & +4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{7}{2} & +3 \\ 0 & -\frac{5}{3} \end{bmatrix}$$

$$A - B = \begin{bmatrix} -\frac{1}{2} & -1 \\ +1 & +\frac{1}{3} \end{bmatrix} - \begin{bmatrix} -3 & +4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} +\frac{5}{2} & -5 \\ +2 & +\frac{7}{3} \end{bmatrix}$$

$$A * B = \begin{bmatrix} -\frac{1}{2} & -1 \\ +1 & +\frac{1}{3} \end{bmatrix} * \begin{bmatrix} -3 & +4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} \frac{3}{2}+1 & -2+2 \\ -3-\frac{1}{3} & +4-\frac{2}{3} \end{bmatrix} = \begin{bmatrix} +\frac{5}{2} & 0 \\ -\frac{10}{3} & +\frac{10}{3} \end{bmatrix}$$

$$4. \quad A = \begin{bmatrix} -1 & +1 & +1 \\ -1 & -1 & 0 \\ +2 & -1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} +1 & -2 \\ 0 & -1 \\ +1 & -1 \end{bmatrix}$$

$$A * B = \begin{bmatrix} -1 & +1 & +1 \\ -1 & -1 & 0 \\ +2 & -1 & -1 \end{bmatrix} * \begin{bmatrix} +1 & -2 \\ 0 & -1 \\ +1 & -1 \end{bmatrix} = \begin{bmatrix} -1+0+1 & +2-1-1 \\ -1+0+0 & +2+1+0 \\ +2+0-1 & -4+1+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -1 & +3 \\ +1 & -2 \end{bmatrix}$$

$$5. \quad A = \begin{bmatrix} +2 & +1 & +3 \\ -2 & -3 & 0 \\ 0 & +4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & -1 & -1 \\ +1 & -2 & -4 \\ +1 & 0 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} +2 & +1 & +3 \\ -2 & -3 & 0 \\ 0 & +4 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -1 & -1 \\ +1 & -2 & -4 \\ +1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} +1 & 0 & +2 \\ -1 & -5 & -4 \\ +1 & +4 & 0 \end{bmatrix}$$

$$A - B = \begin{bmatrix} +2 & +1 & +3 \\ -2 & -3 & 0 \\ 0 & +4 & 0 \end{bmatrix} - \begin{bmatrix} -1 & -1 & -1 \\ +1 & -2 & -4 \\ +1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} +3 & +2 & +4 \\ -3 & -1 & +4 \\ -1 & +4 & 0 \end{bmatrix}$$

$$A * B = \begin{bmatrix} +2 & +1 & +3 \\ -2 & -3 & 0 \\ 0 & +4 & 0 \end{bmatrix} * \begin{bmatrix} -1 & -1 & -1 \\ +1 & -2 & -4 \\ +1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} +2 & -4 & -6 \\ -1 & +8 & +14 \\ +4 & -8 & +16 \end{bmatrix}$$

$$6. \quad A = \begin{bmatrix} +2 & -3 \\ 0 & 0 \\ -1 & +4 \end{bmatrix}, \quad B = \begin{bmatrix} +6 & -1 \\ -1 & 3 \\ -2 & +2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} +2 & -3 \\ 0 & 0 \\ -1 & +4 \end{bmatrix} + \begin{bmatrix} +6 & -1 \\ -1 & 3 \\ -2 & +2 \end{bmatrix} = \begin{bmatrix} +8 & -4 \\ -1 & +3 \\ -3 & +6 \end{bmatrix}$$

$$A - B = \begin{bmatrix} +2 & -3 \\ 0 & 0 \\ -1 & +4 \end{bmatrix} - \begin{bmatrix} +6 & -1 \\ -1 & 3 \\ -2 & +2 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ +1 & -3 \\ +1 & +2 \end{bmatrix}$$

$$7. \quad A = [-2 \quad +1 \quad -5] \quad , \quad B = \begin{bmatrix} +1 & -3 \\ 0 & +1 \\ -1 & +3 \end{bmatrix}$$

$$A * B = [-2 \quad +1 \quad -5] * \begin{bmatrix} +1 & -3 \\ 0 & +1 \\ -1 & +3 \end{bmatrix} = [-2+0+5 \quad +6+1-15] = [+3 \quad -8]$$

$$8. \quad A = \begin{bmatrix} +1 & +1 & 0 \\ -5 & -5 & 1 \\ +1 & +1 & -2 \end{bmatrix} \quad , \quad B = \begin{bmatrix} -2 & +1 & -2 \\ +4 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix}$$

$$A + B = \begin{bmatrix} +1 & +1 & 0 \\ -5 & -5 & 1 \\ +1 & +1 & -2 \end{bmatrix} + \begin{bmatrix} -2 & +1 & -2 \\ +4 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix} = \begin{bmatrix} -1 & +2 & -2 \\ -1 & -7 & 0 \\ +1 & 0 & -5 \end{bmatrix}$$

$$A - B = \begin{bmatrix} +1 & +1 & 0 \\ -5 & -5 & 1 \\ +1 & +1 & -2 \end{bmatrix} - \begin{bmatrix} -2 & +1 & -2 \\ +4 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix} = \begin{bmatrix} +3 & 0 & +2 \\ -9 & -3 & +2 \\ +1 & +2 & +1 \end{bmatrix}$$

$$A * B = \begin{bmatrix} +1 & +1 & 0 \\ -5 & -5 & 1 \\ +1 & +1 & -2 \end{bmatrix} * \begin{bmatrix} -2 & +1 & -2 \\ +4 & -2 & -1 \\ 0 & -1 & -3 \end{bmatrix} = \begin{bmatrix} +2 & -1 & -3 \\ -10 & +4 & +12 \\ +2 & +1 & +3 \end{bmatrix}$$

$$9. \quad A = \begin{bmatrix} -1 & +2 & 0 & 0 \\ -1 & +1 & +1 & 0 \end{bmatrix} \quad , \quad B = \begin{bmatrix} -1 & -2 & +1 & +3 \\ 0 & -1 & 0 & -1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -1 & +2 & 0 & 0 \\ -1 & +1 & +1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & -2 & +1 & +3 \\ 0 & -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -2 & 0 & +1 & +3 \\ -1 & 0 & +1 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -1 & +2 & 0 & 0 \\ -1 & +1 & +1 & 0 \end{bmatrix} - \begin{bmatrix} -1 & -2 & +1 & +3 \\ 0 & -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 & +4 & -1 & -3 \\ -1 & +2 & +1 & +1 \end{bmatrix}$$

Calcolare il determinante e , quando possibile , la matrice inversa delle seguenti matrici :

$$10. \quad A = \begin{bmatrix} -1 & +2 \\ +\frac{1}{2} & -1 \end{bmatrix} \quad , \quad B = \begin{bmatrix} -4 & +3 \\ +1 & -1 \end{bmatrix}$$

$$A = \begin{vmatrix} -1 & +2 \\ +\frac{1}{2} & -1 \end{vmatrix} = (1-1) = 0 \quad B = \begin{vmatrix} -4 & +3 \\ +1 & -1 \end{vmatrix} = (4-3) = 1$$

$$\begin{cases} b_{11} = \frac{+1 \cdot |-1|}{1} = -1 \\ b_{12} = \frac{-1 \cdot |3|}{1} = -3 \\ b_{21} = \frac{-1 \cdot |+1|}{1} = -1 \\ b_{22} = \frac{+1 \cdot |-4|}{1} = -4 \end{cases} \Rightarrow B^{-1} = \begin{bmatrix} -1 & -3 \\ -1 & -4 \end{bmatrix}$$

infatti come da verifica :  $B \cdot B^{-1} = \begin{bmatrix} -4 & +3 \\ +1 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -3 \\ -1 & -4 \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$

$$11. \quad A = \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & +1 \\ +7 & -1 \end{bmatrix}$$

$$A = \begin{vmatrix} 0 & -2 \\ 3 & -1 \end{vmatrix} = (0+6) = +6 \quad B = \begin{vmatrix} -1 & +1 \\ +7 & -1 \end{vmatrix} = (1-7) = -6$$

$$\begin{cases} a_{11} = \frac{+1 \cdot |-1|}{6} = -\frac{1}{6} \\ a_{12} = \frac{-1 \cdot |-2|}{6} = \frac{1}{3} \\ a_{21} = \frac{-1 \cdot |3|}{6} = -\frac{1}{2} \\ a_{22} = \frac{+1 \cdot |0|}{6} = 0 \end{cases} \Rightarrow A^{-1} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{2} & 0 \end{bmatrix}, \quad \begin{cases} b_{11} = \frac{+1 \cdot |-1|}{-6} = \frac{1}{6} \\ b_{12} = \frac{-1 \cdot |+1|}{-6} = \frac{1}{6} \\ b_{21} = \frac{-1 \cdot |7|}{-6} = \frac{7}{6} \\ b_{22} = \frac{+1 \cdot |-1|}{-6} = \frac{1}{6} \end{cases} \Rightarrow B^{-1} = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{7}{6} & \frac{1}{6} \end{bmatrix}$$

infatti come da verifica :

$$A \cdot A^{-1} = \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{6} & \frac{1}{3} \\ -\frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix} \quad B \cdot B^{-1} = \begin{bmatrix} -1 & +1 \\ +7 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{6} & \frac{1}{6} \\ \frac{7}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$$

$$12. \quad A = \begin{bmatrix} 0 & -1 \\ 1 & +2 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & +4 \\ -1 & +1 \end{bmatrix}$$

$$A = \begin{vmatrix} 0 & -1 \\ 1 & +2 \end{vmatrix} = (0+1) = +1 \quad B = \begin{vmatrix} -4 & +4 \\ -1 & +1 \end{vmatrix} = (-4+4) = 0$$

$$\begin{cases} a_{11} = \frac{+1 \cdot |2|}{1} = 2 \\ a_{12} = \frac{-1 \cdot |-1|}{1} = 1 \\ a_{21} = \frac{-1 \cdot |1|}{1} = -1 \\ a_{22} = \frac{+1 \cdot |0|}{1} = 0 \end{cases} \Rightarrow A^{-1} = \begin{bmatrix} +2 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\text{infatti come da verifica : } A \cdot A^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & +2 \end{bmatrix} \cdot \begin{bmatrix} +2 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$$



$$13. \quad A = \begin{bmatrix} -2 & +2 & +3 \\ +2 & -1 & +1 \\ +1 & -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & +1 & -1 \\ 0 & +1 & +3 \\ -1 & +1 & -1 \end{bmatrix}$$

$$A = \begin{vmatrix} -2 & +2 & +3 \\ +2 & -1 & +1 \\ +1 & -1 & -2 \end{vmatrix} = -2 \begin{vmatrix} -1 & +1 \\ -1 & -2 \end{vmatrix} - 2 \begin{vmatrix} +2 & +1 \\ +1 & -2 \end{vmatrix} + 3 \begin{vmatrix} +2 & -1 \\ +1 & -1 \end{vmatrix} = -2(2+1) - 2(-4-1) + 3(-2+1) = 1$$

$$\left\{ \begin{array}{l} a_{11} = \frac{+1 \cdot \begin{vmatrix} -1 & +1 \\ -1 & -2 \end{vmatrix}}{1} = 3, \quad a_{12} = \frac{-1 \cdot \begin{vmatrix} +2 & +3 \\ -1 & -2 \end{vmatrix}}{1} = 1, \quad a_{13} = \frac{+1 \cdot \begin{vmatrix} +2 & +3 \\ -1 & +1 \end{vmatrix}}{1} = 5 \\ a_{21} = \frac{-1 \cdot \begin{vmatrix} +2 & +1 \\ +1 & -2 \end{vmatrix}}{1} = +5, \quad a_{22} = \frac{+1 \cdot \begin{vmatrix} -2 & +3 \\ +1 & -2 \end{vmatrix}}{1} = +1, \quad a_{23} = \frac{-1 \cdot \begin{vmatrix} -2 & +3 \\ +2 & +1 \end{vmatrix}}{1} = 8 \\ a_{31} = \frac{+1 \cdot \begin{vmatrix} +2 & -1 \\ +1 & -1 \end{vmatrix}}{1} = -1, \quad a_{32} = \frac{-1 \cdot \begin{vmatrix} -2 & +2 \\ +1 & -1 \end{vmatrix}}{1} = 0, \quad a_{33} = \frac{+1 \cdot \begin{vmatrix} -2 & +2 \\ +2 & -1 \end{vmatrix}}{1} = -2 \end{array} \right.$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & 1 & 5 \\ 5 & 1 & 8 \\ -1 & 0 & -2 \end{bmatrix}$$

infatti come da verifica :  $A \cdot A^{-1} = \begin{bmatrix} -2 & +2 & +3 \\ +2 & -1 & +1 \\ +1 & -1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 1 & 5 \\ 5 & 1 & 8 \\ -1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$B = \begin{vmatrix} -1 & +1 & -1 \\ 0 & +1 & +3 \\ -1 & +1 & -1 \end{vmatrix} = 0 \quad \text{non esiste quindi l'inversa.}$$

$$14. \quad A = \begin{bmatrix} +1 & -1 & +3 \\ -2 & -1 & +1 \\ 0 & +4 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 & -1 \\ 1 & -2 & -4 \\ 0 & -1 & 0 \end{bmatrix}$$

$$A = \begin{vmatrix} +1 & -1 & +3 \\ -2 & -1 & +1 \\ 0 & +4 & 0 \end{vmatrix} = -4 \begin{vmatrix} +1 & +3 \\ -2 & +1 \end{vmatrix} = -28$$

$$\left\{ \begin{array}{l} a_{11} = \frac{+1 \cdot \begin{vmatrix} -1 & +1 \\ +4 & 0 \end{vmatrix}}{-28} = \frac{1}{7}, \quad a_{12} = \frac{-1 \cdot \begin{vmatrix} -1 & +3 \\ +4 & 0 \end{vmatrix}}{-28} = -\frac{3}{7}, \quad a_{13} = \frac{+1 \cdot \begin{vmatrix} -1 & +3 \\ -1 & +1 \end{vmatrix}}{-28} = -\frac{1}{14} \\ a_{21} = \frac{-1 \cdot \begin{vmatrix} -2 & +1 \\ 0 & 0 \end{vmatrix}}{-28} = 0, \quad a_{22} = \frac{+1 \cdot \begin{vmatrix} +1 & +3 \\ 0 & 0 \end{vmatrix}}{-28} = 0, \quad a_{23} = \frac{-1 \cdot \begin{vmatrix} +1 & +3 \\ -2 & +1 \end{vmatrix}}{-28} = \frac{1}{4} \\ a_{31} = \frac{+1 \cdot \begin{vmatrix} -2 & -1 \\ 0 & +4 \end{vmatrix}}{-28} = \frac{2}{7}, \quad a_{32} = \frac{-1 \cdot \begin{vmatrix} +1 & -1 \\ 0 & +4 \end{vmatrix}}{-28} = \frac{1}{7}, \quad a_{33} = \frac{+1 \cdot \begin{vmatrix} +1 & -1 \\ -2 & -1 \end{vmatrix}}{-28} = \frac{3}{28} \end{array} \right.$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{7} & -\frac{3}{7} & -\frac{1}{14} \\ 0 & 0 & \frac{1}{4} \\ \frac{2}{7} & \frac{1}{7} & \frac{3}{28} \end{bmatrix}$$

infatti come da verifica :

$$A \cdot A^{-1} = \begin{bmatrix} +1 & -1 & +3 \\ -2 & -1 & +1 \\ 0 & +4 & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{7} & -\frac{3}{7} & -\frac{1}{14} \\ 0 & 0 & \frac{1}{4} \\ \frac{2}{7} & \frac{1}{7} & \frac{3}{28} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{vmatrix} 0 & -1 & -1 \\ +1 & -2 & -4 \\ 0 & -1 & 0 \end{vmatrix} = -1 \begin{vmatrix} -1 & -1 \\ -1 & 0 \end{vmatrix} = +1$$

$$\left\{ \begin{array}{l} b_{11} = \frac{+1 \cdot \begin{vmatrix} -2 & -4 \\ -1 & 0 \end{vmatrix}}{1} = -4, \quad b_{12} = \frac{-1 \cdot \begin{vmatrix} -1 & -1 \\ -1 & 0 \end{vmatrix}}{1} = 1, \quad b_{13} = \frac{+1 \cdot \begin{vmatrix} -1 & -1 \\ -2 & -4 \end{vmatrix}}{1} = 2 \\ b_{21} = \frac{-1 \cdot \begin{vmatrix} +1 & -4 \\ 0 & 0 \end{vmatrix}}{1} = 0, \quad b_{22} = \frac{+1 \cdot \begin{vmatrix} 0 & -1 \\ 0 & 0 \end{vmatrix}}{1} = 0, \quad b_{23} = \frac{-1 \cdot \begin{vmatrix} 0 & -1 \\ +1 & -4 \end{vmatrix}}{1} = -1 \\ b_{31} = \frac{+1 \cdot \begin{vmatrix} +1 & -2 \\ 0 & -1 \end{vmatrix}}{1} = -1, \quad b_{32} = \frac{-1 \cdot \begin{vmatrix} 0 & -1 \\ 0 & -1 \end{vmatrix}}{1} = 0, \quad b_{33} = \frac{+1 \cdot \begin{vmatrix} 0 & -1 \\ +1 & -2 \end{vmatrix}}{1} = 1 \end{array} \right.$$

$$\Rightarrow B^{-1} = \begin{bmatrix} -4 & +1 & +2 \\ 0 & 0 & -1 \\ -1 & 0 & +1 \end{bmatrix}$$

infatti come da verifica :  $B \cdot B^{-1} = \begin{bmatrix} 0 & -1 & -1 \\ +1 & -2 & -4 \\ 0 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & +1 & +2 \\ 0 & 0 & -1 \\ -1 & 0 & +1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

15.  $A = \begin{bmatrix} -1 & +1 & -1 & +2 \\ 0 & -1 & -1 & 0 \\ +1 & -1 & 0 & -1 \\ +2 & -1 & -1 & +1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & +1 & -2 & 0 \\ -1 & +1 & +3 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} -1 & +1 & -1 & +2 \\ 0 & -1 & -1 & 0 \\ +1 & -1 & 0 & -1 \\ +2 & -1 & -1 & +1 \end{bmatrix} \cong \begin{bmatrix} -1 & +2 & -1 & +2 \\ 0 & 0 & -1 & 0 \\ +1 & -1 & 0 & -1 \\ +2 & 0 & -1 & +1 \end{bmatrix} \Rightarrow 2^{\text{col.}} \rightarrow 2^{\text{col.}} - 3^{\text{col.}}$$

e quindi : 
$$A = \begin{vmatrix} -1 & +2 & -1 & +2 \\ 0 & 0 & -1 & 0 \\ +1 & -1 & 0 & -1 \\ +2 & 0 & -1 & +1 \end{vmatrix} \Rightarrow +1 \cdot \begin{vmatrix} -1 & +2 & +2 \\ +1 & -1 & -1 \\ +2 & 0 & +1 \end{vmatrix} = -1$$

$$\left. \begin{aligned} a_{11} &= \frac{+1 \cdot \begin{vmatrix} -1 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & -1 & +1 \end{vmatrix}}{-1} = +1, & a_{12} &= \frac{-1 \cdot \begin{vmatrix} +1 & -1 & +2 \\ -1 & 0 & -1 \\ -1 & -1 & +1 \end{vmatrix}}{-1} = -1 \\ a_{13} &= \frac{+1 \cdot \begin{vmatrix} +1 & -1 & +2 \\ -1 & -1 & 0 \\ -1 & -1 & +1 \end{vmatrix}}{-1} = +2, & a_{14} &= \frac{-1 \cdot \begin{vmatrix} +1 & -1 & +2 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \end{vmatrix}}{-1} = 0 \\ a_{21} &= \frac{-1 \cdot \begin{vmatrix} 0 & -1 & 0 \\ +1 & 0 & -1 \\ +2 & -1 & +1 \end{vmatrix}}{-1} = +3, & a_{22} &= \frac{+1 \cdot \begin{vmatrix} -1 & -1 & +2 \\ +1 & 0 & -1 \\ +2 & -1 & +1 \end{vmatrix}}{-1} = -2 \\ a_{23} &= \frac{-1 \cdot \begin{vmatrix} -1 & -1 & +2 \\ 0 & -1 & 0 \\ +2 & -1 & +1 \end{vmatrix}}{-1} = +5, & a_{24} &= \frac{+1 \cdot \begin{vmatrix} -1 & -1 & +2 \\ 0 & -1 & 0 \\ +1 & 0 & -1 \end{vmatrix}}{-1} = -1 \\ a_{31} &= \frac{+1 \cdot \begin{vmatrix} 0 & -1 & 0 \\ +1 & -1 & -1 \\ +2 & -1 & +1 \end{vmatrix}}{-1} = -3, & a_{32} &= \frac{-1 \cdot \begin{vmatrix} -1 & +1 & +2 \\ +1 & -1 & -1 \\ +2 & -1 & +1 \end{vmatrix}}{-1} = +1 \\ a_{33} &= \frac{+1 \cdot \begin{vmatrix} -1 & +1 & +2 \\ 0 & -1 & 0 \\ +2 & -1 & +1 \end{vmatrix}}{-1} = -5, & a_{34} &= \frac{-1 \cdot \begin{vmatrix} -1 & +1 & +2 \\ 0 & -1 & 0 \\ +1 & -1 & -1 \end{vmatrix}}{-1} = +1 \end{aligned} \right\}$$

$$\left\{ \begin{array}{l} a_{41} = \frac{-1 \cdot \begin{vmatrix} 0 & -1 & -1 \\ +1 & -1 & 0 \\ +2 & -1 & -1 \end{vmatrix}}{-1} = -2, \quad a_{42} = \frac{+1 \cdot \begin{vmatrix} -1 & +1 & -1 \\ +1 & -1 & 0 \\ +2 & -1 & -1 \end{vmatrix}}{-1} = +1 \\ a_{43} = \frac{-1 \cdot \begin{vmatrix} -1 & +1 & -1 \\ 0 & -1 & -1 \\ 2 & -1 & -1 \end{vmatrix}}{-1} = -4, \quad a_{44} = \frac{+1 \cdot \begin{vmatrix} -1 & +1 & -1 \\ 0 & -1 & -1 \\ +1 & -1 & 0 \end{vmatrix}}{-1} = +1 \end{array} \right.$$

$$\text{e di qui : } A^{-1} = \begin{bmatrix} +1 & -1 & +2 & 0 \\ +3 & -2 & +5 & -1 \\ -3 & +1 & -5 & +1 \\ -2 & +1 & -4 & +1 \end{bmatrix}$$

$$\text{infatti come da verifica : } A \cdot A^{-1} = \begin{bmatrix} -1 & +1 & -1 & +2 \\ 0 & -1 & -1 & 0 \\ +1 & -1 & 0 & -1 \\ +2 & -1 & -1 & +1 \end{bmatrix} \cdot \begin{bmatrix} +1 & -1 & +2 & 0 \\ +3 & -2 & +5 & -1 \\ -3 & +1 & -5 & +1 \\ -2 & +1 & -4 & +1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{vmatrix} 0 & -1 & -1 & -1 \\ 0 & 0 & -1 & 0 \\ -1 & +1 & -2 & 0 \\ -1 & +1 & +3 & 0 \end{vmatrix} = 0 \quad \text{non esiste quindi l'inversa.}$$

$$16. \quad A = \begin{bmatrix} -1 & +2 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} +1 & -3 \\ 0 & +1 \\ -1 & +3 \end{bmatrix}$$

$$A = \begin{vmatrix} -1 & +2 \\ 0 & -1 \end{vmatrix} = (+1-0) = +1$$

$$\begin{cases} a_{11} = \frac{+1 \cdot |-1|}{1} = -1 \\ a_{12} = \frac{-1 \cdot |+2|}{1} = -2 \\ a_{21} = \frac{-1 \cdot |0|}{1} = 0 \\ a_{22} = \frac{+1 \cdot |-1|}{1} = -1 \end{cases} \Rightarrow A^{-1} = \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}$$

infatti come da verifica :  $A \cdot A^{-1} = \begin{bmatrix} -1 & +2 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$B = \begin{bmatrix} +1 & -3 \\ 0 & +1 \\ -1 & +3 \end{bmatrix} \quad \text{non esiste il determinante}$$

$$17. \quad A = \begin{bmatrix} 0 & +1 & 0 \\ -2 & -1 & +4 \\ +2 & +1 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & +1 \\ +3 & 0 & -1 \\ 0 & -5 & -1 \end{bmatrix}$$

$$A = \begin{vmatrix} 0 & +1 & 0 \\ -2 & -1 & +4 \\ +2 & +1 & -4 \end{vmatrix} = -1 \begin{vmatrix} -2 & +4 \\ +2 & -4 \end{vmatrix} = 0 \quad \text{non esiste l'inversa.}$$

$$B = \begin{vmatrix} -1 & 0 & +1 \\ +3 & 0 & -1 \\ 0 & -5 & -1 \end{vmatrix} = +5 \begin{vmatrix} -1 & +1 \\ +3 & -1 \end{vmatrix} = -10$$

$$\left\{ \begin{array}{l} b_{11} = \frac{+1 \cdot \begin{vmatrix} 0 & -1 \\ -5 & -1 \end{vmatrix}}{-10} = \frac{1}{2}, \quad b_{12} = \frac{-1 \cdot \begin{vmatrix} 0 & +1 \\ -5 & -1 \end{vmatrix}}{-10} = \frac{1}{2}, \quad b_{13} = \frac{+1 \cdot \begin{vmatrix} 0 & +1 \\ 0 & -1 \end{vmatrix}}{-10} = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} b_{21} = \frac{-1 \cdot \begin{vmatrix} +3 & -1 \\ 0 & -1 \end{vmatrix}}{-10} = -\frac{3}{10}, \quad b_{22} = \frac{+1 \cdot \begin{vmatrix} -1 & +1 \\ 0 & -1 \end{vmatrix}}{-10} = -\frac{1}{10}, \quad b_{23} = \frac{-1 \cdot \begin{vmatrix} -1 & +1 \\ +3 & -1 \end{vmatrix}}{-10} = -\frac{1}{5} \end{array} \right.$$

$$\left\{ \begin{array}{l} b_{31} = \frac{+1 \cdot \begin{vmatrix} +3 & 0 \\ 0 & -5 \end{vmatrix}}{-10} = \frac{3}{2}, \quad b_{32} = \frac{-1 \cdot \begin{vmatrix} -1 & 0 \\ 0 & -5 \end{vmatrix}}{-10} = \frac{1}{2}, \quad b_{33} = \frac{+1 \cdot \begin{vmatrix} -1 & 0 \\ +3 & 0 \end{vmatrix}}{-10} = 0 \end{array} \right.$$

$$\Rightarrow B^{-1} = \begin{bmatrix} +\frac{1}{2} & +\frac{1}{2} & 0 \\ -\frac{3}{10} & -\frac{1}{10} & -\frac{1}{5} \\ +\frac{3}{2} & +\frac{1}{2} & 0 \end{bmatrix}$$

infatti come da verifica :  $B \cdot B^{-1} = \begin{bmatrix} -1 & 0 & +1 \\ +3 & 0 & -1 \\ 0 & -5 & -1 \end{bmatrix} \cdot \begin{bmatrix} +\frac{1}{2} & +\frac{1}{2} & 0 \\ -\frac{3}{10} & -\frac{1}{10} & -\frac{1}{5} \\ +\frac{3}{2} & +\frac{1}{2} & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$18. \quad A = \begin{bmatrix} -1 & 0 & -1 & +1 & -1 \\ -1 & 0 & +1 & -1 & 0 \\ -1 & -1 & +1 & -1 & 0 \\ -2 & 0 & -1 & +1 & -1 \\ +1 & 0 & -1 & +2 & +1 \end{bmatrix}$$

con opportune operazioni elementari tra linee :

$$A = \begin{bmatrix} -1 & 0 & -1 & +1 & -1 \\ -1 & 0 & +1 & -1 & 0 \\ -1 & -1 & +1 & -1 & 0 \\ -2 & 0 & -1 & +1 & -1 \\ +1 & 0 & -1 & +2 & +1 \end{bmatrix} \cong \begin{bmatrix} 0 & -1 & -1 & +1 & -1 \\ 0 & -1 & +1 & -1 & 0 \\ -1 & -1 & +1 & -1 & 0 \\ 0 & -2 & -1 & +1 & -1 \\ 0 & +1 & -1 & +2 & +1 \end{bmatrix} \cong \begin{bmatrix} -1 & -1 & +1 & -1 & 0 \\ 0 & -1 & +1 & -1 & 0 \\ 0 & -1 & -1 & +1 & -1 \\ 0 & -2 & -1 & +1 & -1 \\ 0 & +1 & -1 & +2 & +1 \end{bmatrix} \cong$$

$$\cong \begin{bmatrix} -1 & +1 & -1 & -1 & 0 \\ 0 & +1 & -1 & -1 & 0 \\ 0 & -1 & -1 & +1 & -1 \\ 0 & -1 & -2 & +1 & -1 \\ 0 & -1 & +1 & +2 & +1 \end{bmatrix} \cong \begin{bmatrix} -1 & +1 & 0 & -1 & -1 \\ 0 & +1 & 0 & -1 & -1 \\ 0 & 0 & -2 & 0 & -1 \\ 0 & 0 & -3 & 0 & -1 \\ 0 & 0 & 0 & +1 & +1 \end{bmatrix} \cong \begin{bmatrix} -1 & +1 & -1 & -1 & 0 \\ 0 & +1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & 0 & -3 \\ 0 & 0 & +1 & +1 & 0 \end{bmatrix} \cong$$

$$\cong \begin{bmatrix} -1 & +1 & -1 & -1 & 0 \\ 0 & +1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & +1 & -2 \end{bmatrix} \cong \begin{bmatrix} -1 & +1 & -1 & -1 & 0 \\ 0 & +1 & -1 & -1 & 0 \\ 0 & 0 & -1 & 0 & -2 \\ 0 & 0 & 0 & +1 & -2 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \Rightarrow \det A = 1$$

procedendo come nei precedenti esercizi otteniamo infine :

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & +1 & -1 & 0 & 0 \\ +4 & +3 & 0 & -3 & +1 \\ +3 & +2 & 0 & -2 & +1 \\ -3 & -1 & 0 & +2 & 0 \end{bmatrix}$$