

Stabilire se il seguente campo vettoriale, definito in tutto lo spazio R^3 , e' conservativo:

1. $(X, Y, Z) = (e^{2+xy} y z^2, e^{2+xy} x z^2, 2 e^{2+xy} x y)$
Risposta : NO $X_y = e^{2+xy} z^2 + e^{2+xy} x y z^2$ $Y_x = e^{2+xy} z^2 + e^{2+xy} x y z^2$
 $X_z = 2 e^{2+xy} y z$ $Z_x = 2 e^{2+xy} y + 2 e^{2+xy} x y^2$
 $Y_z = 2 e^{2+xy} x z$ $Z_y = 2 e^{2+xy} x + 2 e^{2+xy} x^2 y$
2. $(X, Y, Z) = \left(2x \cos(z), \frac{-2yz}{1+y^4 z^2}, -\frac{y^2}{1+y^4 z^2} - x^2 \sin(z) \right)$
Risposta : SI $X_y = Y_x = 0$ $X_z = Z_x = -2x \sin(z)$ $Y_z = Z_y = \frac{4y^5 z^2}{(1+y^4 z^2)^2} - \frac{2y}{1+y^4 z^2}$
3. $(X, Y, Z) = \left(1, -\frac{yz}{1+y^2 z^2} - \arctg(yz), -\frac{y^2}{1+y^2 z^2} \right)$
Risposta : SI $X_y = Y_x = 0$ $X_z = Z_x = 0$ $Y_z = Z_y = \frac{2y^3 z^2}{(1+y^2 z^2)^2} - \frac{2y}{1+y^2 z^2}$
4. $(X, Y, Z) = (e^{xy+z} y, e^{xy+z} x, e^{xy+z} z)$
Risposta : NO $X_y = e^{xy+z} + e^{xy+z} x y$ $Y_x = e^{xy+z} + e^{xy+z} x y$
 $X_z = e^{xy+z} y$ $Z_x = e^{xy+z} y z$
 $Y_z = e^{xy+z} x$ $Z_y = e^{xy+z} x z$
5. $(X, Y, Z) = (\arctg(z), -2yz \cos(y^2 z), y^2 \cos(y^2 z))$
Risposta : NO $X_y = 0$ $Y_x = 0$
 $X_z = \frac{1}{1+z^2}$ $Z_x = 0$
 $Y_z = -2y \cos(y^2 z) + 2y^3 z \sin(y^2 z)$ $Z_y = 2y \cos(y^2 z) - 2y^3 z \sin(y^2 z)$
6. $(X, Y, Z) = \left(-(y \cos(xy)) + \log(2+z^2), -(x \cos(xy)), \frac{2xz}{2+z^2} \right)$
Risposta : SI $X_y = Y_x = -\cos(xy) + xy \sin(xy)$ $X_z = Z_x = \frac{2z}{2+z^2}$ $Y_z = Z_y = 0$
7. $(X, Y, Z) = \left(z + y \sin(xy), x \sin(xy), x - \frac{2z}{2+z^2} \right)$
Risposta : SI $X_y = Y_x = xy \cos(xy) + \sin(xy)$ $X_z = Z_x = 1$ $Y_z = Z_y = 0$
8. $(X, Y, Z) = (1, -\arctg(yz), 1 + y^2 z^2)$
Risposta : NO $X_y = 0$ $Y_x = 0$
 $X_z = 0$ $Z_x = 0$
 $Y_z = -\frac{y}{1+y^2 z^2}$ $Z_y = 2y z^2$