ESERCITAZIONE DEL 24 GENNAIO 2018: RISPOSTE

1. Determinare il rango delle seguenti matrici

$$A = \begin{pmatrix} 1 & 4 & 3 & -2 \\ -2 & 1 & 3 & -5 \\ 0 & -2 & -2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 1$

$$A = \begin{pmatrix} 1 & 3 & 2 & -1 \\ -2 & 1 & 3 & -5 \\ 0 & -2 & -2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 3 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 2$

$$A = \begin{pmatrix} 1 & 3 & 2 & -1 \\ -2 & 0 & 2 & -4 \\ 0 & -2 & -2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 1$

$$A = \begin{pmatrix} 1 & 3 & 2 & -1 \\ -2 & 0 & 2 & -4 \\ 0 & -1 & -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 1$

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ -2 & 0 & 2 & -4 \\ 0 & -1 & -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 3 & 2 & 2 & 2 & 2 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 2$

$$A = \begin{pmatrix} 2 & 2 & 0 & 2 \\ -2 & 0 & 2 & -4 \\ 0 & -1 & -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 5 & 5 & 5 & 5 & 5 & 5 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 1$

$$A = \begin{pmatrix} 2 & 2 & 0 & 2 \\ -1 & 1 & 2 & -3 \\ 0 & -1 & -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 4 & 4 & 4 & 4 & 4 & 4 & 4 \\ 3 & 3 & 3 & 3 & 3 & 3 & 3 \\ 2 & 2 & 2 & 2 & 2 & 2 & 2 \end{pmatrix}$$

Risposta: $r(A) = 2$, $r(B) = 2$

2. Risolvere il seguente sistema; se si tratta di un sistema compatibile, controllare la correttezza del risultato ottenuto.

$$\begin{cases} 2x + 5y + 4z = 3 \\ x + 2y - z = 2 \end{cases}$$

$$\textbf{Risposta}: \quad (4 + 13z, -1 - 6z, z)$$

$$\begin{cases} x + y + z = 1 \\ 3x + 3y = 0 \\ x + 2y = 0 \end{cases}$$

$$\textbf{Risposta}: \quad (0, 0, 1)$$

$$\begin{cases} x + 3y + 4z = 3 \\ x + 4y + z = 2 \end{cases}$$

$$\textbf{Risposta}: \quad (6 - 13z, -1 + 3z, z)$$

$$\begin{cases} 3x + y = 1 \\ x + 2y = 0 \\ 4x + 8y = 0 \end{cases}$$

$$\textbf{Risposta}: \quad (2/5, -1/5)$$

$$\begin{cases} x - y - z = 1 \\ -2x + 3y + 5z = 4 \end{cases}$$

$$\textbf{Risposta}: \quad (7 - 2z, 6 - 3z, z)$$

$$\begin{cases} x + 5y - 2z = 2 \\ 4x - 3z = 4 \\ x - 3y = 1 \end{cases}$$

$$\textbf{Risposta}: \quad \textit{nessuna} \quad \textit{soluzione}$$

$$\begin{cases} 5x - 2y - z = 0 \\ -2x + y + 4z = 3 \end{cases}$$

$$\textbf{Risposta}: \quad (6 - 7z, 15 - 18z, z)$$

$$\begin{cases} 3x + 2y - z = 2 \\ x - 3z = 1 \\ -4x - 3y = 1 \end{cases}$$

$$\textbf{Risposta}: \quad \textit{nessuna} \quad \textit{soluzione}$$

$$\begin{cases} x - y + z = 0 \\ -x + 2y + 3z = 2 \end{cases}$$

$$\textbf{Risposta}: \quad (2 - 5z, 2 - 4z, z)$$

$$\begin{cases} 3x + 3y - z = 2 \\ x - 3z = 1 \\ -4x - 3y = 1 \end{cases}$$

$$\textbf{Risposta}: \quad (-2, 7/3, -1)$$

$$\begin{cases} x - 4y + z = 0 \\ -x + y = 2 \end{cases}$$

$$\textbf{Risposta}: \quad \left(\frac{z - 8}{3}, \frac{z - 2}{3}, z\right)$$

$$\begin{cases} 3x + y - z = 0 \\ x - 2z = 1 \\ -2x + 4z = -2 \end{cases}$$

$$\textbf{Risposta}: \quad (1 + 2z, -3 - 5z, z)$$

$$\begin{cases} 2x - y = 0 \\ -3x + y + z = 2 \end{cases}$$

$$\textbf{Risposta}: \quad (z - 2, 2z - 4, z)$$

3. Assegnati i seguenti vettori $\mathbf{u}$, $\mathbf{v} \in \mathbf{R}^3$, determinare il modulo di $\mathbf{u}$, il versore di $\mathbf{u}$, il vettore $\mathbf{u}+3\mathbf{v}$, il prodotto scalare $\mathbf{u}\cdot\mathbf{v}$ e l'angolo $\widehat{\mathbf{uv}}$:

$\mathbf{u}=(2, 0, 2) \qquad \mathbf{v}=\left(1, 1, \frac{1}{2}\right)$

Risposta: $|\mathbf{u}| = 2\sqrt{2}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=\left(5, 3, \frac{7}{2}\right)$, $\mathbf{u}\cdot\mathbf{v}=3$, $\widehat{\mathbf{uv}}=\frac{\pi}{4}$

$\mathbf{u}=\left(\frac{2}{3}, 0, \frac{2}{3}\right) \qquad \mathbf{v}=\left(3, 3, \frac{3}{2}\right)$

Risposta: $|\mathbf{u}| = 2\sqrt{2}/3$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=\left(\frac{29}{3}, 9, \frac{31}{6}\right)$, $\mathbf{u}\cdot\mathbf{v}=3$, $\widehat{\mathbf{uv}}=\frac{\pi}{4}$

$\mathbf{u}=\left(\frac{3}{2}, 0, \frac{3}{2}\right) \qquad \mathbf{v}=\left(\frac{4}{3}, \frac{4}{3}, \frac{2}{3}\right)$

Risposta: $|\mathbf{u}| = 3/\sqrt{2}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=\left(\frac{11}{2}, 4, \frac{7}{2}\right)$, $\mathbf{u}\cdot\mathbf{v}=3$, $\widehat{\mathbf{uv}}=\frac{\pi}{4}$

$\mathbf{u}=\left(\frac{1}{2}, 0, \frac{1}{2}\right) \qquad \mathbf{v}=(-4, -4, -2)$

Risposta: $|\mathbf{u}| = 1/\sqrt{2}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=\left(-\frac{23}{2}, -12, -\frac{11}{2}\right)$, $\mathbf{u}\cdot\mathbf{v}=-3$, $\widehat{\mathbf{uv}}=\frac{3\pi}{4}$

$\mathbf{u}=(3, 0, 3) \qquad \mathbf{v}=\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right)$

Risposta: $|\mathbf{u}| = 3\sqrt{2}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=(1, -2, 2)$, $\mathbf{u}\cdot\mathbf{v}=-3$, $\widehat{\mathbf{uv}}=\frac{3\pi}{4}$

$\mathbf{u}=(4, 0, 4) \qquad \mathbf{v}=\left(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{4}\right)$

Risposta: $|\mathbf{u}| = 4\sqrt{2}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\mathbf{u}+3\mathbf{v}=\left(\frac{5}{2}, -\frac{3}{2}, \frac{13}{4}\right)$, $\mathbf{u}\cdot\mathbf{v}=-3$, $\widehat{\mathbf{uv}}=\frac{3\pi}{4}$

$\mathbf{u}=(2, 3, 1) \qquad \mathbf{v}=(2, -1, 3)$

Risposta: $|\mathbf{u}| = \sqrt{14}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(\frac{\sqrt{2}}{\sqrt{7}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$, $\mathbf{u}+3\mathbf{v}=(8, 0, 10)$, $\mathbf{u}\cdot\mathbf{v}=4$, $\widehat{\mathbf{uv}}=\arccos\left(\frac{2}{7}\right)$

$\mathbf{u}=(-2, 3, 1) \qquad \mathbf{v}=(2, -1, -3)$

Risposta: $|\mathbf{u}| = \sqrt{14}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(-\frac{\sqrt{2}}{\sqrt{7}}, \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}\right)$, $\mathbf{u}+3\mathbf{v}=(4, 0, -8)$, $\mathbf{u}\cdot\mathbf{v}=-10$, $\widehat{\mathbf{uv}}=\arccos\left(-\frac{5}{7}\right)$

$\mathbf{u}=(0, -3, 1) \qquad \mathbf{v}=(-2, -1, -3)$

Risposta: $|\mathbf{u}| = \sqrt{10}$, $\frac{\mathbf{u}}{|\mathbf{u}|} = \left(0, -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$, $\mathbf{u}+3\mathbf{v}=(-6, -6, -8)$, $\mathbf{u}\cdot\mathbf{v}=0$, $\widehat{\mathbf{uv}}=\frac{\pi}{2}$

$\mathbf{u}=(1,-3,0) \quad \mathbf{v}=(-3,-1,-2)$

Risposta: $|\mathbf{u}|=\sqrt{10},\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{1}{\sqrt{10}},-\dfrac{3}{\sqrt{10}},0\right),\ \mathbf{u}+3\mathbf{v}=(-8,-6,-6),\ \mathbf{u}\cdot\mathbf{v}=0,\ \widehat{\mathbf{uv}}=\dfrac{\pi}{2}$

$\mathbf{u}=(2,-3,-1) \quad \mathbf{v}=(-2,3,1)$

Risposta: $|\mathbf{u}|=\sqrt{14},\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{\sqrt{2}}{\sqrt{7}},-\dfrac{3}{\sqrt{14}},-\dfrac{1}{\sqrt{14}}\right),\ \mathbf{u}+3\mathbf{v}=(-4,6,2),\ \mathbf{u}\cdot\mathbf{v}=-14,\ \widehat{\mathbf{uv}}=\pi$

$\mathbf{u}=(8,-2,2) \quad \mathbf{v}=\left(\dfrac{1}{2},\dfrac{3}{2},-\dfrac{1}{2}\right)$

Risposta: $|\mathbf{u}|=6\sqrt{2},\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{2\sqrt{2}}{3},-\dfrac{1}{3\sqrt{2}},\dfrac{1}{3\sqrt{2}}\right),\ \mathbf{u}+3\mathbf{v}=\left(\dfrac{19}{2},\dfrac{5}{2},\dfrac{1}{2}\right),\ \mathbf{u}\cdot\mathbf{v}=0,\ \widehat{\mathbf{uv}}=\dfrac{\pi}{2}$

$\mathbf{u}=\left(1,1,-\frac{1}{2}\right) \quad \mathbf{v}=(2,8,2)$

Risposta: $|\mathbf{u}|=\dfrac{3}{2},\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{2}{3},\dfrac{2}{3},-\dfrac{1}{3}\right),\ \mathbf{u}+3\mathbf{v}=\left(7,25,\dfrac{11}{2}\right),\ \mathbf{u}\cdot\mathbf{v}=9,\ \widehat{\mathbf{uv}}=\dfrac{\pi}{4}$

$\mathbf{u}=(6,6,-3) \quad \mathbf{v}=\left(\dfrac{1}{3},\dfrac{4}{3},\dfrac{1}{3}\right)$

Risposta: $|\mathbf{u}|=9,\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{2}{3},\dfrac{2}{3},-\dfrac{1}{3}\right),\ \mathbf{u}+3\mathbf{v}=(7,10,-2),\ \mathbf{u}\cdot\mathbf{v}=9,\ \widehat{\mathbf{uv}}=\dfrac{\pi}{4}$

$\mathbf{u}=\left(5,5,-\dfrac{5}{2}\right) \quad \mathbf{v}=\left(\dfrac{2}{5},\dfrac{8}{5},\dfrac{2}{5}\right)$

Risposta: $|\mathbf{u}|=\dfrac{15}{2},\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{2}{3},\dfrac{2}{3},-\dfrac{1}{3}\right),\ \mathbf{u}+3\mathbf{v}=\left(\dfrac{31}{5},\dfrac{49}{5},-\dfrac{13}{10}\right),\ \mathbf{u}\cdot\mathbf{v}=9,\ \widehat{\mathbf{uv}}=\dfrac{\pi}{4}$

$\mathbf{u}=(-4,-4,2) \quad \mathbf{v}=(1,4,1)$

Risposta: $|\mathbf{u}|=6,\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(-\dfrac{2}{3},-\dfrac{2}{3},\dfrac{1}{3}\right),\ \mathbf{u}+3\mathbf{v}=(-1,8,5),\ \mathbf{u}\cdot\mathbf{v}=-18,\ \widehat{\mathbf{uv}}=\dfrac{3\pi}{4}$

$\mathbf{u}=(2,2,-1) \quad \mathbf{v}=(2,8,2)$

Risposta: $|\mathbf{u}|=3,\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{2}{3},\dfrac{2}{3},-\dfrac{1}{3}\right),\ \mathbf{u}+3\mathbf{v}=(8,26,5),\ \mathbf{u}\cdot\mathbf{v}=18,\ \widehat{\mathbf{uv}}=\dfrac{\pi}{4}$

$\mathbf{u}=(2,8,2) \quad \mathbf{v}=(1,-2,-2)$

Risposta: $|\mathbf{u}|=6\sqrt{2},\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(\dfrac{1}{3\sqrt{2}},\dfrac{2\sqrt{2}}{3},\dfrac{1}{3\sqrt{2}}\right),\ \mathbf{u}+3\mathbf{v}=(5,2,-4),\ \mathbf{u}\cdot\mathbf{v}=-18,\ \widehat{\mathbf{uv}}=\dfrac{3\pi}{4}$

$\mathbf{u}=\left(-1,0,-\sqrt{3}\right) \quad \mathbf{v}=\left(\sqrt{3},0,1\right)$

Risposta: $|\mathbf{u}|=2,\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(-\dfrac{1}{2},0,-\dfrac{\sqrt{3}}{2}\right),\ \mathbf{u}+3\mathbf{v}=\left(-1+3\sqrt{3},0,3-\sqrt{3}\right),\ \mathbf{u}\cdot\mathbf{v}=-2\sqrt{3},\ \widehat{\mathbf{uv}}=\dfrac{5\pi}{6}$

$\mathbf{u}=\left(-\dfrac{1}{2},0,-\dfrac{\sqrt{3}}{2}\right) \quad \mathbf{v}=\left(2\sqrt{3},0,2\right)$

Risposta: $|\mathbf{u}|=1,\ \dfrac{\mathbf{u}}{|\mathbf{u}|}=\left(-\dfrac{1}{2},0,-\dfrac{\sqrt{3}}{2}\right),\ \mathbf{u}+3\mathbf{v}=\left(-\dfrac{1}{2}+6\sqrt{3},0,6-\dfrac{\sqrt{3}}{2}\right),\ \mathbf{u}\cdot\mathbf{v}=-2\sqrt{3},\ \widehat{\mathbf{uv}}=\dfrac{5\pi}{6}$

4. Assegnati i seguenti vettori **u**, **v** $\in \mathbf{R}^3$, determinare il prodotto vettoriale $\mathbf{u}\times\mathbf{v}$ e l'area del parallelogramma determinato da **u** e **v**. Verificare, inoltre, che il prodotto vettoriale ottenuto sia ortogonale ai due vettori assegnati.

$\mathbf{u}=(1,1,4)$ $\quad \mathbf{v}=(0,2,1)$

Risposta: $\mathbf{u}\times\mathbf{v} = (-7,-1,2)$; $area = |\mathbf{u}\times\mathbf{v}| = 3\sqrt{6}$

$\mathbf{u}=(2,0,2)$ $\quad \mathbf{v}=(1,1,1)$

Risposta: $\mathbf{u}\times\mathbf{v} = (-2,0,2)$; $area = |\mathbf{u}\times\mathbf{v}| = 2\sqrt{2}$

$\mathbf{u}=(1,3,2)$ $\quad \mathbf{v}=(1,-2,1)$

Risposta: $\mathbf{u}\times\mathbf{v} = (7,1,-5)$; $area = |\mathbf{u}\times\mathbf{v}| = 5\sqrt{3}$

$\mathbf{u}=(0,-2,2)$ $\quad \mathbf{v}=(1,-2,2)$

Risposta: $\mathbf{u}\times\mathbf{v} = (0,2,2)$; $area = |\mathbf{u}\times\mathbf{v}| = 2\sqrt{2}$

$\mathbf{u}=(3,-1,0)$ $\quad \mathbf{v}=(1,-2,-2)$

Risposta: $\mathbf{u}\times\mathbf{v} = (2,6,-5)$; $area = |\mathbf{u}\times\mathbf{v}| = \sqrt{65}$

$\mathbf{u}=(4,-2,0)$ $\quad \mathbf{v}=(1,-2,-2)$

Risposta: $\mathbf{u}\times\mathbf{v} = (4,8,-6)$; $area = |\mathbf{u}\times\mathbf{v}| = 2\sqrt{29}$

$\mathbf{u}=(-3,-2,0)$ $\quad \mathbf{v}=(1,-2,2)$

Risposta: $\mathbf{u}\times\mathbf{v} = (-4,6,8)$; $area = |\mathbf{u}\times\mathbf{v}| = 2\sqrt{29}$

$\mathbf{u}=(3,-2,0)$ $\quad \mathbf{v}=(3,-2,2)$

Risposta: $\mathbf{u}\times\mathbf{v} = (-4,-6,0)$; $area = |\mathbf{u}\times\mathbf{v}| = 2\sqrt{13}$

$\mathbf{u}=(3,-1,0)$ $\quad \mathbf{v}=(3,-2,4)$

Risposta: $\mathbf{u}\times\mathbf{v} = (-4,-12,-3)$; $area = |\mathbf{u}\times\mathbf{v}| = 13$

5. Date le seguenti rette r, r', rispondere alle seguenti domande:

a) r e r' sono parallele?

b) r e r' sono incidenti?

c) r e r' sono ortogonali?

$$r: 2x-3y+5=0\,, \quad r': 2x-3y+7=0$$

Risposta: *sono parallele, non sono incidenti, non sono ortogonali*

$$r: x-2y+4=0\,, \quad r': 2x+y+3=0$$

Risposta: *non sono parallele, sono incidenti e ortogonali*

$$r: x-3y+4=0\,, \quad r': x-y+7=0$$

Risposta: *non sono parallele, sono incidenti, non sono ortogonali*

$$r: 4x-y+4=0\,, \quad r': 8x-2y+7=0$$

Risposta: *sono parallele, non sono incidenti, non sono ortogonali*

$$r: x+y+9=0\,, \quad r': x-y-3=0$$

Risposta: *non sono parallele, sono incidenti e ortogonali*

$$r: x+2y+4=0\,, \quad r': -2x-3y+3=0$$

Risposta: *non sono parallele, sono incidenti, non sono ortogonali*

$$r: 3x-2y+4=0\,, \quad r': x-y+3=0$$

Risposta: *non sono parallele, sono incidenti, non sono ortogonali*

$$r: x-2y+4=0\,, \quad r': x+2y+4=0$$

Risposta: *non sono parallele, sono incidenti, non sono ortogonali*