Esercizi su integrali indefiniti

$$\int \frac{\cos(\log x)}{x}\, dx = ?$$

per calcolare questo integrale bisogna preventivamente sapere:

① $D(\operatorname{sen} x) = \cos x$ (tabella delle derivate)

② $D(\log x) = \frac{1}{x}$ (tabella delle derivate)

③ $D(g(f(x))) = g'(f(x)) \cdot f'(x)$ (teorema)

④ $D(\operatorname{sen}(\log x)) = \cos(\log x) \cdot \frac{1}{x}$ (applicazione del teorema)

⑤ $\int \cos x\, dx = \operatorname{sen} x + c$ (tabella degli integrali)

$$\int \frac{\cos(\log x)}{x}\, dx = \int \frac{\cos(\log x)}{x}\, dx = \int \cos(\log x) \cdot \frac{1}{x}\, dx = \operatorname{sen}(\log x) + c$$

$$\int \frac{\cos x}{\cos^2(\operatorname{sen} x)}\,dx = \int \frac{\cos x}{\cos^2(\operatorname{sen} x\ )}\,dx = \int \frac{1}{\cos^2(\operatorname{sen} x)} \cdot \cos x \,dx = \operatorname{tg}(\operatorname{sen} x) + c$$

$$\int \frac{1}{(1+x^2)\operatorname{arctg} x}\,dx = \int \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{1+x^2}\,dx$$

$$= \log|\operatorname{arctg} x| + c$$

$$\int e^{\cos x} \operatorname{sen} x \, dx = \int (-1)\, e^{\cos x} (-\operatorname{sen} x)\, dx$$

$$= -\int e^{\cos x} (-\operatorname{sen} x)\, dx = -e^{\cos x} + c$$

$$\int \frac{1}{\cos^2(7x)}\, dx = \int \frac{1}{7} \cdot \frac{1}{\cos^2(7x)} \cdot 7\, dx$$

$$= \frac{1}{7} \int \frac{1}{\cos^2(7x)} \cdot 7\, dx = \frac{1}{7} \operatorname{tg}(7x) + c$$

$$\int \frac{1}{\sqrt{1-9x^2}}\,dx = \int \frac{1}{3} \frac{1}{\sqrt{1-(3x)^2}} \cdot 3\,dx$$

$$= \frac{1}{3} \int \frac{1}{\sqrt{1-(3x)^2}} \cdot 3\,dx = \frac{1}{3} \operatorname{arcsen}(3x) + c$$

$$\int \sqrt{3x}\,dx = \sqrt{3} \int x^{\frac{1}{2}}\,dx = \sqrt{3}\,\frac{1}{\frac{1}{2}+1}\,x^{\frac{1}{2}+1} + c$$

$$= \sqrt{3} \cdot \frac{2}{3} x^{\frac{3}{2}} + c = \frac{2}{\sqrt{3}} x^{\frac{3}{2}} + c$$

$$\int tg\,x\,dx = \int \frac{sen\,x}{cos\,x}\,dx = \int \frac{1}{cos\,x}\cdot sen\,x\,dx$$

$$= -\int \frac{1}{cos\,x}\cdot(-sen\,x)\,dx = -\log|cos\,x| + c$$

$$\int e^x\,sen(e^x)\,dx = \int sen(e^x)\cdot e^x\,dx$$

$$= -cos(e^x) + c$$

$$\int \frac{\cos(\log x)}{x}\,dx = \int \cos(\log x) \cdot \frac{1}{x}\,dx =$$

$$= \operatorname{sen}(\log x) + c$$

$$\int \frac{1}{\sqrt{1-x^2}\ \operatorname{arcsen} x}\,dx = \int \frac{1}{\operatorname{arcsen} x} \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= \log|\operatorname{arcsen} x| + c$$

$$\int \frac{e^{tg x}}{\cos^2 x}\,dx = \int e^{tg x} \cdot \frac{1}{\cos^2 x}\,dx = e^{tg x} + c$$

$$\int \frac{e^x}{(e^x)^2 + 1}\,dx = \int \frac{1}{1 + (e^x)^2} \cdot e^x\,dx$$

$$= \operatorname{arctg}(e^x) + c$$

$$\int \frac{2x-3}{x^2-3x+5}\,dx = \int \frac{1}{x^2-3x+5}(2x-3)\,dx =$$

$$\log|x^2-3x+5| + c$$

$$\int x\sqrt{2x^2+5}\,dx = \frac{1}{4}\int (2x^2+5)^{\frac{1}{2}} \cdot 4x\,dx$$

$$= \frac{1}{4} \cdot \frac{1}{\frac{1}{2}+1} (2x^2+5)^{\frac{1}{2}+1} + c$$

$$= \frac{1}{4} \cdot \frac{2}{3} (2x^2+5)^{\frac{3}{2}} + c = \frac{1}{6} (2x^2+5)^{\frac{3}{2}} + c$$

$$\int \frac{x}{\sqrt{4x^2-1}}\,dx = \frac{1}{8}\int (4x^2-1)^{-\frac{1}{2}} \cdot 8x\,dx$$

$$= \frac{1}{8} \cdot \frac{1}{-\frac{1}{2}+1}\,(4x^2-1)^{-\frac{1}{2}+1} + c$$

$$= \frac{1}{8} \cdot 2\,(4x^2-1)^{\frac{1}{2}} + c = \frac{1}{4}\sqrt{4x^2-1} + c$$

$$\int \frac{\log x}{x}\,dx = \int (\log x)\cdot \frac{1}{x}\,dx = \frac{1}{2}\log^2 x + c$$