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# R2R-EU: Software for fragility fitting and evaluation of estimation uncertainty in seismic risk analysis

Roberto Baraschino [a,b], Georgios Baltzopoulos [b,*], Iunio Iervolino [b]

[a] National Technical University of Athens, Athens, Greece
[b] Università degli Studi di Napoli Federico II, Naples, Italy

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ABSTRACT

Analytical methods in performance-based earthquake engineering (PBEE) typically employ suites of ground motion records to run dynamic analysis of a structure's computer model. The sample size of ground motions used in this context, affects the estimation uncertainty that underlies the seismic risk metrics thus obtained. This article presents R2R-EU (record-to-record estimation uncertainty), which is a PBEE software tool that numerically implements various schemes for estimating structure-specific seismic fragility and for the quantification of the estimation uncertainty behind seismic risk estimates, emanating from record-to-record variability in structural response. The software accepts as input the results of structural dynamic analysis to a set of accelerograms and seismic hazard curves. Estimation uncertainty is quantified by providing statistics, such as mean and variance, of the estimators of the failure rate and the fragility parameters (where applicable) and possibly their distribution. The user can choose the analysis method among some resampling and/or simulation schemes belonging to the bootstrap family, the delta method and other solutions from probability and statistics theory.

## 1. Introduction

Seismic assessment or design of a structure according to the performance-based earthquake engineering paradigm (PBEE) typically involves the evaluation of the rate of earthquakes causing failure of the structure, $\lambda_f$, via Equation (1):

$$\lambda_f = \int_{IM} P[f|IM = im] \cdot |d\lambda_{im}|, \tag{1}$$

where $P[f|IM = im]$ is the *structural fragility*. It is a function providing the conditional probability of failure given a certain value *im* of a ground motion intensity measure *IM*. The term $|d\lambda_{im}|$ depends on the derivative of the *hazard curve*, which provides the annual rate $\lambda_{im}$, of exceeding each *im* value at a the site of interest.

Both terms under the integral of Equation (1), can be affected by so-called *estimation uncertainty*, since both functions have to be deduced from available (i.e., limited) data. The work presented herein primarily deals with estimation uncertainty affecting the fragility function. In fact, the evaluation of the fragility term is often based on the results of nonlinear dynamic analysis of a numerical model, which is subjected to a set (i.e., a limited *sample*) of ground motions, in order to capture the *record-to-record variability* of structural response (e.g. Ref. [1]). Thus, when a structure's probability of failure-given-intensity is inferred from a sample of structural responses from dynamic analysis, that only constitutes an estimate of the fragility function. As a consequence, any seismic risk metric calculated on the basis of that fragility, such as the failure rate, is also an estimate, henceforth indicated as $\widehat{\lambda}_f$, of the unknown *true* value $\lambda_f$. In other words, any probabilistic model for structural fragility that is based on that limited sample of structural responses, will be affected by estimation uncertainty and that uncertainty will be propagated to the estimator of the failure rate $\widehat{\lambda}_f$ (e.g. Ref. [2]).

The focus of this article is the presentation of the PBEE software tool R2R-EU (*record-to-record estimation uncertainty*), which was developed in MATLAB® with a dual purpose: (i) estimating structure-specific seismic fragility, based on dynamic analysis, and (ii) quantifying estimation uncertainty, emanating specifically from record-to-record variability of seismic structural response, and the extent to which that uncertainty propagates unto risk metrics, such as the failure rate. The R2R-EU tool considers various consolidated nonlinear analysis strategies used in PBEE, such as *incremental dynamic analysis* (IDA [4]), *multi-stripe analysis* (MSA; e.g. Ref. [5,6]) or *cloud analysis*, in the context of Cornell's seismic

* Corresponding author.
*E-mail address:* georgios.baltzopoulos@unina.it (G. Baltzopoulos).

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reliability method [7]. The software (available via a page on the website www.reluis.it) offers several alternatives for the definition of the fragility function, and implements a series of statistical inference methods based on resampling and/or simulation schemes belonging to the bootstrap family [8], the *delta method* [9] and direct application of formulas from statistical inference theory. In the remainder of this article, first a brief overview of the methodologies for quantification of estimation uncertainty used in R2R-EU is provided, in tandem with a review of the corresponding alternative approaches used for conducting dynamic analysis and for evaluating structural fragility. Subsequently, a brief operational description of the software capabilities is given, and an example application is provided, followed by some concluding remarks.

## 2. Estimation uncertainty in fragility functions and seismic risk

As highlighted in the introductory discussion, the R2R-EU tool deals with the definition of structure-specific seismic fragility and with the quantification of that part of estimation uncertainty in the failure rate, that can be attributed to the record-to-record variability of structural response. The latter can be provided by one of the methods mentioned in the introduction; i.e., IDA, MSA and cloud. IDA involves progressively scaling each ground motion in a set, so as to cover a broad range of IM levels, and running dynamic analysis, ideally until the numerical model experiences instability that can be interpreted as side-sway structural collapse [10]. A measure of structural response, often termed an *engineering demand parameter* or EDP, is being registered at each IM level. The output of this analysis is a set of EDP-IM curves, equal in number to the number of ground motion records used (Fig. 1a). On the other hand, MSA involves the use of different sets of – ideally unscaled – accelerograms per IM level, chosen to represent the seismic scenarios causing that level of shaking at the construction site as indicated by *disaggregation* of seismic hazard [11]. The output of MSA is a set of EDP responses at fixed IM values (Fig. 1b). Cloud analysis uses a set of unscaled accelerograms to perform dynamic analysis so that at each record represents a single IM value and corresponds to a single EDP response. The output is a cloud of points (Fig. 1c), hence the name.

Given the output of dynamic analysis, the strategy for analytically evaluating a fragility function often branches into one of two approaches: the *IM-based* approach and the *EDP-based* approach; IM-based fragility estimation is suitable within the IDA framework, while EDP-based can be applied in both IDA and MSA settings. In both cases, it is assumed that a threshold EDP, indicated as $edp_f$, can be defined, so that its exceedance will be tantamount to failure, that is, $P[f|IM = im] = P[EDP > edp_f|IM = im]$. In the IM-based approach, a new random variable (RV) needs to be introduced: the IM-value causing failure, denoted as $IM_f$, which is, in principle, different for each record. After the analysis has been concluded and the IDA curves become available, a sample of the RV can be obtained by finding the intersection, $im_{f,i}$ , between the vertical line passing through $edp_f$ and the $i$-th IDA curve, $i = \{1, 2, ..., n\}$ (Fig. 2a). The fragility function may then be considered as the probability of $IM_f$ being equal or lower than the level of seismic intensity possibly occurring at the site; i.e., $P[f|IM = im] = P[IM_f \leq im]$. It is also possible to assign a parametric model to the distribution of $IM_f$ and a typical choice is the lognormal model, which is completely defined by mean $\eta$ and standard deviation $\beta$ of the logarithm of $IM_f$. In that case, fragility can be expressed using the standard Gaussian function, $\Phi(\cdot)$:

$$P[f|IM = im] = P[IM_f \leq im] = \Phi[(\ln(im) - \widehat{\eta}) / \widehat{\beta}]. \quad (2)$$

The two parameters $\{\eta, \beta\}$ are generally unknown and one way to obtain estimates of these parameters, $\{\widehat{\eta}, \widehat{\beta}\}$, is by using the sample of responses resulting from IDA according to Equation (3):

$$\widehat{\eta} = n^{-1} \cdot \sum_{i=1}^{n} \ln(im_{f,i}), \quad \widehat{\beta} = \sqrt{(n-1)^{-1} \cdot \sum_{i=1}^{n} \left[\ln(im_{f,i}) - \widehat{\eta}\right]^2}, \quad (3)$$

where $n$ represents the number of IDA curves and is therefore equal to the number of records used and $im_{f,i}$ is the intensity that one needs to scale the $i$-th record (out of $n$ in total), in order to cause failure of the structure. Of course, it is not necessary to assume a parametric model for IM-based fragility; in fact, a non-parametric representation can be obtained directly from the sample of $IM_f$ values, according to Equation (4):

$$P[f|IM = im] = n^{-1} \cdot \sum_{i=1}^{n} I_{(im_{f,i} \leq im)}, \quad (4)$$

where $I_{(im_{f,i} \leq im)}$ is an indicator function that returns 1 if $im_{f,i} \leq im$ or 0 if $im_{f,i} > im$. The use of estimation uncertainty as a means for determining the number of records to use in IM-based fragility derivation was explored in Ref. [3].

Structural fragility can also be computed by following an EDP-based approach. In fact, the EDP-based method works both for IDA and MSA; in this case, EDP responses are obtained at discrete (fixed) IM levels. When these EPD responses are plotted against the corresponding IMs, they are arranged in horizontal stripes (e.g., Fig. 1b), one for each level of shaking intensity considered. By counting the fraction of records in each stripe that cause the exceedance of the limit state threshold, $edp_f$, the estimation of the fragility parameters $\{\widehat{\eta}, \widehat{\beta}\}$ can be carried out via maximum likelihood, according to Equation (5), which is from Ref. [12]:

$$\{\widehat{\eta}, \widehat{\beta}\} = \underset{\eta,\beta}{\operatorname{argmax}} \left[ \sum_{j=1}^{u} \left( \ln \binom{n}{q_j} + q_j \cdot \ln\left\{ \Phi\left[\frac{\ln(im_j) - \eta}{\beta}\right] \right\} + (n - q_j) \cdot \ln\left\{ 1 - \Phi\left[\frac{\ln(im_j) - \eta}{\beta}\right] \right\} \right) \right] \quad (5)$$

where $u$ is the number of IM levels considered (i.e., the number of stripes, each stripe containing responses from $n$ records), and $q_j$ is the number of failures observed at the stripe corresponding to $IM = im_j$ (Fig. 2b), when $edp_{ij}$, $i = \{1, .., n\}$, $j = \{1, .., u\}$ represents the single structural response recorded at the $i$-th record of the $j$-th stripe. In this formulation, cases of non-convergent analysis (referred to as *collapse* cases) due to the numerical model coming too close to highly-nonlinear behaviour associated with incipient instability, say $c_j$ in number, are also counted in $q_j$ and are therefore accounted for, despite the potential lack

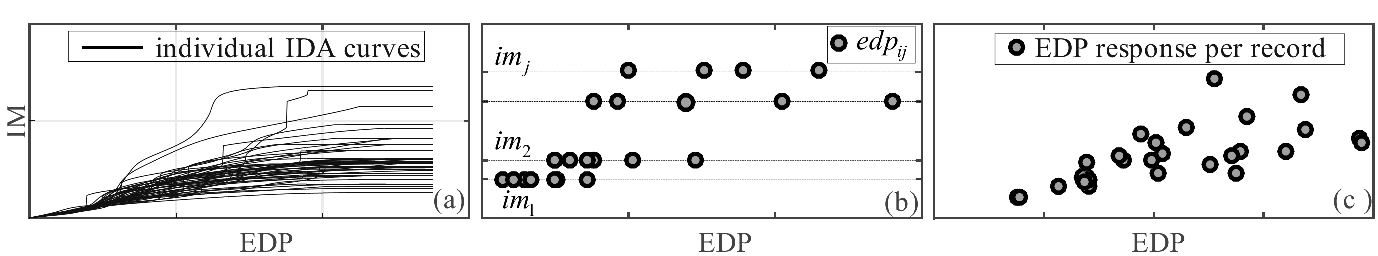


**Fig. 1.** Example of IDA curves of an inelastic frame structure (a); EDP responses of a non-linear structure at four IM levels obtained via MSA (b); EDP-IM response for the same structure obtained via cloud analysis (c).

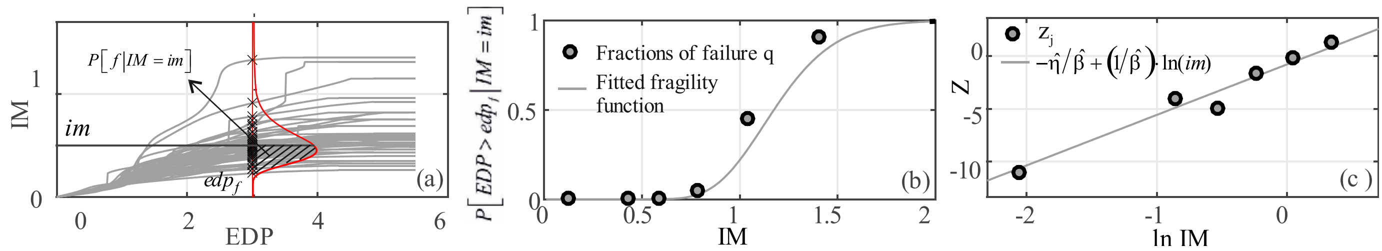


**Fig. 2.** Intersection of threshold line with IDA curves and fitted probability density function (a); maximum likelihood fit of a lognormal fragility function to the results of MSA (b); fit of lognormal fragility on normal probability paper (c).

of a meaningful EDP value [10]. At this point it should be noted that, in cases where the observed numbers of failure $q_j$ remain excessively low over all stripes considered, the maximum likelihood estimates implied in Equation (5) can suffer from numerical problems. One viable alternative for considering fragility, consistent with the EDP-based approach, is the three-parameter-per-intensity model adopted by Shome and Cornell [10], given by Equation (6):

$$P[f|IM = im_j] = \left(1 - \frac{c_j}{n}\right)\cdot\left\{1 - \Phi\left[\frac{\ln(edp_f) - \widehat{\eta}_{\ln(EDP_j)}}{\widehat{\beta}_{\ln(EDP_j)}}\right]\right\} + \frac{c_j}{n} \tag{6}$$

where $c_j$ is the number of observed collapse cases at the $j$-th stripe, according to the previous definition, and $\widehat{\eta}_{\ln(EDP_j)} = (n - c_j)^{-1}\cdot\sum_{i=1}^{(n-c_j)}\ln\left(edp_{ij}\right)$ and $\widehat{\beta}^2_{\ln(EDP_j)} = (n - c_j - 1)^{-1}\cdot\sum_{i=1}^{(n-c_j)}\left[\ln\left(edp_{ij}\right) - \widehat{\eta}_{\ln(EDP_j)}\right]^2$ are the mean and variance of the logarithm of EDP, at $IM = im_j$, respectively. They are provided by structural analysis and not affected by numerical instability (no-collapse cases). Note that this approach provides fragility at the discrete intensities $IM = im_j$, rather than as a continuous function of IM.

Another alternative procedure is to plot the failure probabilities per stripe, $P[f|IM = im_j]$ obtained from Equation (6), on a normal probability paper and estimate the parameters $\{\widehat{\eta}, \widehat{\beta}\}$ via least squares fitting of a line. For this procedure, the values of the standard normal variable, $Z$, corresponding to the failure probabilities are calculated as $z_j = \Phi^{-1}(P[f|IM = im_j])$, for which it can be assumed that, on a *normal probability paper* [13], a linear relationship of the form $Z = -\widehat{\eta}/\widehat{\beta} + (1/\widehat{\beta})\cdot\ln(im)$ should hold; e.g., Fig. 2c. In this case, the least squares estimate for $\{\widehat{\eta}, \widehat{\beta}\}$ is given directly from the normal equations of the regression problem.

All of the aforementioned approaches for estimating a fragility function (which may entail assigning a parametric model or not) have been implemented in R2R-EU. They also allow to quantify the estimation uncertainty in the failure rate, used in conjunction with methodologies of statistical inference. These methodologies are: parametric or non-parametric resampling plans (generally belonging to the bootstrap family), the application of known results for the distribution of the estimators of the lognormal parameters, and the delta method, which is based on Taylor series expansion of either the risk integral or the formula from Cornell's seismic reliability method.

### 2.1. Estimators of the Gaussian distribution's parameters

If structural fragility is assumed lognormal, the estimators of the parameters logarithmic mean $(\widehat{\eta})$ and variance $(\widehat{\beta}^2)$, obtained according to Equation (3), have known distributions. The estimator $\widehat{\eta}$ is distributed as a Gaussian with mean and variance equal to $\eta$ and $\beta^2/n$ respectively (but assumed equal to $\widehat{\eta}$ and $\widehat{\beta}^2/n$; i.e., $\eta$ and $\beta^2$ are substituted by the available point estimates), while $\widehat{\beta}^2\cdot(n-1)/\beta^2$ is *chi-squared* distributed with $n-1$ degrees of freedom. Since the failure rate is a function of these two stochastically independent RVs, shown here as Equation (7):

$$\widehat{\lambda}_f(\widehat{\eta}, \widehat{\beta}) = \int_{IM}\Phi[(\ln(im) - \widehat{\eta})/\widehat{\beta}]\cdot|d\lambda_{im}|, \tag{7}$$

it follows that the mean and variance of $\widehat{\lambda}_f$ can be evaluated (in R2R-EU) using their known densities.

### 2.2. Bootstrap

The bootstrap is a statistical inference process, which is based on taking an original data set and generating, so-called, *bootstrap samples* by resampling the original data with replacement. The bootstrap samples have the same size as the original. This resampling process is implemented in R2R-EU for three cases: non-parametric IM-based fragility derived from IDA, EDP-based fragility using the three-parameter model of Equation (6) and EDP-based fragility with parameter estimation via the normal probability paper procedure.

In the case of IM-based non-parametric fragility, the bootstrap implementation takes the original $n$-size sample of $IM_f$ realizations already available from IDA, $\{im_{f,1}, im_{f,2}, .., im_{f,n}\}$, and generates an arbitrary number, $m$, of bootstrap samples $\{im^*_{f,1k}, im^*_{f,2k}, .., im^*_{f,nk}\}$, where $k = \{1, .., m\}$. Subsequently, a Monte Carlo simulation is performed, where, for each bootstrap sample, Equation (1) is used to compute a *bootstrap replication* of the failure rate, $\widehat{\lambda}^*_{f,k}$. Then, the mean and variance of the failure rate estimator (denoted via the operators $E[\cdot]$ and $VAR[\cdot]$, respectively) are evaluated using the simulations results according to Equation (8):

$$E\left[\widehat{\lambda}_f\right] = m^{-1}\cdot\sum_{k=1}^{m}\widehat{\lambda}^*_{f,k} \quad ,$$
$$VAR\left[\widehat{\lambda}_f\right] = (m-1)^{-1}\cdot\sum_{k=1}^{m}\left(\widehat{\lambda}^*_{f,k} - E\left[\widehat{\lambda}_f\right]\right)^2. \tag{8}$$

In the case of EDP-based fragility, the bootstrap process starts from a set of $n \times u$ EDP responses, available from either MSA or IDA and denoted as previously by $edp_{ij}$, obtained from $n$ records $(i = \{1, .., n\})$ at each one of $u$ IM levels $j = \{1,..,u\}$, denoted as $\{im_1, im_2, .., im_u\}$. As a first step, the EDP responses at each IM level (stripe), are resampled with replacement $m$ times, resulting in new sets of responses at the $j$-th stripe (i.e., bootstrap samples) $\{edp^*_{1j,k}, edp^*_{2j,k}, .., edp^*_{nj,k}\}$, $k = \{1, .., m\}$. Subsequently, at each and every $j$-th, $j = \{1,..,u\}$, stripe of the $k$-th bootstrap sample, $k = \{1,..,m\}$, the responses $c^*_{j,k}$, corresponding to collapse cases, are identified, and the probabilities of failure, $P^*_k[f|IM = im_j]$, are calculated according to Equation (6).

In the case of the three-parameter model, the $k$-th bootstrap replication of the failure rate, $\widehat{\lambda}^*_{f,k}$ is evaluated according to Equation (1), while the mean and variance of the estimator are again evaluated according to Equation (8). In the case of EDP-based lognormal fragility whose parameters are estimated via linear fit on normal probability paper, new parameter estimates $\{\widehat{\eta}_k^*, \widehat{\beta}_k^*\}$, $k = \{1,..,m\}$, are obtained for each bootstrap sample via least squares. Then bootstrap replications $\widehat{\lambda}_{f,k}^*$ are calculated by substituting the parameters $\{\widehat{\eta}_k^*, \widehat{\beta}_k^*\}$ into Equations (2) and (1). Finally, Equation (8) is used to obtain the statistics of the failure rate estimator, same as before.

### 2.3. Parametric bootstrap

When a parametric model is assumed for the fragility function, the mean and variance of the failure rate's estimator can be inferred via a parametric version of the bootstrap. In the parametric version, bootstrap samples can be extracted directly from the assumed fragility model, rather than by means of resampling the original dataset. In R2R-EU this is implemented for both cases of IM- and EDP-based lognormal fragility (in the EDP-based case, when parameter estimation occurs via maximum likelihood). In the IM-based case, the $n$ values of $IM_f$ obtained from IDA, $\{im_{f,1}, im_{f,2}, .., im_{f,n}\}$, are used to derive the reference lognormal fragility parameters $\{\widehat{\eta}, \widehat{\beta}\}$ via Equation (3). Subsequently, an arbitrary number $m$ of new bootstrap samples, $\{im_{f,1k}^*, im_{f,2k}^*, .., im_{f,nk}^*\}$, $k = \{1,..,m\}$, is extracted from the reference distribution defined by $\{\widehat{\eta}, \widehat{\beta}\}$, with each new sample being of size $n$. Then, for the $k$-th out of $m$ bootstrap samples, a new fragility function is evaluated via Equation (3), having parameters $\{\widehat{\eta}_k^*, \widehat{\beta}_k^*\}$, and the bootstrap replication of the failure rate $\widehat{\lambda}_{f,k}^*$ is computed using these parameters and Equation (1). Finally, the mean and variance of the failure rate estimator are calculated via Equation (8).

In the EDP-based case, the reference structural fragility parameters $\{\widehat{\eta}, \widehat{\beta}\}$ are obtained from the available responses via the binomial maximum likelihood of Equation (5). Then, at the $j$-th stripe, corresponding to $IM = im_j$, $j = \{1,..,u\}$, a number of $m$ bootstrap samples is extracted from the binomial distribution with parameter equal to $p_j = \Phi\{[\ln(im_j) - \widehat{\eta}]/\widehat{\beta}\}$ (i.e., the parameter of binomial distribution is the probability of failure). Each sample consists of $n$ Bernoulli trials, resulting in $q_{j,k}^*$ failures and $n - q_{j,k}^*$ survivals of the structure at the $j$-th stripe of the $k$-th bootstrap sample. Subsequently, new lognormal parameters $\{\widehat{\eta}_k^*, \widehat{\beta}_k^*\}$ are obtained from the $q_{j,k}^*$ failures, via Equation (5). It is assumed that, during the bootstrap replications, the maximum likelihood estimate may run into numerical problems for a number of $m_o$ bootstrap samples, out of a total $m$. With this assumption in mind, the bootstrap replication of failure rate, $\widehat{\lambda}_{f,k}^*$, is computed via Equation (1) and then the mean and variance of the estimator can be evaluated according to:

$$E\left[\widehat{\lambda}_f\right] = (m - m_o)^{-1} \cdot \sum_{k=1}^{(m-m_o)} \widehat{\lambda}_{f,k}^* \quad ,$$
$$VAR\left[\widehat{\lambda}_f\right] = (m - m_o - 1)^{-1} \cdot \sum_{k=1}^{(m-m_o)} \left(\widehat{\lambda}_{f,k}^* - E\left[\widehat{\lambda}_f\right]\right)^2, \tag{9}$$

which only differs from Equation (8) in the fact that the simulation-based statistics are calculated using a number of $(m - m_o)$ bootstrap replications of the failure rate; i.e., only those that did not encounter numerical issues.

### 2.4. Delta method

An alternative method for evaluating the mean and variance of $\widehat{\lambda}_f$, besides the bootstrap and the properties of the Gaussian function, is the delta method. The delta method uses a Taylor series expansion to approximate the expectation and variance of a RV and has been applied in the context of Cornell's seismic reliability method in Ref. [2]. The latter can be implemented using output from cloud analysis, which entails a set of $n$ ground motion records with variable intensities and the corresponding sample of EDP responses. By performing linear regression of $\ln(EDP)$ against $\ln(IM)$ and assuming that the logarithm of the hazard curve, $\ln(\lambda_{im})$, can be approximately considered locally linear, the annual failure rate can be estimated in closed-form as $\widehat{\lambda}_f \approx k_0 \cdot (edp_f/\widehat{a})^{-\ k/b} \cdot \exp[(k^2/2 \cdot b^2) \cdot (\widehat{\beta}_D^2 + \beta_C^2)]$, where $k_0$ and $k$ are, respectively, the slope and intercept of the $\ln(\lambda_{im})$ curve linearized around the IM corresponding to $edp_f$, $\widehat{a}$ and $\widehat{b}$ are the slope and intercept from the linear regression of $\ln(EDP)$ against $\ln(IM)$, $\widehat{\beta}_D$ is the standard deviation of the residuals of $\ln(EDP)$ given $\ln(IM)$, which is estimated from the regression and $\beta_C$ is the logarithmic standard deviation of $edp_f$, which is assumed to follow a lognormal distribution. In this context, the statistics of $\widehat{\lambda}_f$ can be approximated via Taylor series expansion, which are given in Ref. [2]. The delta method can also be applied for the failure rate estimator from Equation (7), under the assumption of lognormal fragility. In this case the failure rate is regarded as a function of the fragility parameters, that can undergo a Taylor series expansion, which can also be found in Ref. [2] along with the necessary derivatives of $\widehat{\lambda}_f$. The advantage of the delta method, over the other procedures implemented in R2R-EU, is that the closed-form expressions need only be evaluated analytically once, after which the statistics of $\widehat{\lambda}_f$ can be obtained with less computational effort with respect to the bootstrap.

## 3. R2R-EU, operational outline and illustrative application

R2R-EU runs behind a Mathworks MATLAB®-based graphical user interface (see Fig. 3) which implements all of the methods illustrated in the previous section for evaluating a fragility model and for quantification of estimation uncertainty in the fragility parameters and in the failure rate. Two sets of input data are needed to run R2R-EU: one containing the hazard curve, and another containing the structural responses. For the hazard curve, there is the additional possibility of directly importing output files from the REASSESS software [14]. In all supported cases, after elaborating the hazard and dynamic analysis data, R2R-EU provides the fragility model and the point estimates of the fragility parameters (where applicable), the point estimate of the failure rate and the mean and variance of the failure rate estimator. For the cases where one of the bootstrap schemes is applicable, a simulation-based approximation for the distribution of $\widehat{\lambda}_f$ is also provided, in the form of the histogram of requested bootstrap replications. In cases where the chosen fragility model is parametric (lognormal) and inference is conducted via a bootstrap process, the approximate distributions of $\widehat{\eta}$ and $\widehat{\beta}$ are likewise provided, in the form of the histograms of the corresponding bootstrap replications. The R2R-EU tool allows to save and export results, in either MATLAB data file or ASCII text file formats. More details on the workflow and input/output options, as well as additional application examples omitted here for reasons of space, can be found in R2R-EU user's manual.

### 3.1. Illustrative application

In this section an application of R2R-EU is presented, using as case study structure a four-story, plane, code-conforming, steel perimeter moment resisting frame designed to ASCE-SEI 7-05 criteria [15]. The structure is ideally located at a site near the town of Amatrice (central Italy; lat. 42.53°, lon. 13.29°), for which the hazard curve (Fig. 3), in terms of five-percent-damped spectral acceleration at the frame's first-mode vibration period $Sa(T = 1.25\ \text{s})$ was obtained using REASSESS, considering the Italian seismic source model used in the hazard

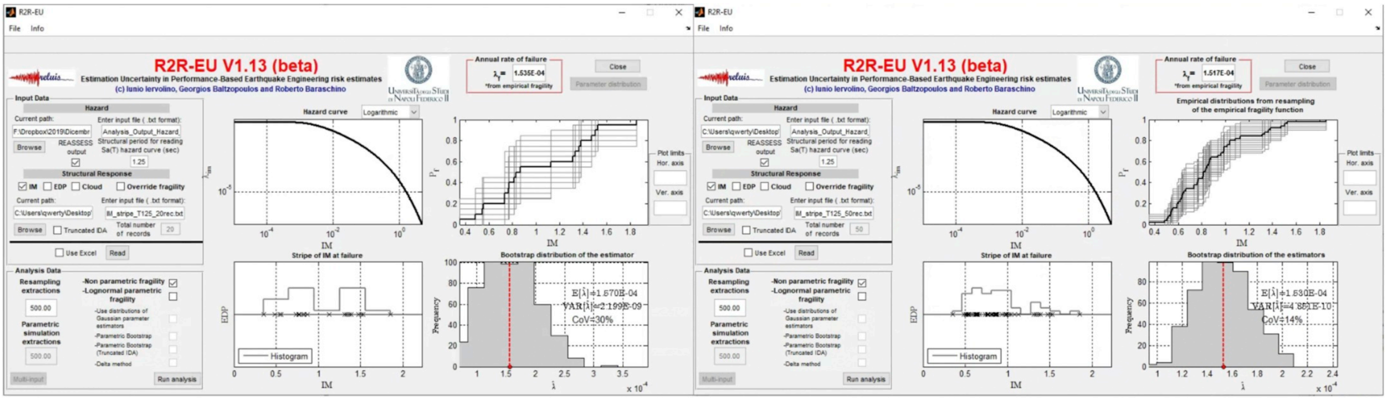


**Fig. 3.** Graphical user interface for the application using IDA results from twenty (left) and fifty records (right).

assessment at the basis of the building code.

IDA is performed for the structure, using a set of fifty records, which are scaled upwards until side-sway collapse. By considering a generic limit state, which is nominally exceeded when a maximum interstorey drift ratio (IDR) above 3.5% is recorded, the R2R-EU software is used for the quantification of the failure rate's estimation uncertainty. To this end, the example uses two of the available strategies[1]: the bootstrap for non-parametric IM-based fragility (shown in Fig. 3) and the delta method for lognormal fragility. The exercise is repeated in two versions: the first only uses a randomly selected subset of twenty-out-of-fifty IDA curves, while the entire set of fifty is used on the second go.

For the application of the bootstrap resampling method, first twenty (and later fifty) records are used to build the empirical fragility curves. In this case, five-hundred bootstrap extractions of the failure rate are requested from R2R-EU, which leads to calculating the mean and variance of the estimator, according to the methodology outlined in paragraph 2.2. These statistics are also calculated by means of the delta method, under the lognormal assumption, and the whole process is repeated using the structural response results from all fifty records; the results provided by R2R-EU are summarized in Table 1, where the coefficient of variation reported in the last column is calculated as $CoV_{\widehat{\lambda}_f} = VAR[\widehat{\lambda}_f]^{1/2}/E[\widehat{\lambda}_f]$. The similarity of the results obtained via the two approaches is evident in this example, along with the effect of the number of records used.

## 4. Concluding remarks

This article dealt with R2R-EU, that is an interactive PBEE software tool that can be used for quantifying the estimation uncertainty in seismic structural risk assessment, due to record-to-record variability of response. R2R-EU takes as input a hazard curve, the results of dynamic analysis, which can be incremental dynamic analysis, multiple stripe analysis or cloud analysis, and a threshold engineering demand parameter that defines the demarcation line for failure. With this input, the software first evaluates the structure-specific seismic fragility function. Subsequently, R2R-EU goes on to calculate a point estimate for the annual failure rate and to evaluate the expected value and variance of the rate's estimator. These calculations can be performed while assuming either a non-parametric representation for structural fragility or a lognormal model or even when employing Cornell's seismic reliability formulation. R2R-EU is available at www.reluis.it.

[1] Applications of the other strategies described above, not given here due to space constraints, can be found in R2R-EU user's manual.

**Table 1**
Statistics of the failure rate estimator for the considered example.

| Method | No. of records | $\widehat{\lambda}_f$ [year$^{-1}$] | $E[\widehat{\lambda}_f]$ [year$^{-1}$] | $VAR[\widehat{\lambda}_f]$ [year$^{-2}$] | $CoV_{\widehat{\lambda}_f}$ |
|---|---|---|---|---|---|
| Non-parametric bootstrap (IM-based) | 20 | $1.53 \cdot 10^{-4}$ | $1.57 \cdot 10^{-4}$ | $2.20 \cdot 10^{-9}$ | 30% |
| | 50 | $1.52 \cdot 10^{-4}$ | $1.53 \cdot 10^{-4}$ | $4.85 \cdot 10^{-10}$ | 14% |
| Delta method | 20 | $1.36 \cdot 10^{-4}$ | $1.41 \cdot 10^{-4}$ | $1.82 \cdot 10^{-9}$ | 30% |
| | 50 | $1.48 \cdot 10^{-4}$ | $1.48 \cdot 10^{-4}$ | $4.68 \cdot 10^{-10}$ | 15% |

## Declaration of competing interest

No conflict of interest exists.

## CRediT authorship contribution statement

**Roberto Baraschino:** Software, Validation, Writing - review & editing. **Georgios Baltzopoulos:** Writing - original draft, Methodology, Software. **Iunio Iervolino:** Conceptualization, Writing - review & editing, Project administration, Supervision, Methodology.


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## References


[1] Shome N, Cornell CA, Bazzurro P, Carballo JE. Earthquakes, records, and nonlinear responses. Earthq Spectra 1998;14:469–500.
[2] Iervolino I. Assessing uncertainty in estimation of seismic response for PBEE. Earthq Eng Struct Dynam 2017;46:1711–23.
[3] Baltzopoulos G, Baraschino R, Iervolino I. On the number of records for structural risk estimation in PBEE. Earthq Eng Struct Dynam 2019;48:489–506.
[4] Vamvatsikos D, Cornell CA. Incremental dynamic analysis. Earthq Eng Struct Dynam 2002;31:491–514.
[5] Jalayer F, Cornell CA. Alternative non-linear demand estimation methods for probability-based seismic assessments. Earthq Eng Struct Dynam 2008;13:1834–49.
[6] Bazzurro P, Cornell CA, Shome N, Carballo JE. Three proposals for characterizing MDOF nonlinear seismic response. J Struct Eng 1998;124(11):1281–9.
[7] Cornell CA, Jalayer F, Hamburger RO, Foutch DA. Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines. J Struct Eng 2002;128:526–33.
[8] Efron B. The jackknife, the bootstrap and other resampling plans. Philadelphia, Pennsylvania: Society For Industrial And Applied Mathematics; 1982.
[9] Oehlert GW. A note on the delta method. Am Statistician 1992;46:27–9.

[10] Shome N, Cornell CA. Structural seismic demand analysis: consideration of collapse. In: 8th ACSE spec conf probabilistic mech struct reliab; 2000. PMC2000-P2119.
[11] Lin T, Haselton CB, Baker JW. Conditional spectrum-based ground motion selection. Part I: hazard consistency for risk-based assessments. Earthq Eng Struct Dynam 2013;42:1847–65.
[12] Baker JW. Efficient analytical fragility function fitting using dynamic structural analysis. Earthq Spectra 2015;31.
[13] Cornell AC, Benjamin JR. Probability, statistics and decision for Civil engineers. New York: McGraw-Hill; 1970.
[14] Chioccarelli E, Cito P, Iervolino I, Giorgio M. REASSESS V2.0: software for single- and multi-site probabilistic seismic hazard analysis. Bull Earthq Eng 2019;17:1769–93.
[15] American Society of Civil Engineers (ASCE). Minimum design loads for buildings and other structures. 2006. 7-05.