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STATE-DEPENDENT SEISMIC FRAGILITY VIA PUSHOVER ANALYSIS

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Abstract

Earthquakes are clustered in space and time. This means that structures in seismically active regions can be subjected to multiple consecutive instances of base acceleration, with insufficient in-between time for repair operations to take place. In such situations, buildings may experience degradation of their lateral-force-resisting capacity due to damage accumulation. Consequently, the use of seismic fragility functions developed for the intact structure may not be enough, in the context of seismic risk assessment studies that consider the effect of seismic clusters. In these cases, one may employ state-dependent fragility curves, which are separate fragility functions assigned to the same structure, depending on distinct damage states that it may be brought to by prior shocks.

State-of-the-art analytical estimation of structure-specific fragility entails the use of dynamic analysis of a numerical model of the structure, for example, incremental dynamic analysis (IDA), which can be computationally laborious, thus motivating the development of simplified, less time-consuming methods, often based on substituting the structural model by equivalent single-degree-of-freedom (SDOF) systems that can be defined via pushover analysis. In fact, existing procedures in the literature, such as back-to-back IDA, that can be used to estimate state-dependent fragility curves, tend to increase computational costs, rendering the development of simplified methodologies for this case a topical issue.

In this context, the present paper presents a method for estimating state-dependent seismic fragility functions, based on pushover analysis and a predictive model for constant-ductility residual displacement ratio. This ratio is defined as the absolute value of the of residual-to-peak-transient seismic displacement ratio of an equivalent SDOF structure. The residual displacement model, which considers yielding SDOF systems that exhibit stiffness and strength degradation, with natural periods between 0.3 s and 2.0 s and post-yield hardening ratios from 0 % to 10%, is outlined first. The model also estimates the joint probability distribution of normalized elongated period and strength degradation, for a given ductility demand. This information allows for a probabilistic evaluation of the pushover curve characterizing a damaged structural system, which is then used to obtain state-dependent fragility, when damage states are defined via ductility demand thresholds. The state-dependent fragility curves are estimated via IDA of SDOF oscillators with pushovers that were previously determined from the model. An illustrative application showcases the ability of the proposed methodology to provide state-dependent fragility estimates in an expedient manner.

Keywords: sequence-based seismic reliability; damage accumulation; residual displacements.

1. Introduction

Seismic risk analysis, in its classical form, does not consider structural failure that is reached progressively due to damage accumulation in multiple events. This can be justified by considering that, for example, after some seismic event damages the structure of interest, enough time will elapse until the next earthquake for the stakeholders to repair it back to its initial state. However, earthquakes are known to be clustered in both space and time and this means that the necessary repair time between seismic shocks may not be available. One such typical case is that of short-term emergency management, during the aftershock sequence that follows an earthquake characterized as the mainshock. In that case, the possibility of aftershock-induced ground shaking exacerbating any damage caused by the main event, must be taken into account in risk assessment [1,2].

Fragility functions are well-established tools, used in seismic risk analyses to probabilistically quantify structural vulnerability (discussion to follow). Traditionally, one fragility per structure is assigned, assuming that earthquake-induced shaking will find the structure in the absence of seismic damage. In order to extend the use of this tool to sequence-based risk assessment, the concept of a set of state-dependent seismic fragility functions must be introduced. State-dependent fragilities provide a full picture of the seismic vulnerability of a structure in which damage can accumulate due to transitions across *damage states* (DSs). State-of-the-art analytical estimation of structure-specific fragility involves the use of dynamic analysis of a numerical model of the structure; e.g., incremental dynamic analysis (IDA) [3,4]. For the evaluation of state-dependent fragility curves, an extended version of IDA has been suggested in several studies [5–11], referred to as *back-to-back* IDA. The main disadvantage of deriving fragility functions based on nonlinear dynamic analysis is the high computational cost involved, which includes both the time investment required for effectively modelling nonlinear structural behavior and computer time needed to run multitudes of analyses and post-process the results. This has motivated the development of simplified procedures for analytical fragility estimation, based on static nonlinear analysis, which is often termed *pushover analysis*. These methods make recourse to a surrogate structure in the form of an equivalent inelastic single-degree-of-freedom (SDOF) system, whose definition is based on the original structure's pushover curve. One such example, used in the case of traditional fragility estimation, is the method proposed in [12], which has been recently streamlined into a dedicated software tool [13].

Herein a preliminary version of a simplified pushover-based methodology is discussed, adapted specifically for the estimation of state-dependent fragility functions. While traditional fragility estimation requires a large number of non-linear runs, governed by the need for obtaining accurate estimates in the face of record-to-record variability of structural response [14,15], this is even more so for state-dependent fragility, when the analysis should ostensibly represent all the possible effects, in terms of damage, of two consecutive earthquakes. Therefore, there is reason for exploring possible simplification in the latter case. In fact, in the case of sequential loading of the structure by consecutive instances of base-acceleration, without the possibility of intermediate remedial measures, the first shaking determines an intermediate damaged state of the structure, which will be called upon to sustain the second shock. This intermediate incarnation of the damaged structure is itself subject to some variability in terms of the fundamental dynamic structural properties, such as loss of stiffness and strength against lateral loads, and also residual displacements due to plastic deformation. In this context, a possible shortcut could be to account for the variability in structural properties of the damaged system via an analytical stochastic model, eschewing the need for dynamic runs representing the first shock, which brings the system to the damage state of interest.

The present study discusses exactly such a simplification, by considering this variability in structural properties, at the given damage state, directly on the static pushover; i.e., on the backbone curve of the equivalent SDOF. This can be achieved by using a semi-empirical predictive model for constant-ductility residual displacement ratios proposed in [16,17]; this model provides the joint probability distribution of residual displacement and other parameters necessary for the definition of the post-(first-)shock static pushover of an inelastic SDOF system, conditional on the attainment of a specific displacement demand during that shock. Thus, in lieu of executing sequential dynamic runs in order to represent a succession of damaging events within a sequence, the damaged structural configuration is obtained via Monte-Carlo simulation and analyses are executed only to account for the second shock, further reducing the computational cost.

The remainder of this paper is organized as follows: first comes a discussion on the analytical derivation of fragility functions, both for the case of an intact structure and for an already-damaged structure. Subsequently the procedure for simulating the damaged structures' pushovers is outlined, starting from a brief presentation of the residual displacement model and going on to describe the stochastic generation of backbone curves, given that the structure is in a specific damage state. Finally, the simplified methodology for state-dependent fragility derivation is illustrated via an application, whose results are then compared to those of a more rigorous procedure that involves sequential dynamic analysis. The article closes with some concluding remarks.

2. State-dependent structure-specific seismic fragility

A structure-specific seismic fragility function defines the conditional probability that, given a ground-shaking intensity measure (IM) is at a specific level (im), the structure fails to meet some performance objective. This failure is often termed exceedance of a *limit*- or *damage*-state and traditionally considers an intact structure that experiences a single seismic event. In the simplest of cases, fragility can be defined considering an appropriate measure of structural response, often termed an engineering demand parameter (EDP), and a threshold value thereof, edp_{DS}, whose exceedance is taken to signify transition of the structure from its initial state to the generic damage state DS, as expressed by Eq. (1):

$$P\left[DS|IM=im\right]=P\left[EDP>edp_{DS}|IM=im\right]. \quad (1)$$

One of the possible strategies for fragility assessment, via dynamic analysis of a structure's non-linear numerical model, is the so-called *IM-based* approach [18], which employs the results of IDA [3,4]. IDA scales a set of acceleration records to progressively higher im values, for which the numerical model provides the corresponding EDP responses. For every record used, the obtained EDPs can be plotted against the corresponding im level that the record had been scaled to – a graph which is usually designated as an IDA curve (Fig. 1).

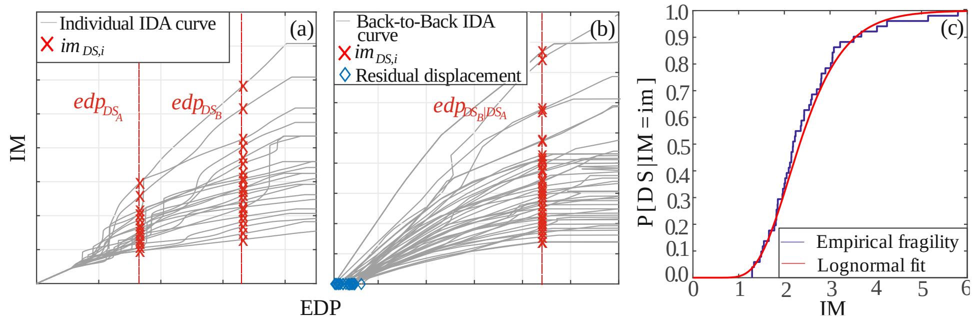

Fig. 1 – Example of IDA curves used for the evaluation of fragility curves for an intact structure (a); example of back-to-back IDA curves for the evaluation of state-dependent fragility curves (b); fragility curve estimation obtained by means of the IM-based approach (c).

The IM-based method entails finding the intersections of the IDA curves , im_{DS}, with the vertical line passing through the threshold edp_{DS} value (Fig. 1). These im_{DS} values can be regarded as realizations of a random variable (RV), IM_{DS}, which is the seismic intensity to which one needs to scale the ground motion in order for the structure to reach damage state DS. It is common practice to assume that IM_{DS} follows a lognormal distribution [13,19], in which case the fragility function can be estimated according to Eq. (2):

$$\begin{cases} P\left[DS \mid IM = im\right] = P\left[IM_{DS} \le im\right] = \Phi\left[\left(\ln(im) - \eta\right)/\beta\right] \\ \eta = \frac{1}{n} \cdot \sum_{i=1}^{n} \ln\left(im_{DS,i}\right) \\ \beta = \sqrt{\frac{1}{n-1} \cdot \sum_{i=1}^{n} \left[\ln\left(im_{DS,i}\right) - \eta\right]^2} \end{cases}, \quad (2)$$

where η and β are the parameter estimates (median and logarithmic standard deviation) of the assumed lognormal distribution of IM_{DS}, $im_{DS,i}$ is the realization of the RV coming from to the *i*-th record and $\Phi(\cdot)$ is the standard Gaussian (cumulative) function.

When seismic reliability calculations are expected to account for earthquake clusters, the need arises to evaluate the probability that an already-damaged structure transitions from one damage state, say DS_A, to another more severe one, DS_B, in one seismic event. A state-dependent fragility function will provide that probability, conditional on occurrence of a shaking intensity im during one of the shocks in the cluster, which can be expressed as $P\left[EDP > edp_{DS_B|DS_A} \middle| DS_A \cap IM = im\right]$. In this case, the notation $edp_{DS_B|DS_A}$ denotes the EDP threshold for DS_B when the structure is already found in DS_A and the state-dependent fragility can simply denoted as $P\left[DS_B \middle| DS_A \cap IM = im\right]$.

As already mentioned, one way of analytically estimating a state-dependent version of a fragility function, is my means of a variant of IDA, which is termed by some authors back-to-back IDA. In this type of dynamic analysis, the structural model is first subjected to a set of records hitting the structure at its intact (or initial) state, each scaled in amplitude to the lowest im value that results in $EDP = edp_{DS_A}$. At the end of each run, a different realization of the structure is produced, which can be considered to have made the transition to DS_A. Subsequently, each damaged incarnation of the structure is subjected to a second set of accelerograms representing a subsequent event of the same cluster. These records of the second set are scaled to progressively increasing im levels, similar to the traditional IDA procedure, until $EDP = edp_{DS_B|DS_A}$ is verified for the damaged structure, at an intensity of the shock which can be noted as $im_{DS_B,i}$ for the *i*-th succession of base accelerations. These intensity values can be used for the estimation of the parameters of a lognormal model for the state-dependent fragility, according to Eq. (2), in the same manner as in the case of traditional fragility. In Fig. 1b an example of back-to-back IDA curves is provided, where it can be seen that at zero intensity, the curves start from a residual EDP value that the damaged structure has inherited from the first event.

3. Simulating the static pushover of an earthquake-damaged structure

3.1 Predictive model for constant-ductility residual displacement ratio

As mentioned previously, this study introduces a further simplification in pushover-based state-dependent fragility assessment; i.e., apart from use on an equivalent SDOF substitute structure, in the form of analytical probabilistic definition of the possible pushover curves that characterize the structure that has been damaged by a previous shock. This can be achieved by random sampling of the parameters that define a set of pushover curves, which represent different realizations of the damaged system. In this case, the chosen parameters are the residual displacement δ_{res}, the relative period elongation ΔT, and the loss of lateral strength ΔR (to follow). The analytical arsenal for performing this simulation is provided by a predictive model for the constant-ductility residual displacement ratio, C_μ, developed by the same authors; a preliminary version of this model, limited to non-degrading systems, was presented in [17], while the complete model that includes cyclic strength degradation in the hysteresis is given in [16] but is also briefly outlined here, since the focus of the present article is to propose an application of said model.

The constant-ductility residual displacement ratio is defined as $C_\mu = |\delta_{res}/\delta_{\max}|$, that is the absolute value of the ratio of residual displacement δ_{res} to peak transient displacement $\delta_{\max}$, corresponding to a certain ductility μ . Relative period elongation is a measure of the loss of lateral stiffness of the structure during ground shaking and is defined as $\Delta T = (T' - T)/T$, where T' is the elongated post-shock period and T is the initial period of the SDOF structure. The elongated period is calculated as $T' = 2 \cdot \pi \cdot \sqrt{m/k'}$, where k' is the post-shock reloading stiffness (Fig. 2a). Finally, loss of lateral strength is defined as $\Delta R = (F_{\max} - F'_{\max})/F_{\max}$, where $F_{\max} = F_y \cdot [1 + \alpha_h \cdot (\mu - 1)]$ is the restoring force reached along the hardening branch of the initial backbone when pushed at ductility μ under static regime (i.e., in the absence of cyclic strength deterioration) , F_y and α_h are, respectively, the yield force and hardening slope of the intact structure and $F'_{\max}$ represents the restoring force that can be reached at the same ductility on the backbone of the damaged SDOF system, when it exhibits cyclic strength degradation. As shown in Fig. 2b, cyclic strength degradation entails a gradual offset of the force-displacement envelope towards the horizontal axis, due to progressive deterioration of structural elements. This is often modeled analytically by updating the backbone each time a hysteretic half-cycle is completed, with a reduction in resistance that is proportional to the dissipated energy [20], by a factor that can be calibrated to represent a certain range of structural behavior, in terms of susceptibility deterioration phenomena. In this case, four deterioration levels are considered, termed as no degradation and low-, medium-, high-degradation cases, with the first being representative of modern code-conforming structural elements and the last of structural elements with poor dissipative characteristics. In analytical terms, these lateral strength degradation levels are represented by the dummy variable $DL = \{0,1,2,3\}$, with $DL = 0$ corresponding to no degradation, $DL = 3$ to high-degradation level etc.

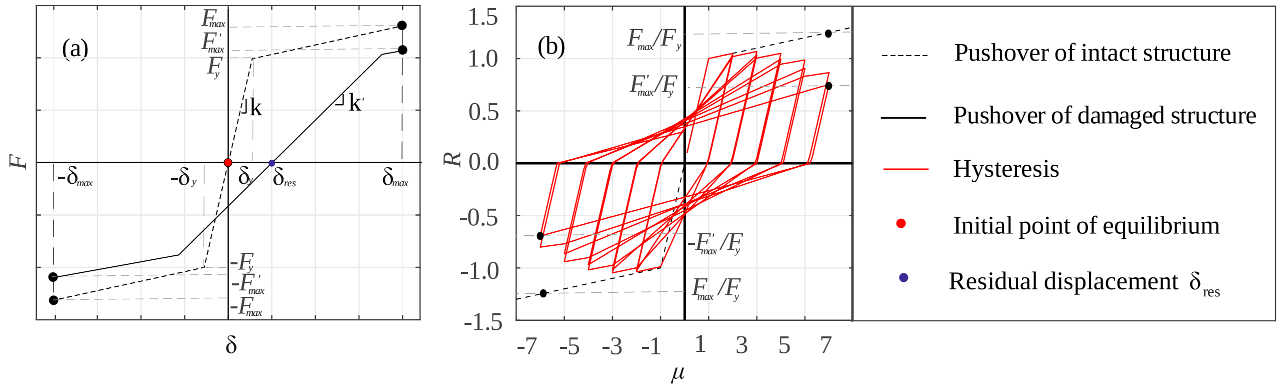

Fig. 2 - Examples of an SDOF structure's monotonic pushover (backbone) curve before and after incurring seismic damage (a); example of peak-oriented hysteresis at medium strength degradation level (b).

For a given ductility demand, that can represent the threshold edp_{DS} of some DS , the parameters C_μ , ΔT and ΔR are RVs whose joint distribution is provided by the predictive model given in [16,17], and which is reported herein as Eq. (3):

$$\begin{cases} C_\mu = |\delta_{res}/\delta_{\max}| = |\ \theta_1 \cdot \ln(\Delta T) + \beta_1 + \varepsilon_o \cdot \sigma_\delta\ | \\ \ln(\Delta T) = \theta_2 \cdot \ln(\mu - 1) + \beta_2 + \varepsilon_1 \\ \ln(\Delta R) = \theta_3 \cdot \ln(\mu - 1) + \beta_3 + \varepsilon_2 \end{cases} , \qquad (3)$$

where θ_i and β_i represent the slope and intercept of each linear regression model, $\mu = \delta_{\max}/\delta_y$ is the ductility demand, ε_o is a standard Gaussian variable and $\{\varepsilon_1, \varepsilon_2\}$ is a bivariate zero-mean Gaussian vector with covariance matrix provided by Eq. (4):

$$\begin{cases} \Sigma = \begin{bmatrix} \sigma^2_{\ln(\Delta T)} & \sigma_{\ln(\Delta T)} \cdot \sigma_{\ln(\Delta R)} \cdot \rho_{\ln(\Delta T),\ln(\Delta R)} \\ \sigma_{\ln(\Delta T)} \cdot \sigma_{\ln(\Delta R)} \cdot \rho_{\ln(\Delta T),\ln(\Delta R)} & \sigma^2_{\ln(\Delta R)} \end{bmatrix} \\ \rho_{\ln(\Delta T),\ln(\Delta R)} = e_1 + e_2 \cdot \left[\ln(\mu-1)\right]^2 + e_3 \cdot \left[\ln(\mu-1)\right]^3 + e_4 \cdot \left[\ln(\mu-1)\right] \cdot \alpha_h \end{cases} \quad (4)$$

where $\rho_{\ln(\Delta T),\ln(\Delta R)}$ is the correlation coefficient, which is modelled as a function of ductility demand and hardening slope, with $\{e_1, e_2, e_3, e_4\}$ being model coefficients. Additionally, σ_δ, $\sigma_{\ln(\Delta T)}$ and $\sigma_{\ln(\Delta R)}$ are the standard deviations of regression residuals, which, along with the regression parameters θ_i, β_i, are also modelled as functions of the variables T, μ, α_h and DL, which is expressed, in compact form, in Eq. (5):

$$\begin{cases} \theta_i = \sum_j b_{ij} \cdot f_{ij}\left[\alpha_h, \mu, \ln(\Delta R)\right] \\ \beta_i = \sum_j c_{ij} \cdot f_{ij}\left[\alpha_h, \mu, \ln(\Delta R)\right] \\ \left\{\sigma_\delta, \sigma_{\ln(\Delta T)}, \sigma_{\ln(\Delta R)}\right\} = \sum_j d_{ij} \cdot f_{ij}\left[\mu, T, \alpha_h, DL\right] \end{cases}, \quad (5)$$

where f_{ij} are functions of the variables in the brackets and $b_{ij}, c_{ij}, d_{ij}, e_j$ are model coefficients, with indices $i = \{1,2,3\}$ corresponding respectively to the functions/coefficients for $\{\delta_{res}/\delta_{\max}, \ln(\Delta T), \ln(\Delta R)\}$ as they appear in Eq. (3) and j counts the number of functional terms f_{ij} used in each part of the model. These functional forms and coefficient values can be found in [16,17], but are also available in the supplemental material to the present article (http://wpage.unina.it/georgios.baltzopoulos/papers/17WCEE_Esupp.pdf). This model was developed considering the modified Ibarra-Medina-Krawinkler (mIMK) hysteretic model [21] with peak-oriented response. The model's range of applicability is for vibration periods T between 0.3 s and 2.0 s, post-yield hardening ratios α_h ranging from 0 to 10% and ductility demands μ along the hardening branch between 1.5 and 9. Examples of the model are given in Fig. 3&4, where Fig. 3 shows the model for the expected value of $\delta_{res}/\delta_{\max}$ (denoted by the overbar) and the model of standard deviation σ_δ in presence of strength deterioration, while Fig. 4 shows a graph of logarithmic mean of period elongation, $\overline{\ln(\Delta T)}$, (Fig. 4a) and standard deviation $\sigma_{\ln(\Delta T)}$ in case of high DL (Fig. 4b). On the other hand, Fig. 4c and Fig. 4d show the models for central tendency and standard deviation of $\ln(\Delta R)$ in the case of $DL = 1$ (low stregth degradation level).

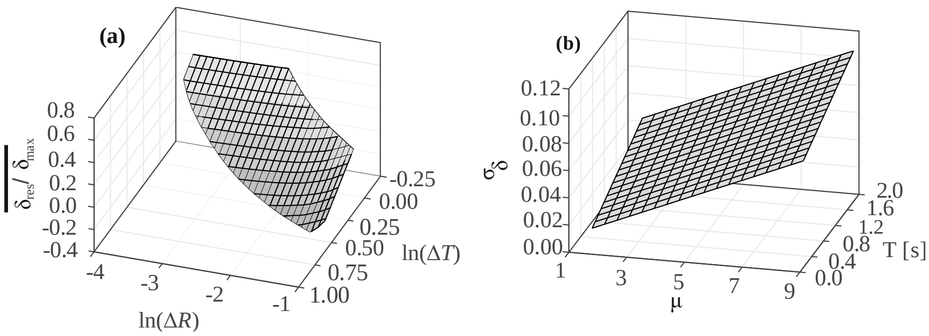

Fig. 3 - Central tendency and standard deviation of the model for the residual displacements in case of strength deterioration.

3.2 Simulation of the damaged structure's backbone curve

A single realization of the pushover curve characterizing a damaged structural system, can be obtained by random sampling triplets of values for elongated period, strength degradation and residual displacement from their joint distribution, given ductility demand. In this context, the conditioning ductility demand μ represents the threshold edp_{DS} that defines transition to the damage state under consideration. At this point, it should be noted that Eq. (3) implies that, given a certain ductility demand, the random vector $\{\ln(\Delta T), \ln(\Delta R)\}$ is conditionally independent of the residual displacement and follows a bivariate normal distribution. This means that a sample $\{x, y, z\}$, of the random vector $\{\ln(\Delta T), \ln(\Delta R), \delta_{res}/\delta_{\max}\}$, can be obtained by the following procedure: first, given μ defining the DS, the level of strength deterioration and the characteristics of the initial structure T and α_h, a random value of $\ln(\Delta T) = x$ is extracted from a Gaussian distribution with mean $\overline{\ln(\Delta T)}$ and standard deviation $\sigma_{\ln(\Delta T)}$ given by Eqs.(3) and (5), respectively. Subsequently, a value of $\ln(\Delta R) = y$ is randomly sampled from the conditional distribution of $\ln(\Delta R)$ given $\ln(\Delta T) = x$, which is also a Gaussian with mean and standard deviation given by Eq. (6):

$$\begin{cases} \mathrm{E}\left[\ln(\Delta R) \middle| \ln(\Delta T) = x\right] = \overline{\ln(\Delta R)} + \rho_{\ln(\Delta T),\ln(\Delta R)} \cdot \left(\sigma_{\ln(\Delta R)} / \sigma_{\ln(\Delta T)}\right) \cdot \left[x - \overline{\ln(\Delta T)}\right] \\ \sigma_{\ln(\Delta R)|\ln(\Delta T)} = \sqrt{\left(1 - \rho^2_{\ln(\Delta T),\ln(\Delta R)}\right) \cdot \sigma^2_{\ln(\Delta R)}} \end{cases}, \quad (6)$$

where $\mathrm{E}\left[\ln(\Delta R) \middle| \ln(\Delta T) = x\right]$ represents the conditional mean of $\ln(\Delta R)$, $\sigma_{\ln(\Delta R)|\ln(\Delta T)}$ its conditional standard deviation, $\overline{\ln(\Delta R)}$ the marginal mean from Eq. (3) and $\rho_{\ln(\Delta T),\ln(\Delta R)}$ is from Eq. (4). Finally, by substituting $\{\ln(\Delta T), \ln(\Delta R)\} = \{x, y\}$ into Eqs. (3),(5) and thus evaluating the conditional mean and standard deviation of the ratio $\delta_{res}/\delta_{\max}$, a value of $\delta_{res}/\delta_{\max} = z$ is randomly sampled from the corresponding normal distribution, which is, however, truncated between -1 and 1 in order to respect the physical constraints of the problem.

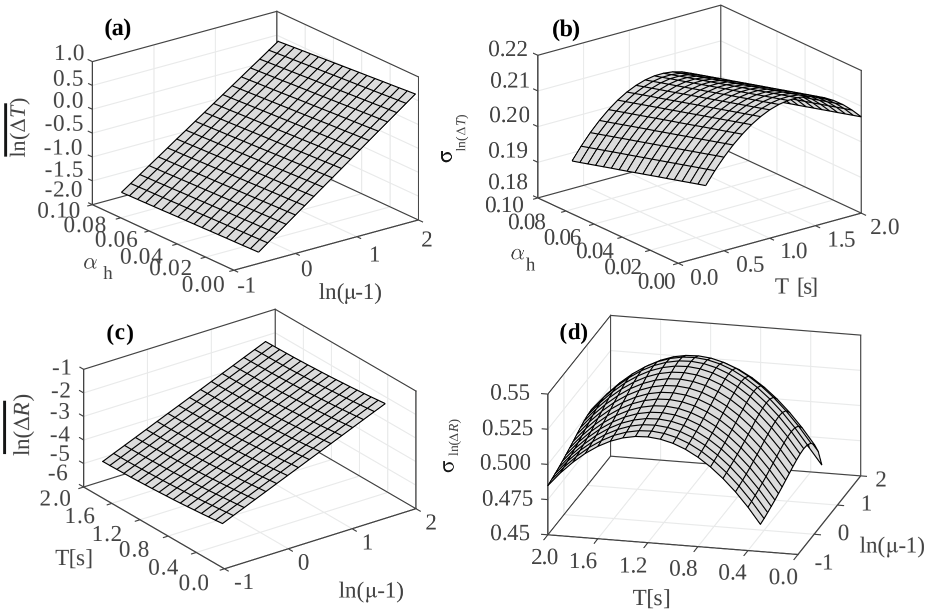

Fig. 4 - Model for the central tendency of period elongation (a) and model of standard deviation $\sigma_{\ln(\Delta T)}$ (b) in case of high strength deterioration; model for the central tendency of $\ln(\Delta R)$ in case of $\alpha_h = 0.01$ (c) and model of standard deviation $\sigma_{\ln(\Delta R)}$ (d) in case of low strength degradation level.

From this random sample of $\{\ln(\Delta T) = x, \ln(\Delta R) = y, \delta_{res}/\delta_{\max} = z\}$, it is straightforward to obtain the corresponding residual displacement δ_{res}, elongated period T' and $F'_{\max}$, all of which were defined previously. From this triplet of parameters, it is then possible to univocally define the realization of the damaged system's pushover curve by means of mechanical and geometric considerations. This is illustrated in

Fig. 5, where the coordinates of the points defining the initial and post-shock curve are given in the displacement-force plane. In the figure, the notation with primes represents the value of the corresponding parameter in the damaged system and the signed subscripts indicate the direction; e.g., δ'_{y+} and δ'_{y-} denote yield displacements of the post-shock backbone in the positive and negative direction, respectively. In fact, the elastic branch of the damaged system's pushover can be determined by evaluating the yield force and displacement in the both directions as reported in Eq. (7):

$$\begin{cases} F'_{y\pm} = \left[F'_{max} - F'_{max}\cdot\dfrac{\alpha_h}{\delta_y}\cdot\dfrac{F_y}{F_{max}}\cdot\left(\mu\cdot\delta_y \mp \delta_{res}\right)\right] \Big/ \left[1-\dfrac{F'_{max}}{F_{max}}\cdot\alpha_h\cdot\left(\dfrac{T'}{T}\right)^2\right] \\ \delta'_{y\pm} = \delta_y\cdot\dfrac{F'_{y\pm}}{F_y}\cdot\left(\dfrac{T'}{T}\right)^2 \pm\delta_{res} \end{cases} \tag{7}$$

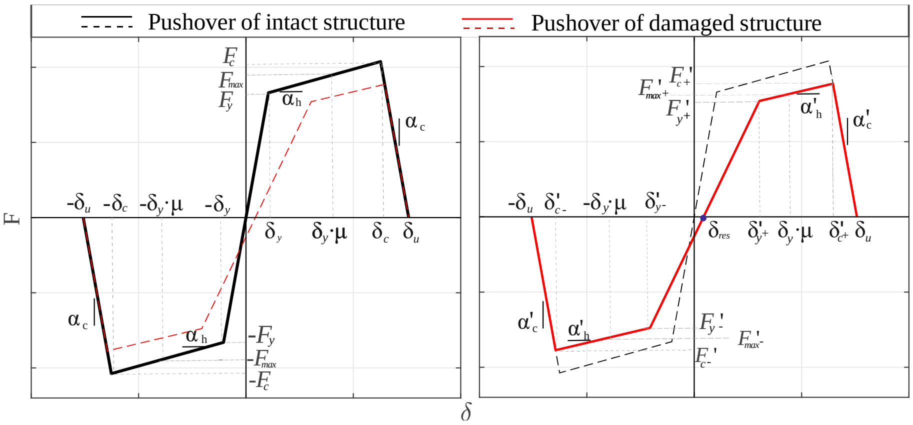

Fig. 5 – Parameters defining the pushover curves; parameters for the definition of the intact structure's pushover curve (a); and the damaged structure's pushover curve (b).

On the other hand, the degradation of the hardening branch's slope due to cyclic strength deterioration mode, which is implicit in the hysteretic model of [21], is given by Eq. (8):

$$\alpha'_h = \alpha_h\cdot\frac{F'_{max}}{F_{max}}\cdot\left(\frac{F_y}{F'_{y\pm}}\cdot\frac{\delta'_{y\pm}\mp\delta_{res}}{\delta_y}\right). \tag{8}$$

Apart from α'_h , the other parameters, which are needed to define the post-yield branch, are the capping point displacements in the two directions, $\delta'_{c\pm}$ and corresponding forces $F'_{c\pm}$. These can be calculated as the intersection points of the damaged structure's hardening branch of the and softening branch, whose slope is assumed to remain invariant, according to Eq. (9):

$$\begin{cases} \delta'_{c\pm} = \left(F'_{y\pm} - \dfrac{F'_{y\pm}\cdot\alpha'_h\cdot\delta'_{y\pm}}{\delta'_{y\pm}\mp\delta_{res}} + \dfrac{F_c}{\delta_c-\delta_u}\delta_u\right) \Big/ \left(\dfrac{F_c}{\delta_c-\delta_u} - \dfrac{F'_{y\pm}}{\delta'_{y\pm}\mp\delta_{res}}\cdot\alpha'_h\right). \\ F'_{c\pm} = \left[F_c\cdot\left(\delta'_{c\pm}-\delta_u\right)\right]/\left(\delta_c-\delta_u\right) \end{cases} \tag{9}$$

4. Simplified evaluation of state-dependent fragility curves

The SDOF structure's backbone curves, sampled using the predictive model for C_μ, can be assumed to represent the pushover curves that correspond to different realizations of the structure, when that structure has transitioned to a certain damage state DS_A due to one shock within an earthquake cluster. Each realization has an asymmetric backbone due to the residual displacement δ_{res}, exhibiting elongated period T' and lateral resistance at yield and capping points in the two directions, $F'_{y^{\pm}}$ and $F'_{c^{\pm}}$.

In order to estimate the state-dependent fragility of the damaged structure, which is already at DS_A, each SDOF realization from Monte-Carlo simulation is subjected to incremental dynamic analysis, performed using a single record, randomly selected from a pool of available ground motions meant to simulate ground shaking due to a subsequent shock of the same cluster. The use of a single record per realization of the structure has been used before in the past, in the context of accounting for model uncertainty in seismic risk analysis [22]. The records used in this phase are scaled to increasing im levels until the structural response of each realization reaches the threshold $edp_{DS_B|DS_A}$ defining the transition from damage state DS_A to DS_B. The final result of this procedure is a set of IDA curves, which constitute a more expedient substitute of the back-to-back IDA curves, and that can be used to evaluate the state-dependent fragility via the IM-based approach. In other words, the intensity values $im_{DS_B,i}$, causing the *i*-th simulated realization of the damaged system to reach $edp_{DS_B|DS_A}$, can be used for the estimation of the parameters defining a lognormal model for the state-dependent fragility, according to Eq(2). This simplified procedure for state-dependent seismic fragility estimation is showcased by means of an illustrative application, which follows.

4.1 Illustrative application

For this application, a simple yielding SDOF structure is considered; although in pushover-based based methods the SDOF system is proxy for the actual structure, which introduces additional sources of approximation, this illustrative example directly considers an SDOF system, so as to isolate the consequences of the proposed procedure from effects stemming from the multi- to single-DOF substitution. The vibration period of the SDOF structure is $T = 1.0\,s$, the hysteresis is assumed non-degrading and its symmetric backbone curve is defined by the parameters reported in Table 1.

Table 1 – Parameters defining the backbone curve of the intact structure used in the example.

T [s]	F_y [kN]	δ_y [m]	α_h	δ_c [m]	δ_u [m]	α_c
1.0	1000	0.11	0.01	0.99	1.105	-1

In order to showcase the simplified procedure, two generic damage states are arbitrarily defined, denoted as DS_A and DS_B, with the latter being the most severe of the two. Transition of the intact structure to damage state DS_A is considered to occur when the structural response of the system exceeds the threshold edp_{DS_A} defined by a seismic ductility demand μ, equal to three, which corresponds to $\delta_{\max} = 0.33$ m in this case. Along the same lines, it is considered that the direct transition of the intact structure into DS_B, occurs when μ exceeds the value of six, or $\delta_{\max} = 0.66$ m. It should be noted that the threshold EDP values, edp_{DS_A} and edp_{DS_B}, considered for the *direct* transition of the intact system into one of these two generic damage states; i.e., when the transition from intact to each DS is due to a single earthquake shock, are defined solely on the basis of transient maximum inelastic displacement. Although the exact value of the threshold displacement should take into account the nature of the DS and structural typology, the practice of using displacement demand alone to mark the exceedance of limit states on the pushover of the intact structure, is common in earthquake engineering; e.g., [23].

On the other hand, ductility demand alone may not carry enough information about structural response and damage accumulation to also serve as the threshold EDP defining the transition of the already-damaged structure from DS_A to DS_B. Although this is an open issue, for the sake of this illustrative example, it will be considered that the DS_A to DS_B transition occurs whenever ductility demand due to the second shock exceeds six or when the system exhibits a cumulative reduction in elastic stiffness, Δk, greater than 70%. This can be expressed analytically by introducing as EDP a demand-over- capacity (D/C) ratio according to Eq. (10):

$$D/C = \max\left\{\frac{\delta_{\max}}{0.66}, \frac{\Delta k}{0.70}\right\}, \tag{10}$$

where $\Delta k = 1 - k'/k$ represents the normalized loss of lateral stiffness and $edp_{DS_B|DS_A}$ is simply unity.

For this application, the conditioning value of $\mu = 3$, i.e., edp_{DS_A}, is used to simulate a set of one-hundred backbone curves, according to the sampling procedure previously described. These backbones represent one hundred possible realizations of the pushover of the structure having reached damage state DS_A. An example of the backbone curves, thus extracted from the predictive model, is given in Fig. 6a, where it can be seen that, in the absence of strength degradation, they differ among themselves only in residual displacement and elastic stiffness. Subsequently, each realization of the damaged system is subjected to IDA, using one record per extracted pushover, which is scaled upwards until the transition from damage state DS_A to DS_B occurs, defined by $edp_{DS_B|DS_A} = 1$. The IM considered during IDA is the spectral acceleration at the period of the intact structure, $Sa(T)$. Fig. 6b compares the backbone curve of the intact structure with one realization of the system damaged by the first shock and shows the hysteretic behavior of the damaged system following the application of a record representing the second shock. These analyses were run using the OPENSees finite-element platform [24], where a custom-made version of the mIMK hysteretic model was implemented, which also allows for user-defined unloading stiffness.

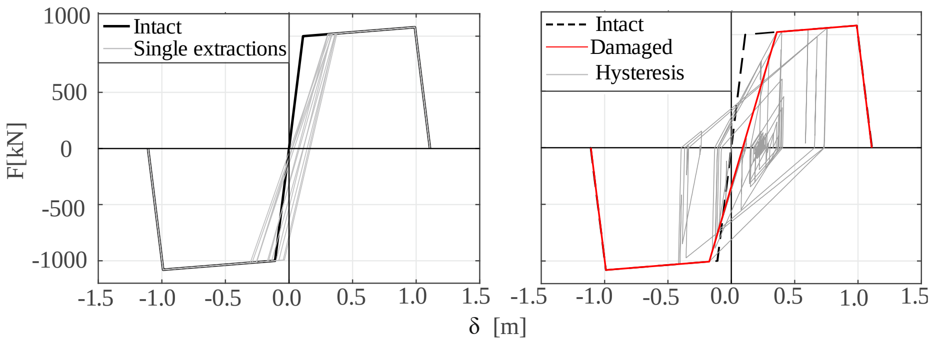

Fig. 6 Examples of backbone curves representing different realizations of the damaged system (a); comparison of the intact system and a single realization of the post-shock damaged structure with the hysteresis obtained applying a subsequent-shock record to the damaged system (b).

The IDA curves obtained in this manner are shown in Fig. 7a, where the im_{DS_B} points, obtained from their intersection with the $D/C = 1$ line, are shown as red crosses. At this point, these im_{DS_B} values can be used to estimate the parameters of a lognormal model for the state-dependent fragility $P[DS_B | DS_A \cap IM = im]$, according to Eq. (2).

In order to obtain some points of reference for comparing the results of this procedure, the same state-dependent fragility was estimated by means of back-to-back IDA, using a set of twenty records to represent the first damaging shock of the cluster, scaled so as to cause a ductility demand of three, and another five subsequent-shock accelerograms per initial shock, for a total of one-hundred curves. Additionally, a twenty-record IDA was used to estimate the scaled intensity causing a ductility demand of six, that is, incurring the condition $EDP > edp_{DS_B}$, for the intact structure; the latter analysis was therefore used for the derivation of the intact structure's traditional fragility, $P[DS_B | IM = im]$. These runs were performed using an OPENSees user interface developed to streamline the back-to-back IDA [11]. The resulting logarithmic means and standard deviations of the intensities causing transition to DS_B, obtained for the three cases of analysis, are reported in Table 2; the corresponding cumulative probability functions are given in Fig. 7b. The comparison shows that the simplified procedure provided a state-dependent fragility estimate which is similar to the one coming from the more rigorous back-to-back IDA. On a side-note, the fragility function of the intact structure serves as a reference, showcasing the characteristic shift-to-the-left of the state-dependent curves, due to the drop in median capacity caused by the transition to DS_A due to the damage induced by the first shock.

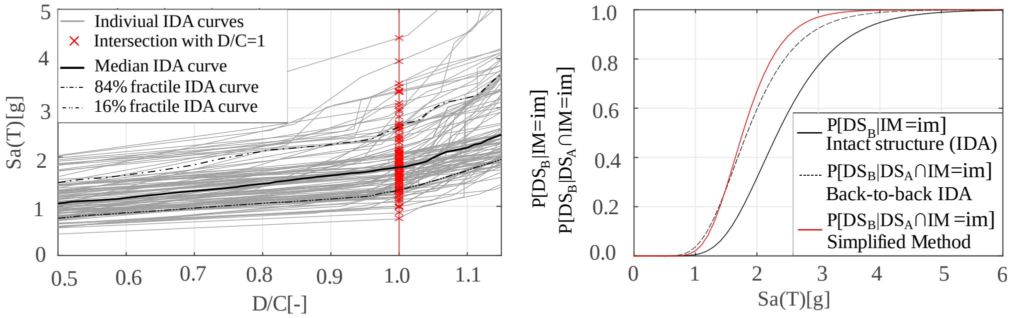

Fig. 7 – IDA curves obtained from the application of the simplified methodology (a); state-dependent fragility curves evaluated with the simplified methodology and the back-to-back IDA approach and fragility curve of intact structure evaluated at the DS_B damage state by means of IDA (b).

Table 2 - Logarithmic mean and standard deviation defining the fragility curves.

State-dependent fragility via simplified method		**State-dependent fragility via back-to-back IDA**		**Fragility for the intact structure via IDA**	
η	β	η	β	η	β
0.57	0.28	0.61	0.34	0.85	0.33

5. Conclusions

The main objective of this article was to present a preliminary version of a simplified pushover-based procedure aimed at the estimation of state-dependent seismic fragility curves. The proposed methodology uses a semi-empirical predictive model for constant-ductility displacement ratios to obtain, through Monte-Carlo simulation, a set of realizations of the damaged structure's pushover curve. The usefulness of this shortcut lies in the fact that, due to the record-to-record variability of structural response to strong earthquakes, a structure subjected to a single instance of base-acceleration may fall under a generic damage state while exhibiting different permutations of basic dynamic properties, such as resistance to inertial load, stiffness and residual displacement. Such variability is typically accounted for via sequential runs to accelerogram couples that represent the alternation of two damaging shocks within an earthquake cluster, as in the case of back-to-back incremental dynamic analysis. In the simplified proposal, the first part of the sequential analysis is avoided, replaced by simulation of the principal characteristics of the equivalent SDOF system at a given damage state. The illustrative application presented as part of this article, shows that the proposed methodology can represent a viable alternative to the more computationally intensive procedures, at least for regular structures for whom pushover-based procedures are a viable approximation.

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