
On the fractal nature of volcano morphology detected via SAR image analysis: the case of Somma–Vesuvius Volcanic Complex

Gerardo Di Martino, Antonio Iodice, Daniele Riccio, Giuseppe Ruello and Ivana Zinno*

Dipartimento di Ingegneria Biomedica, Elettronica e delle Telecomunicazioni,
Università degli Studi di Napoli Federico II, Via Claudio 21, 80125 Napoli

*Corresponding author, e-mail address: ivana.zinno@unina.it.

Abstract

In this paper an innovative technique for the extraction of natural surfaces geomorphologic parameters from SAR data is presented and applied for volcano monitoring purposes. The observed surface is modeled as a fractal two-dimensional stochastic process. A theoretical framework for the analysis of the SAR imaging process is outlined. An algorithm founded on this imaging model is presented, allowing the retrieving of the point by point fractal dimension of the imaged surface. Significant results regarding the application of the proposed technique to the case study of the Somma-Vesuvius volcanic complex are shown, along with preliminary comments regarding the comparison with ground truth maps of the surface fractal dimension.

Keywords: Fractals, Synthetic Aperture Radar.

Introduction

The analysis of Synthetic Aperture Radar (SAR) images of natural surfaces is becoming more and more relevant thanks to the development of remote sensing systems in the last few years. The new generation of sensors marked a huge increase in the resolution of microwave images of the Earth. TerraSAR-X and COSMO-SkyMed are providing SAR data with the remarkable resolution of $1 \times 1 \text{ m}^2$ in the high resolution spotlight operational mode. Owing to this development of remote sensing systems, new possibilities arise with regard to the extraction of geomorphologic surface parameters from SAR images. In particular, as for natural surfaces, until now it was only possible to identify macroscopic topological features (mountains, rivers, seas, etc.) of the observed areas, roughly distinguishing them from urban ones: with the new generation sensors the extraction of meaningful stochastic parameters of the observed surface at microscopic level is now in order. In this paper we present an innovative technique for the analysis and interpretation of SAR images of natural areas. The observed natural surface is modeled as one realization of a fractal two-dimensional stochastic process [Mandelbrot, 1983; Falconer, 1990]: for such a process, the key parameter to be estimated from the image is the fractal dimension D of the imaged surface. In fact, this parameter is strictly related to the roughness and geomorphologic characteristics of the surface and its knowledge can be of key importance for a wide range of applications as the prevention and monitoring of environmental disasters, land classification, the rural and

urban planning and so on [Turcotte, 1997], [Peitgen and Saupe, 1988]. In the open literature, fractal models and tools are gradually spreading throughout the SAR community. A pioneering work on such a type of modeling was afforded for the optical case in [Kube and Pentland, 1988]. In [Di Martino et al., 2008] some of the authors proposed a change detection technique based on the variogram analysis applied to simulated SAR images. In this case, however, the SAR image parameters retrieving was performed in an heuristic way. From that work the need to develop an analytical model to link the fractal surface parameters directly to the SAR image characteristics arose. The modeling of the radar imaging of a topologically one dimensional (1-D) fractal profile was proposed in [Di Martino et al., 2010] and its extension to the two-dimensional case was outlined in [Di Martino et al., 2009; Di Martino et al., 2010a]. It is worth stressing that extension from the 1-D to the 2-D case is definitively not trivial: as a matter of fact, SAR images - due to the particular side-looking radar acquisition system - exhibit an intrinsic asymmetry in the range and azimuth directions. In this paper we present a novel technique for the retrieval of the fractal dimension D of the observed surface based on an appropriate spatial filtering of the amplitude SAR image, whose theoretical rationale was presented by the authors in [Di Martino et al., 2010a]. Moreover, the application of such a type of processing to an actual SAR image of a volcanic complex is presented for the first time; the same type of fractal analysis is performed on the DEM (Digital Elevation Model) of the area of interest and the fractal dimension maps relevant to the SAR image and to the DEM are compared. This type of analysis is here performed for the first time on actual data.

In the following sections we present the theoretical framework and the methodological setup of the proposed approach. Furthermore, a completely automatic SAR image processing aimed to the extraction of the point by point fractal dimension of the observed scene is detailed. The retrieving algorithm is applied to a TerraSAR-X image of the Somma - Vesuvius volcanic complex and a map of the fractal dimension of the observed scene is obtained. Moreover, in the final section of the paper a comparison with the results obtained performing the same type of fractal analysis on the ground truth (DEM) data relevant to the area of interest is presented for the first time. The performed analysis, besides providing geomorphologic information useful for the study of geodynamic phenomena, shows the reliability of such a type of fractal processing that could be used also to verify and improve the accuracy of Digital Elevation Model generation procedures.

Theoretical Framework

It is widely recognized that fractal models represent the best way to describe the irregularity of natural scenes [Mandelbrot, 1983; Franceschetti and Riccio, 2007]. Among this kind of models, we choose the regular stochastic fBm (fractional Brownian motion) process that completely describes natural surfaces by means of two independent parameters: the Hurst coefficient, H (which is linked to the fractal dimension by the simple relation $D=3-H$) and the standard deviation of surface increments at unitary distance, s [m^{1-H}]. The process $z(x, y)$ is an fBm if, for every x, y, x', y' , it satisfies the following relationship:

$$Pr\{z(x, y) - z(x', y') < \bar{\zeta}\} = \frac{1}{\sqrt{2\pi s \tau^H}} \int_{-\infty}^{\bar{\zeta}} \exp\left(-\frac{\zeta^2}{2s^2 \tau^{2H}}\right) d\zeta \quad [1]$$

$$\tau = \sqrt{(x - x')^2 + (y - y')^2} \quad [2]$$

The power density spectrum of the isotropic two dimensional fBm process exhibits an appropriate power-law behavior [Mandelbrot, 1983; Falconer, 1990; Franceschetti and Riccio, 2007]:

$$S(k) = S_0 k^{-\alpha} \quad [3]$$

where in S_0 and α are functions of the fractal parameters.

In order to retrieve the fractal dimension of a natural scene from its SAR image we need a model which describes the relation between the surface and its final amplitude image. This kind of imaging model was presented by the authors in [Di Martino et al. 2009]. The proposed model is based on the assumption of a small slope regime for the observed surface: if this is the case, the image intensity comes out to be a linear function of the partial derivative of the surface evaluated along the range direction.

The expressions of the autocorrelation functions of the SAR image and of the Power Spectral Densities (PSDs) of two cuts of the image in the range and azimuth directions respectively, have been evaluated by the authors in [Di Martino et al., 2009]. The PSDs of the azimuth and of the range cut of the SAR image show very different behaviors, thus highlighting an intrinsic asymmetry in the structure of SAR data, which is also intuitively referable to the particular acquisition geometry typical of a side looking mono-static radar. In particular, the spectrum of the image range cut, in an appropriate range of sufficiently low spatial frequencies, presents a power law behavior - thus showing on a log - log plane a linear behavior with a slope related to the Hurst coefficient H of the observed surface. In fact, the expression of the PSD of the range cut of a SAR image, for adequately low wavenumbers, turns out to be [Di Martino et al., 2009]:

$$S_0(k_y) = s^2 \Gamma(1 + 2H) \sin(\pi H) \frac{1}{|k_y|^{2H-1}} \quad [4]$$

where k_y is the wavenumber of the range cut of the image and $\Gamma(\cdot)$ is the Euler Gamma function. Note that the expression in Equation 4 is in accordance with the PSD obtained in [Di Martino et al., 2010] for the image of a one-dimensional fractal profile, where analogous hypothesis were assumed about the surface slopes and the considered range of spatial frequencies.

Comparing Equation 4 with the expression of the PSD of the surface in Equation 3, it can be inferred that the slope of the spectrum relevant to a range cut of a SAR image is equal to that of the imaged surface, assuming that the Hurst coefficient is decreased by one.

Methodological Setup

In order to retrieve the fractal parameters starting from a SAR image, we can perform the analytical inversion of the presented theoretical model.

In particular, starting from Equation 4, it is possible to implement linear regression algorithms on the spectrum of range cuts of the image in a log - log plane, thus retrieving the fractal dimension, according to the scheme presented in Figure 1.

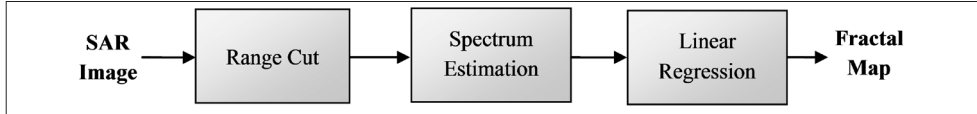


Figure 1 - Block diagram of the extraction of the fractal dimension.

In order to implement the proposed procedure, we developed a software that, by means of a sliding window spanning the entire image, provides the corresponding fractal map, i.e. a matrix of the point by point fractal dimension relevant to the observed surface.

The implemented algorithm extracts the local fractal dimension of the imaged surface working on homogenous patches of the SAR image and iterating the procedure on the whole image, through a moving window, whose dimension can be set by the user according to its specific needs, resulting from a trade-off between accuracy and resolution of the output fractal dimension map. In particular, the algorithm selects in each window a fixed number of range cuts that are sufficiently spaced from each other to be considered uncorrelated. Then the spectra of these cuts (whose number can be again chosen by the user, as a trade-off between accuracy and computation time) are evaluated using a Capon estimator [Austin T. et al., 1994]. Finally, these spectra are averaged and a linear regression is performed on this mean PSD. The question of the spectrum estimation is not a trivial one: as a matter of fact, power-law spectra introduce unique difficulties in the spectral estimation as they suffer leakage effects and high variance problems, yielding a spectral estimate which can deeply modify the original spectral slope. The Capon estimator strongly reduces the above-mentioned negative effects and is particularly well suited when facing short data records: this characteristic is particularly important in our case, in which the number of samples for the spectral estimation is limited by the sliding window dimension. The goal is to reduce as much as possible the sliding window dimension in order to improve the resolution of the final map, preserving at the same time the accuracy of the estimation. To this aim, an experimental analysis to find the best values for the Capon estimator parameters (filter length and range of frequencies to use) has been performed. For a sliding window shortened up to 35x35 pixel the best Capon filtering length is found to be equal to $0.3*N$ (where N is the number of considered samples which in this case is equal to 35) and the frequency cut is done at the Nyquist frequency [Di Martino et al., 2011].

The final result of the proposed elaboration is a map of the fractal dimension of the observed scene: the resolution of this map depends on both the resolution of the input image (the higher the resolution of the image, the better the resolution of the map) and the dimension of the estimation window.

Finally, some comments about the speckle phenomenon, which is responsible for the well-known *salt and pepper* effect on SAR amplitude images, are in order. As a matter of fact, for the fractal case of interest the spatial scales involved by the speckle are mainly those of the order of the sensor resolution, hence in the wavenumber domain the high frequency range of the image spectrum is degraded. In fact, looking to the PSD in Equation 4 it is easily

recognized that in this region of the spectrum the power of the signal is significantly lower than in the low frequency region. However, our algorithm performs the linear regression in a range of spatial frequencies in which the spectrum is not significantly affected by the speckle.

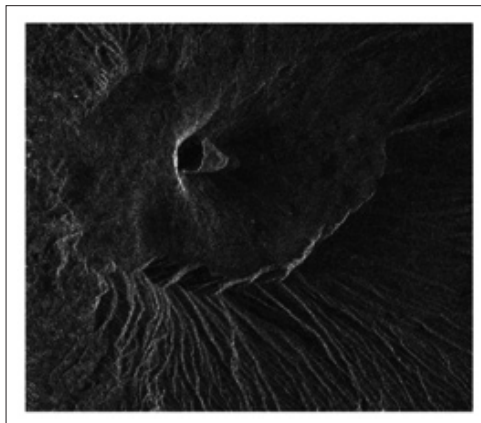
By means of such a type of filtering different applications can be carried out. In the next section, an interesting study case relevant to the Somma- Vesuvius volcanic complex area is shown. In fact, concerning this natural scenario, both a TerraSAR image and a Digital Elevation Model (DEM) of the Somma - Vesuvius volcanic complex are available. Therefore, it is possible to apply to the SAR image the described algorithm in order to retrieve the fractal map of the area and compare it to the fractal map obtained from the DEM.

The case study: the Somma - Vesuvius volcanic complex area

In this section the results of the application of the processing described in the previous section to a SAR image of the Somma - Vesuvius volcanic complex (close to Naples in Italy) are presented, together with the comparison with the relevant ground truth. This second task is not a trivial one: as a matter of fact, the geometrical differences (resolution and scene orientation) between the SAR image and the DEM of the area of interest should be taken into account. Furthermore, the presence of artifacts in the DEM, probably due to the particular interpolation technique used for its generation, significantly complicates the analysis.

Application to the TerraSAR Image of Somma - Vesuvius volcanic complex

In order to show the potentialities of the innovative SAR image electromagnetic and fractal based post-processing proposed in this paper, in the following paragraph the algorithm is applied to a TerraSAR image of the Somma - Vesuvius volcanic complex area.



**Figure 2 - TerraSAR Image of the Somma-
Vesuvius volcanic complex.**

The characteristics of the starting stripmap SAR image (Fig. 2) are the following: dimensions 3251x2820, resolution 3m x 3m, VV polarization. As said in the previous section, the choice

of the sliding window dimension used for the spectral estimation arises from a trade-off between the resolution of the final map, the accuracy of the fractal dimension (D) estimation and the computational complexity. In this case, being the resolution of the TerraSAR image high, the sliding window dimension is set equal to 51×51 pixel, i.e. not very small, in order to guarantee the accuracy of the estimation of D .

Some considerations about the obtained fractal map (Fig. 3) are in order.

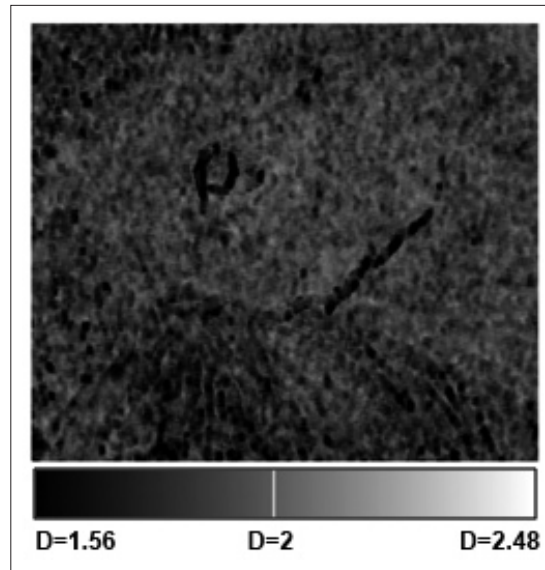


Figure 3 - Fractal Map relevant to the Somma-Vesuvius volcanic complex.

In Figure 3 the fractal map relevant to the SAR image of the Somma - Vesuvius volcanic complex (Fig. 2) is shown. It contains the point by point estimated D values. The fractal map present a range of values of the fractal dimension equal to $1.56 < D < 2.48$ and a mean value equal to $D_{mean} = 2.2$ with a standard deviation equal to 0.08. As seen in the first section, a fractal object has a fractal dimension D : $2 < D < 3$, but natural surfaces usually show a persistent behavior that is $2 < D < 2.5$ [Franceschetti and Riccio, 2007]. Therefore, the estimation of D is consistent with the theoretical assumptions. Concerning the inferior limit, in some areas of the fractal map D presents values smaller than 2 (in the grey level palette of Figure 3 the grey levels associated from the minimum D value to the maximum D value are reported and a white line is set in correspondence to $D=2$). This can be explained considering that, as we can see in Figure 2, some layover effects are present in the TerraSAR image. Obviously, this phenomenon generates non-fractal features on the amplitude image, and, accordingly, in these zones the algorithm recognizes non fractal areas. Moreover, at the left upper corner of the fractal map, also some buildings, that in the SAR image appear as brilliant points, are identifiable as dark spots on the fractal map. In other cases, the areas where the fractal dimension D is less or equal to 2 could be also interpreted as the surface

signature of particular tectonic processes as faulting or caldera structural formation. So, the occurrence of different fractal dimensions could be used as an indicator in order to discriminate the occurrence of different geodynamic processes during the natural evolution of a volcanic complex.

Comparison with the ground truth

In order to compare the fractal map obtained from the TerraSAR image with the ground truth relevant to the scene under survey, a Digital Elevation Model (DEM) of the Somma - Vesuvius volcanic complex area is used.

The DEM, obtained through aerophotogrammetry, has a resolution of 5m x 5m and is a mosaic of 4 pieces with dimensions 711 x 564 pixel.

The first task consists in matching the TerraSAR image and the DEM. To this end, the TerraSAR image (already cut out so that the covered geographic area is the same) must be rotated and resampled. The rotation is deduced from the geographic coordinates, while for the sampling is used the nearest neighbor method, as other interpolation algorithms could significantly invalidate the fractal features of the image. Finally, the modified SAR image has the same resolution of the DEM. To this one, our post-processing is applied first with the same statements of the previous case (same sliding window dimensions and same filtering) in order to compare the results and then with a smaller sliding window in order to obtain a finer resolution for the fractal map.

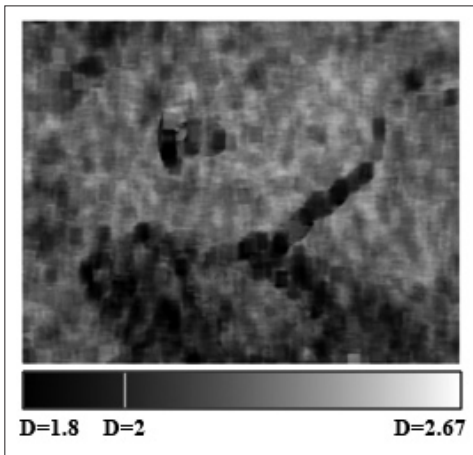


Figure 4 - Fractal map relevant to the resampled SAR image of Somma-Vesuvius volcanic complex with a sliding windows of 51x51 pixel.

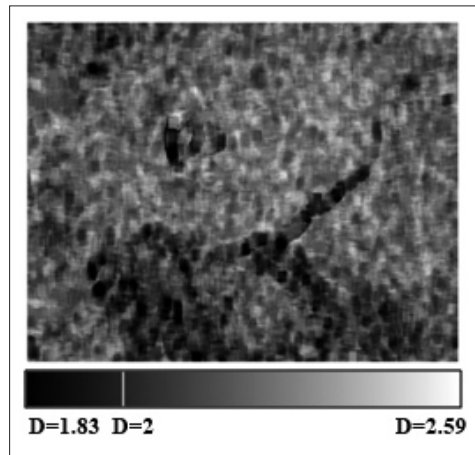


Figure 5 - Fractal map relevant to the resampled of Somma-Vesuvius volcanic complex with a sliding windows of 35x35 pixel.

The fractal statistics of the map in Figure 4 are: $1.8 < D < 2.67$, $D_{mean} = 2.3$, standard deviation equal to 0.1. Comparing the last ones with those of the previous case, it can be deduced that a small variation of the fractal features of the SAR image is present, because of the geometrical transformation carried out. In Figure 5 the fractal map obtained using a smaller sliding window (35x35 pixels) is shown. As the resolution is better and the fractal statistics

remain essentially the same ($1.83 < D < 2.59$, $D_{mean} = 2.3$, *standard deviation* equal to 0.1), leading us to use this last dimension of the sliding window for the comparison with the ground truth.

The presented fractal dimension estimation algorithm is applied to the DEM (Fig. 6), taking into account that the regression in this case is done on the spectrum in Equation 3. Hereafter, the obtained fractal map of the DEM is presented together with some considerations.



Figure 6 - DEM of the Somma-Vesuvius volcanic complex.

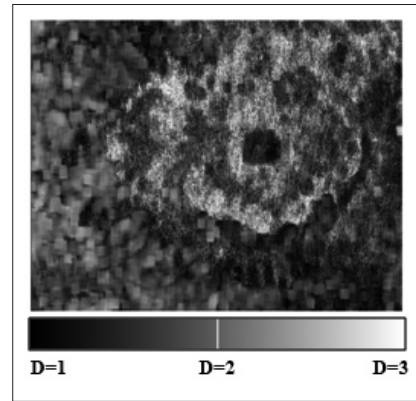


Figure 7 - Fractal relevant to the previous DEM.

The fractal map shown in Figure 7 presents, in some points, characteristics rather different from those obtained from the SAR image and from those that it would have if the DEM were perfectly fractal. This can be explained considering that the DEM is not fractal everywhere: the interpolation used in the DEM generation process is not known, but several artifacts (perfectly flat areas) are clearly recognizable. In particular, especially in the areas of large height variations, the fractal map in Figure 7 presents an anomalous dotted effect, whereas the variation of D should be smoother. Viewing the spectra relevant to those points, they appear definitely non fractals and the estimation seems to be strongly unstable. This confirms the presence of artificial features in the DEM.

The range of values of the fractal dimension of the DEM fractal map is: $-0.7 < D < 3.5$. In Figure 7 is shown the relevant map with values set between: $1 < D < 3$. The mean value of the map of Figure 7 is $D_{mean} = 1.7$ and the *standard deviation* is equal to 0.4. This confirms that the non-fractal characteristics of the DEM significantly alter the retrieved fractal statistic.

In order to demonstrate that the estimation problem is due to the particular technique used to obtain the DEM, the algorithm has been applied to a canonical artificial fractal DEM. As a matter of fact, if this kind of DEM is considered, the algorithm perfectly works. In [Di Martino et al., 2010a] the authors presented the application of such a processing to a canonical simulated SAR image obtained by means of the SARAS [Franceschetti et al., 1992]. Moreover, hereafter the fractal map relevant to a DEM of a canonical fractal surface of parameters $s = 0.1 \text{ m}^{0.2}$, $D = 2.2$ is presented. The surface is synthesized using the Weierstrass-Mandelbrot function [Franceschetti and Riccio, 2007] and is represented in Figure 8.

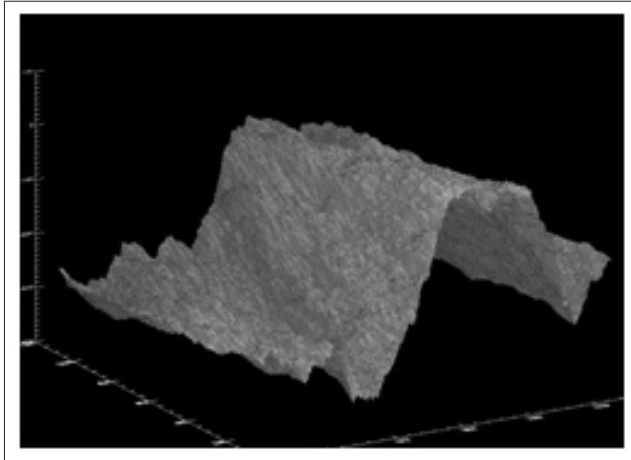


Figure 8 - DEM of a canonical fractal surface with parameters.

In Figure 9 the map of the fractal dimension D (computed with a sliding window of dimension 35×35 pixels) of the considered canonical surface is shown.

The range of values for D is: $1.8 < D < 3$, with a mean value $D_{mean} = 2.24$ and a *standard deviation* equal to 0.1: in this case the D mean value properly matches with the fractal dimension of the surface, thus confirming the effectiveness of the proposed algorithm.

Currently, some studies are in progress in order to exactly identify the artifacts present in the DEM of the Somma - Vesuvius volcanic complex. The idea is to recognize, from the behavior of the spectrum, the points of DEM presenting these artifacts in order to adequately process the DEM, thus obtaining in the fractal map the correct values of the fractal dimension. Fractal interpolation techniques [Yokoya et al., 1989] are possible candidates to obtain this goal.

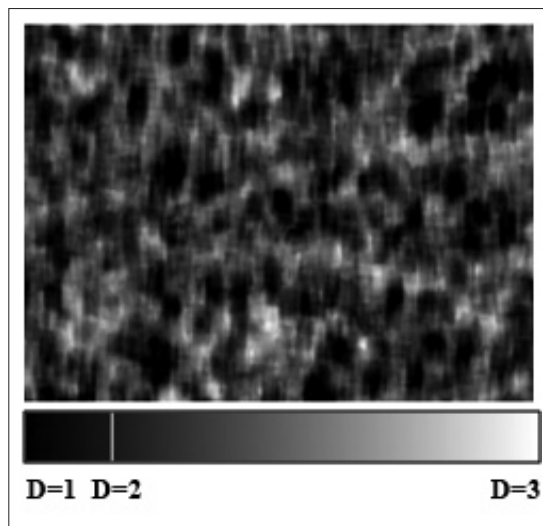


Figure 9 - Fractal map relevant to the previous DEM.

Conclusion

In this paper an innovative fractal based post-processing of SAR images of natural surfaces has been presented, along with its application to the case of volcano monitoring. The proposed technique is based on a complete imaging model developed by the authors: fractal models are employed for the description of the surface. This sound theoretical foundation allows the development of an automatic algorithm for the retrieval of significant geomorphologic parameters of the observed surface from its SAR image. In particular, maps of the point by point fractal dimension of natural scenes can be obtained using SAR data. The description of the proposed processing has been presented together with its application to the case of study of the Somma - Vesuvius volcanic complex. The presented fractal maps are suitable for geomorphologic interpretation. The occurrence of different fractal dimensions could be utilized as indicator to discriminate the presence of different geodynamic process active during the natural evolution of a volcanic complex. In this context, for example, the areas where the fractal dimension D is less or equal to 2 could be also interpreted as the surface expression (signature) of particular tectonic processes as faulting or caldera structural formation. Moreover, the comparison with the ground truth (represented in this case by the fractal map of the DEM of the observed area) highlighted the presence of artifacts and non-fractal characteristics on the available DEM. This issue will be object of future investigation regarding the development of adequate processing techniques to obtain DEM suitable for comparison with SAR data.

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