The Effects of Acquisition Geometry on SAR Images of Natural Scenes

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***Abstract*— In this paper we present novel results regarding the modeling of the SAR imaging process of natural scenes. The proposed model represents the extension to the two-dimensional case of the model previously proposed by some of the authors for the case of a one-dimensional fractal profile. Assuming a fractional Brownian model (fBm) for the observed surface and under a small slope hypothesis, we evaluate here, in closed form, the power density spectrum of the corresponding radar image. The proposed model effectively accounts for the effect of finite sensor resolution and for the peculiarity of SAR acquisition geometry.**

A numerical setup is implemented, based on sound physical models, allowing, on one hand, the validation of the small slope model and, furthermore, the empirical study of the general slope case.

I. INTRODUCTION

In the past decades, the interpretation of SAR images has been prerogative of a small number of specialized people. In fact, the geometrical distortions and the coherent character of the SAR system strongly limited the use of this kind of data and the development of automatic or semi-automatic techniques for the extraction of value-added information from this images. In the last years, however, the advent of new generation sensors marked a strong increase in the resolution of SAR acquisitions. Anyway, this technological improvement has not yet been followed by an increase in the comprehension of the mechanisms of SAR image formation, which is a pre-requisite for the development of un-supervised analysis techniques.

Furthermore, the passage to high resolution not always implies an easier interpretation of the images. This is particularly evident in the case of SAR images of urban areas, where many new features begin to appear as the resolution increase: this situation claims for an effective modeling of the data in support of information extraction. Anyway, thanks to the man-made character of the scene, these images are often speckle-free and the information of interest is related to deterministic and punctual characteristics of the scene. Conversely, in this paper we deal with high resolution images of natural scenes: in this case value-added information is no longer related to deterministic and punctual features of the image but much more to its stochastic and global properties. Hence, any image processing technique aimed to the extraction of significant geophysical parameters or to segmentation and classification issues should be based on the development of a model able to relate the statistics of the image to those of the observed scene.

In this paper we present a novel model for the SAR imaging process of natural surfaces. In fact, it is widely recognized in the literature that fractal models represent the best way to describe the irregularity of this type of surfaces [1], [2]. In order to minimize the number of independent parameters used for the characterization of the surface, a fractional Brownian model (fBm) for the observed surface is assumed [1]. Then, assuming a small slope regime for the observed surface, a novel imaging model is developed. The proposed model is focused on the extension to the case of a two-dimensional fractal surface of the results obtained by some of the authors for one-dimensional fBm profiles [3]. With respect to the one-dimensional problem, in this case the effects of SAR acquisition geometry have to be taken into account. In particular, as detailed in the following sections, in the small slope model the characteristics of the image present a dependence only on the partial derivative of the surface evaluated along the range direction. Furthermore, the difference between range and azimuth resolutions is considered. Relevant analytical results for the small slope case are presented in Sections II and III.

Finally, in Section IV the analytical results are validated via a numerical setup, based on a fully fractal framework. Note that this allows also the study of the general slope case. In fact, an SPM scattering model [2] is used to compute the signal backscattered from the fBm surface.

II. FRACTAL MODELS

Fractal models are widely considered the most appropriate ones to qualitatively and quantitatively describe natural surfaces [1]. In order to describe natural surfaces, we use the regular stochastic fBm process, which is characterized through the pdf of its increments [1], [2]. The two-dimensional stochastic process $z(x, y)$ describes an isotropic fBm surface if, for every x, y, x', y', it satisfies the following relation:

$$\Pr\left\{z(x)-z(x')<\bar{\zeta}\right\}=\frac{1}{\sqrt{2\pi}s\tau^{H}}\int_{-\infty}^{\bar{\zeta}}\exp\left(-\frac{\zeta^2}{2s^2\tau^{2H}}\right)d\zeta \qquad (1)$$
$$\tau=\sqrt{(x-x')^2+(y-y')^2}$$

where H is the *Hurst coefficient* ($0 < H < 1$), related to the fractal dimension D ($D=3-H$), and s is the standard deviation of surface increments at unitary distance measured in $\mathrm{m}^{(1-H)}$.

The power density spectrum of the isotropic two dimensional fBm process exhibits an appropriate power-law behavior [1], [2]:

$$S(k)=S_0k^{-\alpha} \qquad (2)$$

wherein S_0 and α are the spectral parameters [2].

From the equation (2) it's possible to establish that the power spectrum holds a linear behavior in a log-log plane, allowing the use of linear regression techniques for the retrieving of the fractal parameters of the surface from measured data.

Concerning the analytical part of this work, under the hypothesis of a small slope regime, the intensity of the backscattered signal is related to the partial derivative of the surface in the *range* direction. The fractional Gaussian noise (fGn) process is defined as the derivative process of the fBm; in our case, considering the lack of derivative of the fBm process, we introduce a smoothed version of it, obtained by convolving the original surface with an adequate kernel with support in $[0,\varepsilon_x]$ x $[0,\varepsilon_y]$.

Therefore, the autocorrelation function of the range derivative process $z_p(x,y)$ can be evaluated and expressed as a finite difference, turning out to be stationary.

The spectra evaluation of this process is performed on one-dimensional cuts:

- for the *range* cut, the autocorrelation and the power density spectrum of the derivative process exactly match those relevant to results introduced for a one-dimensional profile [3]:

$$R_p(\tau_y;\varepsilon_y)=\frac{1}{2}s^2\varepsilon_y^{\,2H-2}\left[\left(\frac{|\tau_y|}{\varepsilon_y}+1\right)^{2H}-2\left|\frac{\tau_y}{\varepsilon_y}\right|^{2H}+\left|\frac{\tau_y}{\varepsilon_y}-1\right|^{2H}\right] \qquad (3)$$

and, applying the Wiener-Kintchine theorem:

$$S_p(k_y;\varepsilon_y)=2s^2\varepsilon_y^{-2}\Gamma(1+2H)sin(\pi H)(1-\cos(k_y\varepsilon_y))\,|\,k_y\,|^{-(2H+1)} \qquad (4)$$

- for the azimuth cut the following expression is obtained for the autocorrelation function:

$$R_p(\tau_x;\varepsilon_y)=s^2\varepsilon_y^{-2}\left[\left|\tau_x^2+\varepsilon_y^2\right|^{H}-\left|\tau_x\right|^{2H}\right] \qquad (5)$$

The evaluation of the power spectrum needs the resort to generalized Fourier transform; we finally obtain a closed form expression for the power density spectrum of the range derivative process for an azimuth cut of the surface:

$$S_p(k_x;\varepsilon_y)=s^2\varepsilon_y^{-2}$$
$$\left\{\frac{2^{\left(\frac{3}{2}+H\right)}\sqrt{\pi}\,\varepsilon_y^{\left(\frac{1}{2}+H\right)}K_{H+\frac{1}{2}}\left(|k_x|\varepsilon_y\right)\dfrac{1}{|k_x|^{\left(\frac{1}{2}+H\right)}}}{\Gamma(-H)}+2\Gamma(1+2H)\sin(\pi H)\frac{1}{|k_x|^{1+2H}}\right\} \qquad (6)$$

where $K_\nu(.)$ is a modified Bessel function of second type of fractional order v.

III. Imaging Model

Taking into account the particular geometry of radar acquisition, as well as the trigonometric relationships connecting the incidence angle to the directional derivatives of the surface in the two directions of *range* and *azimuth*, respectively p and q, we can evaluate the intensity of the backscattered field as a function of the aforementioned parameters.

In our case, among the fractal scattering functions, the Small Perturbation Method (SPM) model is considered [2]. Expanding the SPM expression of the backscattered coefficient into a Mc Laurin series – in hypothesis of small slopes regime - the intensity of the backscattered field $i(x,y)$ is found to be a linear function only of the partial derivative p, whereas the dependence on q is negligible our approximation:

$$i(x,y)\cong a_0+a_1p(x,y)+o(p,q) \qquad (7)$$

where a_0 and a_1 are the coefficients that depend on the look angle of the sensor and on the fractal parameters of the observed surface.

Therefore, the imaging process can be seen as a system in which the input is the surface and the output is the radar image, which depends on the partial derivative $p(x,y)$ of the surface, as shown in Fig. 1.

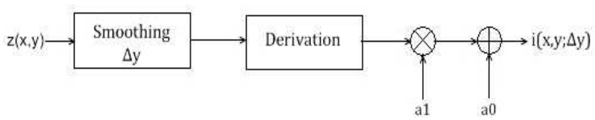

Fig. 1 Block diagram of the linear imaging process.

This allows us to use the results obtained in the previous section, assuming that, in this case, the support of the kernel used in the smoothing operation is connected to finite resolutions of the sensor that perform a sort of law-pass filtering on the surface. In particular we obtain $\Delta x=\varepsilon_x$, $\Delta y=\varepsilon_y$, where Δx and Δy are the resolution in the azimuth and range direction, respectively.

Hence, we can estimate the power spectral density of the image intensity:

- the image spectrum of a range cut of the surface is:

$$S_I(k_y;\Delta y)=a_1^2 S_p(k_y;\Delta y) \quad (8)$$

- the image spectrum of an azimuth cut of the surface is:

$$S_I(k_x;\Delta y)=a_1^2 S_p(k_x;\Delta y) \quad (9)$$

Starting from the closed form expressions of the spectra for the two cuts of the image, we can implement inversion techniques to retrieve the fractal parameters of the correspondent observed surface. In particular, it has been shown [3] that, at least for sufficiently low frequencies, the range cut image spectrum shows a linear behavior in a log-log plane, thus providing the possibility to retrieve the fractal parameters of the observed surface via linear regressions on measured values of the image spectrum. The plots relevant to Eqs. (8) and (9) are presented in Fig. 2 and 3, respectively.

IV. Numerical Setup

To validate the theoretical results shown in the previous section, hereafter we present a numerical framework, based on effective direct geometric and electromagnetic models. Note that use is made of a fully fractal approach: this is one of the key strengths of this work.

We implemented the complete chain that carries out the following steps: starting from fractal parameters arbitrarily chosen by the user, the corresponding two-dimensional fractal surface is generated by means of the Weierstrass–Mandelbrot function (WM) [2], [4]. Let us note that, in this case, we worked in the hypothesis that the observed surface shows the same fractal parameters at all the scales of interest. Then, we evaluated the field backscattered from the synthesized surface, using an SPM scattering model [2].

First of all, we verified the linear dependence of the backscattered signal on the range derivative p of the surface, and, vice versa, the lack of such a dependence on the partial derivative q, in case o a small-slope regime. For this purpose we compared the behaviors of the signal backscattered from two arbitrary cuts of the surface, one in *range* and one in *azimuth*, with the behaviors of the partial derivatives $p(x,y)$ and $q(x,y)$ of the selected profiles.

Then, in order to validate the results obtained in the previous section, we estimated the power density spectra of the two profiles and those of the corresponding backscattered signals.

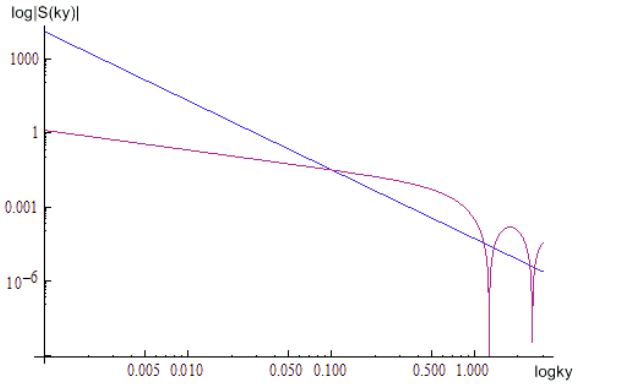

Fig. 2 Log – log plots of range cut power density spectra of the observed surface (blue) and of the corresponding image (red)) for H= 0.9, s=0.01 m^{1-H}, $\Delta x=\Delta y$=5 m, a_1=10.

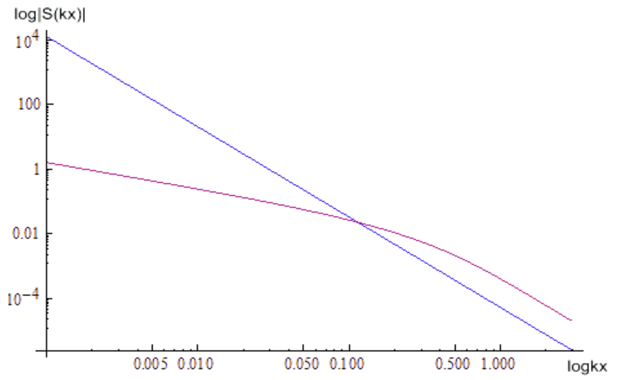

Fig. 3 Log – log plots of azimuth cut power density spectra of the observed surface (blue) and of the corresponding image (red) for H= 0.9, s=0.01 m^{1-H}, $\Delta x=\Delta y$=5 m, a_1=10.

In Fig.4 and in Fig.5 a comparison between the plots of theoretical and estimated spectra of the surface and of the image is presented. The considered range of frequencies is such that the spectra hold a power law behavior (i.e., the frequencies are sufficiently low). In Fig. 6 and 7 the possibility to set different resolutions for range and azimuth is shown. Values of the involved parameters for all the presented figures are shown in Table I.

TABLE I
PARAMETERS USED IN THE NUMERICAL SETUP.

	H	s [m^{1-H}]	Δx [m]	Δy [m]	a_1
Fig.4-5	0.9	0.01	5	5	1
Fig.6-7	0.9	0.01	10	5	1
Fig.8-9	0.9	1	5	5	1

The good agreement between the behavior of theoretical and experimental spectra is due to the specific choice of the parameters, both those relative to the surface (H and s) and to the system (Δx and Δy), that ensure the validity of the linear

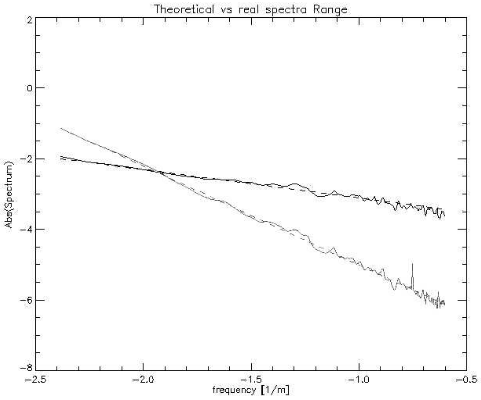

Fig. 4

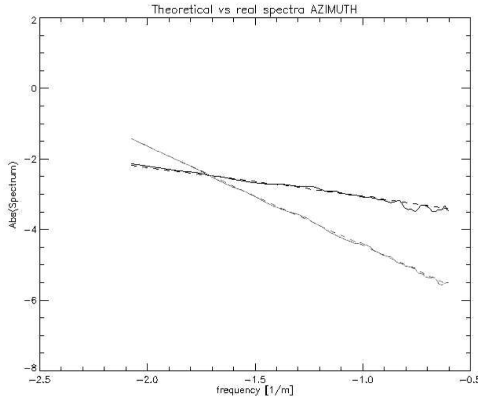

Fig. 7

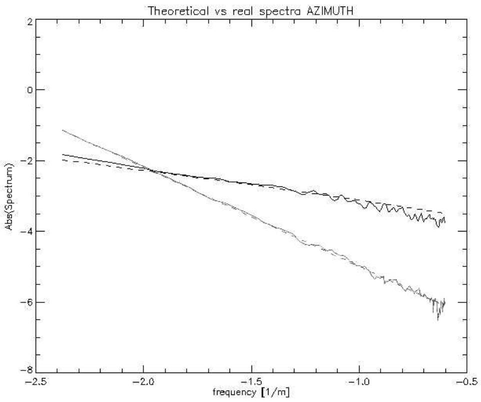

Fig. 5

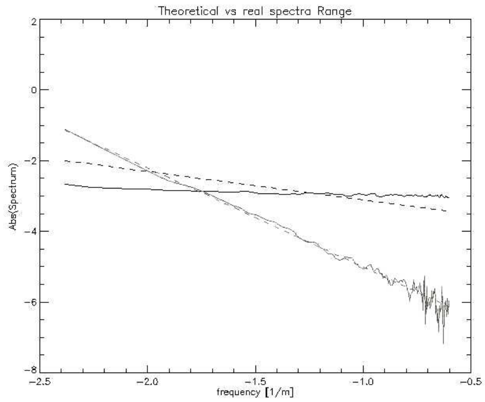

Fig. 8

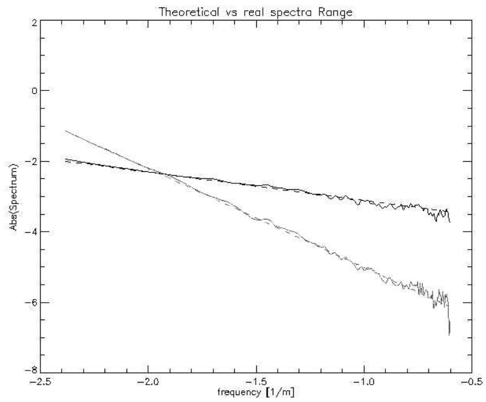

Fig. 6

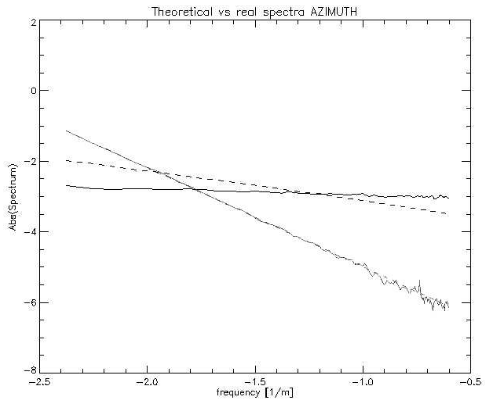

Fig. 9

Fig. 4, 5, 6, 7, 8, 9 Log-log plots of theoretical spectra (broken lines) and estimated ones (full lines) of the profile relevant to range and azimuth cuts of the surface (grey) and of the correspondent Image (black).

hypothesis on which the theoretical results are based. If those parameters are changed, such a degree of agreement is not reached. In Fig. 8 and 9, we show the same plots of Fig. 4 and 5 for different values of the fractal parameters. In particular, in this case the small slope hypothesis for the surface begins not to hold any longer and, as can easily be noticed from the figures, the fit between the spectrum estimated on the image and the theoretical one is not as good as in the previous cases.

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