FRACTAL BASED FILTERING OF SAR IMAGES

Gerardo Di Martino, Antonio Iodice, Daniele Riccio, Giuseppe Ruello, Ivana Zinno

Università di Napoli "Federico II", Via Claudio 21, 80125, Napoli, Italy
{gerardo.dimartino, iodice, daniele.riccio, ruello, ivana.zinno}@unina.it

ABSTRACT

In this paper an innovative fractal based filtering for the analysis of SAR images of natural surfaces is presented. Its definition is based on a complete direct imaging model developed by the authors. The application of this innovative algorithm to SAR images makes it possible to obtain a complete map of the fractal dimension of the observed scene. Significant results obtained on actual SAR data are shown in the last section of the paper.

Index Terms— Fractals, Synthetic Aperture Radar

1. INTRODUCTION

The new generation of Synthetic Aperture Radar (SAR) sensors marked a huge increase in the resolution of microwave images of the Earth. TerraSAR-X and COSMO-SkyMed are providing SAR data with the remarkable resolution of 1 x 1 m^2 in the high resolution spotlight operational mode. Owing to this development of remote sensing systems, innovative models, techniques and tools are required to adequately deal with this new scenario. As a matter of fact, the availability of very high resolution data calls for an update in the models and techniques used for the analysis and elaboration of microwave images of both urban areas and natural zones. In particular, regarding the latter ones, until now it was only possible to identify macroscopic topological features (mountains, rivers, seas, etc.) of the observed areas, roughly distinguishing them from urban ones: with the new generation sensors, the extraction of meaningful stochastic parameters of the observed surface at microscopic level is now in order.

In [1] the authors present a direct model for the imaging of natural surfaces which takes into account the interaction between the surface and the incident electromagnetic wave, through appropriate fractal scattering models [3], as well as the effect of the SAR impulse response. Our analysis is based on a sound direct modeling of the observed surface and of the SAR imaging process [1]. In particular, the observed natural surface is modeled as a fractal two-dimensional stochastic process [2], which is exhaustively described by its independent fractal parameters. In particular, the fractal dimension D of an observed surface is strictly related to the roughness and geophysical characteristics of the surface and its knowledge can be of key importance for a wide range of applications as the prevention and monitoring of environmental disasters, land classification, rural and urban planning and so on. For this reason, the estimation of this parameter from a SAR image would be of key importance for a wide range of geophysical applications. Unfortunately, in the open literature no reliable technique for the estimation of the fractal dimension of an observed surface from its radar image is known. In this paper we present a novel technique for the retrieval of the fractal dimension D of the observed surface based on an appropriate spatial filtering of the amplitude SAR image, whose rationale comes out from the inversion of the models presented in [1]. In the following sections we present first of all the theoretical and methodological framework of the proposed approach (Sect. 2 and 3), and finally some significant results concerning the elaboration of actual SAR data (Sect. 4).

2. THEORETICAL FRAMEWORK

It is widely recognized that fractal models represent the best way to describe the irregularity of natural scenes [2], [3]. Among this kind of models, we choose the regular stochastic fBm (fractional Brownian motion) process that completely describes natural surfaces by means of two independent parameters: the Hurst coefficient, H (which is linked to the fractal dimension by the simple relation $D=3-H$) and the standard deviation of surface increments at unitary distance, s $[m^{1-H}]$. The process $z(x,y)$ is an fBm if, for every x, y, x', y', it satisfies the following relationship:

$$Pr\{z(x,y)-z(x',y')<\bar{\zeta}\}=\frac{1}{\sqrt{2\pi}s\tau^H}\int_{-\infty}^{\bar{\zeta}} exp\left(-\frac{\zeta^2}{2s^2\tau^{2H}}\right)d\zeta,$$
$$\tau=\sqrt{(x-x')^2+(y-y')^2} \qquad (1)$$

The power density spectrum of the isotropic two dimensional fBm process exhibits an appropriate power-law behavior [1], [2]:

$$S(k)=S_0k^{-\alpha} \qquad (2)$$

wherein S_0 and α are functions of the fractal parameters [2].

Besides the surface model, in order to retrieve the fractal dimension of a natural scene starting from its SAR image we need a direct model relating the surface to its final amplitude image.

Fig. 1: Block diagram of the extraction of the fractal dimension.

In [1] the authors presented a complete imaging model based on the assumption of a small slope regime for the observed surface: if this is the case, the image intensity comes out to be a linear function of the partial derivative of the surface evaluated along the range direction.

The expressions of the autocorrelation functions of the SAR image and of the Power Spectral Densities (PSDs) of two cuts of the image in the range and azimuth directions respectively, have been evaluated by the authors in [1]. The PSDs of the azimuth and of the range cut of the SAR image show very different behaviors, thus highlighting an intrinsic asymmetry in the structure of SAR data, that is also intuitively referable to the particular acquisition geometry of a side looking mono-static radar. In particular, the spectrum of the image range cut, in an appropriate range of sufficiently low spatial frequencies, presents a power law behavior - thus showing on a log - log plane a linear behavior with a slope related to the Hurst coefficient H of the observed surface. In fact, the expression of the PSD of the range cut of a SAR image, for adequately low wavenumbers, turns out to be [1]:

$$S_p(k_y) = s^2 \Gamma(1+2H) sen(\pi H) \frac{1}{|k_y|^{2H-1}} \quad (3)$$

where k_y is the wavenumber of the range cut of the image and Γ is the Euler Gamma function.

Comparing Eq. 3 with the expression of the PSD of the surface in Eq. 2, it can be inferred that the slope of the spectrum relevant to a range cut of a SAR image is equal to that of the imaged surface, assuming that the Hurst coefficient is decreased by one.

3. METHODOLOGICAL SETUP

In order to retrieve the fractal parameters starting from a SAR image, we can perform the analytical inversion of the presented theoretical model.

In particular, starting from Eq. 3, it is possible to implement linear regression algorithms on the spectrum of range cuts of the image in a log - log plane, thus retrieving the fractal dimension, according to the scheme presented in Fig. 1. Hence, we developed a software that, by means of a sliding window spanning the entire image, provides the corresponding fractal map, i.e. a matrix of the point by point fractal dimension relevant to the observed surface.

The implemented algorithm extracts the local fractal dimension of the imaged surface working on patches of the SAR image and iterating the procedure on the whole image, through a moving window, whose dimension can be set by the user according to its specific needs, resulting from a trade-off between accuracy and resolution of the output fractal dimension map. In particular, the algorithm performs this estimation selecting in each window range cuts that are sufficiently spaced from each other to be considered uncorrelated. Then the spectra of these cuts (whose number can be again chosen by the user, as a trade-off between accuracy and computation time) are evaluated using a Capon estimator [4]. Finally, these spectra are averaged and a linear regression is performed on this mean PSD. The question of the spectrum estimation is not trivial: as a matter of fact, power-law spectra introduce unique difficulties in the spectral estimation as they suffer leakage effects and high variance problems, yielding a spectral estimate which can deeply modify the original spectral slope. The Capon estimator strongly reduces the above-mentioned negative effects and is particularly well suited when facing short data records: this characteristic is also important in our case, in which the record size is limited by the use of a relatively small sliding window.

As a result, we obtain a map of the fractal dimension of the observed scene: the resolution of this map depends on both the resolution of the input image (the higher the resolution of the image, the better the resolution of the map) and the dimension of the estimation window.

Furthermore, we cannot set completely aside the speckle phenomenon, which is responsible for the well-known *salt and pepper* effect on SAR amplitude images. As a matter of fact, the spatial scales involved by the speckle are mainly those of the order of the sensor resolution, hence in the wavenumber domain the high frequency range of the image spectrum is degraded. However, our algorithm performs the linear regression in a range of spatial frequencies in which the spectrum is not significantly affected by this phenomenon.

By means of such a type of filtering different applications can be carried out. In the next section, we show some study cases that attest the goodness of the theoretical basis and of the software operation. The algorithm is applied on simulated SAR images of canonical surfaces of controlled parameters first, then on real SAR data.

4. EXPERIMENTAL RESULTS

In this section significant experimental results with regard to the application of the developed algorithm to SAR images are presented.

4.1. Application to simulated SAR images

First of all the algorithm is applied to simulated SAR images obtained by means of the SARAS simulator [5]: as input is given a DEM of a fractal surface realized implementing a Weierstrass-Mandelbrot function [2], [3].

Let us note that, in this case, we work in the hypothesis that the observed surface shows the same fractal parameters, chosen and controlled by the user, at all the scales of interest. As scattering model we select the fractal Small Perturbation Method (SPM) [3], which is consistent with the description used for the surface, adequately describing the interaction between an incident electromagnetic field and a fractal surface. The selected sensor is an ERS-1 C sensor providing a SAR image with an azimuth resolution Δx=3.986 m and a ground range resolution Δy=19.928 m and a total dimension of 1060x2530 pixels.

The algorithm, implemented with a sliding windows, whose size is set to 50x50 pixels, provides a fractal map of the imaged surface giving the point by point fractal dimension, D.

In Fig. 2, 3, 4 we show the DEM of the surface given as input to the SARAS, the simulated SAR image obtained and the relevant fractal map, respectively. The surface fractal parameters are set to D=2.2, s=0.1 m$^{0.2}$ and the fractal map presents the following statistics:

- mean: 2.16
- standard deviation: 0.05.

Hence, the retrieved fractal dimension is satisfactory.

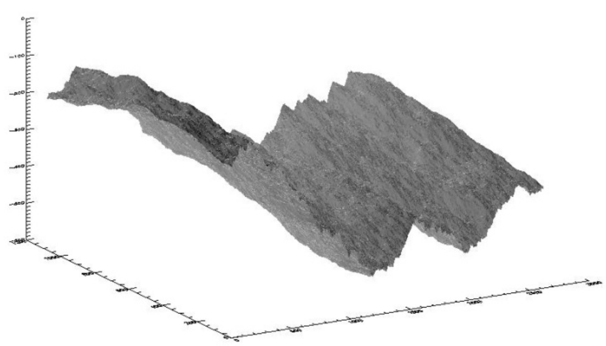

Fig. 2: Fractal Surface of parameters D=2.2, s=0.1 m$^{0.2}$.

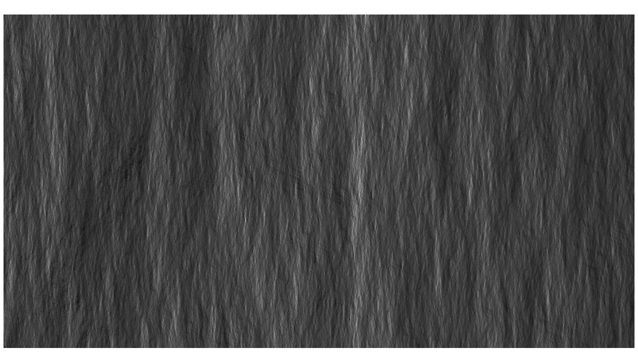

Fig. 3: simulated SAR image relevant to the fractal surface in Fig. 2.

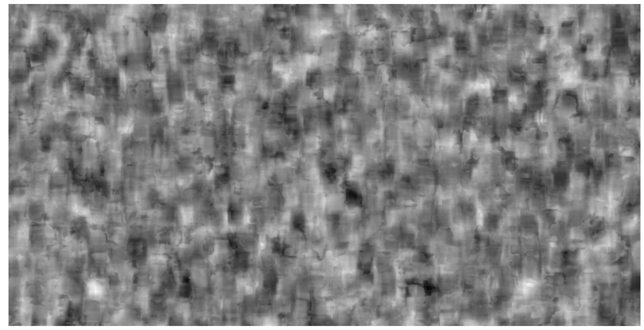

Fig. 4: Fractal map relevant to the image in Fig. 3.

4.2. Application to actual SAR data

In order to show the potentialities of this new type of filtering, a case study relevant to an actual SAR image is for the first time here presented.

The input data is a COSMO-SkyMed image (Fig. 5) of an area near to L'Aquila (Abruzzo-Italy) which is a mix of rural and built-up areas.

Before applying our algorithm, a 2x2 multilook has been performed on the image, and the dimension of the window is set 20x20 pixels. Such a choice has the twofold goals of preserving the map resolution and reducing the occurrence of average on inhomogeneous scenes.

Some considerations on the obtained fractal map, shown in Fig. 6, are now in order. As first, we obtained a point-wise retrieval of the fractal dimension of the imaged scene. The range of values of the recovered fractal dimension is: $1.6 \leq D \leq 2.7$. The lower values correspond to the lighter shade of grey, while the higher values of D are represented with darker shade of grey. Actually, the D range of fractality of a natural surface is $2 < D < 2.5$; pixels with $D < 2$ represent non-fractal objects; pixels with $D > 2.5$ represent fractal scatterers whose fractal dimension does not match the range of fractality of a natural surface.

Note that the obtained map allows the automatic identification of man-made objects (urban areas as well as the motorway in the centre and so on) that appear brighter with respect to the darker natural zones.

Such a type of filtering can be used as a support to image classification and segmentation. It turns out to be extremely simple, effective and reliable because it is enforceable to a single SAR image and it does not require supervision.

5. CONCLUSIONS

In this paper an innovative fractal based filtering of SAR images of natural surfaces is presented. It is based on a complete imaging model: adequate fractal models are employed for the description of the surface and the electromagnetic problem. This sound theoretical foundation allows the development of automatic techniques for the retrieval of significant geophysical parameters of the observed surface from its SAR image.

Fig. 5: COSMO-SkyMed Image of an area close to L'Aquila (Abruzzo-Italy).

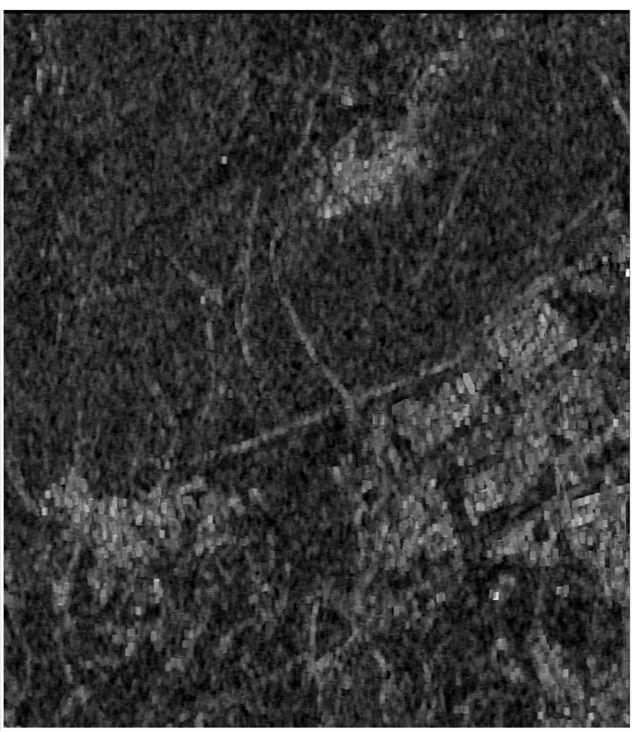

Fig. 6: Fractal map relevant to the SAR image in Fig. 6.

As an example of the potentialities of the proposed approach, the application of this new filtering to simulated and actual SAR images has been presented. A complete map of the fractal dimension of the observed scenes has been obtained. In addition, for simulated images, a validation of the technique has been presented.

In case of actual SAR images the obtained fractal dimension map clearly shows the potentialities of the proposed framework. In particular, it is shown that rural and urban areas can be automatically discriminated.

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6. REFERENCES

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