PHYSICAL-BASED MODELS OF SPECKLE FOR HIGH RESOLUTION SAR IMAGES

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ABSTRACT

In this paper we present a physical model for the description of the speckle in Synthetic Aperture Radar (SAR) images. The proposed model highlights the analytical dependence of the speckle characteristics on the surface roughness and the sensor parameters. The illuminated surface is represented using the fractal geometry. In addition, a SAR raw signal simulator able to generate SAR images whose speckle characteristics are coherent with the observed surface properties is presented.

Index Terms— Synthetic aperture radar, speckle

1. INTRODUCTION

Synthetic Aperture Radar (SAR) single look images present the well-known phenomenon of speckle, which is the reason of the *salt and pepper* effect and it is due to the fact that a resolution cell is usually larger than the wavelength of the incident electromagnetic field. In fact, the received SAR signal is the coherent sum of the elementary returns from the resolution cell: these returns present a phase difference dependent on the difference in the electromagnetic path covered by different returns.

In this paper a SAR raw signal simulator able to generate images whose speckle characteristics are coherent with the illuminated surface properties is presented. The proposed simulator requires as input the geometrical description of the surface and the sensor and orbital parameters. In general, the characteristics of speckle depend in non-linear way on the surface roughness, the signal wavelength, the look angle, the sensor resolution and the synthetic antenna dimension. Therefore, in the scientific literature these images are usually seen as a realization of a stochastic process, formalized as a random walk in the complex plane: the received signal is described as the coherent sum of the returns coming from independent scatterers randomly distributed in the resolution cell [1]-[2].

According to this kind of models, the key parameter for the statistical characterization of the speckle is the number of independent scatterers per resolution cell. Under the hypothesis that this number of scatterers is much larger than unity, the central limit theorem can be applied giving rise to a Gaussian complex circular field, with a Rayleigh distributed amplitude and a phase uniformly distributed in $[0,2\pi]$. In this case the speckle is defined as fully developed. For low resolution SAR sensors, whose resolution cell is of the order of tenth of meters (very large compared to the centimeter wavelength), the above mentioned hypothesis can be safely assumed to hold and, in fact, the Rayleigh model well matches with actual data. Note that this model is very attractive because in this case for the speckle description only the knowledge of the mean square value of the cell return is required, regardless of the accurate characterization of the scatterers.

In the past decades, the K-distribution has been successfully used to model the ocean SAR images, whose statistics do not match with those predicted by the Rayleigh model. In fact, the ocean surface is highly correlated, leading to a number of scatterers per resolution cell of the order of unity. In this case, by describing the return from a single scatterer as Gamma distributed, the sum of a finite number of these returns turns out to be K-distributed [3], [4].

For new generation SAR sensors reaching resolutions of the order of one meter or less, the hypothesis of a resolution cell size much larger than the wavelength cannot be always assumed. In this case, a physical and statistical characterization of the scatterers is required in order to adequately describe the speckle phenomenon.

In this paper we present a new model of speckle, based on a physical description of the scatterers in the resolution cell. In particular, a relation between the number of independent scatterers per resolution cell and the physical properties of the surface (roughness) and of the sensor (wavelength, look angle, resolution) is provided. The roughness of the illuminated surface is modeled as a fractional Brownian motion (fBm). The phase contributions of the returns coming from each point in the resolution cell are evaluated: the difference in these phase contributions characterize the speckle of the image. Thanks to this model the presented simulator is able to assign to the simulated images the correct statistical behavior.

In Sect. 2 the fundamentals of the SAR simulation procedure are detailed. Sect. 3 is devoted to recall the classical Rayleigh and K-distribution speckle models. In Sect. 4 we present the fractal model employed to describe

the cell roughness. Sect. 5 is devoted to present the electromagnetic approach that allowed the evaluation of the number of scatterers per resolution cell as a function of the fractal and the sensor parameters. Relevant results are discussed in the conclusion section.

2. SAR SIGNAL SIMULATION

In Fig. 1 the block diagram describing the proposed simulation procedure is presented. Given the radar (RD) and orbital (OD) data and the fractal surface $z(x,y)$, with its electromagnetic parameters $\varepsilon(x,y)$ and $\sigma(x,y)$, the mean scattering is evaluated via the KA [2] and the speckle is accounted for as a realization of K- or Rayleigh-distributed process, according to the number N of independent scatterers per resolution cell (see Sect. 3 for details). This leads to the reflectivity function $\gamma(x,r)$, which is convolved with the SAR transfer function (STF) by means of a proper frequency domain elaboration in order to evaluate the raw signal $h(x,r)$ [2]. A standard processing is employed to retrieve the simulated image $i(x,r)$.

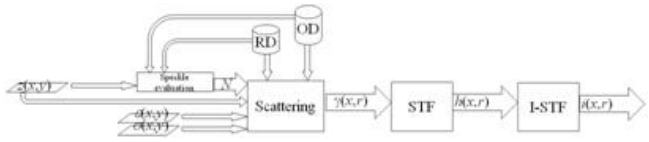

Fig. 1: Block diagram of the simulation procedure.

The main contribution of the proposed paper is the inclusion of the *speckle evaluation* procedure in a SAR raw signal simulator (SARAS) developed in the past decade at the University of Napoli, Italy [2]. The new algorithm requires as input the fractal parameters that describe the roughness of the observed surface along with the orbital and radar data, and provides as output the number N of independent scatterers per resolution cell. This parameter can be used to define the stochastic properties of speckle on the final image, as detailed in the following section.

3. CLASSICAL SPECKLE MODELS

Most of the models available in literature describe the return from a resolution cell as the coherent sum of N electromagnetic returns [1], [2]:

$$E = Ve^{j\phi} = \sum_{i=1}^{N} V_i e^{j\phi_i}, \qquad (1)$$

where $V_i e^{j\phi_i}$ is the contribution due to the i-th scatterer. Hence, the field E is a function of the number N of scatterers and, according to this value, the speckle will be Rayleigh or K-distributed, as detailed in the following sub-sections.

3.1. Rayleigh speckle

In the hypothesis that the size of the resolution cell is much larger than the incident wavelength and the number N of independent scatterers per resolution cell is very large, the central limit theorem can be used to evaluate the phase and amplitude of the field in terms of their pdfs [1]. In this case, the speckle is defined as fully developed and E turns out to be a Gaussian complex circular field, whose amplitude and phase are described by a Rayleigh distribution and an uniform distribution in $[0,2\pi]$, respectively.

3.2. K-distributed speckle

If the hypothesis of a large number of independent scatterers per resolution cell is no longer valid, we have to face the problem of studying the coherent sum of a finite number of fields. If we assume that the amplitudes V_i are statistically independent and identically distributed, the radar cross section $w=|\mathrm{E}(V)|^2$ presents a pdf that can be expressed as [3]:

$$p_N(w) = \frac{1}{2}\int_0^\infty U \mathrm{J}_0\left(U\sqrt{w}\right)\langle \mathrm{J}_0[UV_i]\rangle^N dU, \qquad (2)$$

where $\langle\bullet\rangle$ stands for the statistical mean and J_0 is the zeroth order Bessel function. Equation (2) expresses the pdf of the radar cross section as a function of the pdf of V_i and of the number of independent scatterers N. Unfortunately, it cannot be evaluated analytically for an arbitrary $p(V_i)$. However, Jakeman and Pusey [3] presented a case in which an exact solution for the integral (2) can be found, leading to a new family of distributions based on the modified Bessel functions K. In fact, let assume:

$$p(V_i) = \frac{2b}{\Gamma(\nu+1)}\left(\frac{bV_i}{2}\right)^{\nu+1} K_\nu(bV_i),\ \nu > -1 \qquad (3)$$

where b and ν are parameters depending on $|\mathrm{E}(V)|$, $\mathrm{K}_\nu(\bullet)$ is the second kind modified Bessel function and $\Gamma(\bullet)$ is the Gamma function. With this choice, the integral (2) can be solved in closed form, leading to the following expression:

$$p_N(w) = \frac{b/\sqrt{w}}{\Gamma(M)}\left(\frac{b/\sqrt{w}}{2}\right)^M \mathrm{K}_{M-1}\left(b/\sqrt{w}\right) \qquad (4)$$

where the parameter M is related to the number of scatterers per resolution cell through the relation $M = N(1+\nu)$.

It can be shown that for $M \gg 1$, the distribution (4) reduces to a negative exponential function, which is in accordance with the Rayleigh model.

The presented methods do not provide a physical-based definition of the scatterers, whose number cannot be automatically inferred by the physical parameters characterizing the observed surface.

4. SURFACE DESCRIPTION

The use of an adequate model for the description of natural surfaces is crucial for the success of the whole procedure. To this purpose, we use the fBm which is an always continuous, nowhere differentiable process, described in terms of its increment pdf. A stochastic process $z(x,y)$ is an fBm surface if, for every x, x', y, y' it satisfy the following relation:

$$\Pr\{z(x,y)-z(x',y')<\bar{\zeta}\}= \\ =\frac{1}{\sqrt{2\pi}s\tau^{H}}\int_{-\infty}^{\bar{\zeta}}\exp\left(-\frac{\zeta^{2}}{2s^{2}\tau^{2H}}\right)d\zeta \tag{5}$$

where τ is the distance between the points (x,y) and (x',y'), Pr stands for "probability" and the two parameters that control the fBm behavior are:

- H : the *Hurst coefficient* ($0<H<1$), related to the fractal dimension D by means of the relationship $D=3-H$.
- s : the standard deviation of surface increments at unitary distance, measured in [m^{1-H}].

5. ELECTROMAGNETIC MODEL

The SAR raw signal is the superposition of the reflectivity function $\gamma(\bullet)$, weighted by the SAR unit response $g(\bullet)$:

$$h(x',r')=\iint\gamma(x,r)g(x'-x,r'-r,r)dx\,dr \tag{6}$$

The reflectivity function $\gamma(x,r)$ can be expressed as the product of a polarization factor S_{pq} and an integral I that accounts for the roughness of the resolution cell:

$$\gamma_{pq}=S_{pq}\iint e^{j2\mathbf{k}\cdot\boldsymbol{\rho}}\,dA=S_{pq}I \tag{7}$$

In order to evaluate quantitatively the effect of the surface roughness on the backscattered signal, and then on the SAR image, we need to evaluate the integral I.

In absence of roughness, the position vector is $\boldsymbol{\rho}=\tau\hat{x}$. Therefore, the integral I reduces to a sinc function and it does not bring a random phase shift.

If we consider a rough cell, the position vector is $\boldsymbol{\rho}=\tau\hat{x}+(z+dz)\hat{z}$, where z_0 is the height of the point in the cell center and dz is the height difference between z_0 and a point at distance τ from the center, see Fig. 2. In this case, the γ_{pq} function can be written as:

$$\gamma_{pq}=S_{pq}\iint e^{j2k_{x}x}e^{j2k_{z}z_{0}}e^{j2k_{z}dz}\,dA \tag{8}$$

The points in correspondence of which the phase term $2k_z dz$ is negligible belong to an area from which the electromagnetic return does not present a random phase shift. In order to quantify the size of this area, we have to evaluate the probability that the phase term $2k_z dz$ is much lower than unity.

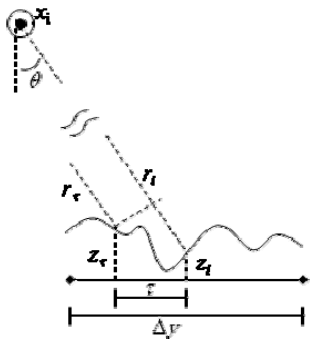

Fig. 2: Geometry of the scattering problem.

By choosing a shift phase of $\pi/5$ as a threshold phase difference, the points that belong to the above described area are determined by the probability that the following condition is fulfilled:

$$|dz|<\frac{\lambda}{20\cos\theta} \tag{9}$$

Such a probability can be evaluated by means of the definition of the fBm process, see eq. (6):

$$p=\Pr\left\{|dz|<\frac{\lambda}{20\cos\theta}\right\}= \\ =\frac{1}{\sqrt{2\pi}s\tau^{H}}\int_{-\frac{\lambda}{20\cos\theta}}^{\frac{\lambda}{20\cos\theta}}\exp\left(-\frac{\zeta^{2}}{2s^{2}\tau^{2H}}\right)d\zeta \tag{10}$$

The p value is a function of the distance τ from the center of the resolution cell, and it allows to determine the dimension of the area from which the return can be considered correlated.

In the following, we present a set of simulations developed for a sensor with a look angle of $\pi/6$ and the incident electromagnetic wavelength of 3 cm.

In Fig. 3 the probability p is shown for a surface with assigned H value (H=0.8), as a function of s ranging from $s=10^{-2}$ $m^{0.2}$ (continuous line) to $s=3\cdot10^{-2}$ $m^{0.2}$ (dotted line).

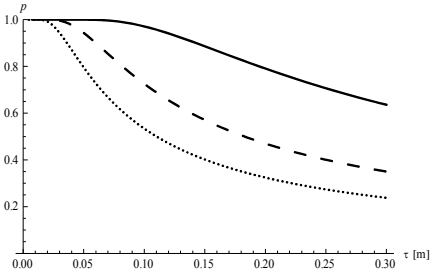

Fig. 3: The probability function is plotted as a function of τ for H=0.8 and for s=10^{-2} $m^{0.2}$ (continuous line), s=$2\cdot10^{-3}$ $m^{0.2}$ (dashed line) and s=$3\cdot10^{-3}$ $m^{0.2}$ (dotted line).

As expected, the dimension of the coherence area decreases as a function of the surface roughness. In Fig. 4 the probability p is shown for a surface with assigned s value (s=10^{-2} m^{1-H}) as a function of H ranging from H=0.7 (continuous line) to H=0.9 (dotted line).

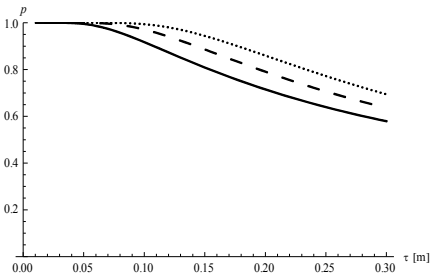

Fig. 4: The probability function is plotted as a function of τ for s=10^{-2} m^{1-H} and for H=0.6 (continuous line), H=0.7 (dashed line) and H=0.8 (dotted line).

Again, the dimension of the coherence area decreases as a function of the surface roughness.

In order to define the dimension of the independent scatterers, we evaluate the correlation of the returns as:

$$\left\langle e^{j2k_z z(r)}\, e^{-j2k_z z(r+dr)} \right\rangle = \left\langle e^{-j2k_z (z(r+dr)-z(r))} \right\rangle = e^{\frac{-(2k_z)^2}{2} s^2 \tau^{2H}} \tag{11}$$

The distance over which, the correlation falls below an assigned threshold value (here we choose e^{-1}), can be considered as the dimension of the scatterer. Such a dimension can be expressed as:

$$\tau_M = \left(\frac{1}{2k_z^2 s^2} \right)^{\frac{1}{2H}} \tag{12}$$

From the dimension of the scatterer we can evaluate the equivalent number of scatterers for resolution cell as the ratio between the cell area and τ_M^2. In Fig. 5 and 6 we show the equivalent number of scatterers as a function of the cell roughness for a resolution cell of 1m x 1m. The obtained results show the range of the number of the independent scatters. Such an algorithm was implemented in the simulation procedure in order to simulate the appropriate speckle.

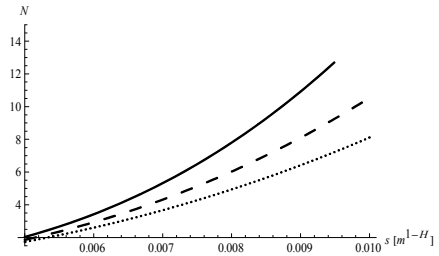

Fig. 5: The number of scatterers N is plotted as a function of s for H=0.6 (continuous line), H=0.7 (dashed line) and H=0.8 (dotted line).

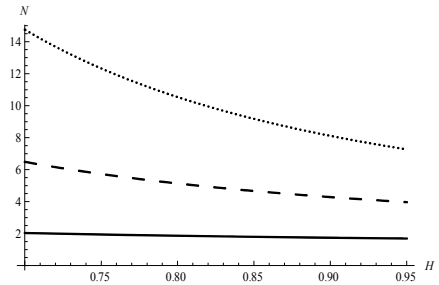

Fig. 6: The number of scatterers N is plotted as a function of H for s=$5\cdot10^{-3}$ m^{1-H} (continuous line), s=$7.5\cdot10^{-3}$ m^{1-H} (dashed line) and s=10^{-2} m^{1-H} (dotted line).

6. CONCLUSIONS

In this paper a new model for describing the speckle in SAR images is presented. The model is based on the use of fractals for the surface description. The equivalent number of scatterers for resolution cell is analytically evaluated as a function of the surface roughness and the sensor parameters.

7. REFERENCES

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