PHYSICAL MODELS FOR SAR SPECKLE SIMULATION

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ABSTRACT

In this paper a SAR simulator able to provide images with the appropriate speckle statistics is introduced. It requires as inputs the radar and surface parameters and it is able to evaluate the number of equivalent scatterers per resolution cell, *N*. The statistics relevant to each single equivalent scatterer in the resolution cell are effectively generated and the global statistics of the return from the resolution cell are obtained as the coherent sum of the contributions pertaining to each equivalent scatterer. The rationale of the proposed simulation framework is presented, along with significant results regarding the analysis of the simulated images.

Index Terms— Synthetic aperture radar (SAR), SAR simulation, speckle.

1. INTRODUCTION

It is well known that Synthetic Aperture Radar (SAR) single look images present the phenomenon of speckle, whose occurrence is due to the fact that a resolution cell is usually much larger than the wavelength of the incident electromagnetic field. Due to the lack of deterministic knowledge of the structure of the observed surface at wavelength scale, a statistical description of SAR images is usually introduced and, accordingly, the received signal is described as the coherent sum of the returns coming from independent scatterers randomly distributed in the resolution cell [1]-[2].

According to this model, a key parameter for the statistical characterization of the speckle is the number of independent scatterers per resolution cell, *N*. As a matter of fact, under the hypothesis that N>>1, the central limit theorem can be applied giving rise to a Gaussian complex circular field, with a Rayleigh distributed amplitude and a phase uniformly distributed in $[0,2\pi]$. In this case the speckle is defined as fully developed. For low resolution SAR sensors, whose resolution cell area is of the order of tenth of square meters, the Rayleigh model well matches with actual data. However, with the increasing availability of space-borne very high resolution SAR data, for which the hypothesis of a resolution cell size much larger than the wavelength cannot be always assumed, in many actual cases the statistics of SAR images can depart from those predicted by the Rayleigh model. In the past decades, in many actual situations the K-distribution has been successfully used to model the statistical behavior of SAR images, as a function of the effective number of scatterers per resolution cell [2].

In [3] the authors presented a theoretical framework which, after introducing a physical definition for the concept of "equivalent scatterer", allows an analytical evaluation of the number of equivalent scatterers present in resolution cells with fixed size, when a fractal fBm model for the observed surface is assumed [4]. In particular, the number of scatterers *N* is a function of the roughness of the surface, which can be described through its fractal parameters, and of the sensor parameters (operating frequency, resolution, look angle). The ability to compute the number of equivalent scatterers per resolution cell for a given sensor and for a surface presenting prescribed fractal parameters can be exploited to introduce new techniques for the simulation of the speckle in SAR images.

In this paper a SAR simulator able to provide images with the appropriate speckle statistics is introduced. It requires as inputs the radar and surface parameters and, using the results of [3], it is able to compute the number of scatterers per resolution cell, *N*. Then, the statistics relevant to each single scatterer in the resolution cell are effectively synthesized [5] and the final statistics of the return from the resolution cell are obtained as the coherent sum of the scatterers contributions. The presented simulator is largely based on the SAR raw signal simulator (SARAS) developed [6] and positively tested [7] by some of the authors, which is able to provide images presenting fully developed speckle statistics. The approach adopted in the present work for the simulation of speckle is analogous to the one used in the SARAS simulator and is aimed at generating spatial statistics in accordance with acquisition parameters of the selected SAR sensor [8].

Based on the proposed simulation framework, it is possible to implement the inversion of the proposed model, testing the performance of estimation algorithms [9]-[10] on simulated data. In the present paper meaningful sample results regarding the inversion step are presented, along with a preliminary study on the effects of SAR system parameters on the estimation of the number of equivalent scatterers.

The paper is organized as follows. In Section 2 the considered mathematical, stochastic model of speckle is introduced. In Section 3 the simulation framework used to generate the image with the prescribed statistics is described. Meaningful results are presented and discussed in Section 4. Finally, Section 5 bears some concluding remarks.

2. SPECKLE MATHEMATICAL MODEL

Most of the models available in literature describe the return from a resolution cell as the coherent sum of N electromagnetic returns [1]-[2]:

$$E = Ve^{j\varphi} = \sum_{i=1}^{N} V_i e^{j\varphi_i} \quad , \qquad (1)$$

where $V_i e^{j\phi_i}$ is the contribution due to the i-th scatterer. Hence, the field E is a function of the number N of scatterers and, according to this value, the speckle can be Rayleigh or K-distributed. In particular, the Rayleigh distribution can be regarded as a particular case of the K-distribution for N>>1 [2]. If the hypothesis of a large number of independent scatterers per resolution cell is no longer valid, we have to face the problem of studying the coherent sum of a finite number of terms. In order to obtain a closed form pdf for the return intensity, we can make the following assumptions [2]:
1) the amplitudes V_i and the phases ϕ_i are statistically independent from each other and from V_j and ϕ_j if $i \neq j$;
2) the V_i are K-distributed, i.e.:

$$p(V_i) = \frac{2b}{\Gamma(1+\upsilon)}\left(\frac{bV_i}{2}\right)^{\upsilon+1} \mathrm{K}_\upsilon(bV_i), \qquad (2)$$

where $\upsilon > -1$ and b are parameters depending on the scene, $K_v(\bullet)$ is the second kind modified Bessel function of order v and $\Gamma(\bullet)$ is the Euler Gamma function;
3) the ϕ_i are uniformly distributed in $[0,2\pi]$.

Under these hypotheses, the return intensity $w=|E|^2$ presents a pdf that can be expressed as [2]:

$$p_N(w) = \frac{b/\sqrt{w}}{\Gamma(M)}\left(\frac{b\sqrt{w}}{2}\right)^M \mathrm{K}_{M-1}(b\sqrt{w}), \qquad (3)$$

where the parameter M is related to the number of scatterers per resolution cell by the relation $M = N(1+\nu)$.

The distribution in (3) is a two parameter distribution, whose parameters are related to the mean and the normalized variance [2]. In fact, the distribution mean is

$$\langle w \rangle = \left(\frac{2}{b}\right)^2 M \qquad (4)$$

and the normalized variance is

$$\frac{\langle w^2 \rangle}{\langle w \rangle^2} - 1 = 1 + \frac{2}{M}, \qquad (5)$$

where <•> stands for statistical mean.

3. SIMULATION FRAMEWORK

The new simulation approach proposed in this paper is largely based on the SARAS simulator [6] and upgrades its capabilities to the simulation of non-fully developed speckle. The proposed simulator is based on sound physical (geometric and electromagnetic) models [4], allowing the evaluation of the reflectivity function of the scene, and on a model for the transfer function of the system, which is used for the evaluation of the SAR raw signal [6]. It requires as input a Digital Elevation Model (DEM), a synthetic description of the roughness within the resolution cell, the electromagnetic parameters of the surface and radar and orbital data of the sensor.

In particular, speckle simulation is performed in such a way that the spatial statistics of the output image are in accordance with the acquisition parameters of the selected SAR sensor [8]. In each resolution cell the speckle component is multiplied by the mean square value of the scattered field, which is computed using sound direct electromagnetic models [4], [7]. The speckle component is evaluated according to (1), as the sum of the stochastic contributions due to each single scatterer, and, as mentioned in the previous section, the amplitude factors V_i of each scatterer are distributed according to (3). We generate K-distributed pseudo-random numbers through a 2-D scale mixture of Gaussian model expressed as [5]:

$$\boldsymbol{Y} = \sqrt{Z}\boldsymbol{X}, \qquad (6)$$

where $\boldsymbol{X}$ is a two-dimensional zero-mean Gaussian variable with covariance matrix equal to the identity matrix and Z is a scalar Γ-distributed random variable independent from $\boldsymbol{X}$. With this assumptions the distribution of the modulus of $\boldsymbol{Y}$ turns out to be K-distributed, presenting the distribution in (3) with parameters related to the parameters of the considered Γ distribution [5].

4. RESULTS

In this section some of the obtained results are presented. In particular, we simulated four SAR images relevant to a flat surface with constant electromagnetic parameters in order to obtain a homogeneous scene. The acquisition parameters of the three images are the same and are relevant to an airborne SAR sensor. On the four images the number of equivalent scatterers per resolution cell N is assumed to be 1, 5, 10 and, finally, in the last case a fully developed Exponentially-distributed speckle is generated, using the algorithm already available in the SARAS simulator [6]. In all the non-developed speckle scenarios v is set to 1.

In Fig. 1 the four simulated intensity images are shown. As a matter of fact, it is very hard to draw significant conclusions only by visual inspection of the presented images. Actually, one thing can be immediately noted, i.e. a significant variation of the intensity from right to left: this is due to the change of the incidence angle between near and far range, which occurs also for completely homogeneous scenes. As discussed in the following, this behavior, typical of SAR images, strongly affects the statistics of the simulated images, at least in case of homogeneous areas.

The proposed simulation framework can be effectively used in order to test the ability in retrieving the number of equivalent scatterers per resolution cell from intensity data. In particular, we can use the expression of the normalized variance (5) in order to estimate the value of M, and, hence, assuming as known the value of v (which in our test cases is always set to 1), the value of N. From (5) M can be estimated as

$$M = \frac{2}{\frac{\langle w^2 \rangle}{\langle w \rangle^2} - 2}, \tag{7}$$

where the mean and the mean square value can be evaluated through spatial averaging on the simulated data and, since we assumed v=1, M=2N. Using (7) we estimated N from the images presented in Fig. 1. The obtained results are reported in Tab. I, along with synthetic statistical parameters of the simulated images.

Looking at Tab. I it is evident that the values of N are strongly underestimated using (7). This behavior can be partly ascribed to the above mentioned near range-far range effect, related to the variation of the incidence angle of the scene between near and far range also in presence of a flat surface. As a matter of fact, it would be interesting to correct our estimate taking into account this effect, in order to draw a more accurate assessment of the potentialities of the proposed simulation framework. As a first approximation, in our case, i.e. in presence of a fully homogeneous area, we can partially compensate this effect normalizing each range line with respect to its value. In fact, in our case study the incidence angle remains constant on each range line and, through the described procedure, we compensate heuristically the difference in the mean square values of the scattered field along different range lines. The resulting corrected simulated images are shown in Fig. 2, where it is possible to note that the near range-far range effect is no longer visible and the speckle pattern can be appreciated much better than in Fig. 1.

In Fig. 3 the histograms relevant to the corrected images are presented. From Fig. 3 it is evident that the behavior of the graphs tend to converge very quickly to the exponential distribution, i.e. to the distribution expected in case of fully developed speckle, as can also be appreciated looking at the images in Fig. 2. In fact, with N=10 scatterers the speckle can be said to be fully developed.

TABLE I
PARAMETERS ESTIMATED ON THE IMAGES

# figure	Estimated N without correction	Estimated N with correction	Expected N
1-2 (a)	0.93	1.6	1
1-2 (b)	1.8	8.4	5
1-2 (c)	2.1	20	10
1-2 (d)	2.5	∞	∞

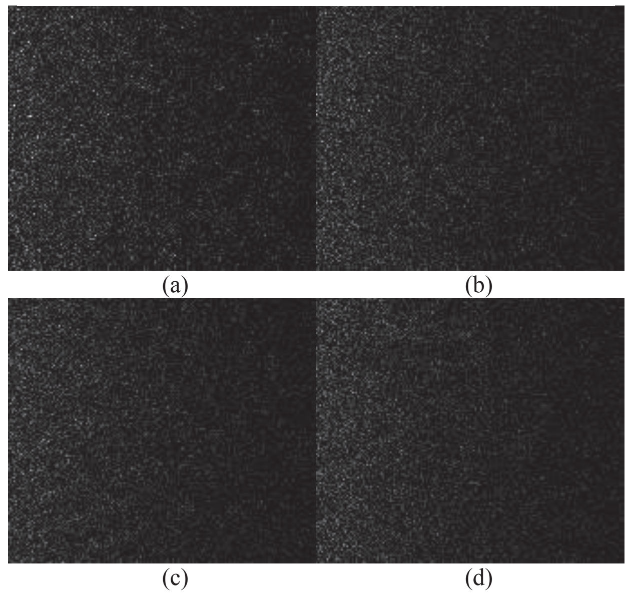

Fig. 1 Simulated images: (a) N=1; (b) N=5; (c) N=10; (d) Rayleigh.

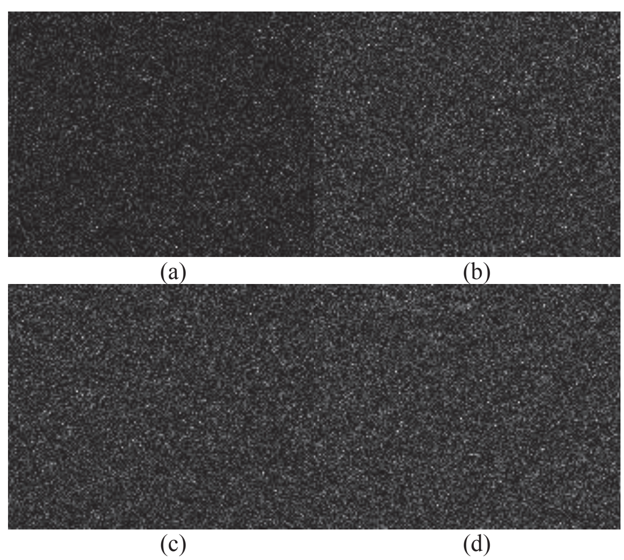

Fig. 2 Simulated images: (a) N=1; (b) N=5; (c) N=10; (d) Rayleigh.

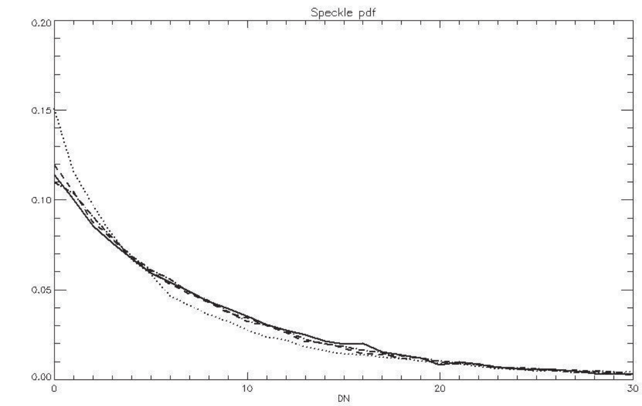

Fig. 3 Histograms of the simulated images when correction of the near range-far range effect is corrected: *N*=1 (dotted line), *N*=5 (dashed line) and *N*=10 (dash dot line), Rayleigh (solid line).

In Tab. I we report also the value of the equivalent number of scatterers estimated from the corrected images of Fig. 2. In this case the value of *N* is overestimated, i.e. it presents a behavior opposite to the one relevant to the previous case. Anyway, this second result can be considered very positive, because it shows that the different statistical behaviors of the speckle can be potentially discriminated: in fact, the difference in the estimated values of *N* for the various cases is significant. Conversely, the difference in the previously estimated values were too small, thus making the different speckle characteristics hardly distinguishable from one case to the other. However, further studies are necessary in order to gain a better insight on the influence of sensor parameters on simulated speckle statistics and on the role of the method used for estimating the distribution parameters [10].

5. CONCLUSIONS

In this paper a framework for the simulation of SAR images with K-distributed speckle statistics is introduced. The proposed approach assumes the knowledge of the number of scatterers per resolution cell, which for fBm surfaces has been analytically evaluated by the authors in a recent work. The synthesis of the adequate speckle statistics is accomplished through the introduction of a 2-D scale mixture of Gaussian variables in a SAR raw signal simulator previously developed by some of the authors. In the final section, the presented preliminary results highlight the potentialities of the proposed simulator, through the analysis of the statistical behavior of a small set of simulated images and a comparison with results relevant to the fully developed speckle case.

Finally, first results regarding the inversion of the proposed model are also presented and discussed. As a matter fact, further investigations are necessary in order to better understand the influence of significant sensor parameters on the overall statistical behavior of the simulated images. Moreover, the extension of the proposed simulation technique to non integer values of *N* will be an important step in order to study a wider range of practical situations (e. g., the case of sea surfaces).

6. ACKNOWLEDGMENTS

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7. REFERENCES

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