AN ANALYTICAL FORMULATION FOR THE CORRELATION OF SURFACE-SCATTERED FIELDS AT TWO BISTATIC RADAR RECEIVERS

Gerardo Di Martino, Alessio Di Simone, Antonio Iodice

Università di Napoli Federico II, Napoli, Italy

ABSTRACT

We present an analytical formulation of the correlation of the electromagnetic fields scattered by a rough or gently undulating surface and measured at two spatially separated points. The latter may be the positions of two different bistatic radar sensors receiving the scattered field at the same time, as in bistatic synthetic aperture radar interferometry, or two positions occupied by a single moving receiver at two slightly different times, as in Global Navigation Satellite System Reflectometry. The scattering surface is modeled as randomly rough, and the Kirchhoff Approximation is employed to compute the scattered field.

The obtained closed-form expression of the field correlation is substantially coincident with the one already available in literature for far-from-specular scattering directions; conversely, for close-to-specular directions, the obtained formulation shows that, as the surface correlation length increases, the degree of coherence smoothly increases from the value obtained with the expression available in literature for rough surfaces to a value close to unity for very gently undulating surfaces.

Index Terms— Bistatic radar, GNSS-R, coherence.

1. INTRODUCTION

Bistatic passive radars for remote sensing applications, exploiting transmitters of opportunity, are recently attracting much interest, due to the possibility to use light and cheap instrumentation. For instance, Global Navigation Satellite System Reflectometry (GNSS-R), exploiting navigation satellite signals, is by now a mature technology, especially for ocean observation [1]; recently, application to land monitoring has been also proposed [2]. In addition, bistatic Synthetic Aperture Radar (SAR) systems have been proposed, possibly implementing interferometric techniques [3]. Both in bistatic SAR interferometry and in GNSS-R, it is useful to compute the correlation of the electromagnetic fields scattered by a rough surface and measured at two spatially separated points. In fact, in bistatic SAR interferometry the two considered points are the positions of two different bistatic radar sensors receiving the scattered field at the same time, whereas in GNSS-R they represent two positions occupied by a single moving receiver at two slightly different times.

An analytical expression of the field correlation, in terms of sensor parameters and observation geometry, is available both for monostatic SAR interferometry [4-5] and for GNSS-R [1]: it is in agreement with the van Cittert-Zernike theorem and is obtained by modeling the scattering surface as composed of uncorrelated point-like scatterers. It holds if the surface is sufficiently rough, i.e., if its root mean square (rms) height is at least of the order of wavelength and its rms slope is at least of the order of 0.1, which is often the case for wind-driven sea surfaces at microwave frequency.

Here, by using the Kirchhoff Approximation (KA) we find a more general closed-form analytical expression of the correlation, that also holds for slightly undulating land surfaces, with height variations of the order of wavelength or larger (at least 10 cm at 1.5 GHz, so that the incoherent component still dominates) but over distances of the order of tens or hundreds of meters, so that the rms slope is of the order of 0.01 or even 0.001. At the best of our knowledge, it is the first time that such a closed form expression, also explicitly depending on surface parameters, is obtained for slightly undulating land surfaces. In fact, only numerical evaluations are currently available in literature [6].

2. THEORY

Let us consider a rough surface $z(x,y)$ whose mean plane is the x-y plane, modelled as a stationary zero-mean Gaussian random process with standard deviation σ and normalized (to σ^2) autocorrelation function $C(\Delta x,\Delta y)$, with $\Delta x=x'-x$, $\Delta y=y'-y$, (x,y) and (x',y') being two generic surface points. Although it is not strictly necessary, we will assume that the surface is statistically isotropic, so that $C(\Delta x,\Delta y) = C(\Delta x^2+\Delta y^2)$.

The geometry of the problem is depicted in Fig.1: we consider a single transmitter placed in $T\equiv(x_T,y_T,z_T)$, with $x_T=r_T\sin\vartheta_T\cos\varphi_T$, $y_T=r_T\sin\vartheta_T\sin\varphi_T$, $z_T=r_T\cos\vartheta_T$; and two slightly spaced receivers placed in $R_1\equiv(x_{R1},0,z_R)$ and $R_2\equiv(x_{R2},y_{R2},z_R)$, with $x_{R1}=r_{R1}\sin\vartheta_{R1}$, $z_R=r_{R1}\cos\vartheta_{R1}$, $x_{R2}=r_{R2}\sin\vartheta_{R2}\cos\varphi_{R2}=x_{R1}+B_x$, $y_{R2}=r_{R2}\sin\vartheta_{R2}\sin\varphi_{R2}=B_y$. We assume that $B_x<<r_{R1}$, $B_y<<r_{R1}$, so that

$$\begin{aligned}\vartheta_{R2}-\vartheta_{R1}&=\Delta\vartheta\cong\frac{B_x\cos\vartheta_{R1}}{r_{R1}}\\ \varphi_{R2}&\cong\frac{B_y}{r_{R1}\sin\vartheta_{R1}}\end{aligned}. \quad (1)$$

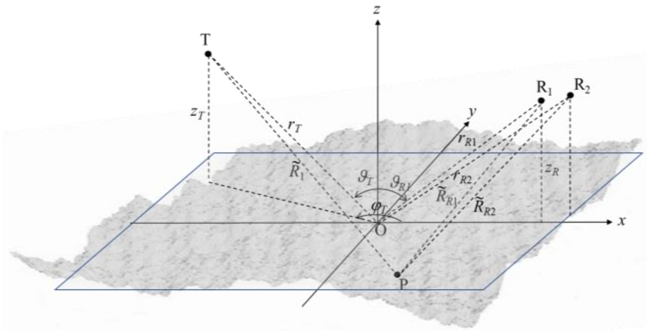

Figure 1: Geometry of the problem

The receiving sensors are pointed toward the origin of the reference system, so that the illuminated area is centered in the origin and its size is related to the sensors' spatial resolutions A_x along x and A_y along y. Note that we have chosen the reference system in such a way that the x-z plane coincides with the scattering plane of the first receiver.

By using the KA, the generic component of the scattered field at R_1 and R_2 can be written as

$$E(\mathrm{R}_1)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(x,y)w(x,y)\frac{\exp\left\{-jk\left[\tilde{R}_T(x,y)+\tilde{R}_{R1}(x,y)\right]\right\}}{\tilde{R}_T(x,y)\tilde{R}_{R1}(x,y)}dxdy$$
$$E(\mathrm{R}_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F(x,y)w(x,y)\frac{\exp\left\{-jk\left[\tilde{R}_T(x,y)+\tilde{R}_{R2}(x,y)\right]\right\}}{\tilde{R}_T(x,y)\tilde{R}_{R2}(x,y)}dxdy \qquad (2)$$

where $F(x,y)$ is a (slowly varying) function, whose expression is of no concern here, $k=2\pi/\lambda$ is the wavenumber, λ is the wavelength,

$$w(x,y)=\exp\left(-\frac{x^2}{2A_x^2}-\frac{y^2}{2A_y^2}\right) \qquad (3)$$

is the sensors' illumination function, and

$$\tilde{R}_X(x,y)=\sqrt{(z_X-z)^2+(x_X-x)^2+(y_X-y)^2}=$$
$$=R_X(x,y)\sqrt{1+\frac{z^2-2z_Xz}{R_X^2(x,y)}}\cong R_X(x,y)-\frac{z_X}{R_X(x,y)}z(x,y) \qquad (4)$$

with $R_X(x,y)=\sqrt{z_X^2+(x_X-x)^2+(y_X-y)^2}$, (5)

and with the subscript X that should be replaced by T, $R1$ or $R2$ as needed.

The correlation between these two fields is

$$<E(\mathrm{R}_1)E^*(\mathrm{R}_2)>\cong$$
$$\cong\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\exp\left\{-jk\left[R_T(x,y)+R_{R1}(x,y)-R_T(x',y')-R_{R2}(x',y')\right]\right\}}{R_T(x,y)R_{R1}(x,y)R_T(x',y')R_{R2}(x',y')}F(x,y)w(x,y)\cdot$$
$$\cdot F(x',y')w(x',y')<\exp\left\{jk\left[u_{z1}(x,y)z(x,y)-u_{z2}(x',y')z(x',y')\right]\right\}>dxdydx'dy' \qquad (6)$$

where <·> stands for statistical mean and

$$u_{z1,2}(x,y)=\frac{z_T}{R_T(x,y)}+\frac{z_R}{R_{R1,2}(x,y)} \quad . \qquad (7)$$

If σ is of the order of λ or larger (actually, it is sufficient that $\sigma\cong\lambda/2$, which for a frequency of 1.5 GHz corresponds to $\sigma\cong$10 cm) we have $k^2\sigma^2>>1$ and a procedure similar to the one of [5] can be followed, thus obtaining

$$<E(\mathrm{R}_1)E^*(\mathrm{R}_2)>\cong$$
$$\cong\frac{4\pi^2}{k^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\left|F(x,y)\right|^2w^2(x,y)\exp\left\{-jk\left[R_{R1}(x,y)-R_{R2}(x,y)\right]\right\}}{u_{z1}^2(x,y)R_T^2(x,y)R_{R1}^2(x,y)}\cdot$$
$$\cdot\exp\left\{-\frac{k^2\sigma^2}{2}\left[u_{z1}(x,y)-u_{z2}(x,y)\right]^2\right\}\cdot$$
$$\cdot\frac{L^2}{2\pi2\sigma^2}\exp\left\{-\frac{1}{2}\frac{u_x^2(x,y)+u_y^2(x,y)}{u_{z1}^2(x,y)2\sigma^2/L^2}\right\}dxdy \quad , \qquad (8)$$

where

$$u_x(x,y)=\frac{\partial(R_T+R_{R1})}{\partial x}\ ,\ u_y(x,y)=\frac{\partial(R_T+R_{R1})}{\partial y} \quad , \qquad (9)$$

L is the surface correlation length and we have assumed a Gaussian autocorrelation function, so that the roughness rms slope is $\sqrt{2}\,\sigma/L$.

The sensor illumination function w in (8) is peaked around the origin and is appreciably different from zero in the resolution cell size A_xA_y. For far-from-specular directions, the sensor resolution is of the order of c/B, where c is the speed of light and B is the bandwidth of the transmitted field, so that the resolution is of few meters for SAR systems and of about 300 m for C/A code GPS signals employed for GNSS-R applications. Therefore, in the resolution cell we can let

$$R_{R1}(x,y)-R_{R2}(x,y)\cong r_{R1}-r_{R2}+\eta_xx+\eta_yy \quad , \qquad (10)$$

where

$$\eta_x=\left.\frac{\partial(R_{R1}-R_{R2})}{\partial x}\right|_{\substack{x=0\\y=0}}\cong\frac{\cos^2\vartheta_{R1}B_x}{r_{r1}}$$
$$\eta_y=\left.\frac{\partial(R_{R1}-R_{R2})}{\partial y}\right|_{\substack{x=0\\y=0}}\cong\frac{B_y}{r_{r1}} \quad , \qquad (11)$$

and assume that all other functions in (8), namely $F(x,y)$, $u_{z1}(x,y)$, $u_x(x,y)$, and $u_y(x,y)$, are constant within the resolution cell and equal to their value in the origin F_0, u_{z10}, u_{x0}, and u_{y0}, respectively:

$$\begin{aligned}&< E(\mathrm{R}_1)E^*(\mathrm{R}_2) > \cong \frac{|F_0|^2 4\pi^2}{u_{z10}^2 k^2 r_T^2 r_{R1}^2}\frac{L^2}{2\pi 2\sigma^2}\exp\left\{-\frac{1}{2}\frac{u_{x0}^2+u_{y0}^2}{u_{z10}^2 2\sigma^2/L^2}\right\}\cdot\\ &\cdot\exp\left[-jk\left(r_{R1}-r_{R2}\right)\right]\exp\left\{-\frac{1}{2}\left[\frac{k\sigma\sin\vartheta_{R1}\cos\vartheta_{R1}B_x}{r_{R1}}\right]^2\right\}\cdot\\ &\cdot\int_{-\infty}^{\infty}\exp\left\{-\frac{x^2}{A_x^2}\right\}\exp\left[-jk\eta_x x\right]dx\int_{-\infty}^{\infty}\exp\left\{-\frac{y^2}{A_y^2}\right\}\exp\left[-jk\eta_y y\right]dy =\end{aligned} \tag{12}$$

$$\begin{aligned}&=< \left|E(\mathrm{R}_1)\right|^2 > \exp\left[-jk\left(r_{R1}-r_{R2}\right)\right]\exp\left\{-\frac{1}{2}\left[\frac{k\sigma\sin\vartheta_{R1}\cos\vartheta_{R1}B_x}{r_{R1}}\right]^2\right\}\cdot\\ &\cdot\exp\left\{-\left[\frac{kA_x\cos^2\vartheta_{R1}B_x}{2r_{R1}}\right]^2\right\}\exp\left\{-\left[\frac{kA_yB_y}{2r_{R1}}\right]^2\right\}\end{aligned} \tag{13}$$

This is the generalization to the bistatic case of the results of [5] for monostatic SAR interferometry (but here the illumination is a Gaussian instead of a sinc function). If, as it is always the case, $A_x >> \sigma$, the second exponential in (13) can be assumed unitary, and the critical baseline along x can be computed by equating the argument of the third exponential in (13) to minus one:

$$B_{xc} = \frac{\lambda r_{R1}}{\pi A_x \cos^2\vartheta_{R1}} \quad , \tag{14}$$

in agreement with available literature.

However, in specular direction, i.e., $\vartheta_T=\vartheta_{R1}$ and $\varphi_T=\pi$, which is usually of interest for GNSS-R, we have that $u_x(0,0)=u_y(0,0)=0$, so that also the last exponential in (8) is peaked around the origin. In addition, the sensor resolution is of the order of $\sqrt{r_{R1}c/B}$, i.e., of the order of tens of km for spaceborne GNSS-R receivers or of one km for airborne ones. Accordingly, (12) is replaced by

$$\begin{aligned}&< E(\mathrm{R}_1)E^*(\mathrm{R}_2) > \cong \frac{|F|^2 4\pi^2}{k^2 4\cos^2\vartheta_{R1} r_T^2 r_{R1}^2}\frac{L^2}{2\pi 2\sigma^2}\cdot\\ &\cdot\exp\left[-jk\left(r_{R1}-r_{R2}\right)\right]\exp\left\{-\frac{1}{2}\left[\frac{k\sigma\sin\vartheta_{R1}\cos\vartheta_{R1}B_x}{r_{R1}}\right]^2\right\}\cdot\\ &\cdot\int_{-\infty}^{\infty}\exp\left\{-\left(\frac{1}{A_x^2}+\frac{1}{G_x^2}\right)x^2\right\}\exp\left[-jk\eta_x x\right]dx\cdot\\ &\cdot\int_{-\infty}^{\infty}\exp\left\{-\left(\frac{1}{A_y^2}+\frac{1}{G_y^2}\right)y^2\right\}\exp\left[-jk\eta_y y\right]dy\end{aligned} \quad , \tag{15}$$

where

$$\begin{aligned}G_x &= \frac{4\sigma}{L\cos\vartheta_{R1}}\frac{r_T r_{R1}}{r_T + r_{R1}} \cong \frac{4\sigma r_{R1}}{L\cos\vartheta_{R1}}\\ G_y &= \frac{4\sigma\cos\vartheta_{R1}}{L}\frac{r_T r_{R1}}{r_T + r_{R1}} \cong \frac{4\sigma\cos\vartheta_{R1} r_{R1}}{L}\end{aligned} \tag{16}$$

are the x and y sizes of the "glistening area", i.e., of the surface area which provides most of the energy scattered in the specular direction, and we have used in (8)

$$\begin{aligned}\left(\frac{u_x}{u_{z1}}\right)^2 &\cong \left(\frac{1}{u_{z1}}\frac{\partial u_x}{\partial x}\bigg|_{\substack{x=0\\y=0}}\right)^2 x^2 = \left(\cos\vartheta_{R1}\frac{r_T + r_{R1}}{2r_T r_{R1}}\right)^2 x^2\\ \left(\frac{u_y}{u_{z1}}\right)^2 &\cong \left(\frac{1}{u_{z1}}\frac{\partial u_y}{\partial y}\bigg|_{\substack{x=0\\y=0}}\right)^2 y^2 = \left(\frac{r_T + r_{R1}}{2\cos\vartheta_{R1} r_T r_{R1}}\right)^2 y^2\end{aligned} \quad . \tag{17}$$

Accordingly, (13) is replaced by

$$\begin{aligned}&< E(\mathrm{R}_1)E^*(\mathrm{R}_2) > = < \left|E(\mathrm{R}_1)\right|^2 > \exp\left[-jk\left(r_{R1}-r_{R2}\right)\right]\cdot\\ &\cdot\exp\left\{-\frac{1}{2}\left[\frac{k\sigma\sin\vartheta_{R1}\cos\vartheta_{R1}B_x}{r_{R1}}\right]^2\right\}\exp\left\{-\left[\frac{kA'_x\cos^2\vartheta_{R1}B_x}{2r_{R1}}\right]^2\right\}\exp\left\{-\left[\frac{kA'_yB_y}{2r_{R1}}\right]^2\right\}\end{aligned} \tag{18}$$

where

$$A'_{x,y} = \frac{A_{x,y}G_{x,y}}{\sqrt{A_{x,y}^2 + G_{x,y}^2}} \quad . \tag{19}$$

For wind-driven sea surfaces, σ/L is of the order of 0.1, so that, especially for spaceborne receivers, $A_{x,y} << G_{x,y}$ and we recover (17) and, hence, (18), that we here rewrite in terms of correlation time τ:

$$\tau = \frac{\lambda r_{R1}}{\pi v A_x \cos^2\vartheta_{R1}} \quad , \tag{20}$$

where v is the receiver velocity, assumed along x (although the formula for an arbitrary velocity direction in the horizontal plane can be easily obtained).

However, for very gently undulating soil surfaces, σ/L may be of the order of 0.01 or even 0.001 (for instance, for σ = 0.1 m and L=100 m) so that the glistening area size $G_{x,y}$ may be of the same order, or even much smaller, than the resolution $A_{x,y}$. For $G_{x,y} << A_{x,y}$ we have $A'_{x,y} \cong G_{x,y}$, so that (20) is replaced by

$$\tau = \frac{\lambda r_{R1}}{\pi v G_x \cos^2\vartheta_{R1}} = \frac{\lambda L}{4\pi\sigma v\cos\vartheta_{R1}} \quad , \tag{21}$$

and it can be verified that the last expression in (21) holds for an arbitrary velocity direction in the horizontal plane. This expression shows that, for large L, correlation time linearly increases with L.

Finally, when $A_{x,y}$ and $G_{x,y}$ are of the same order, the correlation time is

$$\tau = \frac{\lambda r_{R1}}{\pi v A_x' \cos^2 \vartheta_{R1}} \quad . \tag{22}$$

3. NUMERICAL RESULTS

In Fig. 2 we show the plots of the correlation coefficient

$$\rho = \frac{< E(\mathrm{R}_1) E^*(\mathrm{R}_2) >}{\sqrt{<|E(\mathrm{R}_1)|^2 >< |E(\mathrm{R}_2)|^2 >}} \cong \frac{< E(\mathrm{R}_1) E^*(\mathrm{R}_2) >}{<|E(\mathrm{R}_1)|^2 >} \tag{23}$$

as a function of the surface correlation length L for σ=10 cm, for B_x varying from 2 m to 10 m, B_y=0 and for the system parameters of a spaceborne GNSS-R receiver, listed in Table I. The plots are obtained by using (18). They clearly show that as the surface correlation length increases, the correlation coefficient smoothly increases from the value obtained with the expression available in literature for rough surfaces to a value close to unity for gently undulating surfaces.

Finally, in Fig. 3, by using (22), we plot the correlation time as a function of L for σ=10 cm, for the system parameters of Table I.

4. CONCLUSION

A closed-form expression of the correlation of the fields scattered by a rough or gently undulating surface towards two slightly spaced receivers has been obtained. The proposed expression generalizes to the bistatic case the available result for classic SAR interferometry, and it generalizes to the case of gently undulating surfaces the available result for GNSS-R. This last result finds an important application in the evaluation of correlation time for flat land surfaces, which can be modeled as slightly undulating surfaces with extremely small rms slope.

5. REFERENCES

[1] C. Zuffada, T. Elfouhaily, S. Lowe, “Sensitivity analysis of wind vector measurements from ocean reflected GPS signals,” *Remote Sens. Environ.*, vol. 88, no. 3, pp- 341-350, May 2003.

[2] D. Comite, F. Ticconi, L. Dente, L. Guerriero, N. Pierdicca, “Bistatic coherent scattering from rough soils with application to GNSS reflectometry,” *IEEE Trans. Geosci. Remote Sens.*, vol. 58, no. 1, pp. 612-625, Jan. 2020.

[3] G. Krieger, A. Moreira, “Spaceborne bi- and multistatic SAR: Potential and challenges,” *Proc. Inst. Elect. Eng.- Radar, Sonar Navig.*, vol.153, no. 3, pp. 184–198, Jun. 2006.

[4] H. A. Zebker, J. Villasenor, “Decorrelation in interferometric radar echoes,” *IEEE Trans. Geosci. Remote Sens.*, vol. 30, no. 5, pp. 950-959, Sep. 1992.

[5] G. Franceschetti, A. Iodice, M. Migliaccio, D. Riccio, “The effect of surface scattering on IFSAR baseline decorrelation,” *J. Electrom. Waves Applications*, vol. 11, no. 3, pp. 353-370, 1997.

[6] D. Comite, N. Pierdicca, "Decorrelation of the Near-Specular Land Scattering in Bistatic Radar Systems," *IEEE Trans. Geosci. Remote Sens.*, vol. 60, pp. 1-13, 2022.

Table I: Spaceborne GNSS-R system parameters

Parameter	Symbol	Value
Wavelength	λ	20 cm
Sensors' height	z_R	540 km
Incidence angle	ϑ_T	45 deg
Signal bandwidth	B	1 MHz
Sensors' velocity	v_x	7 km/s

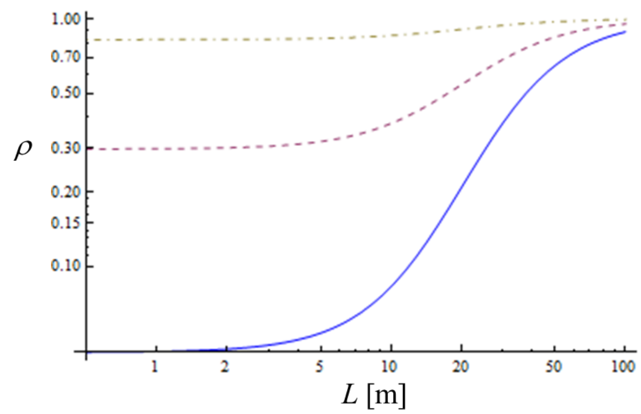

Figure 2: Correlation coefficient vs. correlation length for the system parameters of Table I and spatial baseline B_x equal to 2 m (dot-dashed line), 5 m (dashed line) and 10 m (solid line).

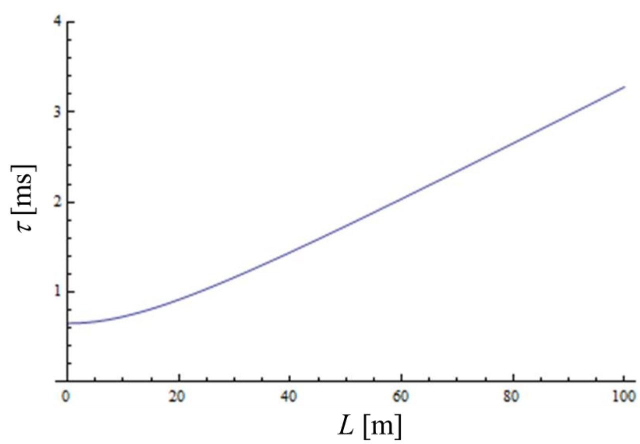

Figure 3: Correlation time vs. correlation length for for the system parameters of Table I.