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SCATTERING ALONG THE SPECULAR DIRECTION FROM THE SEA MODELED AS A FRACTAL SURFACE

Gerardo Di Martino, Alessio Di Simone, Antonio Iodice, Daniele Riccio, Giuseppe Ruello

Università di Napoli Federico II
Dipartimento di Ingegneria Elettrica e delle Tecnologie dell'Informazione

ABSTRACT

Models linking the electromagnetic field scattered from the sea surface along the specular direction to the speed of the wind blowing over the surface are of fundamental importance for wind speed retrieval via Global Navigation Satellite System Reflectometry (GNSS-R). In this work, by modelling the sea surface as a fractional Brownian motion (fBm) random process and using the Kirchhoff approximation (KA), we express the sea bistatic normalized radar cross section (NRCS) σ^0 at specular direction directly in terms of sea surface spectrum parameters, and hence of wind speed. This avoids the need of intermediately computing the large-scale sea surface slope variance, which in turn would require the definition of a somewhat arbitrary cut-off surface wavenumber. We show that the obtained theoretical relationship between wind speed and σ^0 is in reasonable agreement with the empirical ones available in literature.

Index Terms— Electromagnetic scattering; fractional Brownian motion; GNSS-R; wind speed retrieval.

1. INTRODUCTION

Evaluation of electromagnetic scattering from the sea surface along the specular direction is of great interest in remote sensing applications, since this quantity is related to the speed of the wind blowing over the sea. For instance, wind speed retrieval is one of the main applications of Global Navigation Satellite System Reflectometry (GNSS-R).

The usual approach employed to evaluate the sea bistatic normalized radar cross section (NRCS) σ^0 at specular direction is the geometrical optics (GO). According to GO, σ^0 is inversely proportional to the sea surface long-wave slope variance, which is in turn related to wind speed via the sea surface power spectrum. However, computation of the long-wave slope variance from the sea surface spectrum requires the truncation of the spectrum at a cut-off wavenumber, whose choice is, to some extent, arbitrary. It must be also noted that the sea surface has an approximately power-law spectrum, so that the computed surface slope variance is significantly dependent on the cut-off wavenumber, and it diverges with the latter. Accordingly, instead of computing the long-wave slope variance from the spectrum, very often empirical relationships are employed to relate wind speed to long-wave slope variance [1] or directly to σ^0 [2].

In this work we show that by modelling the sea surface as a fractional Brownian motion (fBm) random process and using the Kirchhoff approximation (KA) or, equivalently (at specular direction), the first-order small-slope approximation (SSA1), it is possible to express σ^0 directly in terms of sea surface spectrum parameters, so getting rid of the cut-off wavenumber and obtaining a theoretical relationship between wind speed and σ^0 in reasonable agreement with the empirical ones.

2. THEORY

A two-dimensional (2D) fBm is a random process $z(x,y)$ whose increments $z(x,y) - z(x',y')$ over a fixed horizontal distance $\tau = \sqrt{(x-x')^2 + (y-y')^2}$ are zero-mean Gaussian random variables with variance $s^2\tau^{2H}$, where s is a parameter measured in m^{1-H}, and H is the Hurst coefficient, with $0<H<1$ [3-4]. Realizations of the 2D fBm process are fractal surfaces with fractal dimension $D=3-H$. The power spectral density (PSD) of an fBm follows a power law [3-5]:

$$S(\kappa_x, \kappa_y) = S_0 \kappa^{-\alpha} \quad , \tag{1}$$

where κ_x, κ_y are the x and y components of the surface wavenumber vector, $\kappa = \sqrt{\kappa_x^2 + \kappa_y^2}$ is its modulus, and [4-5]

$$\alpha = 2 + 2H \quad , \tag{2}$$

$$S_0 = \pi H 2^{1+2H} \frac{\Gamma(1+H)}{\Gamma(1-H)} s^2 \quad , \tag{3}$$

where $\Gamma(\cdot)$ is the gamma function. According to KA or SSA1, the co-polarized bistatic NRCS of an fBm surface along the specular direction is [5-6]

$$\sigma_{pp}^0 = \left|R_p(\vartheta)\right|^2 2k^2\cos^2\vartheta \int_0^\infty \exp(-2k^2\cos^2\vartheta\, s^2\tau^{2H})\tau d\tau = \frac{k^2\cos^2\vartheta \left|R_p(\vartheta)\right|^2 \Gamma(1/H)}{H\,(2k^2\cos^2\vartheta\, s^2)^{1/H}} \quad , \tag{4}$$

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where $k=2\pi/\lambda$ is the electromagnetic wavenumber, λ is the electromagnetic wavelength, ϑ is the incidence angle, and $R_p(\vartheta)$ is the Fresnel reflection coefficient at vertical (p=v) or horizontal (p=h) polarization. For circular RL polarization, which is the case of interest for GNSS-R applications, in (4) R_p must be replaced by the arithmetic average of Fresnel coefficients at vertical and horizontal polarizations. Note that for $H\rightarrow1$, i.e., $D\rightarrow2$ (non-fractal limit), (4) reduces to $\sigma_{pp}^0 = |R_p|^2/(2s^2)$, which is the usual GO expression, with s^2 assuming the meaning of surface slope variance.

As it can be seen in Fig. 1, the integrand in (4) is non-negligible only for values of τ ranging from about $\tau_0/10$ to about $4\tau_0$, where

$$\tau_0 = \frac{1}{(2k^2\cos^2\vartheta\, s^2)^{\frac{1}{2H}}} \quad . \tag{5}$$

For an incident wave frequency of 1.5 GHz, typical of GNSS-R applications, and for typical values of s^2 for natural surfaces, this corresponds to values of τ ranging from about 2 to about 80 cm, i.e., to surface wavenumbers $\kappa\sim2\pi/\tau$ from about 8 to about 314 m^{-1}. If in this range of wavenumbers the PSD of a surface can be expressed by (1), then (4) can be employed to compute its specular NRCS. This is the case for many natural surfaces [3-6], and in particular for the sea surface: we found that in the considered range of values of κ, for wind speeds from about 5 to about 30 m/s, and for a fetch of at least 150 km, a very good agreement is obtained between the Elfouhaily sea surface spectrum [7] and (1) with

$$\alpha = 3.6 + 0.4\exp\left(-\frac{u_{10}^2}{u_0^2}\right) \quad , \tag{6}$$

$$S_0 = C\alpha_m\left[1 + \exp\left(-\frac{u_{10}^2}{u_0^2}\right)\right] \quad , \tag{7}$$

where u_{10} is the wind speed at 10 m over the sea level, u_0 =10 m/s, C=0.692 m$^{4-\alpha}$, and α_m is the is the generalized Phillips-Kitaigorodskii equilibrium range parameter, whose expression as an increasing function of wind speed is provided in [7]. The agreement is illustrated in Fig. 2 for the cases of $u_{10} = 10$ m/s and $u_{10} = 20$ m/s with a fully developed sea.

Actually, in order to ensure continuity of the derivative of the function $\alpha_m(u_{10})$, the relation in [7] has been slightly modified as follows:

$$\alpha_m = 0.01\left\{\left[1 + 3\ln\left(\frac{u^*}{c_m}\right)\right]\left[1 - \exp\left(-\frac{u^*}{c_m}\right)^3\right] + \left[1 + \ln\left(\frac{u^*}{c_m}\right)\right]\exp\left(-\frac{u^*}{c_m}\right)^3\right\} \tag{8}$$

where $c_m = 0.23$ m/s and [7-8] $u^* = \sqrt{C_d}u_{10}$, with C_d provided in [8].

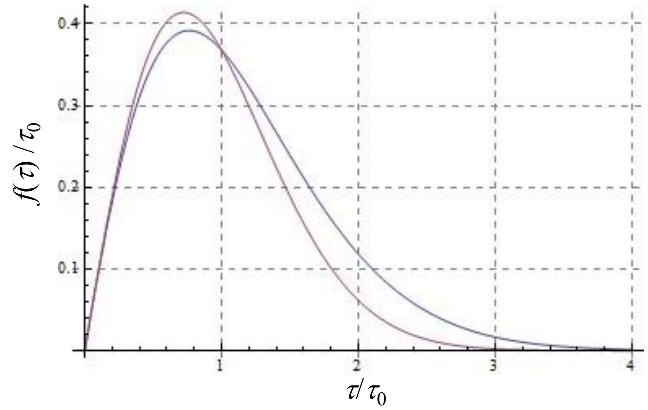

Figure 1: Plots of the integrand of (4), $f(\tau) = \tau\exp[-(\tau/\tau_0)^{2H}]$ and τ_0 given by (5), for H=0.75 (blue line) and H=0.9 (red line).

3. RESULTS

For each value of the wind speed u_{10}, from (6) and (7) it is possible to compute α and S_0, then inverting (2) and (3) H and s^2, and finally from (4) σ^0. In this way a relation between wind speed and σ^0 can be obtained, and it is plotted in Fig. 3 for $\vartheta = 30°$. The result obtained by using GO and the surface slope variance empirical expression of [1] is also plotted for reference in Fig. 3. In both cases, the complex relative dielectric constant is assumed equal to 75–j61.

These results can be compared with the empirical relationship of [2], illustrated in Fig. 4 of [2]. By comparing that figure with our Fig. 3, we can conclude that the proposed relationship, obtained from (4), is in better agreement with the empirical relationship of [2] with respect to the GO result, especially for $u_{10} > 10$ m/s, although both models overestimate the NRCS at high wind velocity.

4. CONCLUSION

We have shown that by modelling the sea surface as an fBm random process and using the KA or, equivalently, the SSA1, it is possible to express the specular NRCS σ^0 directly in terms of sea surface spectrum parameters, with no need of computing the large-scale surface slope variances and so getting rid of the cut-off wavenumber. In particular, we have found that the fBm model accurately describes the sea surface at the scale lengths actually involved in the scattering phenomenon at 1.5 GHz for wind speeds from about 5 to about 30 m/s, and for a fetch of at least 150 km. The corresponding theoretical relationship between wind speed and σ^0 has turned out to be in reasonable agreement with the empirical model of [2].

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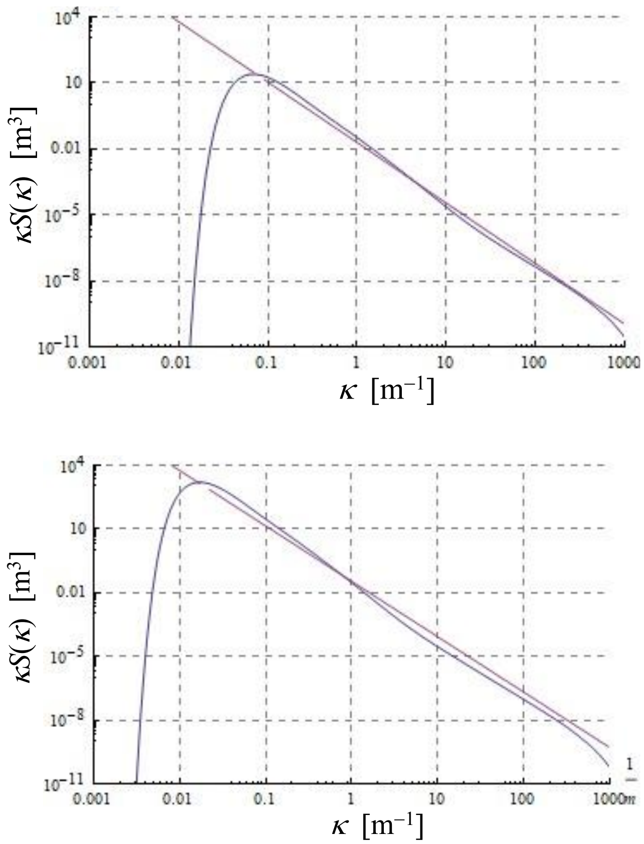

Figure 2: Elfouhaily spectrum (blue line) and its power-law approximation (red line). $u_{10} = 10$ m/s (top) and $u_{10} = 20$ m/s (bottom).

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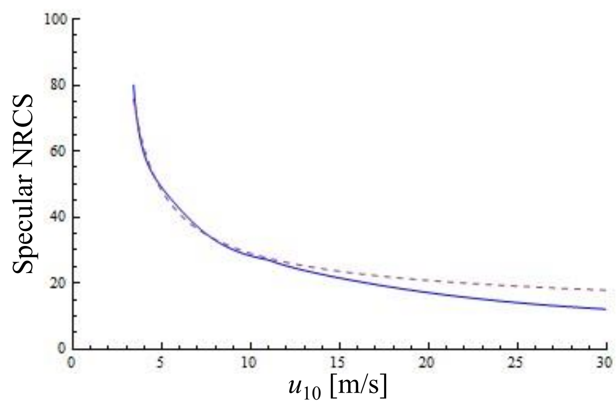

Figure 3: Specular NRCS vs. wind speed. Proposed method (solid line) and GO (dashed line). $\vartheta = 30°$.

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