Coprime Synthetic Aperture Radars

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***Abstract*— In this paper we present a comprehensive discussion regarding the application to Synthetic Aperture Radar of coprime array concepts. The proposed coprime SAR configurations, which we name CopSAR and OrthoCopSAR depending on the used waveforms, can be used to reduce the amount of data to be stored and processed and to increase the range swath, in case of maritime scenes made up of bright targets over a dark background. The costs to be paid are the reduction of the target-to-background ratio and the presence of a (non-stringent) limit on the maximum ship size. Performance indicators and sample results relevant to all the proposed implementations are reported.**

***Keywords*— Coprime arrays, orthogonal waveforms, synthetic aperture radar**

I. Introduction

The requirements of high resolution and coverage of Synthetic Aperture Radar (SAR) systems for maritime applications dictate the need to manage storing and processing of a huge amount of data. Therefore, the development of techniques able to reduce the data rate is of great interest. In addition, methods to extend the size of the range swath of SAR systems are also very relevant for monitoring and surveillance purposes. Accordingly, in recent years different techniques have been proposed to accomplish these goals [1]-[5]. In particular, the possibility to exploit the coprime sensing approach [6]-[7] for both data rate reduction and range swath extension has been recently discussed by the authors [8]-[9]. In this paper, for the first time, we present a comprehensive discussion regarding the ways of applying to SAR the coprime sensing concepts. The proposed approaches are able, in the case of bright targets over a dark (not necessarily homogeneous) background to reduce the amount of data and, in some cases, to increase the range swath, with no geometric resolution loss [8]-[9]. Therefore, the proposed approach is suitable for maritime surveillance applications.

The general approach consists in the transmission of two interlaced sequences of pulses, with sub-Nyquist pulse repetition frequencies (PRFs), where the PRFs or the signal frequencies are related via coprime numbers. Each sequence is separately processed via standard SAR processing, and the two final aliased images are combined in a very simple way to cancel out aliasing. When the two sequences are made up of pulses of the same kind we name the technique CopSAR [8], conversely, when use is made of mutually orthogonal pulses we name it OrthoCopSAR [9].

More specifically, three different CopSAR implementations are discussed in the following: they are aimed at achieving data reduction and/or range swath extension. As for the OrthoCopSAR [9] technique, it can be seen as an enhancement of the CopSAR basic implementation based on the transmission of up- and down-chirps, which are mutually (quasi) orthogonal waveforms [10]. With this implementation, we can achieve both data reduction and range swath extension with no appearance of ghosts and no resolution loss.

All the above-described coprime implementations only imply a reduction of the target-to-background ratio (TBR) and the presence of a (non-stringent) limit on maximum ship size [8]-[9]. In the following, the proposed coprime configurations' performances are discussed. In particular, in Section II the different coprime configurations are presented and their performances are compared. In Section III meaningful experimental results (both on simulated and actual data) are reported and discussed. Finally, in Section IV some conclusions are drawn.

II. Coprime SAR Configurations

In this section, the coprime SAR configurations are analyzed and compared. The first main distinction is based on the type of pulses used in the two sequences. We first present the CopSAR implementations, which make use of the same type of pulses in the two sequences; then we consider the OrthoCopSAR case, which is based on the use of mutually orthogonal pulses in the two sequences.

A. CopSAR

The basic CopSAR configuration is based on the transmission of two interlaced sequences of pulses, one at $PRF_1=PRF_0/N_1$, and the other at $PRF_2=PRF_0/N_2$, where PRF_0 satisfies the Nyquist condition (i.e. $PRF_0 \geq 2v/L$, with v the uniform sensor velocity and L the real azimuth antenna length) and N_1 and N_2 are coprime integers. The two sequences can be individually processed, thus obtaining two SAR images, i.e., $s_1(x,r)$ and $s_2(x,r)$, whit x and r standing for azimuth and range coordinates, respectively. If the entire SAR system bandwidth is processed, the geometric resolution of these images will not change with respect to the standard SAR case, but, of course, the obtained images will be severely aliased. However, if the scene is made up of bright targets placed on a dark background (e.g. in case of ships over the sea), only the true targets will be present on both aliased images at the same location, whereas aliased targets will appear at different locations on the two

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images. In particular, the azimuth displacement of replicas for $s_1(x,r)$ and $s_2(x,r)$ are

$$\Delta x_{i_1} = i_1 \frac{\mathrm{PRF}_0 \lambda r_0}{N_1 2v} \quad \Delta x_{i_2} = i_2 \frac{\mathrm{PRF}_0 \lambda r_0}{N_2 2v}, \tag{1}$$

where λ is the wavelength, and i_1 and i_2 are integers, so that replicas on the two images will not be at the same location unless $i_1/i_2=N_1/N_2$. Since N_1 and N_2 are coprime, this only happens if $i_1=iN_1$ and $i_2=iN_2$, i.e. only at the positions of the replicas in the image that would be obtained in the standard acquisition mode. These replicas will be strongly attenuated by the azimuth antenna pattern and will be neglected in the following discussions. Therefore, using the simple combination rule

$$s(x,r) = \begin{cases} s_1(x,r) & \text{if } |s_1(x,r)| < |s_2(x,r)| \\ s_2(x,r) & \text{otherwise} \end{cases}, \tag{2}$$

the obtained $s(x,r)$ image is not affected by aliasing. However, since the minimum time separation between the pulses of the two sequences is still equal to $1/\mathrm{PRF}_0$, the range swath cannot be increased with respect to standard SAR without the appearance of significant range ambiguities.

A doubling of the range swath can be obtained via the missing pulse implementation, i.e. avoiding to transmit in the sequence of pulses transmitted at rate PRF_0/N_1, those pulses that would be separated by just $1/\mathrm{PRF}_0$ from a pulse of the other sequence. In this case, in the image $s_1(x,r)$ each bright target will have not only replicas displaced according to (1), but also replicas at distances

$$\Delta x_{i_{12}} = i_{12} \frac{\mathrm{PRF}_0 \lambda r_0}{N_1 N_2 2v}, \tag{3}$$

and these replicas will overlap with a replica of the second image for $i_{12}=i_2N_1$, so that several "ghosts" will appear on the final image $s(x,r)$. However, these ghosts will be attenuated with respect to the true target by a factor N_2 due to the reduced number of pulses that contribute to focus them. Moreover, there will be also the attenuation due to the slight replica defocusing [8].

A more significant extension of the range swath, with no appearance of any ghost, can be obtained via the dual frequency implementation. In this case, the two sequences of pulses are both transmitted at rate $\mathrm{PRF}_1=\mathrm{PRF}_0/N_1$, but the first is transmitted at frequency f_1 and the second at frequency $f_2=(N_2/N_1)f_1$, so that wavelengths for the two sequences are related by $\lambda_2=(N_1/N_2)\lambda_1$. According to (1), replicas' locations will be the same of the CopSAR basic implementation. Pulses at the two frequencies can be transmitted at the same time, if two different antennas are employed, or they can be interlaced, if a single antenna is used [8].

B. *OrthoCopSAR*

Until now, we considered the case in which the waveforms used in the two sequences are of the same kind. In the OrthoCopSAR technique, the pulses of the two interlaced subsampled sequences are mutually (quasi) orthogonal: in particular, different waveforms are transmitted in the first and in the second sequence, i.e. up-chirp and down-chirp ones. Since these waveforms are quasi orthogonal [9], non-aliased images can be obtained by processing each sequence via the appropriate up- or down-chirp matched filter in the range compression step. So, in the final focused image the energy of the unfocused target contribution due to the presence of the mismatched chirp will be spread over a length corresponding to two pulse duration intervals [9]. The proposed technique can be implemented also on single-antenna SAR sensors. In this case, "blind ranges", arising since the sensor cannot receive during the transmission interval, are present in the raw signal: however, when doubling the range swath, this only happens for pulses at distance $1/\mathrm{PRF}_0$ (i.e., two out of N_1N_2). Therefore, there will be no blind range on the final image, but only the possible appearance of strongly attenuated azimuth replicas, similar to those of the CopSAR missing-pulse implementation, over a specific slant range interval.

C. *Comparison*

In Table I meaningful performance indicators are reported: data rate reduction, i.e. the ratio between the number of pulses transmitted in coprime mode and in standard SAR mode, and maximum range swath extension, i.e. the ratio between the CopSAR range swath size and the standard SAR one. The maximum azimuth target size to avoid replicas is the same for all the configurations and is obtained from (3), setting $i_{12}=1$. A discussion on TBR reduction can be found in [8].

Each coprime SAR configuration obtains different results in terms of data rate reduction and range swath extension. However, while the CopSAR basic and missing pulse implementations can be devised with standard SAR systems, CopSAR dual frequency and OrthoCopSAR implementations need ad hoc hardware modifications. However, these modifications are not difficult to be implemented, especially in the OrthoCopSAR case, where no change of the radiating element is necessary. This is particularly interesting since the OrthoCopSAR implementation obtains the best performances in terms of maximum attainable range swath extension, whereas the performance in terms of data rate reduction is quite similar to that of the basic CopSAR implementation.

TABLE I. PERFORMANCE INDICATORS

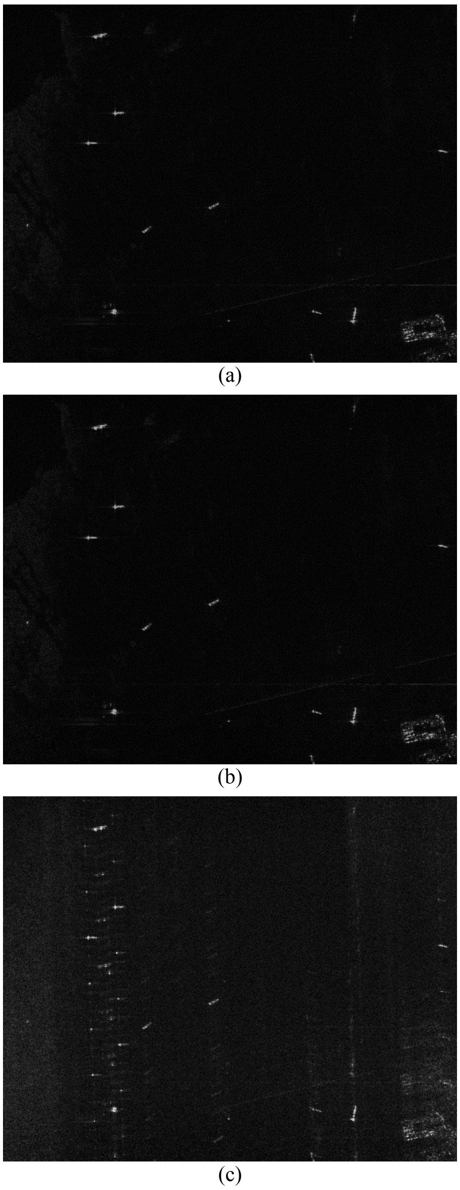
Configuration	Data rate reduction	Maximum range swath extension
CopSAR basic	$\frac{N_1+N_2-1}{N_1N_2}$	1
CopSAR missing pulse	$\frac{N_1+N_2-3}{N_1N_2}$	2
CopSAR dual frequency	$\frac{2}{N_1}$	N_1 (single antenna: $N_1/2$)
OrthoCopSAR	$\frac{N_1+N_2}{N_1N_2}$	N_1

III. Experimental Results

In this section, we present sample results relevant to the different coprime SAR configurations. When possible, i.e. in case of the CopSAR basic and missing pulse implementations, we present the results obtained for actual SAR images. For the other coprime configurations, we provide results obtained on simulated images.

A. Actual data: basic and missing pulse CopSAR

The CopSAR approach has been applied to a real ERS-2 SAR raw signal relative to a marine scene near the Long Beach Harbor, Los Angeles, CA, USA. The scene contains several ships and it is close to the coast; moreover, the sea background is not uniform: therefore, this scene is especially challenging for the coprime approach. The two undersampling factors are chosen to be N_1=5 and N_2=6. In Fig.1 we present a comparison of the images obtained via standard SAR processing and CopSAR basic and missing pulse processing. A spatial multilook over 4 x 1 (azimuth x range) pixel windows has been applied to obtain an approximately square pixel. Several attenuated ghosts are still visible in the missing pulse implementation image, whereas no ghost can be appreciated in the CopSAR basic implementation case.



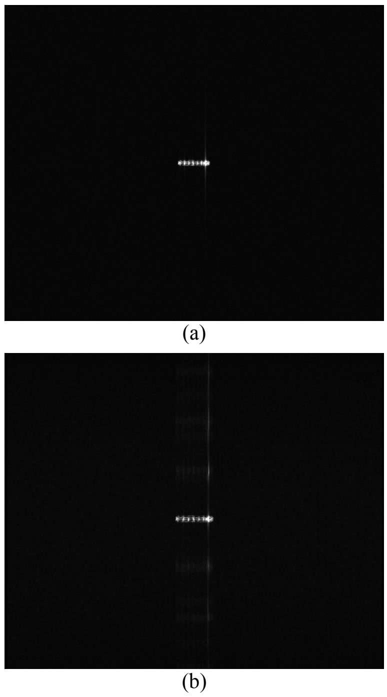
(a) (b) (c)

Fig. 1. ERS experiments: (a) standard SAR; (b) CopSAR basic; (c) CopSAR missing pulse.

B. Simulated data

CopSAR dual frequency implementation requires the adoption of two sequences with coprime carrier frequencies: therefore, it cannot be tested using raw data of existing SAR sensor. Similarly, the OrthoCopSAR implementation requires the use of mutually orthogonal signals for the two sequences, so that it cannot be demonstrated using actual SAR data. In the following, we test these two implementations on data simulated using the SARAS raw signal simulator [11].

1) CopSAR dual frequency implementation: We used the simulated SAR raw signal obtained by employing the system parameters of the Sentinel-1 SAR sensor in stripmap acquisition mode. In particular, we simulate two simultaneous acquisitions using two coprime frequencies such that the ratio between them is equal to N_2/N_1 = 6/5. For both raw signals, the PRF has been reduced by a factor N_1 = 5 with respect to the real sensor's one. The considered scenario is a very large ship (whose reflectivity function has been obtained from a real SAR image), oriented along the range direction, over a uniform speckled dark background. In Fig.2 we show the obtained results: actually, just a very low residual ambiguity, due to sidelobe superposition, can be appreciated.



(a) (b)

Fig. 2. Simulation experiments: (a) standard SAR; (b) CopSAR dual frequency implementation.

2) OrthoCopSAR: In Fig. 3 the results obtained with the OrthoCopSAR configuration are shown. In this case, we consider the presence of two ships with different headings placed in the near range area of the scene, and we simulate a range swath twice the size of the maximum unambiguous range swath of standard SAR. In the standard SAR case in the far range region (reported on the right) the presence of range ambiguities can be appreciated. These range ambiguities are effectively suppresed in the OrthoCopSAR case, whereas only a small residual azimuth ambiguity can be appreciated.

IV. CONCLUSION

In this paper, a comprehensive discussion on coprime Synthetic Aperture Radar implementations has been provided. It has been shown how these radars allow for a significant reduction of the data rate and, in some cases, for an increase of the range swath size, when the imaging of maritime scenes is of interest. In particular, in the OrthoCopSAR configuration a huge increase of the range swath can be obtained, with a very limited hardware complication. The only cost is a (non-stringent) limit on the maximum allowed ship size and a reduction of the TBR. A wide experimental setup has been also presented and the merits and drawbacks of each implementation have been discussed in detail.

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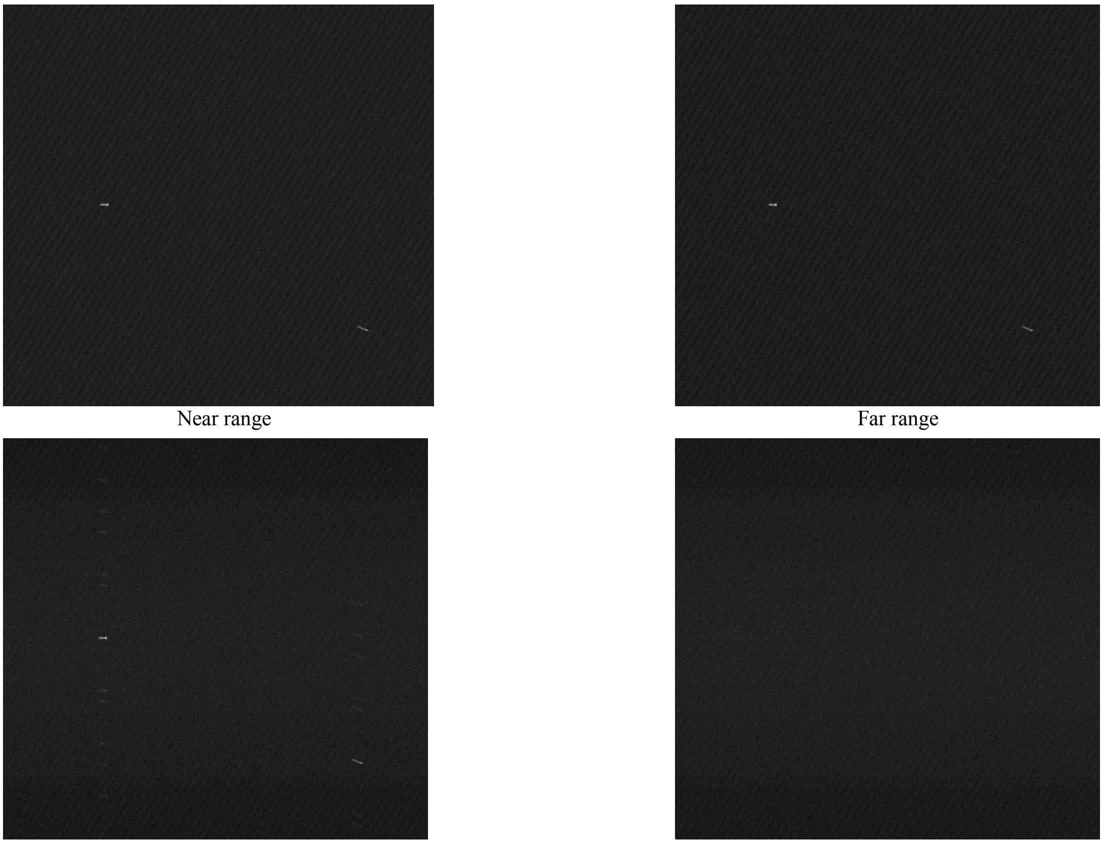

Fig. 3. Simulation Experiments: in the first line, standard SAR, in the second line, OrthoCopSAR.