

SAR Image Post-Processing for the Estimation of Fractal Parameters

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ABSTRACT

In this paper a fractal based processing for the analysis of SAR images of natural surfaces is presented. Its definition is based on a complete direct imaging model developed by the authors. The application of this innovative algorithm to SAR images makes possible to obtain complete maps of the two key parameters of a fractal scene: the fractal dimension and the increment standard deviation. The fractal parameters extraction is based on the estimation of the power spectral density of the SAR amplitude image. From a theoretic point of view, the attention is focused on the retrieving procedure of the increment standard deviation, here presented for the first time. In the last section of the paper, the application of the introduced processing to high resolution SAR images is presented, with the relevant maps of the fractal dimension and of the increment standard deviation.

Keywords: Fractals, Synthetic Aperture Radar, Spectral Estimation

1. INTRODUCTION

The new generation of Synthetic Aperture Radar (SAR) sensors has marked a huge increase in the resolution of microwave images of the Earth and other solar system planets. Therefore, new models, techniques and tools are required to adequately deal with these new available data. Concerning SAR images of natural areas, until now it was only possible to identify macroscopic topological features of the observed areas, roughly distinguishing them from urban ones. With the new available images, the extraction of meaningful stochastic parameters relevant to the roughness and to the shape of the observed surface is now in order. In this paper we present an innovative technique for the analysis and interpretation of SAR images of natural areas. It consists in an electromagnetic post-processing of the image that provides value added information, with a precise physical meaning, of the scene under survey.

In [1] the authors presented a direct model for the imaging of natural surfaces. The developed model takes into account both the interaction between the surface and the incident electromagnetic wave, through the use of appropriate fractal scattering models, and the effect of the SAR impulse response. Our analysis is based on a sound direct modeling of the observed surface and of the SAR imaging process. In particular, the observed natural surface is modeled as a fractal two-dimensional stochastic process [2], [3] which is exhaustively described by its independent fractal parameters: the fractal dimension D and the increment standard deviation s [m^{1-H}]. The two independent parameters D and s provide two different types of contributions to the roughness and to the geophysical characteristics of the surface. Their knowledge can be of key importance for a wide range of applications, including the prevention and monitoring of environmental disasters, land classification, rural and urban planning and so on. The estimation of this parameters from a SAR image can be accomplished, in an appropriate range of frequencies, through a linear regression on the log – log spectrum of a range cut of the SAR image. Up to now the attention has been focalized on the extraction of the fractal dimension D that is related to the slope of the log – log spectrum of a range cut of the SAR image, as shown in [1], [4].

In this paper, for the first time, the retrieving of the increment standard deviation s [m^{1-H}] of an observed scene is investigated and implemented. The s extraction is not a trivial task as it is linked to the amplitude of the log – log spectrum of a SAR image range cut that depends on several parameters besides s . A theoretic treatment directed to the separation of s from the other parameters is presented, together with the implementation of an algorithm that, starting from a SAR image, retrieves, in an automatic way, the point by point fractal parameter s , normalized to the value of a reference spectrum s , of the observed area.

The paper is organized as follows. In Section 2 we first introduce the theoretical aspects of the proposed imaging model and then we describe in detail the extraction of the increment standard deviation s – in relative - starting from the spectrum of a range cut of a SAR image. In Section 3 a numerical setup is presented: the extraction of s is performed on several simulated SAR images obtained using the SARAS [5] (SAR image simulator) starting from fractal surfaces of known fractal parameters generated with the Weierstrass-Mandelbrot function. In Section 4 an algorithm that performs the retrieving of the fractal parameters s and D on actual SAR images providing the relevant fractal maps is presented,

together with its application to COSMO-SkyMed stripmap images of a natural area of Burkina Faso, in West Africa. Finally, in Section 5 significant conclusions are highlighted.

2. THEORETICAL FRAMEWORK

2.1 Imaging model

It is widely recognized that fractal models represent the best way to describe the irregularity of natural scenes [2], [3]. Among this kind of models, we choose the regular stochastic fBm (fractional Brownian motion) process that completely describes natural surfaces by means of two independent parameters: the Hurst coefficient, H (which is linked to the fractal dimension by the simple relation $D = 3 - H$) and the standard deviation of surface increments at unitary distance, s [m^{1-H}]. The process $z(x, y)$ is an fBm if, for every x, y, x', y' , it satisfies the following relationship:

$$Pr\{z(x, y) - z(x', y') < \bar{\zeta}\} = \frac{1}{\sqrt{2\pi s \tau^H}} \int_{-\infty}^{\bar{\zeta}} \exp\left(-\frac{\zeta^2}{2s^2 \tau^{2H}}\right) d\zeta, \quad (1)$$

$$\tau = \sqrt{(x - x')^2 + (y - y')^2}$$

The power spectral density of an fBm profile (that coincides with a one-dimensional cut of an fBm surface) exhibits an appropriate power-law behavior [1], [2], [3]:

$$S(k) = S_0 k^{-\alpha} \quad (2)$$

wherein

$$S_0 = \frac{\pi H}{\cos(\pi H)} \frac{1}{\Gamma(1 - 2H)} s^2, \quad (3)$$

$$\alpha = 1 + 2H = 5 - 2D, \quad (4)$$

are functions of the fractal parameters [3].

Besides the surface model, in order to retrieve the fractal parameters of a natural scene starting from its SAR image we need a direct model relating the surface to its final amplitude image.

In [1] the authors presented a complete imaging model based on the assumption of a small slope regime for the observed surface: if this is the case, the image intensity comes out to be a linear function of the partial derivative of the surface evaluated along the range direction.

The expressions of the autocorrelation functions of the SAR image and of the Power Spectral Densities (PSDs) of two cuts of the image in the range and azimuth directions respectively, have been evaluated by the authors in [1]. The PSDs of the azimuth and of the range cut of the SAR image show very different behaviors, thus highlighting an intrinsic asymmetry in the structure of SAR data, that is also intuitively referable to the particular acquisition geometry of a side looking mono-static radar. In particular, the spectrum of the image range cut, in an appropriate range of sufficiently low spatial frequencies, presents a power law behavior, thus showing on a log - log plane a linear behavior with a slope related to the Hurst coefficient H of the observed surface. In fact, the expression of the PSD of the range cut of a SAR image, for adequately low wavenumbers, turns out to be [1]:

$$S_i(k_y) = a_1^2 s^2 \Gamma(1 + 2H) \text{sen}(\pi H) \frac{1}{|k_y|^{2H-1}} = S_0^i |k_y|^{-\alpha^i}, \quad (5)$$

wherein, k_y is the wavenumber of the range cut of the image, Γ is the Euler Gamma function and a_1 is a coefficient that depends on the assumed scattering model and in the SPM (Small Perturbation Method) case is:

$$a_1 = \sqrt{\frac{S_0 \kappa^{1-2H} |\beta_{mn}|^2}{2^{2H}}} \cos \theta_0 \sin^{-H} \theta_0 [2 + (1 + H) \cos^2 \theta_0 \sin^{-2} \theta_0] \quad (6)$$

wherein κ is the electromagnetic wavenumber; θ_0 is the sensor look angle; β_{mn} , accounting for the incident and reflected fields polarization, is a function of both the dielectric constant of the surface and the local incidence angle θ [4] and it can be considered constant with θ in the angular interval of interest in the co-polarized case; S_0 and H are the surface fractal parameters previously introduced.

Comparing Eq. (5) with the expression of the PSD of the surface in Eq. (2), it can be inferred that the slope of the spectrum relevant to a range cut of a SAR image is equal to that of the imaged surface, assuming that the Hurst coefficient is decreased by one. Hence, the retrieving of the fractal dimension D of an observed surface, recalling that $D=3-H$, can be performed through a linear regression on the SAR image range spectrum in a $\log(k_y)$ - $\log(\text{spectrum})$ plane, extracting the slope of the spectrum [1], [4].

2.2 Extraction of the increment standard deviation

The retrieving of the second fractal parameter, the increment standard deviation s [m^{1-H}], starting from Eq. (5), is more involved with respect to the fractal dimension case. Actually, in this case, if the linear regression is enforced to extract the s value, this value will be retrieved from the intercept, i.e. from a single value extremely influenced by the spectrum slope estimate. Thus, the s estimate would be completely dependent on the goodness of the estimate of the other fractal parameter. Moreover, the SAR image power spectrum amplitude can depend not only on the amplitude coefficient of Eq. (5), but also on calibration parameters of the specific sensor that cannot be always known and controlled. For this reason we look for a relative value of s that is retrieved considering the ratio of two SAR image power spectra according to the following theoretical and analytical considerations.

The spectral amplitude of the SAR image PSD (Power Spectral Density), S_0^i (see Eq. (5)), depends on several parameters, including θ_0 and κ that are known and depend on the sensor characteristics, k_y that depends on the spatial frequencies and is calculable and H , the Hurst coefficient, that can be preliminarily estimated with the linear regression discussed in the previous paragraph. Further incidental calibration coefficients that modify the spectral amplitude are assumed to be constant for SAR images acquired by the same sensor. According to these considerations we can consider the ratio between two SAR image PSDs. Substituting Eq. (6) and Eq. (3) in Eq. (5), we have:

$$\begin{aligned} & \frac{S_{i1}(k_y)}{S_{i2}(k_y)} \\ &= \frac{s_1^4 \frac{\kappa_1^{1-2H_1}}{2^{2H_1}} \frac{\pi H_1}{\cos(\pi H_1)} \frac{\Gamma(1+2H_1)}{\Gamma(1-2H_1)} \cos^2 \theta_{01} \sin^{-2H_1} \theta_{01} [2 + (1 + H_1) \cos^2 \theta_{01} \sin^{-2} \theta_{01}]^2 \sin(\pi H_1) \frac{1}{|k_y|^{2H_1-1}}}{s_2^4 \frac{\kappa_2^{1-2H_2}}{2^{2H_2}} \frac{\pi H_2}{\cos(\pi H_2)} \frac{\Gamma(1+2H_2)}{\Gamma(1-2H_2)} \cos^2 \theta_{02} \sin^{-2H_2} \theta_{02} [2 + (1 + H_2) \cos^2 \theta_{02} \sin^{-2} \theta_{02}]^2 \sin(\pi H_2) \frac{1}{|k_y|^{2H_2-1}}} \\ &= \frac{s_1^4}{s_2^4} \cdot M(\kappa_1, \kappa_2, H_1, H_2, \theta_{01}, \theta_{02}, k_y) \end{aligned} \quad (7)$$

wherein $M(\cdot)$ is a multiplicative function. If we are considering power spectra of SAR images acquired by the same sensor (or of different cuts of the same SAR image) the sensor parameters and the wavenumber κ are the same, so Eq. (7) turns into:

$$\frac{S_{i1}(k_y)}{S_{i2}(k_y)} = \frac{s_1^4}{s_2^4} \cdot M(\kappa, \theta_0, k_y, H_1, H_2) \quad (8)$$

Therefore the estimation of the increment standard deviation s , in relative, with respect to a controlled value of s , can be performed after estimating the PSDs of the SAR image of the region of interest and after estimating, through the last ones, the Hurst coefficients H_1 and H_2 , as follows:

$$s_1 = s_2 \left(\frac{S_{i1}(k_y)}{S_{i2}(k_y)} \cdot \frac{1}{M(\kappa, \theta_0, k_y, H_1, H_2)} \right)^{\frac{1}{4}} \quad (9)$$

3. NUMERICAL SETUP

To validate the theoretical analysis presented in the previous section, we first perform the s parameter extraction on canonical SAR images obtained starting from canonical fractal surface of known parameters. The used SAR images are simulated using the SARAS (SAR image simulator) [5] and using as input a digital elevation model (DEM) of a fractal surface of controlled fractal parameters. The surface DEM is generated implementing a Weierstrass-Mandelbrot function (WM) [2], [3]. Let us note that, in this case, we work in the hypothesis that the observed surface shows the same fractal parameters, chosen and controlled by the user, at all the scales of interest. The SARAS simulator - receiving as inputs the surface DEM, the type of SAR sensor to simulate and the scattering model to use, in our case the SPM one [3] - provides as output the correspondent SAR image. Starting from this image, we estimate the PSDs on image cuts along the ground-range direction, through appropriate filtering and estimation techniques. More in detail, for the spectra estimation, we use the Capon estimator [6], particularly suited to power-law spectra, that minimizes the spectral variance and reduces the leakage phenomenon. As the theoretical formula stands for averaged spectra, the estimation of the range PSD is performed averaging out the spectra relevant to several cuts of the image, sufficiently spaced to be considered uncorrelated. Then, performing a linear regression on the estimated range image spectrum, the Hurst coefficient H of the SAR image is retrieved. Therefore, the multiplicative factor of Eq. (8) can be calculated and - through Eq. (9) - the ratio of the increment standard deviations of two SAR images of constant and controlled fractal parameters can be evaluated and compared with the theoretical one. Note that, in order to make the estimate more robust, the abovementioned operations (evaluation of the multiplicative factor M and calculation of the PSDs ratio) are implemented on the PSDs integrals, so averaging on all the estimated spectra points.

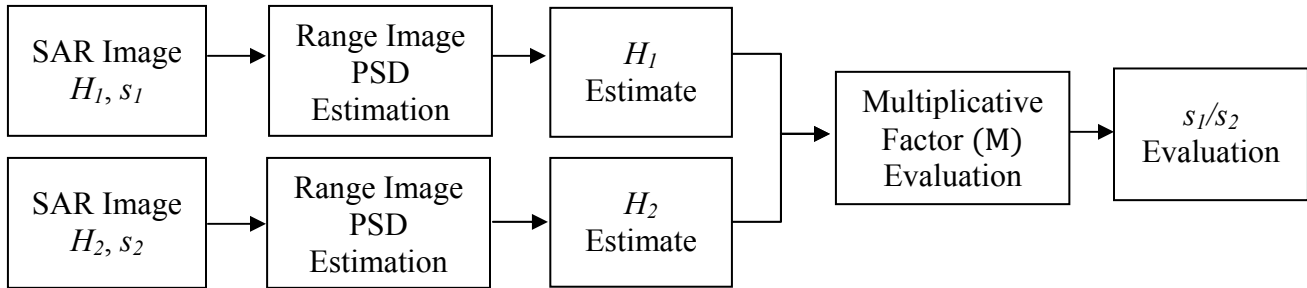


Figure 1. Block diagram of the relative increment standard deviation evaluation.

In Figures 2-5 the theoretical and estimated range cuts PSDs ((a) and (b) respectively) of SAR images of natural surfaces with several fractal parameters are represented. The starting SAR images have 1000x1000 pixels dimensions and the same fractal parameters at all scales of interest. In particular, cases of SAR images of fractal surfaces having:

- same Hurst coefficient and different increment standard deviations (Fig. 2)
- same increment standard deviation and different Hurst coefficients (Fig. 3)
- different Hurst coefficients and different increment standard deviations (Fig. 4, 5)

are considered. The results relevant to the fractal parameters s and H estimations are given in Table 1.

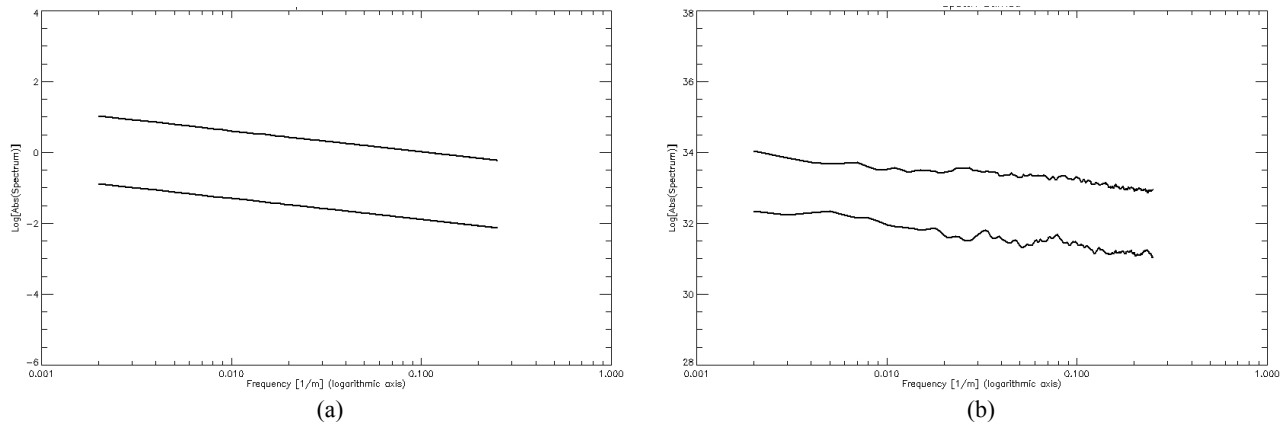


Figure 2. Theoretical (a) and estimated (b) spectra relevant to SAR Images of surfaces having the following fractal parameters:
 $H_1 = H_2 = 0.8$, $s_1 = 0.3 \text{ m}^{0.2}$, $s_2 = 0.1 \text{ m}^{0.2}$

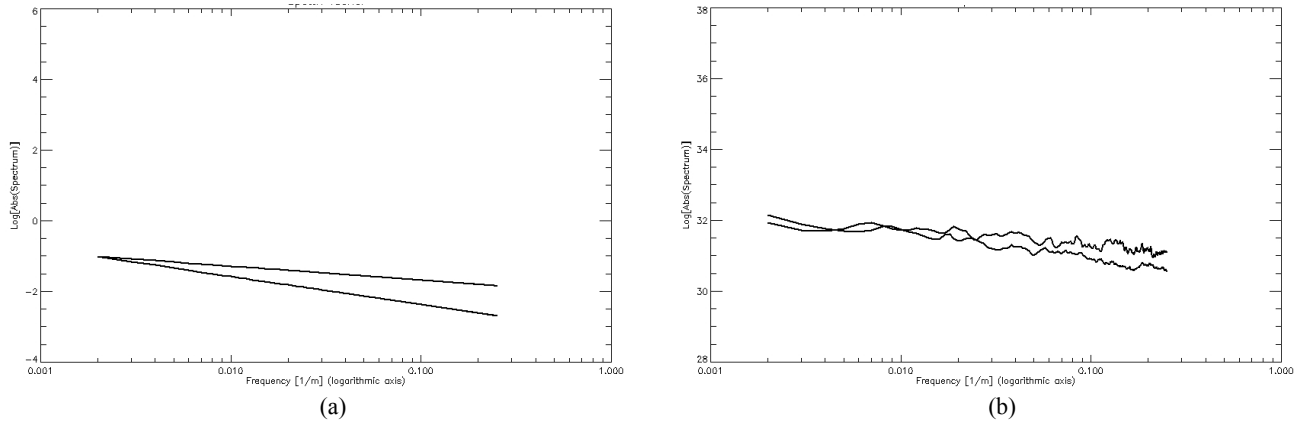


Figure 3. Theoretical (a) and estimated (b) spectra relevant to SAR Images of surfaces having the following fractal parameters:
 $H_1 = 0.9, H_2 = 0.7, s_1 = 0.1 \text{ m}^{0.1}, s_2 = 0.1 \text{ m}^{0.3}$

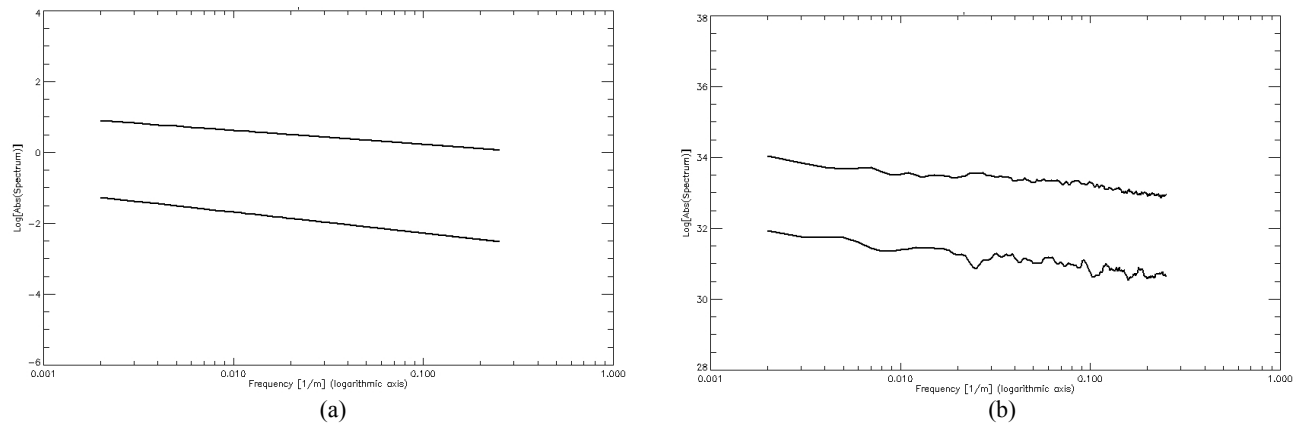


Figure 4. Theoretical (a) and estimated (b) spectra relevant to SAR Images of surfaces having the following fractal parameters:
 $H_1 = 0.7, H_2 = 0.8, s_1 = 0.3 \text{ m}^{0.3}, s_2 = 0.08 \text{ m}^{0.2}$

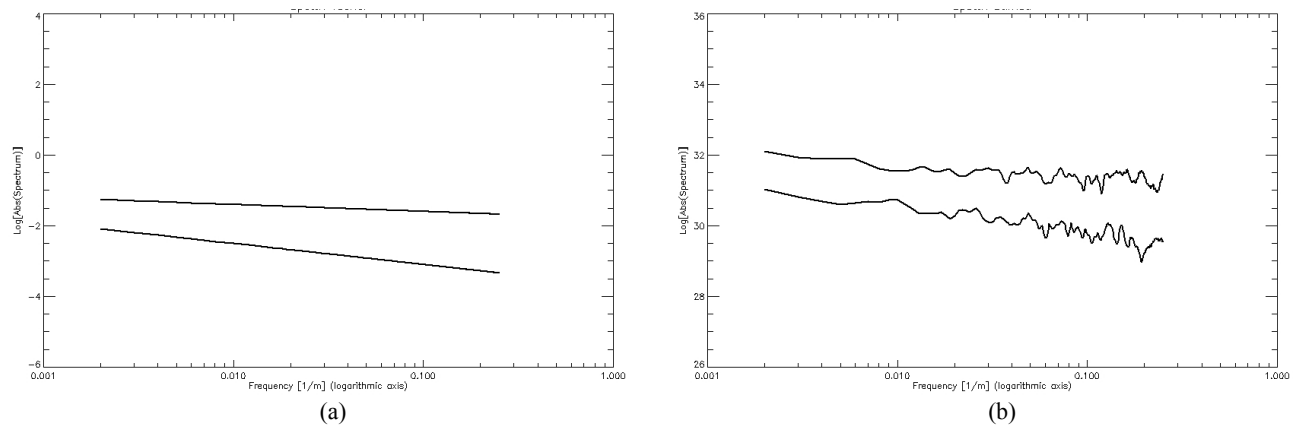


Figure 5. Theoretical (a) and estimated (b) spectra relevant to SAR Images of surfaces having the following fractal parameters:
 $H_1 = 0.6, H_2 = 0.8, s_1 = 0.1 \text{ m}^{0.4}, s_2 = 0.05 \text{ m}^{0.2}$

Table 1. Theoretical and estimated fractal parameters relevant to the PSDs in Figures 2-5.

		theoretical s [m^{1-H}]	theoretical H	estimated H	theoretical s ratio [$m^{H_2-H_1}$]	estimated s ratio [$m^{H_2-H_1}$]
Figure 2	PSD 1	0.3	0.8	0.797	3.000	2.754
	PSD 2	0.1	0.8	0.800		
Figure 3	PSD 1	0.1	0.9	0.889	1.000	1.062
	PSD 2	0.1	0.7	0.754		
Figure 4	PSD 1	0.3	0.7	0.776	3.750	3.636
	PSD 2	0.08	0.8	0.801		
Figure 5	PSD 1	0.1	0.6	0.654	2.000	1.787
	PSD 2	0.05	0.8	0.864		

Further to the presented results, some consideration on the fractal parameters estimation are in order. The H retrieving is a very delicate issue as its accuracy is strongly influenced by the number of samples used for the estimation of the spectrum (in the presented cases 1000x1000 sample range profile have been used to validate the theoretical analysis) and as the model goes out of the hypothesis of small slopes regime the H estimate becomes worse. The s estimation, instead, is more robust both with respect to the number of samples used for the spectrum estimation and for the validity limits of the theoretical model. Moreover, and this is a non slight result, the goodness of the s estimate is not significantly affected by the accuracy of the H estimation: also in cases in which the H estimation is less accurate (see Fig. 3 PSD 2, Fig. 4 PSD 1, Fig 5 PSD 1, 2) the results on the relative s are comparable with the others.

4. EXPERIMENTAL RESULTS

In this section significant experimental results regarding the retrieving of the fractal parameters H and s directly from actual SAR images are presented. To this aim, we developed an algorithm that, by means of a sliding window spanning the entire image, provides the corresponding fractal maps, i.e. two matrixes of the point by point D ($D=3-H$) and of the point by point s relevant to the observed surface.

The implemented algorithm extracts the local fractal parameters of the imaged surface working on patches of the SAR image and iterating the procedure on the whole image, through a moving window, whose dimension can be set by the user according to its specific needs, resulting from a trade-off between accuracy and resolution of the output fractal parameters maps. In particular, the algorithm performs this estimation selecting in each window range cuts that are sufficiently spaced from each other to be considered uncorrelated. Then the spectra of these cuts (whose number can be again chosen by the user, as a trade-off between accuracy and computation time) are evaluated using a Capon estimator [6]. Finally, these spectra are averaged and a linear regression is performed on this mean PSD, for retrieving D . For the s estimation, first of all, a reference PSD is evaluated starting from an homogeneous area of the SAR image having the sliding window dimension; then, with regard to this spectrum, in each window the relative s estimation is performed according to the operations described in the block diagram in Figure 1.

The question of the spectrum estimation is not trivial: as a matter of fact, power-law spectra introduce unique difficulties in the spectral estimation as they suffer leakage effects and high variance problems, yielding a spectral estimate which can deeply modify the original spectral slope. The Capon estimator strongly reduces the above-mentioned negative effects and is particularly well suited when facing short data records: this characteristic is also important in our case, in which the record size is limited by the use of a relatively small sliding window.

As a result, we obtain a map of the fractal parameters D and s of the observed scene: the resolution of this maps depends on both the resolution of the input image (the higher the resolution of the image, the better the resolution of the maps) and the dimension of the estimation window.

Furthermore, we cannot set completely aside the speckle phenomenon, which is responsible for the well-known *salt and pepper* effect on SAR amplitude images. As a matter of fact, the spatial scales involved by the speckle are mainly those of the order of the sensor resolution, hence in the wavenumber domain the high frequency range of the image spectrum

is degraded. However, our algorithm works in a range of spatial frequencies in which the spectrum is not significantly affected by this phenomenon.

Hereafter the results of application of this new SAR image post processing to a COSMO-SkyMed stripmap image of a rural area of Burkina Faso (West Africa) are presented. The starting SAR image is a 9277x9192 pixel image and, being a stripmap SAR image, has a pixel resolution of about 3x3 m².

In Figures 6-8 the COSMO-SkyMed image and the fractal maps of D and s respectively are shown.

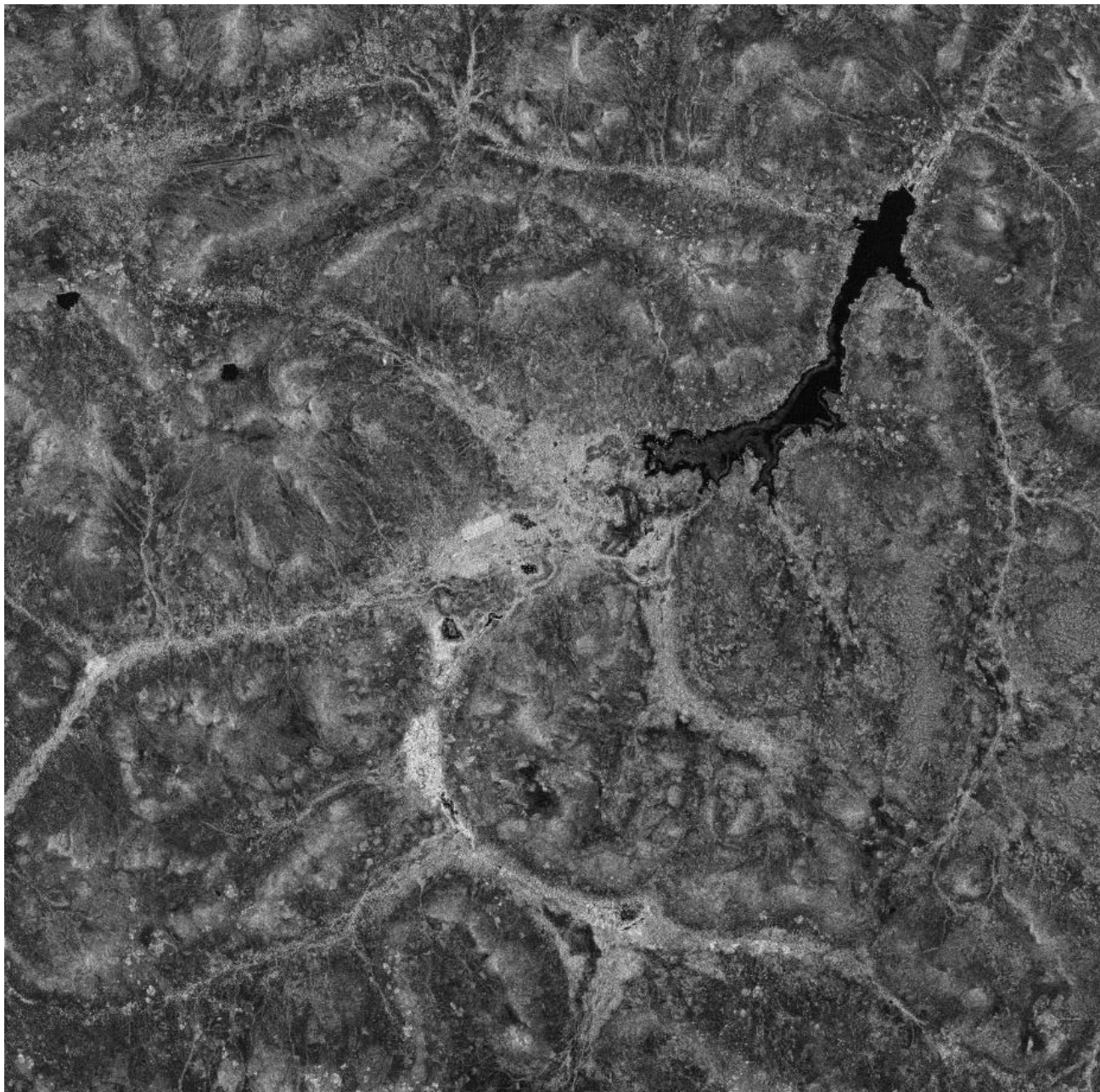


Figure 6. COSMO-SkyMed Stripmap image of a rural area of Burkina (Africa). The dimensions of the image are 9277x9192 pixels. Near range is on the left.

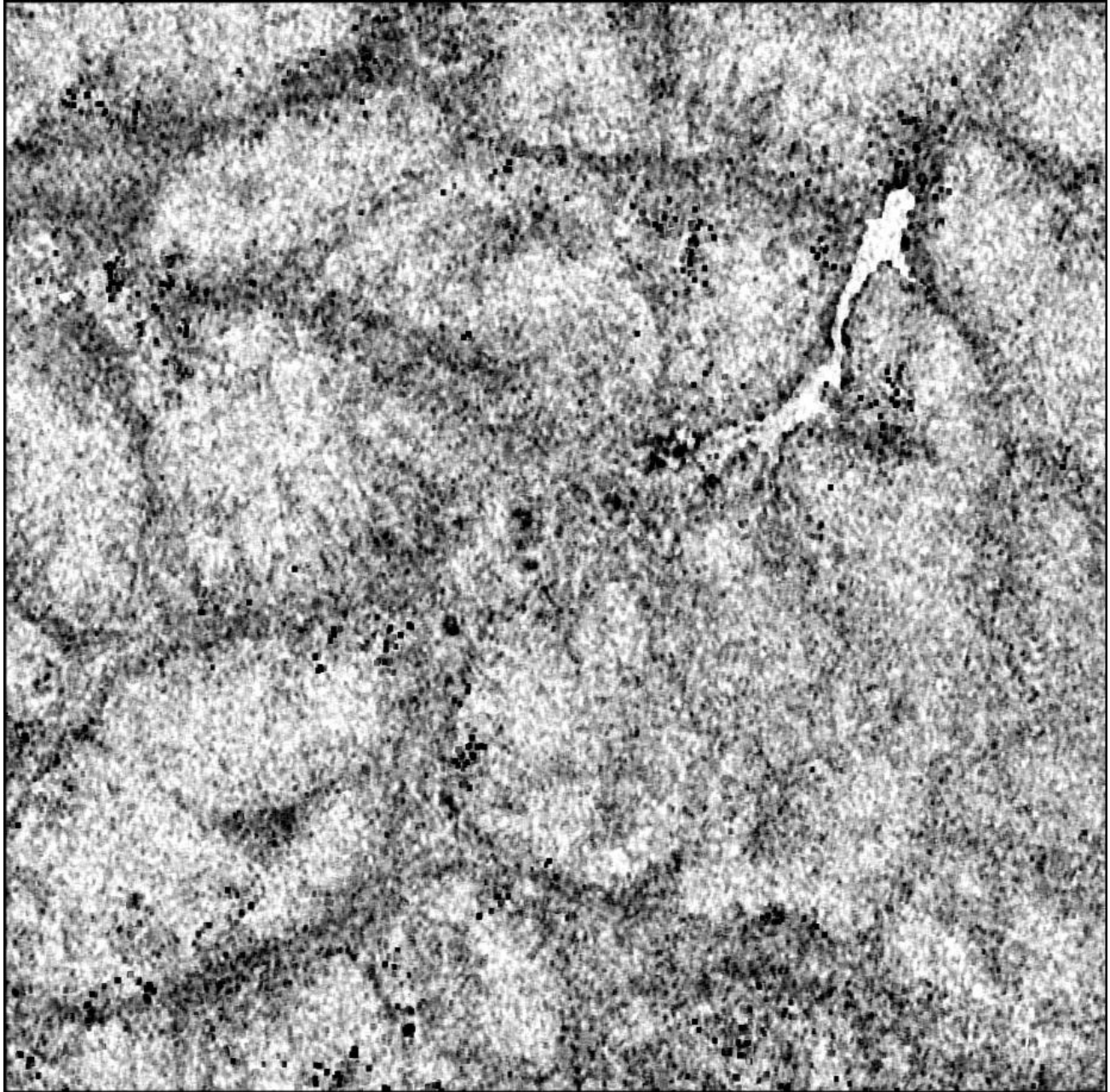


Figure 7. Fractal dimension (D) map relevant to the SAR image in Figure 6.

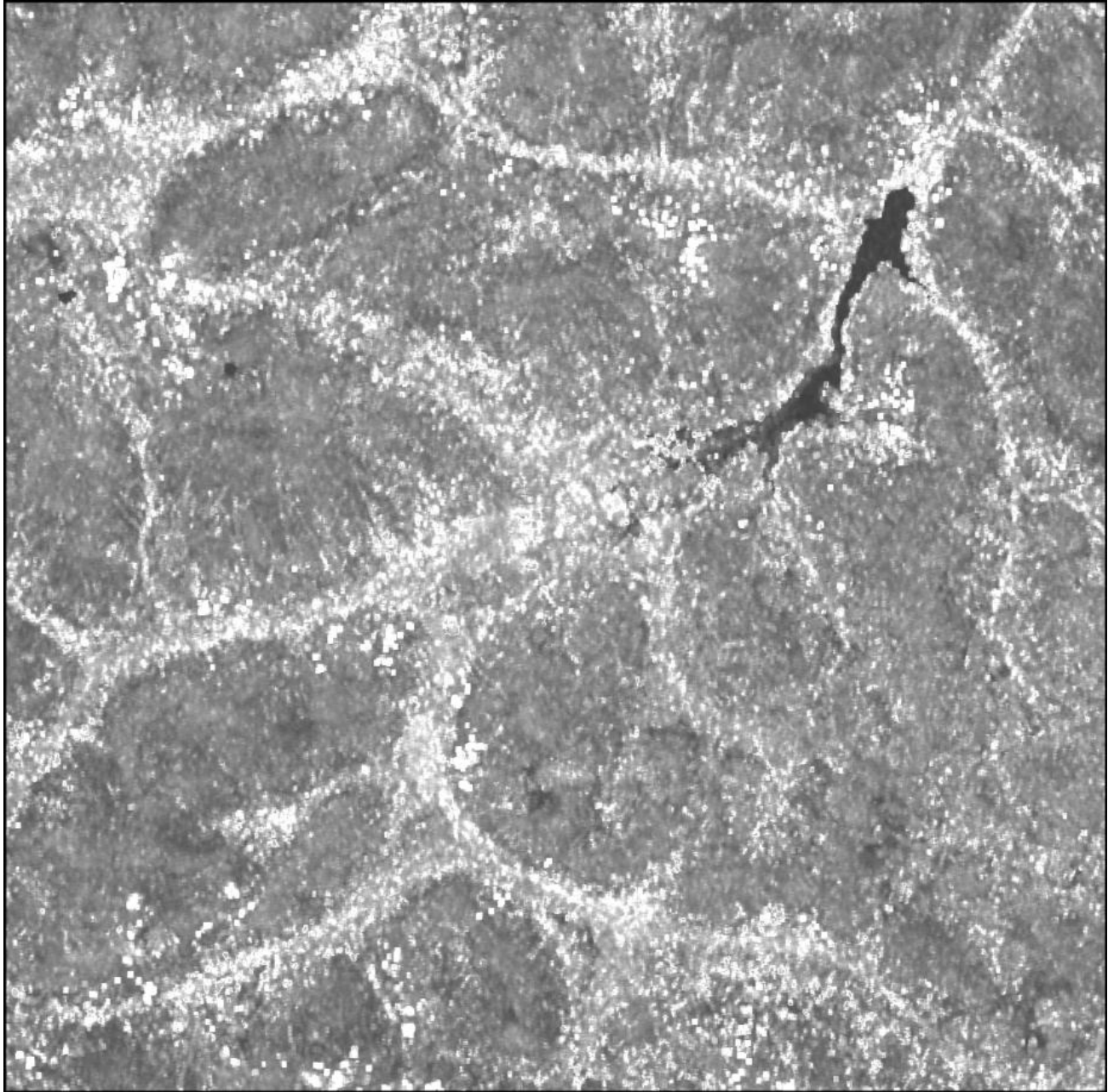


Figure 8. Increment standard deviation (s) map relevant to the SAR image in Figure 6.

The statistics of the fractal maps in Figures 7, 8 are given in Table 2.

Table 2. Statistics of the fractal maps in Figure 7, 8.

	Minimum	Maximum	Mean	Standard Deviation
Fractal dimension (D) map (Figure 7)	1.32	2.49	2.20	0.07
Increment standard deviation (relative s) map [$m^{0.82-H_{stim}}$] (Figure 8)	0.38	313.12		

Concerning the fractal dimension map, it can be observed that the D mean is a value typical for natural surfaces. In fact we recall that, for natural surfaces, D is included between 2 and 3 ($2 < D < 3$) while D values smaller than 2 are typical of non fractal objects (as they can be identified in the map as black squares having the dimension of the sliding window). Regarding the increment standard deviation map, there is a large variation range between minimum and maximum but so high values for the relative s occur only for very few points in which probably the starting SAR image presents features that alter the spectrum estimation. The reference spectrum used is represented in Figure 9 and presents an estimated value of D equal to 2.18.

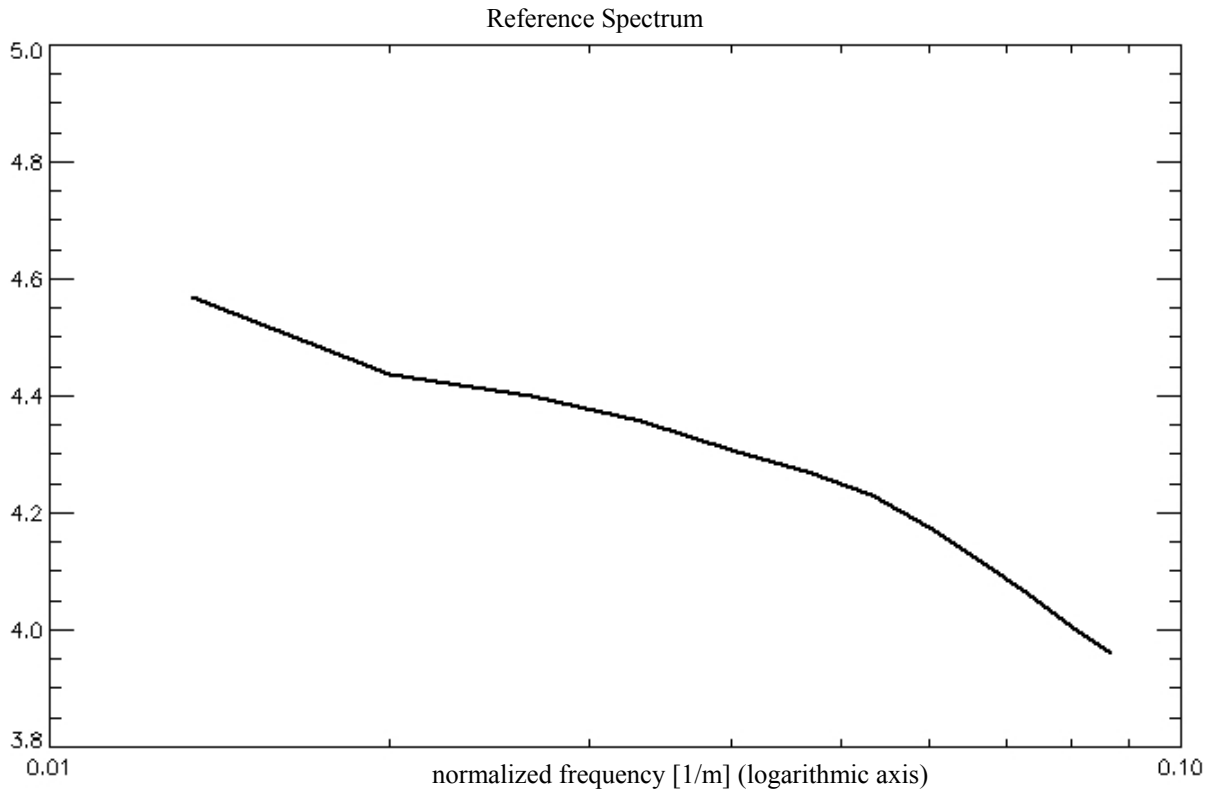


Figure 9. Reference PSD used for the evaluation of the s map in Figure 8. conclusions

In this paper a fractal based processing for the analysis of SAR images of natural surfaces has been presented. Its definition is based on a complete direct imaging model developed by the authors. As a natural surface modeled as a fractal fBm (fractional Brownian motion) is completely described through only two independent fractal parameters: the fractal dimension and the increment standard deviation, the presented innovative algorithm provides the complete maps of these two fractal parameters starting from a single SAR image. The fractal parameters extraction is based on the estimate of the power spectral density of the SAR amplitude image. More details are given on the retrieving procedure of

the increment standard deviation that is here presented for the first time. A numerical setup is provided to validate the theoretical results. Moreover, in the last section of the paper, the application of the introduced processing to high resolution SAR images of a prevalently rural scene is presented, together with the relevant maps of the fractal dimension and of the increment standard deviation.

5. CONCLUSIONS

In this paper a fractal based processing for the analysis of SAR images of natural surfaces has been presented. Its definition is based on a complete direct imaging model developed by the authors. As a natural surface modeled as a fractal fBm (fractional Brownian motion) is completely described through only two independent fractal parameters: the fractal dimension and the increment standard deviation, the presented innovative algorithm provides the complete maps of these two fractal parameters starting from a single SAR image. The fractal parameters extraction is based on the estimation of the power spectral density of the SAR amplitude image. More details are given on the retrieving procedure of the increment standard deviation that is here presented for the first time. A numerical setup is provided to validate the theoretical results. Moreover, in the last section of the paper, the application of the introduced processing to high resolution SAR images of a prevalently rural scene is presented, together with the relevant maps of the fractal dimension and of the increment standard deviation.

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