Geometric Properties for Parabolic and Elliptic PDE's
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ABSTRACTS of the POSTERS

A priori estimates for solutions to fully anisotropic elliptic problems

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(*Joint work with G. di Blasio and F. Feo*)

Abstract: We establish a comparison result for solutions to nonlinear fully anisotropic elliptic Dirichlet problems of the form

$$\begin{cases} -\mathrm{div}\,(\mathrm{a}(\mathrm{x},\mathrm{u},\nabla \mathrm{u})) = \mathrm{f}(\mathrm{x}) - \mathrm{div}\,(\mathrm{g}\,(\mathrm{x})) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded open subset of $\mathbb{R}^n$, $n \geq 2$, $a : \Omega \times \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a Carathéodory function, and the data f and g satisfy a suitable integrability conditions. As an ellipticity condition, we assume that, for a.e. $x \in \Omega$,

$$a(x, \eta, \xi) \cdot \xi \geq \Phi\,(\xi) \quad \text{for } (\eta, \xi) \in \mathbb{R} \times \mathbb{R}^n, \tag{1}$$

where $\Phi : \mathbb{R}^n \to [0, \infty)$ is an $n-$dimensional Young function, namely a convex function, vanishing at the origin, which is neither necessarily radial, nor necessarily of polynomial type. We show that the symmetric rearrangement of a solution to any Dirichlet problem from a certain class, fulfilling the anisotropic ellipticity condition (1), is pointwise dominated by the radial solution to an isotropic problem from the same class, where not only the domain and the data are symmetrized, but also the ellipticity condition is subject to an appropriate symmetrization. Our comparison result requires the use of classical notions of rearrangements and symmetrizations for u and f, as well as a less standard symmetrization of Φ introduced by V.S. Klimov [2]. As a consequence, we deduce optimal bounds for norms of the relevant solutions in terms of norms of the data. Our results will appear in [1].

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A new Steklov-type problem for the biharmonic operator

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Abstract: In this poster we formulate a Steklov-type eigenvalue problem for the biharmonic operator. This problem should not be confused with the well-known problem considered e.g., in [1, 4]. We provide a physical motivation for our problem, highlighting the relations with the Neumann problem for the biharmonic operator via mass concentration arguments. Then we compute Hadamard-type formulas for the shape derivatives of the eigenvalues and use them to prove that balls are critical domains for the eigenvalues under volume constraint. Finally we prove a quantitative isoperimetric inequality and, as a corollary, that the ball maximizes the first non-zero eigenvalue among domains with the same volume. Based on the papers [2, 3].

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Non-trivial translation-invariant valuations on L^∞

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By the expression *valuation* on $L^\infty(\mathbb{R})$ we indicate any functional $\mu : L^\infty(\mathbb{R}) \to \mathbb{R}$ such that $\mu(0) = 0$ which satisfies the following additivity property:

$$\mu(u \vee v) + \mu(u \wedge v) = \mu(u) + \mu(v)$$

for all $u, v \in L^\infty(\mathbb{R})$ (where the symbols $\vee$ and $\wedge$ here denote pointwise maximum and minimum respectively).

Moreover we say that a valuation μ is "translation-invariant" iff $\mu(u) = \mu(\mathcal{T}_t u)$ for all $u \in L^\infty(\mathbb{R})$ and $t \in \mathbb{R}$, where we have set $\mathcal{T}_t u(x) := u(x - t)$ for all real x.

It is fairly easy to show that translation-invariant valuations vanish on functions with compact support. This raises the question of whether there actually exists at least a translation-invariant valuation that does not vanish on every function.

This work shows how the concept of "ultrafilter" (roughly speaking, a way to divide subsets of $\mathbb{R}$ into the families of "small" and "large" sets) can be employed in order to provide some (non-constructive) examples of non-trivial translation-invariant valuations on $L^\infty(\mathbb{R})$.

Blow-up solutions for some nonlinear elliptic equations involving a Finsler-Laplacian

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The aim of the poster is to describe some results, obtained in collaboration with G. di Blasio, regarding existence and asymptotic behavior of strong solutions $u \in W^{2,2}_{\mathrm{loc}}(\Omega)$ of the nonlinear elliptic problem

$$\begin{cases} -\Delta_H u + H(\nabla u)^q + \lambda u = f & \text{in } \Omega, \\ u \to +\infty & \text{on } \partial\Omega, \end{cases} \tag{P}$$

where H is a suitable norm of $\mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$ is a bounded domain, Δ_H is the Finsler Laplacian, $1 < q \leqslant 2$, $\lambda > 0$ and f is a suitable function in L^∞_{loc}. Furthermore, we are interested in the behavior of the solutions when $\lambda \to 0^+$, studying the so-called ergodic problem associated to (P). A key role in order to study the ergodic problem is played by local gradient estimates for (P).

Optimal Szegö-Weinberger type inequalities

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The aim of the poster is to describe some results, obtained in collaboration with F. Brock and F. Chiacchio. We provide sharp Szegö-Weinberger type inequalities for the first non-trivial Neumann eigenvalue $\mu_1(\Omega; e^{h(|x|)})$ of the following class of problems

$$\begin{cases} -\mathrm{div}\left(e^{h(|x|)}\nabla u\right) = \mu e^{h(|x|)} u & \text{in} \quad \Omega \\ \\ \frac{\partial u}{\partial \nu} = 0 & \text{on} \quad \partial\Omega, \end{cases}$$

where Ω is a bounded and Lipschitz domain in $\mathbb{R}^N$ and ν is the outward normal to $\partial\Omega$. Since the degeneracy of the operator is given in terms of the radial function $e^{h(|x|)}$, it appears natural to let Ω vary in the class of sets having prescribed γ_h-measure, where

$$d\gamma_h = e^{h(|x|)}dx, \text{ with } x \in \mathbb{R}^N.$$

Under suitable assumption on h we prove that the ball centered at the origin is the unique set maximizing $\mu_1(\Omega; e^{h(|x|)})$ among all Lipschitz bounded domains Ω of $\mathbb{R}^N$ of prescribed $e^{h(|x|)}dx$-measure and symmetric about the origin. Moreover, an example in the model case $h(|x|) = |x|^2$, shows that, in general, the assumption on the symmetry of the domain cannot be dropped. In the one-dimensional case, i.e. when Ω reduces to an interval (a, b), we consider a wide class of weights (including both Gaussian and anti-Gaussian). We then describe the behavior of the eigenvalue as the interval (a, b) slides along the x-axis keeping fixed its weighted length.

Continuity properties of p-harmonic operators

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Let Ω be a bounded Lipschitz domain of $\mathbb{R}^N$, $N \geqslant 2$. We consider the Dirichlet problem whose model case is given by

$$\begin{cases} \operatorname{div} |\nabla u|^{p-2}\nabla u = \operatorname{div} f \qquad \text{in } \Omega, \\ \\ u = 0 \qquad \text{on } \partial\Omega, \end{cases} \tag{1}$$

with $p > 1$. A solution $u \in W_0^{1,r}(\Omega)$ of (1), with $\max\{1\,,\,p-1\} \leqslant r \leqslant p$, is understood in the weak sense, that is the identity

$$\int_\Omega \left\langle |\nabla u|^{p-2}\nabla u, \nabla\varphi \right\rangle dx = \int_\Omega \langle f, \nabla\varphi\rangle\, dx, \tag{2}$$

holds true for every test function $\varphi \in C_0^\infty(\Omega)$.

We present existence and uniqueness results for problem (1) assuming that f belongs to some Orlicz–Zygmund spaces. Under these assumptions, we also investigate the continuity of the operator $\mathcal{H}\colon f \mapsto \nabla u$ which carries a given vector field f into the gradient field ∇u.

Remark on the stability of the log-Sobolev inequality for the Gaussian measure

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The log-Sobolev inequality asserts that, in any dimension n and for any smooth enough function $f\colon \mathbb{R}^n \to [0, +\infty[$, it holds

$$\mathrm{Ent}_{\gamma_n}(f) \leqslant \frac{1}{2}\int_{\mathbb{R}^n} \frac{|\nabla f|^2}{f}\, d\gamma_n, \tag{0.1}$$

where $\gamma_n(dx) = \varphi_n(x)dx := (2\pi)^{-\frac{n}{2}} \exp\{-\frac{|x|^2}{2}\}dx$, $x \in \mathbb{R}^n$, is the standard Gaussian measure with density φ_n. The constant $1/2$ is optimal. Moreover, equality holds in (0.1) if and only if f is the exponential of a linear function, *i.e.* there exist $a \in \mathbb{R}^n$, $b \in \mathbb{R}$ such that $f(x) = \exp\{a \cdot x + b\}$, $x \in \mathbb{R}^n$.

Very recently there has been some interest in the study of the stability of the log-Sobolev inequality (0.1). Namely the question is: can one bound the difference between the right and left hand side of (0.1) in term of the distance (in a sense to be defined) between f and the set of optimal functions? In other words, can one bound from below the *deficit*

$$\delta_{LS}(f) := \frac{1}{2}\int_{\mathbb{R}^n} \frac{|\nabla f|^2}{f}\, d\gamma_n - \mathrm{Ent}_{\gamma_n}(f) \tag{0.2}$$

in some reasonable way? We refer to [B, FIL, IM] for various results in this direction. What is however currently lacking in the literature is a result stating that $\delta_{LS}(f) \geqslant d(f, \mathcal{O})$, where $\mathcal{O} := \{e^{a\cdot x+b}, a \in \mathbb{R}^n, b \in \mathbb{R}\}$ is the set of functions achieving equality in (0.1) and where d is some distance.

Our aim is to give a result in this direction.

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Name: Giovanni Franzina
Title of the poster: Plateau problem at infinity for hypersurfaces of prescribed mean curvature in the hyperbolic space.
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Abstract: We present a joint work with A. Pisante, M. Ponsiglione and T. Luu concerning the existence and the regularity of hypersurfaces in the hyperbolic space having given mean curvature and prescribed asymptotic values at infinity. By a Γ–convergence result, a Cahn–Hiliard–type phase field ε–approximation can be used to bridge between the geometric problem and the Dirichlet problem at infinity for entire solutions of a semilinear elliptic partial differential equation in the hyperbolic space, for which existence, uniqueness, monotonicity and regularity properties are discussed. The existence of a desired hypersurface, smooth (up to the boundary) out of a small singular set, is inherited in the passage $\varepsilon \to 0^+$ by regularity of geometric measure theory.

AN IMPROVED POINCARÉ-HARDY AND POINCARÉ-RELLICH INEQUALITIES ON THE HYPERBOLIC SPACE

DEBDIP GANGULY(POLITECNICO DI TORINO)

Abstract. In this presentation, Poincare-Hardy and Poincare-Rellich inequalities in the hyperbolic space will be presented. In particular, in the Hardy case we obtain the best constant and a singular remainder term. Generalizations to Riemannian Models will be also presented. This is a joint work with E.Berchio and G.Grillo.

Higher differentiability for solutions of elliptic systems with Sobolev coefficients

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Abstract

We provide higher differentiability results for local solutions of elliptic systems of the form

$$\mathrm{div} A(x, Du) = 0 \quad \text{in } \Omega \subset \mathbb{R}^n,$$

where Ω is a bounded open set in $\mathbb{R}^n$. We allow discontinuous dependence with respect to the x-variable of the operator $A(x, \xi)$, through a suitable Sobolev function.

Power concavity in weakly coupled elliptic systems and parabolic systems

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We consider the weakly coupled elliptic system

$$\begin{cases} -\Delta u = \lambda_1 v^\alpha & \text{in} \quad \Omega, \\ -\Delta v = \lambda_2 u^\beta & \text{in} \quad \Omega, \\ u > 0,\ v > 0 & \text{in} \quad \Omega, \\ u = v = 0 & \text{in} \quad \partial\Omega, \end{cases} \tag{1}$$

and the weakly coupled parabolic system

$$\begin{cases} \partial_t u - \Delta u = \lambda_1 v^\alpha & \text{in} \quad D, \\ \partial_t v - \Delta v = \lambda_2 u^\beta & \text{in} \quad D, \\ u > 0,\ v > 0 & \text{in} \quad D, \\ u = v = 0 & \text{in} \quad \partial D, \end{cases} \tag{2}$$

where $\lambda_1,\ \lambda_2 > 0,\ 0 < \alpha, \beta < 1,\ \Omega$ is a bounded convex domain in $\mathbf{R}^N$ $(N \geq 1)$ and $D := \Omega \times (0, \infty)$. We study power concavity properties of solutions of elliptic system (1) and parabolic system (2) and develop a method of studying concavity properties of elliptic systems and parabolic systems. This is a joint work with Kazushige Nakagawa (Fukushima University, Japan) and Paolo Salani (Università di Firenze, Italy).

A decomposition of the Möbius energy and consequences

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We consider the Möbius energy for closed curves in $\mathbb{R}^n$, so-called since it is invariant under Möbius transformations. Since the energy was introduced for finding the canonical configuration of knots, the energy density contains negative powers of the intrinsic and extrinsic distance between any two points on the curve, and this causes significant difficulty with the analysis.

We can decompose the energy into three parts, each of which is Möbius invariant. The first part characterizes the proper domain of the energy; the second one plays the role of canceling the singularity of the density; and the third one gives us information about the minimal value of the energy.

The decomposition gives us easy-to-analyze components. For example, although the first and second variational formulae of the original energy had already been derived in the sense of Cauchy's principal value without our decomposition, we can more easily derive simpler expressions and hence obtain certain applicable estimates for the variational formulae in fractional Sobolev spaces (as well as other spaces) using our decomposition.

Furthermore, the Möbius invariance of each component gives information concerning the minimizers of the energies.

Serrin's over-determined problem:
(optimal) stability and a new proof of symmetry

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The well-known Serrin's symmetry result states that, if a positive solution of a semi-linear equation in a domain D, that vanishes on its boundary, has constant gradient on the boundary, the it must be radially symmetric and D must be a ball. At the same conclusion one arrives if the gradient condition is replaced by one that requires the solution to be constant on a surface parallel (and sufficiently close) to the boundary — the *parallel surface problem*. I will pinpoint differences and analogies between these two problems and show that symmetry and stability of the symmetric configuration for Serrin's problem can be obtained by proving a sharp stability estimate for the parallel surface problem. The stability thus obtained for Serrin's problem is optimal (Lipschitz) and hence improves the logarithmic estimate obtained by Aftalion, Busca and Reichel and the Hœlder estimate obtained by Brandolini, Nitsch, Salani and Trombetti (for the torsional rigidity problem).

THE MATZOH BALL SOUP PROBLEM:
A COMPLETE CHARACTERIZATION

Michele Marini

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Abstract

We provide a catalog of all possible solutions of the heat equation whose (spatial) equipotential surface do not vary with time, such solutions are either isoparametric or split in space-time. We show that similar results can be obtained by considering solutions of a class of quasi-linear parabolic equations with coefficients which are homogeneous functions of the gradient variable.

On the self-adjointness of the fractional Laplacian in weighted L^2 spaces

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Given a weight $\rho(x)$ (let $x \in \mathbb{R}^d$), we consider the operator formally defined as

$$A(v) := \rho^{-1}(-\Delta)^s(v)\,,$$

namely the fractional Laplacian on $\mathbb{R}^d$ times the inverse of the weight. In particular we investigate its self-adjointness property, and consequently the validity of suitable "integration by parts" formulas, in the corresponding ρ-weighed L^2 space. We prove that if $\rho(x)$ decays at infinity as $|x|^{-\gamma}$, with $\gamma \in (0, 2s)$, then such operator is indeed self-adjoint. We proceed by means of pure cut-off and density arguments, which are however harder in view of the nonlocal nature of the operator. Our approach allows us to establish analogous "integration by parts" formulas in ρ-weighted L^p spaces, with $p > 2$. Finally, with a refinement of the above methods, we show that in the critical case $\gamma = 2s$ we still have self-adjointness. As a consequence, in this framework the corresponding weighted fractional heat-type equation

$$\rho(x)u_t = (-\Delta)^s(u)$$

is always well-posed, with no need for conditions at infinity. Whether or not one can restore well-posedness in the supercritical case $\gamma > 2s$ (imposing suitable conditions at infinity) is an open question. This kind of results has been exploited by the author and collaborators to tackle the problem of existence and uniqueness of weak solutions to weighted fractional porous medium equations with positive finite Radon measures as initial data.

The Kirchhoff type elliptic problem involving the critical exponential growth in $\mathbb{R}^2$

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In this poster, we consider the Kirchhoff type elliptic problem in $\mathbb{R}^2$,

$$\begin{cases} -\left(1+\alpha \int_\Omega |\nabla u|^2 dx\right) \Delta u = f(u) \text{ in } \Omega, \\ u = 0 \text{ on } \partial\Omega, \end{cases} \tag{P}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary and $\alpha > 0$. In addition, we assume the critical exponential growth on f. With the combined effect of the nonlocal coefficient and f, (P) admits new existence results including the multiplicity. Our proof is based on the variational method and the concentration compactness argument. This is the joint work with the Prof. Tarsi at University of Milan.

Hardy-Sobolev Inequalities for the Biharmonic Operator with remainder terms

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Abstract

We first review improvements of (first order) Sobolev and Hardy inequalities by the addition of suitable lower order terms - these improved inequalities have been pioneered by Brezis-Nirenberg (1983) and Brezis-Vázquez (1997). Recently, corresponding results concerning first order Hardy-Sobolev and higher order Sobolev and Hardy inequalities have been proved.

We then consider second order inequalities of Hardy-Sobolev type, which are an interpolation between the second order Sobolev and the second order Hardy inequality. We prove that the corresponding critical Hardy-Sobolev constants C^{τ}_{HS} ($0 \leq \tau \leq 4$, where $\tau = 0$ corresponds to the Sobolev and $\tau = 4$ to the Hardy case) do not depend on all traces of the space $W^{2,2}(\Omega)$, i.e. we prove that the critical Hardy-Sobolev constant with Navier conditions coincides with the constant with Dirichlet conditions. Moreover, under the same assumptions and with both Navier and Dirichlet boundary conditions, we derive lower order improvements of these second order Hardy-Sobolev inequalities.

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Neutrally coated inclusions in three dimensions

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Among coated inclusions each of which consists of a core and a shell with different constant conductivities in three dimensions, we want to determine the neutrally coated inclusions, that is, those which are neutral to all uniform fields in the isotropic medium. We first derive an over-determined boundary value problem in the shell of the inclusion provided that the conductivity of the core is larger than that of the shell. Then we prove that if the over-determined problem admits a solution, then the inclusion must be spherical. This is a joint work with Hyeonbae Kang and Hyundae Lee.

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On the shape of the solution of the damped wave equation

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This poster session is based on the joint work with Yuta Wakasugi.
We consider the *damped wave equation*

$$\begin{cases} \left(\dfrac{\partial^2}{\partial t^2} - \Delta + \dfrac{\partial}{\partial t}\right) u(x,t) = 0, & x \in \mathbb{R}^n,\ t > 0, \\ \left(u, \dfrac{\partial u}{\partial t}\right)(x,0) = (f,g)(x), & x \in \mathbb{R}^n. \end{cases} \tag{1}$$

We can find the heat operator $-\Delta + \partial/\partial t$ in the equation (1). Thus, it is expected that the equation (1) has similar properties to the heat equation. In fact, we can derive the equation (1) in the same manner as the heat equation by using the *time-delayed Fourier's law* instead of the usual Fourier's law (see [Li]). Also, the relation between the damped wave equation and the heat equation has been studied by some researchers. For example, in [HO, MN, Nar, Nis], it was shown that the solution of the equation (1) approaches to that of the corresponding heat equation at infinite time.

In this poster session, referring to the observation in [CK, JS], we investigate the shape of the solution of the equation (1).

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Heat equation with a nonlinear boundary condition and unbounded initial functions

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Abstract

We consider the heat equation with a nonlinear boundary condition,

$$\begin{cases} \partial_t u = \Delta u, & x \in \Omega,\ t > 0, \\ \nabla u \cdot \nu(x) = u^p, & x \in \partial\Omega,\ t > 0, \\ u(x,0) = \varphi(x) \geq 0, & x \in \Omega, \end{cases} \tag{1}$$

where $N \geq 1$, $p > 1$, Ω is a (possibly unbounded) smooth domain in $\mathbf{R}^N$, $\partial_t = \partial/\partial t$ and $\nu = \nu(x)$ is the outer unit normal vector to $\partial\Omega$. In this poster we study the existence of local-in-time solutions of (1) for unbounded initial functions at the space infinity. This is a joint work with Prof. Kazuhiro Ishige.

Time-dependent singularities for a semilinear heat equation with absorption

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We consider solutions with time-dependent singularities for a semilinear heat equation with a superlinear absorption term. By time-dependent singularity, we mean a singularity with respect to the space variable whose position depends on the time variable. It is shown that if the power of the nonlinearity is in some range, there is no time-dependent singular solution. In other range, two types of time-dependent singular solutions exist.

This is a joint work with Eiji Yanagida (Tokyo Institute of Technology).

Dynamics of intefaces in the Fisher-KPP equation

Eiji Yanagida (Tokyo Institute of Technology)

Under appropriate rescaling, solutions of the Fisher-KPP equation exhibit interfaces that correspond to transition layers from the trivial steady state to a stable positive steady state. If initial data decay rapidly in space, then the interface moves with a constant speed that is equal to the minimal speed of traveling fronts in one-dimensional space. On the other hand, for slowly decaying initial data, the interface may expand in a rather irregular way. We show that the dynamics of interfaces for slowly decaying initial data is described as a level set of a first-order equation of Hamilton-Jacobi type, and study some properties of solutions of the equation. This is a joint work with Hirokazu Ninomiya of the Meiji University.

Mean field equations with probability measure in 2D-turbulence

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We outline some results obtained in recent years concerning a class of semilinear elliptic equations of the "mean field" type containing a probability measure, motivated by the statistical mechanics description of $2D$ turbulence. In particular, we exhibit the optimal Moser-Trudinger constants, we study the blow-up of solution sequences, we prove their mass quantization under suitable conditions and we consequently derive the existence of solutions by variational methods. Those results are obtained in collaboration with T. Ricciardi.

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