PREDICTION RELATIONSHIPS FOR A VECTOR-VALUED GROUND MOTION INTENSITY MEASURE ACCOUNTING FOR CUMULATIVE DAMAGE POTENTIAL

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ABSTRACT :

Vector-valued ground motion intensity measures have been extensively investigated recently. Proposed measures are mainly related to pairs of spectral ordinates because the spectral shape has been shown to be useful in the probabilistic assessment of structural and non-structural response of buildings. This is especially appropriate in the case of structures following modern earthquake resistant design principles, in which structural damage is mainly due to displacements experienced in non-linear behavior. Although it is generally believed that, under some hypotheses, integral ground motion parameters associated to duration are less important for structural demand assessment with respect to peak quantities of ground motion, there are cases in which the cumulative damage potential of the earthquake is also of concern. In this paper the joint distribution of peak ground acceleration (PGA) and a parameter which may account for the cumulative damage potential of ground motion, is investigated with respect to some engineering seismology issues. The chosen energy related measure is the so-called *Cosenza and Manfredi index* (I_D). A ground motion prediction relationship has been retrieved for I_D on the basis of an empirical dataset of Italian records already used for well known attenuation laws proposed in the past by other researchers. Subsequently, the residuals have been tested for correlation and for joint normality. The study allowed to obtain distributions of I_D conditional on PGA and on the corresponding design earthquake, in terms of magnitude and distance, from hazard disaggregation.

KEYWORDS: Vector-valued IMs, Cumulative damage, Ground Motion Prediction Relationships.

1. INTRODUCTION

Non-linear dynamic analysis is recognized as the more accurate tool for probabilistic seismic risk assessment of structures. The key issue in performing this kind of analysis is the selection of appropriate seismic input, which should allow for a correct and accurate estimation of the seismic performance on the basis of the seismic hazard at the site where the structure is located. To this aim several procedures have been proposed, they require specific characterization of real ground motion records via the so-called *intensity measures* (IMs) in order to estimate probabilistically the structural seismic performance. An IM is a parameter which is considered to be a proxy for the potential effect of the record on the structure. Typical ground motion IMs are the peaks of the acceleration, velocity and displacement signals (PGA, PGV and PGD, respectively); conventional probabilistic seismic hazard analysis provides the probability of exceeding a specified value of one of these parameters in a given time span. Linear spectral ordinates, especially accelerations at the fundamentals period of the structure, $Sa(T_1)$, are also often used as IMs for probabilistic assessment of structures. This is mainly because $Sa(T_1)$, being the response of a linear single degree of freedom system (SDOF), should be, in principle, more correlated with the structural global non-linear performance with respect to, for example, PGA. More sophisticated IMs are currently the focus of a great deal of research. For example, Baker (2007) discusses vector-valued IMs' potential to reduce the number of non-linear dynamic analyses to get a good estimate of the structural seismic response.

The most of the proposed vector-valued IMs is composed of spectral ordinates or other proxies for the spectral shape in a range of periods believed to be of interest for the non-linear structural behavior. This helps to estimate the peak structural response especially in terms of displacements. Integral signal's parameters, e.g., the Arias intensity or significant ground motion duration, are possible IMs, although they are considered more related to the cyclic energy dissipation rather than to the peak structural response. In fact, some studies (e.g., Iervolino et al., 2006; Bojòrquez et al., 2006) investigated how ground motion duration or energy-based parameters affect nonlinear structural response. It was found that, generally, spectral ordinates are *sufficient* if one is interested in the ductility response, while duration-related measures do play a role only if the cyclic structural response is that to assess, i.e., those cases in which the cumulative damage potential of the earthquake is of concern.
Herein, an intensity measure consisting of peak ground acceleration (PGA), and a parameter which may account for the cumulative damage potential of ground motion is investigated with respect to prediction relationships and joint distribution. The chosen energy-related measure is the so-called *Cosenza and Manfredi index* (I_D) (Cosenza and Manfredi, 1997; Manfredi 2001). This factor (Eqn. 1.1) has been proven to be a good predictor for computation of cyclic structural response. In Eqn. 1.1 a(t) is the acceleration time-history and t_E is the total duration of the seismic event. Therefore, the numerator of I_D is proportional to the Arias Intensity and therefore will be referred to as I_A.

$$I_D = \frac{I_A}{PGA \cdot PGV} = \frac{\int_0^{t_E} a^2(t)dt}{PGA \cdot PGV} \qquad (1.1)$$

A ground motion prediction relationship has been retrieved for I_D on the basis of an empirical dataset of Italian records already used for other well known attenuation laws proposed in the past by Sabetta and Pugliese (1987, 1996). Subsequently, the residuals of the regressions were tested for correlation and for joint normality. The study allowed to obtain distributions of I_D conditional on PGA and the corresponding design earthquake in terms of magnitude and distance from hazard disaggregation, as illustrated in two simple examples at the end of the paper.

2. PREDICTION RELATIONSHIP FOR I_D FROM ITALIAN STRONG-MOTION DATA

The best candidates to be ground motion intensity measures are those for which hazard analysis, and therefore attenuation relationships are easy to compute. A ground motion prediction relationship has been developed for I_D. The dataset consists of 190 horizontal components from 95 recordings of Italian earthquakes used by Sabetta and Pugliese (1987, 1996). For the purposes of the present study the records were obtained by the *European Strong-motion Database* (ESD), whose URL is http://www.isesd.cv.ic.ac.uk (Ambraseys et al. 2000; Ambraseys et al. 2004). The distribution of the records used with respect to magnitude and distance is given in Figure 1.

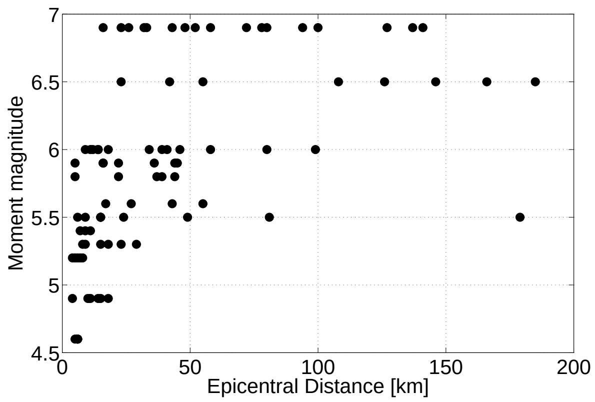

Figure 1 - Distribution of the strong-motion records with respect to moment-magnitude and epicentral distance.

2.1. Functional form and regressions for PGV, PGA and I_A.

Empirical predictive equations for the log of the parameter of interest (Y), as a function of moment magnitude (*M*), epicentral distance in km (*R*) and recording site geology were developed by regression considering the same functional form of Sabetta and Pugliese (1987) and reported in Eqn. 2.1, where *h* is a fictitious depth, *S* = 0 for stiff and deep soil sites and *S* = 1 for shallow soil sites and $\varepsilon_{\log_{10}Y}$, the residual, is a random variable assumed to be Gaussian with zero mean and a standard deviation of $\sigma_{\varepsilon_{\log_{10}Y}}$.

The estimates for the coefficients in Eqn. 2.1, obtained using the ordinary least-squares analysis, are given in Table 1, along with the standard deviation of the residual, for PGA, PGV and I_A. The *h* values were not estimated and assumed to be coincident to those provided by Sabetta and Pugliese (1996).

$$\log_{10}(\mathrm{Y}) = a + b\,\mathrm{M} + c\log_{10}\left(\mathrm{R}^2 + h^2\right)^{\frac{1}{2}} + d\,\mathrm{S} + \varepsilon_{\log_{10}\mathrm{Y}} \tag{2.1}$$

Table 1. Regression coefficients of Eqn. 2.1 for PGA, PGV and I_A.

Y	*a*	*b*	*c*	*d*	*h*	$\sigma_{\varepsilon_{\log_{10}Y}}$
PGA [g]	-1.834	0.336	-0.917	0.155	5.0	0.194
PGV [cm/s]	-1.230	0.547	-0.965	0.080	3.9	0.248
I_A [cm^2/s^3]	0.472	0.921	-1.717	0.193	5.3	0.389

Shapiro-Wilk test (1965) was used to examine the assumption of univariate normality of $\varepsilon_{\log_{10}PGA}$, $\varepsilon_{\log_{10}PGV}$ and $\varepsilon_{\log_{10}I_A}$, based on the considered sample. Results of the tests, summarized in Table 2, indicate that the null hypothesis of normality cannot be rejected, assuming a 0.05 significance level, for the logs of all the parameters considered.

Table 2. Normality test for residuals of regressions of Table 1 using the Shapiro-Wilk (1965) test.

	$\varepsilon_{\log_{10}PGA}$	$\varepsilon_{\log_{10}PGV}$	$\varepsilon_{\log_{10}I_A}$
Test statistic	0.9871	0.9881	0.9932
Significance[1]	0.4796	0.5537	0.9148

For PGA and PGV, the coefficient *c* resulted very close to − 1. The Student's *T-Test* (Mood et al., 1988) was performed to test the null hypothesis H_o: $c = -1$. In both cases it has been possible not to reject H_o at 5% significance level (Table 3). In fact, the regressions of Sabetta and Pugliese (1987, 1996) assume the value of the distance coefficient to be − 1. Regressions were repeated with this constraint and the new estimated coefficients are reported in Table 4; again, *h* values were not estimated.

Table 3. Test on the value of the distance coefficient for the predictions of PGA and PGV.

	PGA	PGV
Test statistic	1.2424	0.4247
Critical value at 5% significance level	± 1.986	± 1.986

Although the results of the regression are slightly different from those obtained by Sabetta and Pugliese (1987, 1996), these differences are expected. The records used come from different databases and therefore may have been subjected to a different processing. Moreover, in this study all the regression analyses (for PGA, PGV and I_A) were performed using the horizontal component of each station having the larger value of PGA, while

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[1] Computed with STATA® ver. 10 software.

Sabetta and Pugliese (1987, 1996) used the component featuring the largest value of the parameter of interest separately for each regression. The tests for univariate normality of the residuals have been repeated for the regressions of Table 4 and confirmed at the same significance level.

Table 4. Regression coefficients of Eqn. 2.1 for PGA, PGV after assuming $c = -1$ for PGA and PGV.

Y	a	b	c	d	h	$\sigma_{\varepsilon_{\log_{10}Y}}$
PGA [g]	-1.917	0.370	-1	0.153	5.0	0.195
PGV [cm/s]	-1.269	0.562	-1	0.079	3.9	0.247

2.2. Derivation of prediction equation coefficients for I_D

In order to obtain an attenuation relation for the log of I_D as a function of M, R and S it is possible to derive its coefficients as linear combinations of those for $\log_{10}\mathrm{PGA}$, $\log_{10}\mathrm{PGV}$ and $\log_{10}I_A$. In fact, the log of I_D is given by the log of I_A minus the logs of PGA and PGV. This leads to the expression of Eqn. 2.2, the coefficients of which are reported in Table 5. Note that a, b and d are obtained combining those of Table 4, while c is that of I_A; subscripts 1, 2 and 3 for h refers to PGA, PGV and I_A respectively.
Shapiro-Wilk test was used again to examine the assumption of univariate normality of $\varepsilon_{\log_{10}I_D}$ and it cannot be rejected at a 0.05 significance level (Table 6).

$$\log_{10}I_D = a + b\,M + \log\left(\frac{\left(R^2+h_2{}^2\right)\left(R^2+h_1{}^2\right)}{\left(R^2+h_3{}^2\right)^c}\right)^{\frac{1}{2}} + d\,S + \varepsilon_{\log_{10}I_D} \qquad (2.2)$$

Table 5. Coefficients of Eqn. 2.2.

Y	a	b	c	d	$\sigma_{\varepsilon_{\log_{10}Y}}$
I_D	0.668	-0.011	1.717	-0.039	0.197

Table 6. Univariate normality test for residuals of the log of I_D.

	$\varepsilon_{\log_{10}I_D}$
Test statistic	0.9875
Significance[1]	0.5102

For I_D, the coefficient b resulted close to zero. Therefore a significance Student's *T-Test* test was performed and this hypothesis could not be rejected at the 5% significance level (Table 7). Then, this coefficient was constrained to the value of 0 and a new model was estimated and reported in Table 8 (note that also the soil coefficient may prove to be non-significant). The normality of the residual after the new regression was tested again and the significance obtained was very similar to that of the test before setting $b = 0$.

Table 7. Test on the value of the magnitude coefficient for the predictions of I_D.

Test statistic	-0.371
Critical value at 5% significance level	± 1.986

Table 8. Coefficients of Eqn. 2.2 after tested non-significance of b.

Y	a	b	c	d	$\sigma_{\varepsilon_{\log_{10}Y}}$
I_D	0.596	0	1.717	-0.032	0.197

3. MULTIVARIATE NORMALITY

3.1. Joint normality of logs of PGA, PGV, I_A and I_D.

Before testing the logs PGA and I_D to be jointly Gaussian, a more general test has been performed on all four parameters considered herein. The skewness and kurtosis' tests of Mardia (1985) were used to test multivariate normality, as they have been also used before to test multivariate normality in ground motion-related studies (e.g. Jarayam and Baker, 2008).

Results of the multivariate normality tests performed for $\left(\varepsilon_{\log_{10}\mathrm{PGA}}, \varepsilon_{\log_{10}\mathrm{PGV}}, \varepsilon_{\log_{10}\mathrm{I_E}}, \varepsilon_{\log_{10}\mathrm{I_D}}\right)$ are given in Table 9. With a given significance level of 0.05, the multivariate skewness and the multivariate kurtosis result non-significant; i.e. there is no statistical evidence to reject the hypothesis of joint normality of the vector's components.

Table 9. Tests for joint normality of the logs of the four parameters.

	Mardia's test of skewness	Mardia's test of kurtosis
Test statistic	20.0397	-0.6131
Critical value at 5% significance level	31.4104	± 1.96

3.2. Joint normality of logs of Id and PGA

As this study aims to investigate the joint and conditional distributions of PGA and I_D, the joint normality of the pair was tested. In fact, because the vector above can be considered normally distributed, the vector made of the logs of I_D and PGA should be still Gaussian (as well as all the possible marginal and conditional distributions obtained from the joint distribution of $\varepsilon_{\log_{10}\mathrm{PGA}}, \varepsilon_{\log_{10}\mathrm{PGV}}$, $\varepsilon_{\log_{10}\mathrm{I_E}}$ and $\varepsilon_{\log_{10}\mathrm{I_D}}$). Therefore, in principle, there would be no need to test the normality of the joint distribution of $\varepsilon_{\log_{10}\mathrm{PGA}}$ and $\varepsilon_{\log_{10}\mathrm{I_D}}$. Nevertheless, joint normality of the above was based on an hypothesis test, therefore it may be prudent to also test the bivariate normality.

Results of the Mardia's normality tests performed for $\left(\varepsilon_{\log_{10}\mathrm{PGA}}, \varepsilon_{\log_{10}\mathrm{I_D}}\right)$ are given in Table 10. With a given significance level of 0.05, the multivariate skewness and the multivariate kurtosis result non-significant.

Table 10. Tests for joint normality of the logs of PGA and I_D.

	Mardia's test of skewness	Mardia's test of kurtosis
Test statistic	1.4078	-0.8429
Critical value at 5% significance level	9.4877	± 1.96

4. CONDITIONAL PROBABILITY OF I_D GIVEN PGA

Finally, the residuals of the prediction relationships for the logs of PGA and I_D have been tested for correlation in order to compute $f(\log_{10}\mathrm{I_D} \mid \log_{10}\mathrm{PGA})$, that is, the conditional probability density function of the log of I_D given the log of PGA.

The estimated correlation coefficient (r) between $\varepsilon_{\log_{10}\mathrm{PGA}}$ and $\varepsilon_{\log_{10}\mathrm{I_D}}$ (equal to –0.2865) has been tested with respect to the the null hypothesis H_o: $\rho = 0$ (ρ is the "true" correlation coefficient), which has been rejected at 5% significance level (Table 11). For the test, the statistic of Eqn. 4.1 has been used, which, under the null hypothesis, is distributed as a Student-T random variable with $n - 2$ degrees of freedom (n is the sample size).

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} \sim T_{n-2} \tag{4.1}$$

Table 11. Test on the significance of the correlation between the logs of PGA and I_D.

Test statistic	-2.884
Critical value at 5% significance level	± 1.986

Then, the joint distribution of $\log_{10} I_D$ and $\log_{10} PGA$ may be defined by the following bivariate normal probability density function:

$$f(\log_{10} I_D, \log_{10} PGA) =$$
$$= \frac{1}{2\pi\sigma_{\log_{10}I_D}\sigma_{\log_{10}PGA}\sqrt{(1-\rho^2)}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(\log_{10}I_D - \mu_{\log_{10}I_D})^2}{\sigma^2_{\log_{10}I_D}} - \frac{2\rho(\log_{10}I_D - \mu_{\log_{10}I_D})(\log_{10}PGA - \mu_{\log_{10}PGA})}{\sigma_{\log_{10}I_D}\sigma_{\log_{10}PGA}} + \frac{(\log_{10}PGA - \mu_{\log_{10}PGA})^2}{\sigma^2_{\log_{10}PGA}}\right]} \tag{4.2}$$

where $\mu_{\log_{10}I_D}$ and $\sigma_{\log_{10}I_D}$ are the mean and the standard deviation of $\log_{10} I_D$ respectively; $\mu_{\log_{10}PGA}$ and $\sigma_{\log_{10}PGA}$ are the mean and the standard deviation of $\log_{10} PGA$ respectively. The variance-covariance matrix for $\varepsilon_{\log_{10}PGA}$ and $\varepsilon_{\log_{10}I_D}$ is reported in Eqn. 4.3.

$$\begin{pmatrix} 0.038 & -0.011 \\ -0.011 & 0.039 \end{pmatrix} \tag{4.3}$$

Because of bivariate normality, the conditional probability density function for one of the variables, given a known value for the other variable, is normally distributed. The conditional mean ($\mu_{\log_{10}I_D|\log_{10}PGA=a}$) and variance ($\sigma_{\log_{10}I_D|\log_{10}PGA=a}$) of $\log_{10} I_D$ given that $\log_{10} PGA = a$ is:

$$\begin{cases} \mu_{\log_{10}I_D|\log_{10}PGA=a} = \mu_{\log_{10}I_D} + \rho\sigma_{\log_{10}I_D} \dfrac{a - \mu_{\log_{10}PGA}}{\sigma_{\log_{10}PGA}} \\ \sigma_{\log_{10}I_D|\log_{10}PGA=a} = \sigma_{\log_{10}I_D}\sqrt{1-\rho^2} \end{cases} \tag{4.4}$$

4.1. Illustrative applications

As an example of the possible use of the results obtained, curves in Figure 2 represent the complementary cumulative density functions of I_D conditional on PGA for nine different cases for the sites of S. Angelo dei Lombardi and Naples in the Campania region (southern Italy). The chosen scenarios, in terms of *M* and *R* (Table 12), refer to the mean values obtained from disaggregation of seismic hazard for PGA (Bazzurro and Cornell, 1999) for different probabilities of exceedance in 50 years (corresponding to 9 return periods, T_R, from 30 to 2475 years). In other words, seismic hazard in terms of PGA was obtained for the two sites from the study of the Italian Istituto Nazionale di Geofisica e Vulcanologia (http://esse1.mi.ingv.it/). Disaggregation, in terms of magnitude and distance, of seismic hazard corresponding to the 9 return periods was also retrieved from the same website. For the mean *M* and *R* values from disaggregation, median PGA, $\mu_{PGA|M,R}$, was computed using the attenuation relationships discussed above. Finally, the distribution of I_D was computed conditional on $\mu_{PGA|M,R}$, *M* and *R*. The curves of Figure 2 give information on the values of I_D which should be taken into account in respect to the hazard in terms of PGA at the site.

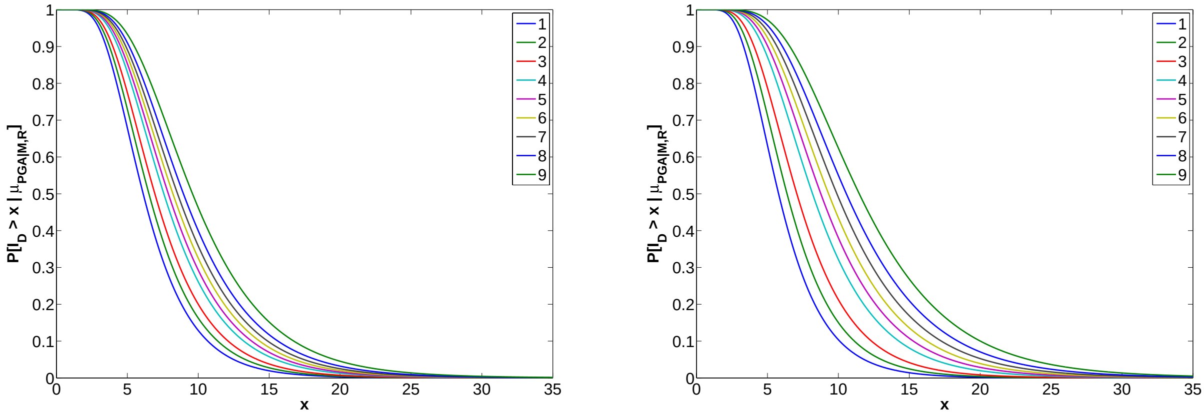

Figure 2. Probability of exceedance of I_D given PGA for nine scenarios for S. Angelo dei Lombardi (left) and Napoli (right).

Table 12. Considered scenarios for S. Angelo dei Lombardi and Naples.

Scenarios	S. Angelo dei Lombardi			Naples		
	M	R [km]	$\mu_{\mathrm{PGA}\vert\mathrm{M,R}}$ [g]	M	R [km]	$\mu_{\mathrm{PGA}\vert\mathrm{M,R}}$ [g]
1 (T_R = 2475 years)	6.39	5.8	0.3669	5.02	4.8	0.1253
2 (T_R = 975 years)	6.20	7.0	0.2769	4.99	6.6	0.1031
3 (T_R = 475 years)	6.04	8.4	0.2121	5.00	8.9	0.0838
4 (T_R = 201 years)	5.84	11.0	0.1451	5.06	14.0	0.0607
5 (T_R = 140 years)	5.77	12.5	0.1227	5.11	17.5	0.0517
6 (T_R = 101 years)	5.72	14.2	0.1051	5.16	21.4	0.0447
7 (T_R = 72 years)	5.66	16.1	0.0892	5.22	26.3	0.0386
8 (T_R = 50 years)	5.61	18.8	0.0741	5.30	32.9	0.0333
9 (T_R = 30 years)	5.54	23.7	0.0561	5.39	43.8	0.0271

5. CONCLUSIONS

In recent years, many IMs were introduced in order to link real ground motion records to the hazard at a given site and to estimate probabilistically the seismic performance of structures. The aim was to obtain a correct and accurate estimation of the structural performance in a way that it does not require an undesirable large number of non-linear dynamic analyses to run. For example, if displacement response is of interest, it is widely accepted that first-mode spectral acceleration is an efficient IM, as it allows to select records without taking care of magnitude, distance and related features as duration. Nevertheless, for structures suffering cumulative damage because of cyclic degradation (especially some kind of under-designed existing structures), the characterization of seismic demand should include the cyclic potential of ground motion.

In this study joint and conditional distributions of a vector of IMs consisting of PGA and I_D, a parameter proven to be a good predictor of cyclic structural response, was investigated. Italian strong-motion data were used to obtain a prediction relationship for I_D. Subsequently, the residuals of the models estimated have been tested for univariate and joint normality. After the bivariate normality of the logs of I_D and PGA has been verified and their correlation has been estimated, the conditional probability density function of the log of I_D given the log of PGA was also obtained in close form.

Finally, results have been used to compute conditional distribution of I_D for different scenarios related to disaggregation of hazard for PGA which may be used for structural design and/or assessment purposes.

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