Seismic Demand Analysis of Steel Storage Tanks

Giovanni Fabbrocino, Iunio Iervolino, and Antonio Dicarluccio

Abstract

Seismic behaviour of steel tanks for oil storage is relevant in the light of industrial risk assessment because collapse of these structures may trigger other catastrophic phenomena, as fires or explosions due to loss containment. Therefore, seismic assessment should be focussed on leakage-based limit states. Damages suffered by storage tanks under seismic actions are generally related to large axial compressive stresses that can induce shell buckling near the base and to large displacements of unanchored structures leading to detachment of piping, liquid. The present paper approaches the analysis of seismic response of sliding, non-uplifting, unanchored liquid storage tanks subject to three-dimensional ground motion. The algorithm to solve the equation of motion for a simplified tank’s model is proposed and a sample estimation of the seismic demand by incremental dynamic analysis is discussed.

Keywords: Seismic risk, storage tanks, dynamic analysis, ground motion, sliding, elephant foot buckling.

1 Introduction

Earthquakes represent an external hazard for industrial plants and may trigger accidents, i.e. fire and explosions resulting in injury to people and to near field equipments or constructions, if structural failures result in release of hazardous material. Quantitative Risk Analysis (QRA) [1] provides a guide for analysis of industrial risk; such an assessment may include the seismic threat if ground motion related malfunctioning (i.e. failure) rates are available for components [2]. From the structural perspective, steel tanks for oil storage are standardized structures both in terms of design and construction [3], [4], [5]. Review of international standards for the construction points out that design evolved slowly; therefore, a large number of post-earthquake damage observations [6] is available and empirical vulnerability functions have been developed [7]. This is a privileged case with respect other
building-like and non-building-like structures, however empirical fragility typically suffers some shortcomings; for example vulnerability data contain also information about site effect which may be hard to disaggregate. Therefore, the development of analytical models able to predict the response of the structural components and systems under seismic loading is worth to be explored.

The present work is aimed at the discussion of an algorithm able to analyze the three-dimensional response accounting for sliding behaviour of unanchored tanks. It is accounts for fluid-structure interaction in a simplified manner, since limitation of computational efforts is a key aspect in seismic reliability evaluations. It takes advantage of the several proposal to approximate the seismic dynamics of tanks available in literature and makes an attempt to extend them in order to including large-displacement limit states.

Then, model and algorithm have been employed for estimation of seismic demand in terms of base plate-ground relative displacement and shell compressive stress, which represent the engineering demand parameters related to the failure of connection piping and shell’s elephant foot buckling (EFB). The method used is the Incremental Dynamic Analysis which has been originally developed for buildings and recently extended to tanks [8].

2 One-Dimensional lumped mass models

Housner was one of the first to investigate the seismic behaviour of tanks presenting in 1963 a simplified model for seismic analysis of anchored tanks with rigid walls [9]. According to this model: in a tank with a free liquid surface subjected to horizontal ground acceleration a given fraction of the liquid is forced to participate in this motion as rigid mass; on the other hand the motion of the tank walls excites the liquid into oscillations which result in a dynamic force on the tank. This force is assumed to be the same of a lumped mass, know as a convective mass, that can vibrate horizontally restrained by a spring [10], [11].

Rosenblueth and Newmark [12] modified the expression suggested by Housner to estimate the convective and rigid masses and gave updated formulations for the evaluation of the seismic design forces of liquid storage tanks. In 1983 Haroun developed a model to evaluate of the seismic response of storage tanks including wall’s deformation [13]. In this model a part of the liquid moves independently of tank’s shell, again convective motion, while another part of the liquid oscillates unison with the tank. If the flexibility of the tank’s wall is considered, a part of this mass moves independently (impulsive mass) while the remaining accelerates back and forth with the tank (rigid mass). Figure 1 shows the idealised structural model of liquid storage tank. The contained continuous liquid mass is lumped as convective, impulsive and rigid masses referred as \( m_c \), \( m_i \) and \( m_r \), respectively. The convective and impulsive masses are connected to the tank’s wall by different equivalent spring having stiffness \( k_c \) and \( k_i \), respectively. This model for anchored storage tank has been extended to analyse unanchored base-isolated liquid storage tanks [14].
The effective masses are defined in terms of liquid mass $m$ in Equations (1-4) where $Y_c$, $Y_i$, and $Y_r$ are the function of the $h/R$ ratio, which is a filling coefficient and $\rho_w$ is the liquid’s specific weight.

\[
\begin{align*}
m_c &= mY_c \left( \frac{h}{R} \right) \\
m_i &= mY_i \left( \frac{h}{R} \right) \\
m_r &= mY_r \left( \frac{h}{R} \right) \\
m &= \pi R^2 h \rho_w
\end{align*}
\]

The natural frequencies of convective mass, $\omega_c$ and impulsive mass, $\omega_i$ are given by expressions (5) and (6).

\[
\begin{align*}
\omega_c &= \sqrt{\frac{1.84g}{R}} \tanh \left( \frac{1.84h}{R} \right) \\
\omega_i &= \frac{P}{H} \sqrt{\frac{h}{\rho_s}}
\end{align*}
\]

Where $E$ and $\rho_s$ are the modulus of elasticity and density of tank’s wall respectively; $g$ is the acceleration due to gravity; and $P$ is a dimensionless parameter also it function of $h/R$. This model is assumed as basis of the seismic demand estimations discussed in the following.

### 3 Unanchored tanks seismic behaviour

Motion of unanchored tanks is characterized by large-displacement phenomena: during the ground motion the tank both can slide relatively to the foundation and the base plate may uplift due to overturning moment.

The sliding depends on the base shear, once it reaches the limit value corresponding to the frictional resistance (Equation 7) relative motion between the tank and the foundation starts. Sliding reduces the max acceleration the tank suffers.
This reduction is dependent upon the frictional factor \( \mu \), but relatively small values of the latter produce large relative displacements.

Different model can be used in a sliding system to describe the frictional force. In fact, together with the conventional frictional relationship, hysteretic model have been proposed [15]; the letter are generally continuous and need the automativ continuity of the hysteretic displacement components. In the following analyses the conventional model is used for the frictional force, but this assumption actually does not represent a limitation of the approach.

In particular, the friction force is evaluated by considering the equilibrium of the base: the system remain in the non-sliding phase if the frictional force in time \( t \) is lower than the limiting frictional force expressed by Equation (7); where \( g \) represent the gravitational acceleration, \( u_{\text{vert}} \).

\[
F_{\text{lim}} = m \mu (u_{\text{vert}} + g)
\]  

(7)

Therefore the motion can be subdivided in non-sliding and sliding phase. Whenever the tank does not slide, the dynamic equilibrium of forces in Equation (8) applies if instance one horizontal component is considered, while Equation (9) fit the case of both horizontal components.

\[
F_x = -\left( m \ddot{x} + m \ddot{x} + M \ddot{x} + M \ddot{y} \right)
\]  

\[
F_y = -\left( m \ddot{y} + M \ddot{y} + M \ddot{y} \right)
\]  

In additional, another large-displacement mechanism can be recognised after real observations of the seismic response of unanchored liquid storage tanks; it is represented by the partial uplift of the base plate [16]. This phenomenon reduces the hydrodynamic forces in the tank, but increases significantly the axial compressive stress in the tank wall. In fact, base uplifting in tanks supported directly on flexible soil foundations does not lead to a significant increase in the axial compressive stress in the tank wall, but many lead to large foundation penetrations and several cycles of large plastic rotations at the plate boundary [17], [18]. Flexibly supported unanchored tanks are therefore less prone to elephant-foot buckling damage, but more prone to uneven settlement of the foundation and fatigue rupture at the plate-shell connection. An aspect particularly interesting is the force-displacement relationship for the plate boundary. The definition of this relationship is complicated by the nonlinearities arising from: 1) the continuous variation of the contact area of the interface between the base plate and the foundation; 2) the plastic yielding of the base plate; and 3) the effect of the membrane forces induced by the large deflections of the plate.

In the following, partially uplifted base plate is not considered even if the automatic procedure implemented can be integrated with uplifting procedure; this task is the next scheduled step in the algorithm development.
4 Equations of motion

The equations of motion of the unanchored tank under a generalised three-dimensional input ground motion are reported in Equation (10) in the case of non-sliding (rest) of the system.

\[
\begin{align*}
    m_c \ddot{x}_c + b_c \dot{x}_c + k_c x_c &= -m_c \ddot{u}_{gx} \\
    m_i \ddot{x}_i + b_i \dot{x}_i + k_i x_i &= -m_i \ddot{u}_{gx} \\
    m_c \ddot{y}_c + b_c \dot{y}_c + k_c y_c &= -m_c \ddot{u}_{gy} \\
    m_i \ddot{y}_i + b_i \dot{y}_i + k_i y_i &= -m_i \ddot{u}_{gy}
\end{align*}
\]  

(10)

Where \(x_c\) and \(y_c\) are the components of displacements of convective masses relative to the base; \(x_i\) and \(y_i\) are the components of displacements of impulsive masses relative to the base; \(x_b\) and \(y_b\) are the component of displacement of the base relative to the ground; \(\ddot{u}_{gx}\) and \(\ddot{u}_{gy}\) are the two horizontal components of ground acceleration. In this case \(m_c\) and \(m_i\) represent two simple oscillators and there is no coupling between two directions of motion (\(x_b\) and \(y_b\) are known).

If the system is sliding the equations can be obtained by the dynamic equilibrium for convective and impulsive masses and for the system as a whole Equation (11).

\[
\begin{align*}
    m_c \ddot{x}_c + m_c \ddot{x}_b + b_c \dot{x}_c + k_c x_c &= -m_c \ddot{u}_{gx} \\
    m_i \ddot{x}_i + m_i \ddot{x}_b + b_i \dot{x}_i + k_i x_i &= -m_i \ddot{u}_{gx} \\
    m_c \ddot{y}_c + m_c \ddot{y}_b + b_c \dot{y}_c + k_c y_c &= -m_c \ddot{u}_{gy} \\
    m_i \ddot{y}_i + m_i \ddot{y}_b + b_i \dot{y}_i + k_i y_i &= -m_i \ddot{u}_{gy} \\
    m_c \ddot{x}_c + m_c \ddot{y}_c + m_{tot} \dot{x}_b + m_{tot} \dot{y}_b + m_{tot} \mu (\ddot{u}_{vert} + g) \cos(\alpha) &= -m_{tot} \ddot{u}_{gx} \\
    m_i \ddot{x}_i + m_i \ddot{y}_i + m_{tot} \dot{x}_b + m_{tot} \dot{y}_b + m_{tot} \mu (\ddot{u}_{vert} + g) \sin(\alpha) &= -m_{tot} \ddot{u}_{gy}
\end{align*}
\]  

(11)

Both the systems of equations for sliding and non-sliding can be expressed in the same matrix format as in Equation (12).

\[
\begin{bmatrix}
    \dddot{x} + B \ddot{x} + K x + F_1 = -M \dddot{u}_{gx} \\
    \dddot{y} + B \ddot{y} + K y + F_1 = -M \dddot{u}_{gy}
\end{bmatrix}
\]

(12)

5 Numerical study

The numerical study consisted of: (1) incremental dynamic analysis with one horizontal and the vertical ground acceleration components; (2) analysis with three acceleration components.

The first step in the IDA analysis procedure consisted of the acquisition of suitable set records, details about ground motions are given in the following sections. Each of the two investigations were repeated for a range of \(S\) (filling ratio) and a set of \(\mu\) (friction coefficient).
To obtain the seismic demand at selected ground motion intensity levels records are scaled in terms of Peak Ground Acceleration (PGA). The scaling factor $\chi$ varies to get the PGA from 0.2g to 1.5g.

For both the analyses the discussed algorithm to solve the equation of motion was implemented in a computer code using the Wilson theta method [19]. The flow chart of the procedure for time history analysis for storage tanks is shown in Figure 2. The first step is the check of the base velocity at time $t$: in case that it is zero than to known the phase of motion is necessary another inspection of the value of base shear. Whenever the base shear in instant $t$ is lower than the limit frictional value than the motion is rest (no-sliding) type in $[t,t+\Delta t]$; otherwise the system, in the same interval, is sliding.

If the system is in non-sliding the check of the value of the base shear at $t+\Delta t$ is also need. In the case it is lower than $F_{lim}$ the integration of the equation of dynamic equilibrium for the next $\Delta t$ is possible; otherwise is necessary the computation of the time $t^*$, intermediate between $t$ and $t+\Delta t$ corresponding to base shear that equates the limit. In other words, between $t$ and $t^*$ the tank is no-sliding while between $t^*$ and $t+\Delta t$ it is in sliding. In order to compute the intermediate time $t^*$ a linear variation of base shear is assumed in the time interval $[t,t+\Delta t]$. If at time $t$ the velocity is not equal to zero the system is in sliding phase and another check at $t+\Delta t$ is necessary; when the velocity has changed its signum between $t$ and $t+\Delta t$ the computation of $t^*$ corresponding to the point zero velocity is performed. Otherwise the integration of the equation of dynamic equilibrium for the next $\Delta t$ can take place. The time $t^*$ is calculated again a linear variation of velocity. Between $t$ and $t^*$ the system is sliding but the type of motion between $t^*$ and $t+\Delta t$ depends on the base shear at $t^*$: if it is lower than the limit then the system is in no-sliding between $t^*$ and $t+\Delta t$ otherwise it is sliding. The number of equations and the number of unknowns depends on the type of motion at time $t$ of (sliding or no-sliding).

5.1 Results and discussion

In the first phase of analysis a parametric IDA with only one horizontal ground acceleration component was carried out. The parameters of the investigated tank are described in Table 1.

<table>
<thead>
<tr>
<th>$t_b$ [m]</th>
<th>$h$ [m]</th>
<th>$\rho_s$ [kgm$^{-3}$]</th>
<th>$\rho_l$ [kgm$^{-3}$]</th>
<th>$E$ [GPa]</th>
</tr>
</thead>
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<tr>
<td>0.008</td>
<td>10</td>
<td>7860</td>
<td>1000</td>
<td>210</td>
</tr>
</tbody>
</table>

Table 1. Tank’s mechanical parameters.

Where $t_b$ is the base plate and shell’s thickness; $h$ is the height of liquid in the tank, $\rho_s$ and $\rho_l$ are the specific weights of steel and liquid respectively and $E$ is the modulus of elasticity of the tank’s structure. In the parametric analysis $S$ (filling coefficient) varies from 2 to 3.5 and $\mu$ (friction factor) varies from 0.1 to 0.8. The earthquake ground motions considered are showed in Table 2.
Figure 2. Algorithm’s flow chart.

Start

$\bar{\chi}(t)$

Yes

$F(t) < F_{lim}$

No

$\dot{x}_s(t) = 0$

Yes

No Sliding

Solving for
\[
\{\dot{x}(t + \Delta t)\}; \{\dot{x}(t + \Delta t)\}; \{\dot{x}(t + \Delta t)\}
\]

No

Yes

$F(t + \Delta t) < F_{lim}$

$\dot{x}^* : F(\dot{x}^*) = F_{lim}$

No sliding in $[t, \dot{x}^*]$

Solving for
\[
\{\dot{x}(\dot{x}^*)\}; \{\dot{x}(\dot{x}^*)\}; \{\dot{x}(\dot{x}^*)\}
\]

Sliding in $[\dot{x}^*, t + \Delta t]$

Yes

No

$\dot{x}_s(t^*) = 0$

Sliding in $[t, \dot{x}^*]$

Solving for
\[
\{\dot{x}(\dot{x}^*)\}; \{\dot{x}(\dot{x}^*)\}; \{\dot{x}(\dot{x}^*)\}
\]

No

Sliding in $[\dot{x}^*, t + \Delta t]$

Solving for
\[
\{\dot{x}(\dot{x}^* + \Delta t)\}; \{\dot{x}(\dot{x}^* + \Delta t)\}; \{\dot{x}(\dot{x}^* + \Delta t)\}
\]

Yes

$F(\dot{x}^*) < F_{lim}$

No Sliding in $[\dot{x}^*, t + \Delta t]$

Solving for
\[
\{\dot{x}(\dot{x}^* + \Delta t)\}; \{\dot{x}(\dot{x}^* + \Delta t)\}; \{\dot{x}(\dot{x}^* + \Delta t)\}
\]

$t = t + \Delta t$

Yes

No

Yes

No

No

Yes

No

No

Yes

No

No

Yes

No
They are all stiff soil records from a broad range of magnitude and distances. All the accelerograms herein employed come from the European Strong Motion Database (http://www.isesd.cv.ic.ac.uk/) and can be easily retrieved from there.

<table>
<thead>
<tr>
<th>Station</th>
<th>Earthquake</th>
<th>Date</th>
<th>Nation</th>
<th>Mw</th>
<th>Epicentral Distance [km]</th>
<th>PGA [m/s²]</th>
<th>Local Geology</th>
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</thead>
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<td>Friuli (aftershock)</td>
<td>11/09/1976</td>
<td>Italia</td>
<td>5.3</td>
<td>21</td>
<td>0.17g</td>
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<td>000120</td>
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<td>11/09/1976</td>
<td>Italia</td>
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<td>Italia</td>
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<td>15</td>
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</tr>
<tr>
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<td>Italia</td>
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<tr>
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</tr>
<tr>
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<tr>
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<td>000536</td>
<td>Erzincan</td>
<td>13/03/1992</td>
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<td>0.03g</td>
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<tr>
<td>000584</td>
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<td>23/05/1994</td>
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<tr>
<td>000595</td>
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<td>Italia</td>
<td>5.7</td>
<td>25</td>
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<td>000947</td>
<td>Potenza</td>
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<td>Italia</td>
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<td>0.06g</td>
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<td>001863</td>
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<td>Italia</td>
<td>5.4</td>
<td>21</td>
<td>0.04g</td>
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</tr>
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<td>Iceland</td>
<td>6.5</td>
<td>52</td>
<td>0.07g</td>
<td>stiff soil</td>
</tr>
</tbody>
</table>

Table 2. Ground motion station used in the analyses.

In the following selected results are presented for sake of brevity. In particular the $S = 1.5$ and $\mu = 0.1$ case is discussed even if the parametric analysis has shown influence $S$ and $\mu$ on the rigid displacement of base plate and on the axial compressive stress. The result underline that increasing $\mu$ produces the increase of the axial compressive stress and the decrease of the rigid displacement. Conversely an increase in $S$ decreases the axial compressive stress increasing the displacement.

The demand curve in term of axial compressive stress [MPa] for the unidirectional model (including vertical acceleration), computed by Equation (13) from AWWA D100-96, is given in Figure 3.

$$
\sigma_c = \left( w_* + \frac{1.273M}{D^2} \right) \frac{1}{1000\nu_s} 
$$

(13)
Curves include media and 1σ bounds (σ is the standard deviation of the logs of the demand not to be confused with the compressive stress σc). The base-displacement demand curve as function of the PGA for the set of ground motions is summarized in Figure 4. Also for this case median ±1σ curves are given.

![Compressive Stress](image1.png)

**Figure 3.** IDA curve for the compressive axial stress for the uni-directional model.

![Rigid Displacement](image2.png)

**Figure 4.** IDA for the sliding-induced displacement for the uni-directional model analysis.

To understand the effects of including in the analysis also the second component of horizontal ground motion uni-directional and bi-directional results in terms of base-displacement are compared in Figure 5. As expected, these two analyses are not equivalent. In fact, the maximum base displacement for the one-dimension analysis is 0.2m while including the second component it is 0.4m.

![Base-Displacement](image3.png)
6 Conclusions

This paper investigates the seismic response of unanchored steel tanks for oil storage in the large displacement regime in terms of those limit states relevant for industrial risk analysis (Elephant Foot Buckling and base-sliding). Algorithms to integrate equations of motions have been formulated for both one-directional (including vertical acceleration) and considering all three ground motion components. The model does not include the base uplifting, which may affect compressive stress demand, but it is ready to. The model has been employed to produce incremental dynamic analysis demand curves as for building-like structures.

Comparison of the two models has also been carried out, results show that the uni-directional results may be un-conservative, at least in terms of base-displacement demand for sliding tanks. IDA curves can also be similarly developed for bi-directional ground motion, for example using as ground motion intensity measure the geometric mean of the PGA in the two directions, and therefore the proposed model may be used for computation of numerical fragility curves and for structural and industrial seismic risk analysis.

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