

WHICH SPECTRAL SHAPE REALLY MATTER TO PREDICT NONLINEAR STRUCTURAL RESPONSE: APPLICATION TO STEEL FRAMES

Eden Bojórquez¹⁾, Iunio Iervolino²⁾, and Alfredo Reyes-Salazar¹⁾

1) Professor, Faculty of Engineering, Universidad Autonoma de Sinaloa, Culiacan, Sinaloa, México

2) Professor, Dept. of Structural Engineering, University of Naples, Naples, Italy

Email: ebojorq@uas.uasnet.mx; iunio.iervolino@unina.it; reyes@uas.uasnet.mx

Abstract: In this paper, several vector-valued ground motion intensity measures based on a parameter to characterize the spectral shape named N_p are analyzed. The parameter N_p is commonly used to represent the spectral acceleration response spectra in a range of periods, and in the present study this parameters was used to discuss which spectral shape is more related with a specific performance parameter corresponding to maximum and energy demands. For this aim, vector-valued ground motion intensity measures based on N_p through different type of response spectra such as: pseudo-acceleration, pseudo-velocity and input energy are analyzed. The efficiency of the vectors to predict the nonlinear response of several steel frames subjected to narrow-band motions from the soft soil of Mexico City is compared. Finally, an approach to predict the nonlinear response of structures dominated by higher modes of vibrations is proposed.

1. INTRODUCTION

In the last years, parameters to represent the ground motion potential of an earthquake named intensity measures (IMs) have been proposed (Housner 1952, Arias 1970, Von-Thun et al. 1988, Cordova et al. 2001, Baker and Cornell 2005, Tothong and Luco 2007, Baker and Cornell 2008, Bojórquez and Iervolino 2010). One of the desirable features of an IM must be the ability to predict the response of structures subjected to earthquakes (small variability of structural response given the IM). This ability is known as *efficiency* (Bazzurro 1998, Shome 1999, Luco 2002). The efficiency is crucial in records selection for nonlinear structural response with seismic design purposes as well as for reliability-based analysis. Due to this, many efforts have been oriented to propose ground motion intensity measures. Most of these parameters have the peculiar characteristic of representing the spectral shape in a range of periods, which is strongly related with the structural response as suggested in many studies (Cordova et al. 2001, Baker and Cornell 2005, Tothong and Luco 2007, Baker and Cornell 2008, Bojórquez and Iervolino 2010). In particular, vector and scalar ground motion intensity measures based on N_p which is representative of the spectral shape have resulted very well correlated with the nonlinear structural response (Bojórquez and Iervolino 2010, Bojórquez et al. 2010a). However, the studies regarding the parameter N_p have been developed considering only the spectral pseudo-acceleration shape, and it is important to observe which spectral shape is more

related with a specific performance parameters as in the case of maximum interstory drift, hysteretic energy, etc. It is the main objective of this paper to analyze the efficiency of several vector-valued ground motion intensity measures, based on N_p and considering different types of response spectra, to predict the structural response of steel frames.

2. THE PARAMETER N_p

2.1 Definition

It has been mentioned that IMs often try to capture the structural response via the spectral shape with different degrees of accuracy. For example, $Sa(T_1)$, is the perfect predictor for the response of elastic single degree of freedom systems, and a good predictor for elastic multi-degree of freedom systems dominated by the first mode of vibration (associated to the T_1 period). Moreover, studies have found the sufficiency of $Sa(T_1)$ with respect to magnitude and distance (Shome 1999, Iervolino and Cornell 2005). Nevertheless, $Sa(T_1)$ does not provide information about the spectral shape in other regions of the spectrum, which may be important for the nonlinear behavior (beyond T_1) or for structures dominated by higher modes (before T_1). In the case of structures with nonlinear behavior, the structural response may be sensitive to different spectral values associated to the whole range of periods, from the fundamental period until a specific period of the structure beyond of T_1 to account for nonlinear behavior here named T_N . This calls for intensity measures providing information

about the spectral shape in different regions of the spectrum (Baker and Cornell 2008). The parameter N_p is inspired and focused in the featuring of the spectral acceleration shape and it can be obtained by the ratio between the geometrical mean in the range of periods from T_1 until T_N $Sa_{avg}(T_1, \dots, T_N)$ divided by $Sa(T_1)$. It is mathematically represented as:

$$N_p = \frac{Sa_{avg}(T_1, \dots, T_N)}{Sa(T_1)} \quad (1)$$

According to this equation, if we have one or n records with a mean N_p value close to one, it is expected that the average spectrum be about flat in the range of periods between T_1 and T_N . For a N_p value lower than one, an average spectrum with negative slope is expected. As an example, the mean value of N_p for a group of ordinary records in the period $T_1=0.6s$ is 0.39. In Figure 1a, the average spectrum of this set is illustrated. In the case of N_p values larger than one, the spectra tend to increase beyond T_1 . As it can be appreciated for a set of narrow-band records, where the mean value of $N_p=1.9$ for $T_1=1.2s$, the average spectrum shows an increasing accelerations zone (see Figure 1b). Finally, the normalization of $Sa(T_1)$ let N_p be independent of the scaling level of the records based on $Sa(T_1)$, but most importantly it helps to improve the knowledge of the path of the spectrum from period T_1 until T_N , which is related with the nonlinear structural response. In the present study, for all the cases analyzed T_N equals to 2.2 times T_1 was used, as suggested by Bojórquez and Iervolino (2010).

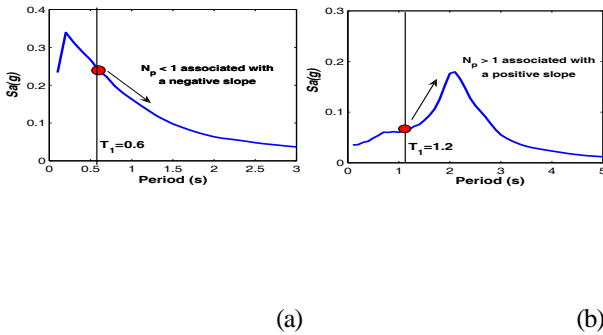


Figure 1 Mean elastic response spectra for a set of records with: (a) $N_p=0.39$, and (b) $N_p=1.9$

3. FEATURING DIFFERENT TYPES OF SPECTRAL SHAPE, AND VECTOR-VALUED GROUND MOTION IMs CONSIDERED

The efficiency to predict the nonlinear response of moment-resisting steel frames subjected to ground motion records is compared by means of different alternatives vector-valued ground motion intensity measures IMs with two parameters. An efficient IM can help to considerably reduce the number of analyses required to estimate the response of structures subjected to earthquakes with accuracy. Because most of the seismic hazard maps around the world were developed for the spectral acceleration at first mode of vibration $Sa(T_1)$, all the vectors here considered are based in $Sa(T_1)$ as the first parameter;

however, other parameters as pseudo-velocity or nonlinear parameters can be used. In the second parameter of the vector, N_p was considered but using different types of response spectra. In total three different response spectra were considered to feature N_p as the second parameter. The first vector considered is $\langle Sa(T_1), N_{pSa} \rangle$ which originally was proposed by Bojórquez and Iervolino (2010). It can be obtained by using Equation 1. The second vector is $\langle Sa(T_1), N_{pSv} \rangle$, where N_{pSv} is obtained from Equation 1, but using the pseudo-velocity response spectrum instead of the pseudo-acceleration response spectrum. Finally, the last vector-valued ground motion intensity measure considered was $\langle Sa(T_1), N_{pEI} \rangle$, where N_{pEI} is based on the input energy response spectrum. The input energy can be defined from the equation of motion of a single degree of freedom system as follows:

$$m \ddot{x}(t) + c \dot{x}(t) + f_s(x, \dot{x}) = -m\ddot{x}_g(t) \quad (2)$$

In equation 2, m is the mass of the system; c , the viscous damping coefficient; $f_s(x, \dot{x})$, the non-linear force; \ddot{x} , the ground acceleration; and x , the displacement with respect to the base of the system. A dot above x indicates a derivative with respect to time. In case of an elastic linear system, $f_s(x, \dot{x}) = k x$, where k is the stiffness of the system.

Integrating each member of Equation 2 with respects to x , yields:

$$\int m \ddot{x}(t) dx + \int c \dot{x}(t) dx + \int f_s(x, \dot{x}) dx = -\int m \ddot{x}_g(t) dx \quad (3)$$

Equation 3 can be written as energy balanced equation as follows:

$$E_K + E_D + E_S + E_H = E_I \quad (4)$$

where E_K , E_D , E_S and E_H represent the kinetic (k), viscous damping (D), deformation (S) and dissipated hysteretic (H) energies, respectively; and E_I is the relative input energy, which will be used to obtain the vector $\langle Sa(T_1), N_{pEI} \rangle$. Note that all the vectors were selected to represent maximum and cumulative potential of a ground motion shaking.

4. STRUCTURAL STEEL FRAME MODELS, SEISMIC RECORDS AND PERFORMANCE PARAMETERS

4.1 Structural steel frame models

Four moment-resisting steel frames having 4, 6, 8 and 10 stories, were considered for the studies reported herein. The frames are denoted as F4, F6, F8 and F10, respectively. As shown in Figure 2, the frames, designed according to the Mexico City Building Code (MCBC), have three eight-meter bays and inter-story heights of 3.5 meters. Each frame was provided with ductile detailing and its lateral strength was established according to the MCBC. A36 steel was used for the beams and columns of the frames. Relevant characteristics for each frame, such as the fundamental period of vibration (T_1), and the seismic coefficient and

displacement at yielding (C_y and D_y) are shown in Table 1 (the latter two values were established from static nonlinear analyses).

A two dimensional, lumped plasticity nonlinear model of each frame was prepared and analyzed. For this purpose, an elasto-plastic model with 3% strain-hardening was used to represent the cyclic behavior (in terms of bending moment and rotation) of the transverse sections located at both ends of the steel beams and columns. As discussed by Bojórquez and Rivera (2008), this model provides a good approximation to the actual hysteretic behavior of steel members. The frames were analyzed considering 3% of critical damping.

Table 1 Relevant characteristics of the steel frames

Frame	Number of Stories	$T_1(s)$	C_y	$D_y(m)$
F4	4	0.90	0.45	0.136
F6	6	1.07	0.42	0.174
F8	8	1.20	0.38	0.192
F10	10	1.37	0.36	0.226

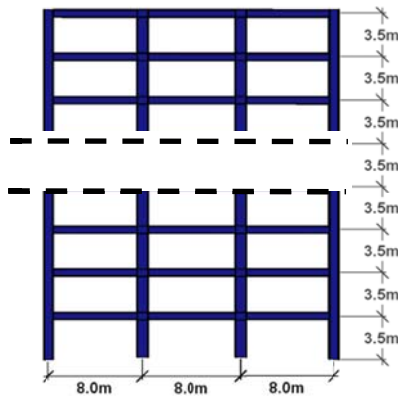


Figure 2 Geometrical characteristics of the steel frames considered

4.2 Seismic records

A set of 30 narrow-banded ground motions recorded at Lake Zone sites of Mexico City was considered. Particularly, all motions were recorded at sites having soil periods of two seconds, during seismic events with magnitudes near of seven or larger and having epicenters located at distances of 300 km or more from Mexico City. Some important characteristics of the records are summarized in Table 2. In this table, PGA and PGV denote the peak ground acceleration and velocity, respectively. It should be mentioned that sites having soil periods of two seconds are fairly common within the Lake Zone, and that the higher levels of shaking (in terms of peak ground acceleration) have been consistently observed at these sites.

4.3 Performance parameters

Two performance parameters were considered. The maximum interstory drift ratio, due to its relevance for design purposes, and the normalized dissipated hysteretic energy (E_N) by yielding displacement (D_y) and strength (F_y), see Equation 5. E_N was selected here as a performance parameter because its direct relationship with the cumulative demands (Iervolino et al. 2006). In fact, currently various damage indexes have been proposed based on hysteretic energy (Terán and Jirsa 2005, Rodríguez and Padilla 2008, Bojórquez et al. 2010b). It is important to say, that F_y and D_y were obtained from a push-over analysis, and E_H corresponds to the total plastic energy dissipated by the structure (the plastic energy dissipated by all the elements).

$$N_{EH} = \frac{E_H}{F_y D_y} \quad (5)$$

Table 2 Ground motion records

Record	Date	Magnitude	PGA (cm/s ²)	PGV (cm/s)
1	19/09/1985	8.1	178.0	59.5
2	21/09/1985	7.6	48.7	14.6
3	25/04/1989	6.9	45.0	15.6
4	25/04/1989	6.9	68.0	21.5
5	25/04/1989	6.9	44.9	12.8
6	25/04/1989	6.9	45.1	15.3
7	25/04/1989	6.9	52.9	17.3
8	25/04/1989	6.9	49.5	17.3
9	14/09/1995	7.3	39.3	12.2
10	14/09/1995	7.3	39.1	10.6
11	14/09/1995	7.3	30.1	9.62
12	14/09/1995	7.3	33.5	9.37
13	14/09/1995	7.3	34.3	12.5
14	14/09/1995	7.3	27.5	7.8
15	14/09/1995	7.3	27.2	7.4
16	09/10/1995	7.5	14.4	4.6
17	09/10/1995	7.5	15.8	5.1
18	09/10/1995	7.5	15.7	4.8
19	09/10/1995	7.5	24.9	8.6
20	09/10/1995	7.5	17.6	6.3
21	09/10/1995	7.5	19.2	7.9
22	09/10/1995	7.5	13.7	5.3
23	09/10/1995	7.5	17.9	7.18
24	11/01/1997	6.9	16.2	5.9
25	11/01/1997	6.9	16.3	5.5
26	11/01/1997	6.9	18.7	6.9
27	11/01/1997	6.9	22.2	8.6
28	11/01/1997	6.9	21.0	7.76
29	11/01/1997	6.9	20.4	7.1
30	11/01/1997	6.9	16.0	7.2

5.0 RELATION BETWEEN VECTOR-VALUED IMs AND THE STRUCTURAL DEMAND OF STEEL FRAMES

Baker and Cornell (2005) and Bojórquez and Iervolino (2010) showed the advantages of using vector-valued ground motion intensity measures instead of scalars. The main advantage is the increment in the efficiency to predict the structural response. Herein with the aim to obtain the relation between the structural response of steel frames and the vectors selected; nonlinear incremental dynamic analysis of the frames subjected to the 30 records by using the first parameter of the vector, in this case $Sa(T_1)$ was performed, and then the relation between the structural response of the steel frames and the second parameter of the vector based on N_p is obtained. Note that it must be developed for a specific level of spectral acceleration and for all the intensity levels considered. While Figure 3a shows an example of the incremental dynamic analysis for $Sa(T_1)$, Figure 3b illustrates the relation obtained for $\langle Sa(T_1), N_{pSa} \rangle$ and the maximum interstory drift demand when $Sa(T_1)=1g$. Note the good relation between N_{pSa} and the maximum interstory drift reflecting the advantage of the vector-valued ground motion intensity measure.

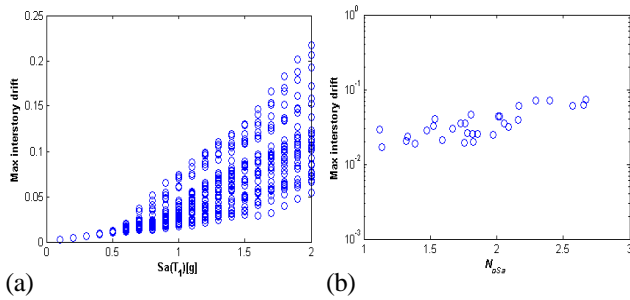


Figure 3 (a) incremental dynamic analysis scaling for $Sa(T_1)$; (b) relation between N_{pSa} and the maximum interstory drift for $Sa(T_1)=1g$

5.1 Prediction of maximum interstory drift and normalized hysteretic energy via vector-valued ground motion intensity measures: Analyses results

The relation between maximum interstory drift and normalized hysteretic energy demand of the frames analyzed is discussed. Figure 4 compares N_{pSa} , N_{pSv} and N_{pEI} with the maximum interstory drift for frame F6, and for the records scaled at $Sa(T_1)=0.9g$. This scaling level corresponds to the point where the median value of the maximum interstory drift achieves 0.03 (structural collapse according to the MCBC). However, other scaling levels are considered as will be illustrated below. A good correlation can be observed between maximum interstory drift and all the parameters based on the spectral shape considered. This observation is valid also to predict the normalized hysteretic energy, see Figure 5; except that in this case, the parameter based on the input energy spectral shape is less effectiveness to predict normalized hysteretic energy demands, which is an

important observation since this vector take into account structural damage potential. All these conclusions are valid also for the other frames under considerations. An important issue fall in the fact that the variation of the N_p based on spectral acceleration is in the range from 1 to 3 while in the other cases this variation is larger; particularly for the input energy the interval is between 0 and 30 which is quite large. Finally, the trend observed for the N_p based on pseudo-acceleration and pseudo-velocity response spectra is very similar due to the dependence of these parameters through the circular frequency.

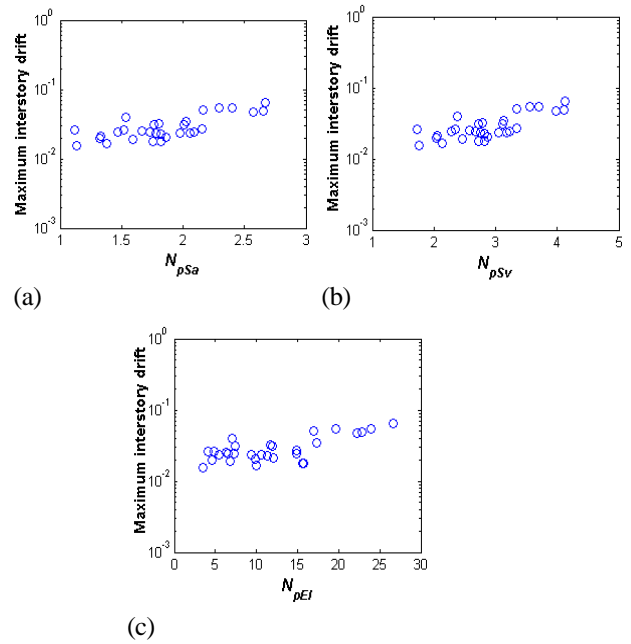


Figure 4 Maximum interstory drift prediction for the steel frame F6 with $T_1=1.07s$ ($Sa(T_1)=0.9g$) for: (a) N_{pSa} , (b) N_{pSv} and (c) N_{pEI}

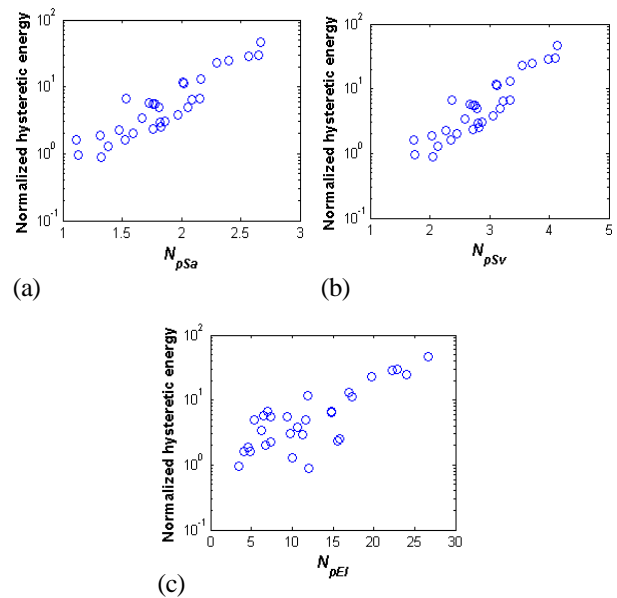


Figure 5 Normalized hysteretic energy prediction for the steel frame F6 with $T_1=1.07s$ ($Sa=0.9g$) for: (a) N_{pSa} , (b) N_{pSv} and (c) N_{pEI}

To demonstrate that all the conclusions obtained before are valid for a wider range of intensity levels. Figure 6 compares the standard deviation of the natural logarithm of maximum interstory drift for frame F4 and for all the IMs: $S_d(T_1)$ and the vectors $\langle Sa, N_{pSa} \rangle$, $\langle Sa, N_{pSv} \rangle$ and $\langle Sa, N_{pEI} \rangle$. As Figure 6 suggests, the vector-valued ground motion intensity measures results in less dispersion for all the scaling level of the records compared with the scaling criteria based on spectral acceleration. Note that the vectors based on pseudo-acceleration and pseudo-velocity results in similar standard deviation. As it has been mentioned before, this is due to the relationship between both parameters, which suggest that there is no significant difference when using spectral acceleration shape or pseudo-velocity spectral shape as the type of vector here presented. The same conclusion is valid for frame F6 illustrated in Figure 7.

The results for normalized dissipated hysteretic energy and all the frames under consideration are presented in Figure 8. It is observed that, the vectors $\langle Sa, N_{pSa} \rangle$ and $\langle Sa, N_{pSv} \rangle$ are similar for frames F4 and F6, but in the case of frames F8 and F10 they results with minimum difference. The figure suggests that $\langle Sa, N_{pSa} \rangle$ is the best correlated with nonlinear structural response of the steel frames subjected to narrow-band motions, and for this reason, it suggests that the spectral acceleration shape is a good indicator to predict maximum and cumulative demands. Regarding the question of which spectral shape really matter to predict nonlinear structural response?, the study conclude that both pseudo-acceleration and pseudo-velocity response spectra really matter; in particular, the spectral pseudo-acceleration shape.

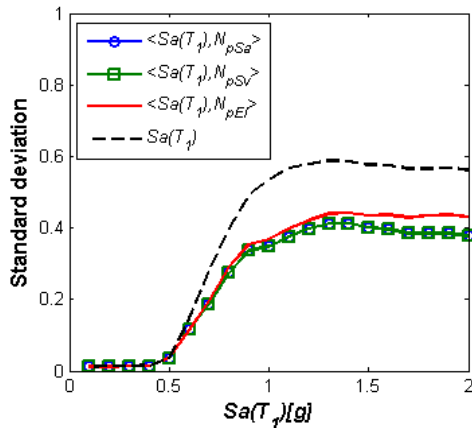


Figure 6 Standard deviation of the natural logarithm of maximum interstory drift given $S_d(T_1)$ and the vectors $\langle Sa, N_{pSa} \rangle$, $\langle Sa, N_{pSv} \rangle$ and $\langle Sa, N_{pEI} \rangle$ at different intensity levels for frame F4

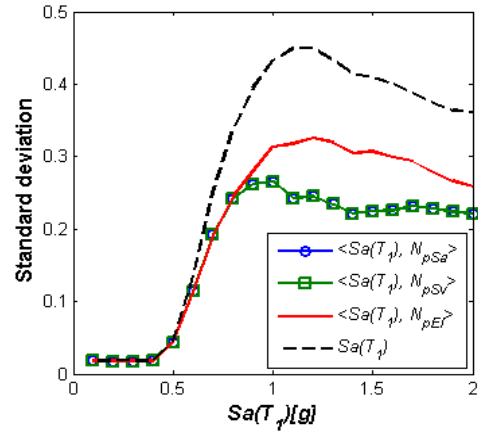


Figure 7 Standard deviation of the natural logarithm of maximum interstory drift given $S_d(T_1)$ and the vectors $\langle Sa, N_{pSa} \rangle$, $\langle Sa, N_{pSv} \rangle$ and $\langle Sa, N_{pEI} \rangle$ at different intensity levels for frame F6

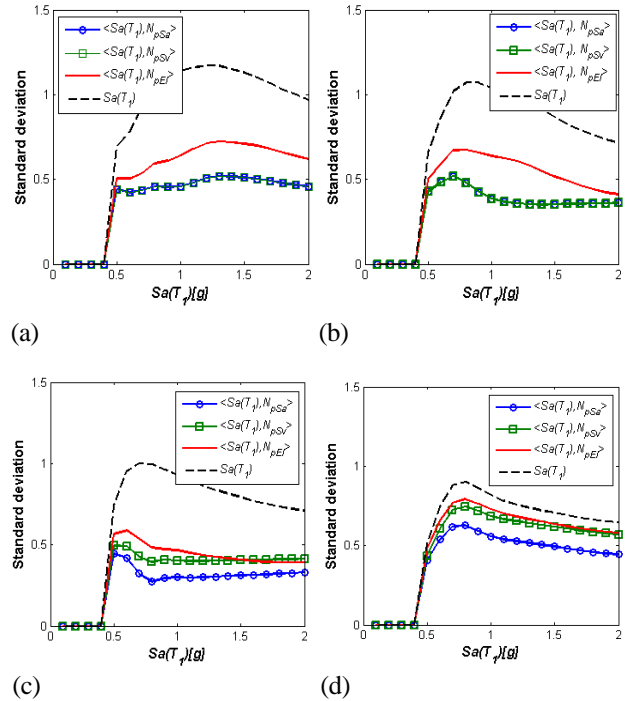


Figure 8 Standard deviation of the natural logarithm of normalized hysteretic energy given $S_d(T_1)$ and the vectors $\langle Sa, N_{pSa} \rangle$, $\langle Sa, N_{pSv} \rangle$ and $\langle Sa, N_{pEI} \rangle$ at different intensity levels for frames (a) F4, (b) F6, (c) F8 and (d) F10

6. INCORPORATING THE EFFECT OF HIGHER MODES OF VIBRATION AND FUTURE STUDIES

The work presented here was limited to predict the nonlinear seismic response of structural steel frames. Nevertheless, an efficient ground motion intensity measure should also be able to predict the nonlinear seismic response of structures dominated by higher modes as in the case of tall buildings. This is out of the scope of this paper; however,

some ideas to incorporate the effect of higher modes are given herein.

The influence of higher modes of vibration in structural response can be developed through a simple modification in the intervals of periods delimiting the parameter N_p . This is, instead assessing N_p between the range of T_1 (initial period) and T_N (final period), one possibility is to evaluate this parameter from the period of the interest mode (e.g. $T_{2\text{modo}}$) until T_N . Note that seismic ground motions selection in FEMA is based on a very similar approach. Moreover, the N_p parameter can be an alternative in record selection for nonlinear dynamic analysis. Furthermore, the inclusion of higher modes can be realized via vector with three parameters, as example: $\langle Sa, R_{T_1, T_2}, R_{T_1, T_{2\text{modo}}} \rangle$, $\langle Sa, N_{pT_1, T_N}, R_{T_1, T_{2\text{modo}}} \rangle$ or, $\langle Sa, N_{pT_1, T_N}, N_{pT_1, T_{2\text{modo}}} \rangle$ (where R_{T_1, T_2} proposed by Cordoba et al. 2001, is the ratio of the spectral acceleration at period T_2 and the spectral acceleration at period T_1 ; T_2 represents a period longer than T_1 , the same is valid for other modes).

Another alternative is by giving a specific weighted to the contribution of the elastic response, nonlinear response or that dominated by higher modes of vibrations.

It would be of interest to develop a similar study but evaluating N_p non as the relation between the geometrical means divided by $Sa(T_1)$, if not by the relation of the weighted geometrical mean divided by $Sa(T_1)$. This can improve considerably the efficiency of the vectors here proposed. As can be observed, there are an ample possible alternatives to be incorporated in similar studies with the aim of increasing the efficiency of ground motion intensity measures to predict the seismic response of structures under earthquakes. Further, although it is necessary to compare other ground motion intensity measure commonly used with no dependence of the spectral shape, a recent study developed by Bojórquez et al. (2010a) showed that fragility assessment of steel buildings results more benefic if spectral shape ground motion intensity measure are used, compared with the traditional IMs as Arias intensity (Arias 1970), peak ground acceleration, peak ground velocity, ground motions durations an others.

7. CONCLUSIONS

A study to observe which spectral shape is more related with the structural response was developed. The study considers spectral shape ground motion intensity measures based on maximum and cumulative damage potential of an earthquake. The results suggest that the use of vector-valued ground motion IMs considerably predict with better approximation the structural response compared with the most commonly scalar IM (spectral acceleration at first mode of vibration). The study concludes that spectral shape ground motion IMs based on the pseudo-acceleration or pseudo-velocity spectra are more effective than those based

on input energy; either to predict maximum interstory drift of steel frames under narrow-band motions, or hysteretic energy demands. In general, the work here presented recommends the use of the vector $\langle Sa, N_{psa} \rangle$ for prediction of nonlinear seismic response of structures.

Acknowledgements:

The support given by la Secretaría de Educación Pública under PROMEP and La Universidad Autónoma de Sinaloa under grant PROFAPI 2010/018 is appreciated.

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