

# MODELLI SEMPLIFICATI PER L'ANALISI DI FRAGILITÀ SISMICA DI SERBATOI NON ANCORATI

## SIMPLIFIED MODELS OF UNANCHORED STEEL TANKS FOR SEISMIC FRAGILITY ANALYSIS

Giovanni Fabbrocino  
Università degli studi del Molise  
Dip.to SAVA  
Campobasso, Italia  
[giovanni.fabbrocino@unimol.it](mailto:giovanni.fabbrocino@unimol.it)

Iunio Iervolino, Antonio Di Carluccio  
Università degli studi di Napoli Federico II  
Dip.to di Analisi e Progettazione Strutturale  
Napoli, Italia  
iunio@stanford.edu, [adicarlu@unina.it](mailto:adicarlu@unina.it)

### ABSTRACT

Seismic behaviour of steel tanks for oil storage is relevant in the light of industrial risk assessment because collapse of these structures may trigger other catastrophic phenomena, as fires or explosions due to loss containment. Damages suffered by storage tanks under seismic actions are generally related to large axial compressive stresses that can induce shell buckling near the base and to large displacements of unanchored structures leading to detachment of piping. The present paper approaches the analysis of seismic response of sliding, non-uplifting, unanchored liquid storage tanks subject to three-dimensional ground motion. A procedure to solve the equation of motion for a simplified tank's model is proposed and a sample estimation of the seismic demand by incremental dynamic analysis is discussed.

### SOMMARIO

La risposta sismica dei serbatoi in acciaio per lo stoccaggio di olio combustibile ha grande importanza nelle valutazioni di rischio industriale per le conseguenze che il collasso strutturale o più in particolare la perdita del contenuto possono generare. L'esame dei danni subiti dai serbatoi non ancorati in occasione di terremoti dimostra la prevalenza di due modalità di rottura: l'instabilità del mantello in prossimità della piastra di base per eccesso di azione assiale e lo spostamento rigido del serbatoio. Il presente lavoro inquadra il problema dell'analisi della risposta dei serbatoi soggetti ad azioni sismiche tridimensionali. Viene proposta una procedura per la risoluzione dell'equazione del moto di un modello semplificato della struttura; esso rappresenta un utile strumento per la valutazione della domanda sismica, della quale viene presentata a titolo di esempio una stima, nelle più generali procedure per le analisi di fragilità.

### 1 INTRODUCTION

Earthquakes represent an *external hazard* for industrial plants and may trigger accidents, *i.e.* fire and explosions resulting in injury to people and to near field equipments or constructions, if structural failures result in release of hazardous material. Quantitative Risk Analysis (QRA)

[1] provides a guide for analysis of industrial risk; such an assessment may include the seismic threat if ground motion related malfunctioning (*i.e.* failure) rates are available for components [2]. From the structural perspective, steel tanks for oil storage are standardized structures both in terms of design and construction [3], [4], [5]. Review of international standards for the construction points out that design evolved slowly; therefore, a large number of post-earthquake damage observations [6] is available and empirical vulnerability functions have been developed [7]. This is a privileged case with respect other building-like and non-building-like structures, however empirical fragility typically suffers some shortcomings; for example vulnerability data contain also information about site effect which may be hard to disaggregate. Therefore, the development of analytical models able to predict the response of the structural components and systems under seismic loading is worth to be explored.

The present work is aimed at the discussion of a procedure able to analyze the three-dimensional response accounting for sliding behaviour of unanchored tanks. It accounts for fluid-structure interaction in a simplified manner, since limitation of computational efforts is a key aspect in seismic reliability evaluations. It takes advantage of the several proposals to approximate the seismic dynamics of tanks available in literature and makes an attempt to extend them in order to including large-displacement limit states. Then, an estimation of the seismic demand in terms of base plate-ground relative displacement and shell compressive stress, which represent the engineering demand parameters related to the failure of connection piping and shell's elephant foot buckling (EFB) is presented. The method used is the Incremental Dynamic Analysis which has been originally developed for buildings and recently extended to tanks [8].

## 2 STRUCTURAL MODEL DEFINITION

In the present section, the definition of the lumped-mass model of the steel tank is reported after a brief review of the main research contributions found in literature. Housner [9] was the author of the first proposal of a simplified model for seismic analysis of anchored tanks with rigid walls. It was based on the assumption that in a tank with a free liquid surface subjected to horizontal ground acceleration a given fraction of the liquid is forced to participate in this motion as rigid mass; on the other hand the motion of the tank walls excites the liquid into oscillations which result in a dynamic force on the tank. This force is assumed to be the same of a lumped mass, known as a convective mass, that can vibrate horizontally restrained by a spring [10], [11].

Rosenblueth and Newmark [12] modified the early formulation by Housner of the convective and rigid masses and gave updated provisions for the evaluation of the seismic design forces of liquid storage tanks. Later, Haroun developed a model to evaluate the seismic response of storage tanks including wall's deformation [13]. In this model a part of the liquid moves independently of tank's shell, again convective motion, while another part of the liquid oscillates unison with the tank. If the flexibility of the tank's wall is considered, a part of this mass moves independently (impulsive mass) while the remaining accelerates back and forth with the tank (rigid mass).

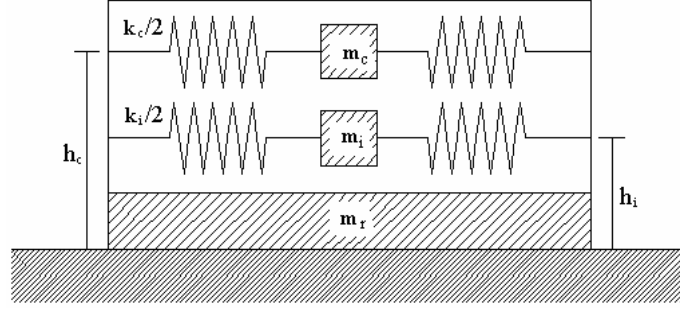
Figure 1 shows the idealised structural model of liquid storage tank. The contained continuous liquid mass is lumped as convective, impulsive and rigid masses are referred as  $m_c$ ,  $m_i$  and  $m_r$ , respectively. The convective and impulsive masses are connected to the tank's wall by different equivalent spring having stiffness  $k_c$  and  $k_i$ , respectively. This model for anchored storage tank has been extended to analyse unanchored base-isolated liquid storage tanks [14]. The effective masses are defined as a function of total liquid mass  $m$  in Equations (1-4) where  $Y_c$ ,  $Y_i$ , and  $Y_r$  are the function of the  $h/R$  ratio, which is a filling coefficient and  $\rho_w$  is the liquid specific weight.

$$m_c = mY_c \left( \frac{h}{R} \right) \quad (1)$$

$$m_i = mY_i \left( \frac{h}{R} \right) \quad (2)$$

$$m_r = mY_r \left( \frac{h}{R} \right) \quad (3)$$

$$m = \pi R^2 h \rho_w \quad (4)$$



**Fig. 1:** One-dimensional dynamic model of tank as in [14]

The natural frequencies of convective mass,  $\omega_c$  and impulsive mass,  $\omega_i$  are given by expressions (5) and (6).

$$\omega_c = \sqrt{1.84 \left( \frac{g}{R} \right) \tanh \left( 1.84 \frac{h}{R} \right)} \quad (5)$$

$$\omega_i = \frac{P}{H} \sqrt{\frac{h}{\rho_s}} \quad (6)$$

Where  $E$  and  $\rho_s$  are the modulus of elasticity and density of tank's wall respectively;  $g$  is the acceleration due to gravity; and  $P$  is a dimensionless parameter also dependent on  $h/R$ . This model is assumed as basis of the seismic demand estimations discussed in the following.

### 3 UNANCHORED TANKS SEISMIC BEHAVIOUR

Motion of unanchored tanks can be characterized by large-displacement phenomena: during the ground motion the tank both can slide relatively to the foundation and the base plate may uplift due to overturning moment. The sliding depends on the base shear, once it reaches the limit value corresponding to the frictional resistance (Equation 7) relative motion between the tank and the foundation starts. Sliding reduces the maximum acceleration suffered by the tank. This reduction is dependent upon the frictional factor ( $\mu$ ), but relatively small values of the latter produce large relative displacements.

Different model can be used in a sliding system to describe the frictional force. In fact, together with the conventional frictional relationship, hysteretic model have been proposed [15]; the latter are generally continuous and need the continuity of the hysteretic displacement components. In the following analyses, the conventional model is used for the frictional force, but this assumption actually does not represent a limitation of the approach. In particular, the friction force is evaluated by considering the equilibrium of the base: the system remain in the non-sliding phase if the frictional force in time  $t$  is lower than the limiting frictional force expressed by Equation (7); where  $g$  represent the gravitational acceleration, and  $\ddot{u}_{vert}$  is the vertical component of the ground motion.

$$F_{lim} = m\mu(\ddot{u}_{vert} + g) \quad (7)$$

Therefore the motion can be subdivided in non-sliding and sliding phase. Whenever the tank does not slide, the dynamic equilibrium of forces in Equation (8) applies if for instance one horizontal component is considered.

$$F_x = -(m_c \ddot{x}_c + m \ddot{x}_i + M \ddot{x}_b + M \ddot{u}_g) \quad (8)$$

In addition, another large-displacement mechanism can be recognized after field observations of the seismic response of unanchored liquid storage tanks; it is represented by the partial uplift of the base plate [16]. This phenomenon reduces the hydrodynamic forces in the tank, but increases significantly the axial compressive stress in the tank wall. In fact, base uplifting in tanks supported directly on flexible soil foundations does not lead to a significant increase in the axial compressive stress in the tank wall, but many lead to large foundation penetrations and several cycles of large plastic rotations at the plate boundary [17], [18]. Flexibly supported unanchored tanks are therefore less prone to elephant-foot buckling damage, but more prone to uneven settlement of the foundation and fatigue rupture at the plate-shell connection.

An aspect particularly interesting is the force-displacement relationship for the plate boundary. The definition of this relationship is complicated by the nonlinearities arising from: 1) the continuous variation of the contact area of the interface between the base plate and the foundation; 2) the plastic yielding of the base plate; and 3) the effect of the membrane forces induced by the large deflections of the plate. In the following, partially uplifting of the base plate is not considered even if the procedure implemented can be integrated with uplifting procedure; this task is the next scheduled step in the algorithm development.

#### 4 EQUATIONS OF MOTION

The equations of motion of the unanchored tank under a generalized three dimensional input ground motion are reported in Equation (9) in the case of non-sliding (rest) of the system and in Equation (10) in the case the system is in sliding.

$$\begin{cases} m_c \ddot{x}_c + b_c \dot{x}_c + k_c x_c = -m_c \ddot{u}_{gx} \\ m_i \ddot{x}_i + b_i \dot{x}_i + k_i x_i = -m_i \ddot{u}_{gx} \\ m_c \ddot{y}_c + b_c \dot{y}_c + k_c y_c = -m_c \ddot{u}_{gy} \\ m_i \ddot{y}_i + b_i \dot{y}_i + k_i y_i = -m_i \ddot{u}_{gy} \end{cases} \quad (9)$$

$$\begin{cases} m_c \ddot{x}_c + m_c \ddot{x}_b + b_c \dot{x}_c + k_c x_c = -m_c \ddot{u}_{gx} \\ m_i \ddot{x}_i + m_i \ddot{x}_b + b_i \dot{x}_i + k_i x_i = -m_i \ddot{u}_{gx} \\ m_c \ddot{x}_c + m_i \ddot{x}_i + m_{tot} \ddot{x}_b - m_{tot} \mu (\ddot{u}_{vert} + g) \cos(\alpha) = -m_{tot} \ddot{u}_{gx} \\ m_c \ddot{y}_c + m_c \ddot{y}_b + b_c \dot{y}_c + k_c y_c = -m_c \ddot{u}_{gy} \\ m_i \ddot{y}_i + m_i \ddot{y}_b + b_i \dot{y}_i + k_i y_i = -m_i \ddot{u}_{gy} \\ m_c \ddot{y}_c + m_i \ddot{y}_i + m_{tot} \ddot{y}_b - m_{tot} \mu (\ddot{u}_{vert} + g) \sin(\alpha) = -m_{tot} \ddot{u}_{gy} \end{cases} \quad (10)$$

Where  $x_c$  and  $y_c$  are the components of displacements of convective masses relative to the base;  $x_i$  and  $y_i$  are the components of displacements of impulsive masses relative to the base;  $x_b$  and  $y_b$  are the component of displacement of the base relative to the ground;  $\ddot{u}_{gx}$  and  $\ddot{u}_{gy}$  are the two horizontal components of ground acceleration. In this case  $m_c$  and  $m_i$  represent two simple oscillators and there is not coupling between two directions of motion ( $x_b$  and  $y_b$  are known). Both the systems of equations for sliding and non-sliding can be expressed in the same matrix format as in Equation (11).

$$[M]\{\ddot{x}\} + [B]\{\dot{x}\} + [K]\{x\} + \{F\} = -[M][r]\{\ddot{u}_g\} \quad (11)$$

The Wilson-theta algorithm is used for numerical integration of the equations of motion.

## 5 NUMERICAL STUDY

The numerical study consisted of: (1) incremental dynamic analysis with one horizontal and the vertical ground acceleration components; (2) analysis with three acceleration components. The first step in the IDA analysis procedure consisted of the acquisition of suitable set records, details about ground motions are given in the following sections. Each of the two investigations were repeated for a range of  $S$  (filling ratio) and a set of  $\mu$  (friction coefficient). To obtain the seismic demand at selected ground motion intensity levels records are scaled in terms of Peak Ground Acceleration (PGA). The scaling factor  $\chi$  varies to get the PGA from 0.2g to 1.5g.

### 5.1 Results and discussion

In the first phase of analysis a parametric IDA with only one horizontal ground acceleration component was carried out. The parameters of the investigated tank are described in Table 1.

**Table 1: Tank's mechanical parameters**

$t_b$ [m]	$h$ [m]	$\rho_s$ [ $kgm^{-3}$ ]	$\rho_l$ [ $kgm^{-3}$ ]	$E$ [GPa]
0.008	10	7860	1000	210

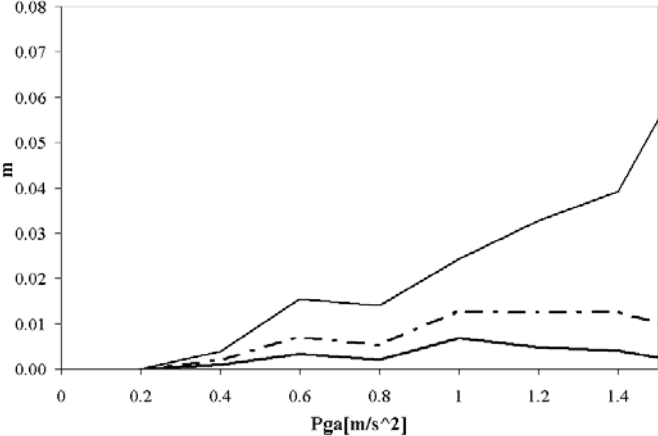
Where  $t_b$  is the base plate and shell's thickness;  $h$  is the height of liquid in the tank,  $\rho_s$  and  $\rho_l$  are the specific weights of steel and liquid respectively and  $E$  is the modulus of elasticity of the tank's structure. In the parametric analysis  $S$  (filling coefficient) varies from 2 to 3.5 and  $\mu$  (friction factor) varies from 0.1 to 0.8. The earthquake ground motions considered [19] are all stiff soil records from a broad range of magnitude and distances; in fact the magnitude ranges between 5.3 and 6.9, while the epicentral distance goes from 6 to 69 km.

All the accelerograms herein employed come from the European Strong Motion Database (<http://www.isesd.cv.ic.ac.uk/>) and can be easily retrieved from there. In the following selected results are presented for sake of brevity. In particular the  $S = 2$  and  $\mu = 0.1$  case is discussed even if the parametric analysis can point out the influence of  $S$  and  $\mu$  on the selected limit states. In particular, the results show that as  $\mu$  increases, the axial compressive stress increases while of the rigid displacement decreases. Conversely an increase in  $S$  reduces the axial compressive stress and increases the relative displacement. The demand curve in term of axial compressive stress [MPa] for the uni-directional model (including vertical acceleration), computed by Equation (12) from AWWA D100-96, is given in Figure 2.

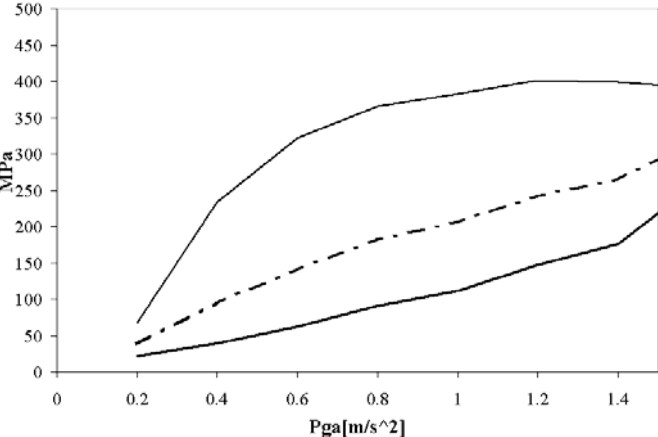
$$\sigma_c = \left( w_t + \frac{1.273M}{D^2} \right) \frac{1}{1000t_s} \quad (12)$$

Curves include media and  $1\sigma$  bounds ( $\sigma$  is the standard deviation of the logs of the demand not to be confused with the compressive stress  $\sigma_c$ ). The base-displacement demand curve as function of the PGA for the set of ground motions is summarized in Figure 3. Also for this case median  $\pm 1\sigma$  curves are given. To understand the effects the second component of horizontal ground motion, uni-directional and bi-directional results in terms of base-displacement are compared in Figure 4. As expected, these two analyses are not equivalent. In fact, the maximum base displacement for the one-dimension analysis is 0.2 m while including the second component it is 0.4m. On the other hand, vertical ground acceleration component can play a relevant role in the estimation of sliding trajectory of the tank, as shown in Figure

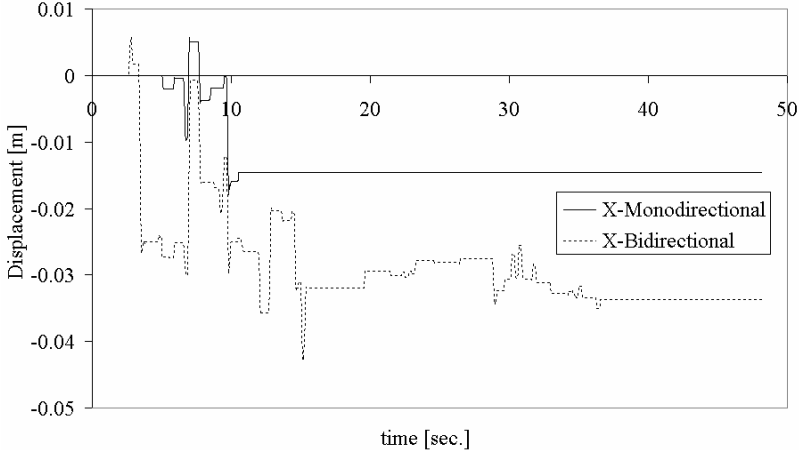
5. According to numerical results, vertical ground acceleration component influences the motion of the tank; in fact, the trajectory changes and affects also the modulus of the maximum base displacement. In particular, an increase of the latter of about 33% is observed, since without vertical acceleration the peak displacement is 0.043 m while accounting for the vertical acceleration the peak displacement is 0.057m.



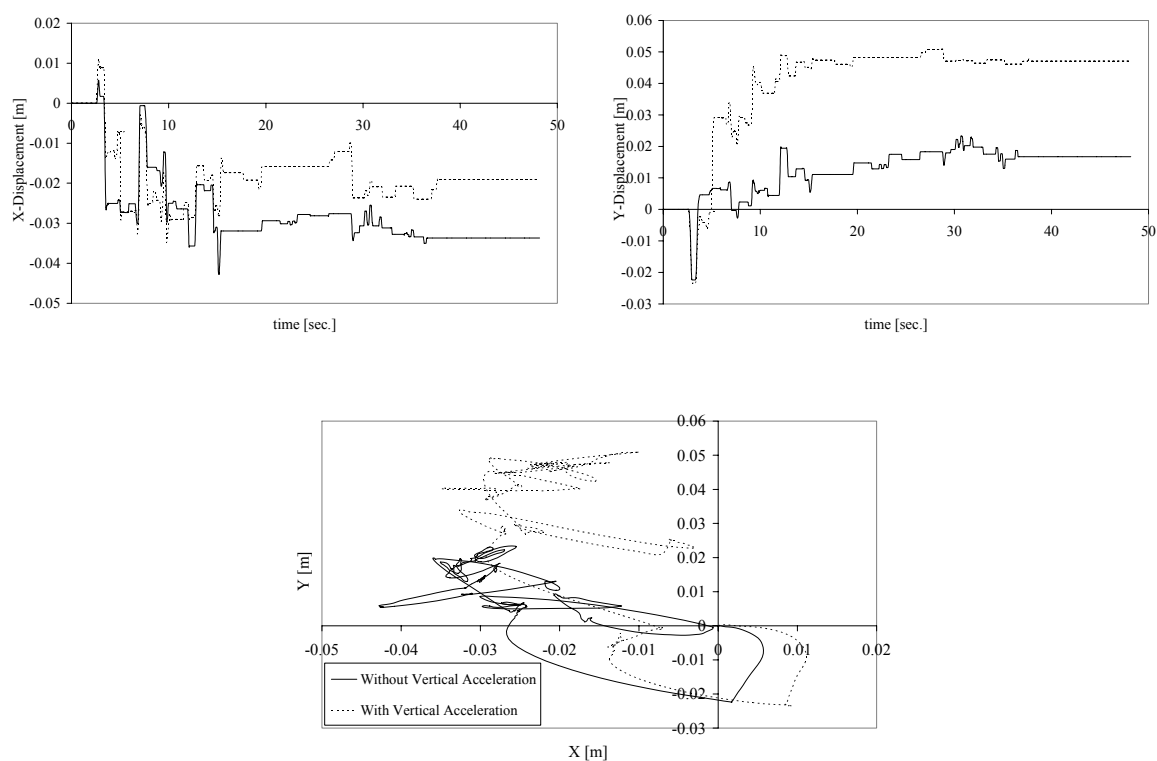
**Fig. 2:** IDA curve for the compressive axial stress for the uni-directional model



**Fig. 3:** IDA curve for the sliding-induced displacement for the uni-directional model



**Fig. 4:** Comparison of uni-directional and bi-directional analyses, along x axis, for the 000197-Montenegro earthquake



**Fig. 5:** Comparison of the component of displacement and of trajectory of the tank calculated with and without the vertical ground acceleration component. 000197-Montenegro earthquake

## 6 CONCLUSIONS

This paper investigates the seismic response of unanchored steel tanks for oil storage in the large displacement regime taking into consideration those limit states relevant for industrial risk analysis (Elephant Foot Buckling and base-sliding). Algorithms to integrate equations of motions have been formulated for both one-directional (including vertical acceleration) and considering all three ground motion components. The model does not include the base uplifting, which may affect compressive stress demand, but it is ready to. The model has been employed to produce incremental dynamic analysis demand curves as for building-like structures. Comparison of the two models has also been carried out, results show that the uni-directional results may be un-conservative, at least in terms of base-displacement demand for sliding tanks. IDA curves can also be similarly developed for bi-directional ground motion, for example using as ground motion intensity measure the geometric mean of the PGA in the two directions, and therefore the proposed model may be used for computation of numerical fragility curves and for structural and industrial seismic risk analysis.

## ACKNOWLEDGMENTS

This study has been conducted in within the activities of the LESSLOSS project subtask 2.3b ([www.lessloss.org](http://www.lessloss.org)), a research endeavour funded by the European Commission within the scope of FP6, the Sixth European Community Framework Programme for Research, Technological Development and Demonstration.

## REFERENCES

- [1] F.P. Lees, Loss Prevention in the Process Industries, second ed., Butterworth-Heinemann, Oxford, 1996.

- [2] Fabbrocino G., Iervolino I., Orlando F., Salzano E., “Quantitative Risk Analysis of Oil storage facilities in seismic areas”. *Journal of Hazardous Materials*, in press, 2005.
- [3] AWWA D100-96, “Welded Steel Tanks for Water Storage”, American Water Works Association, Denver, Colorado, USA, 1996.
- [4] API 620, “Welded Steel Tanks for Oil Storage”, American Petroleum Institute, Washington D.C., USA, 1998.
- [5] API 650, “Design and Construction of Large, Welded, Low-Pressure Storage Tanks”, American Petroleum Institute, Washington D.C., USA, 1998.
- [6] NIST GCR 97-720, “Reliability and restoration of water supply systems for fire suppression and drinking following earthquakes”, National Institute of Standards and Technology, Gaithersburg, MD, USA 1997.
- [7] Salzano E., Iervolino I., Fabbrocino G., “Seismic Risk of Atmospheric Storage Tanks in the Framework of Quantitative Risk Analysis”, *J. Loss Prevent. Proc.*, 16, 403–409, 2003.
- [8] Iervolino I., Fabbrocino G., Manfredi G., “Fragility of Standard Industrial Structures by a Response Surface Based Method”, *Journal of Earthquake Engineering*, 8, 927–946, 2004.
- [9] Housner, G.W., “Dynamic behaviour of water tanks”, *Bulletin of Seismological Society of America*, (53), 381-387, 1963.
- [10] Jacobsen, L.S., “Impulsive hydrodynamics of fluid inside a cylindrical tank”, *Bulletin of Seismological Society of America*, (39), 1949.
- [11] Housner, G.W., “Dynamic pressures on accelerated fluid containers”, *Bulletin of Seismological Society of America*,(1), 15-37, 1957.
- [12] Rosenblueth, E. & Newmark, N.M., “Fundamentals of Earthquake Engineering”, Prentice-Hall: NJ, 1971.
- [13] Haroun, M.A., “Vibration studies and test of liquid storage tanks”, *Earthquake Engineering and Structural Dynamics*, (11), 179-206, 1983.
- [14] Shrimali, M.K., R.S. Jangid, “Seismic response of liquid storage tanks isolated by sliding bearings”, *Engineering Structures*, 24, 909-921, 2002.
- [15] Costantinou, M.C., Mokha, A.S., Reinhorn A.M., “Teflon bearing in base isolation II: modelling”, *Journal of Structural Engineering*, ASCE 1990;116(2):455-74.
- [16] Malhotra, P.K., Veletsos, A.S., “Seismic response of unanchored and partially anchored liquid- storage tanks”, Report TR-105809, Electric Power Research Institute, Palo Alto, 1995.
- [17] Malhotra, P.K., “Base uplifting analysis of flexibly supported liquid-storage tanks”, *Journal of Earthquake Engineering and Structural Dynamics*, 24(12), 1995, 1591-1607.
- [18] Malhotra, P.K., “Seismic response of soil-supported unanchored liquid-storage tanks”, *Journal of Earthquake Engineering*, ASCE, New York, 123(4), 440–450, 1997.
- [19] Fabbrocino, G., Iervolino, I., Di Carluccio, A., Dynamic Analysis of Steel Tanks Subjected To Three-Dimensional Ground Motion, Proceedings of The Tenth International Conference on Civil, Structural and Environmental Engineering Computing, CC2005 & AI2005, Rome, 2005.

## **KEYWORDS**

Seismic risk, storage tanks, dynamic analysis, ground motion, sliding, elephant foot buckling.