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SHORT COMMUNICATION

# On occurrence disaggregation of probabilistic seismic hazard

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**Abstract**

Disaggregation of probabilistic seismic hazard allows to quantify how much one or more earthquake scenarios contribute to the occurrence [exceedance] of a ground motion intensity measure ($IM$) threshold of interest ($x$) at the construction site. The scenario is usually defined in terms of magnitude ($M$), source-to-site distance ($R$), and possibly includes the standardized residual ($\varepsilon$) of the ground motion model considered in the hazard analysis. Analytically, in case occurrence is of interest, disaggregation provides the joint probability density function of $\{M, R\}$ or $\{M, R, \varepsilon\}$ conditional on the $IM = x$ event, that is, $f_{M,R|IM=x}$ or $f_{M,R,\varepsilon|IM=x}$. Occurrence disaggregation is important for a number of earthquake engineering applications, and it is typically addressed in the literature in an approximated manner, considering as the conditioning event $IM \in (x, x + \Delta x)$, with $\Delta x$ being an arbitrary finite width of the interval. This short communication undertakes a deeper examination of occurrence disaggregation clarifying that: (i) no approximation is needed in the case of disaggregation in terms of magnitude and distance (i.e., when $f_{M,R|IM=x}$ is sought); (ii) $f_{M,R,\varepsilon|IM=x}$ is theoretically degenerate, and as such, its approximation via finite $\Delta x$ can lead to misleading results; (iii) if $\Delta x$ is chosen coherently with the discretization of the $\{M, R, \varepsilon\}$ domain used in the hazard integral, it leads to approximated $f_{M,R|IM=x}$, enabling the conclusion that $\{M, R, \varepsilon\}$ occurrence disaggregation does not add information with respect to $\{M, R\}$ disaggregation.

**KEYWORDS**
deaggregation, earthquake scenario, performance-based earthquake engineering, PSHA, record selection

## 1 | INTRODUCTION AND MOTIVATION

Probabilistic seismic hazard analysis (PSHA)[1] allows to derive the rate of earthquakes causing exceedance of a ground motion intensity measure ($IM$) threshold ($x$) at the site of interest. Such a rate ($\lambda_{IM>x}$) is given by the *hazard integral*:

$$\lambda_{IM>x} = \nu \cdot \int_{m_{min}}^{m_{max}} \int_{r_{min}}^{r_{max}} P[IM > x|y,z] \cdot f_{M,R}(y,z) \cdot dy \cdot dz. \tag{1}$$

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In the equation, which is often used to describe the case of one seismic source, $\nu$ is the rate of earthquakes above a minimum magnitude ($M$) of interest, indicated as $m_{min}$. The $f_{M,R}$ term is the joint probability density function (JPDF) of magnitude and source-to-site distance ($R$) random variables (RVs). Magnitude is usually bounded by the maximum magnitude considered possible for the source ($m_{max}$), while the distance varies in the ($r_{min}, r_{max}$) range, determined by the geometry of the source and the position of the site with respect to it. The $P[IM > x|y,z]$ term, which is provided by a ground motion prediction equation (GMPE), is the probability that $IM > x$, conditional on $M = y$ and $R = z$. GMPEs usually model the probability density function (PDF) of $IM$, conditional to $\{M = y, R = z\}$, $f_{IM|M=y,R=z}$, as that of a lognormal RV via the following equation:

$$\log(IM) = \mu_{\log(IM)}(y, z, \underline{\theta}) + \sigma_{\log(IM)} \cdot \varepsilon, \tag{2}$$

where $\mu_{\log(IM)}(y, z, \underline{\theta})$ is the mean of the logarithms of $IM$ at the site for an earthquake with $M = y$ and $R = z$, while $\underline{\theta}$ represents a vector of additional covariates, usually not treated as RVs (e.g., local soil site conditions), and $\sigma_{\log(IM)} \cdot \varepsilon$ is a zero mean and $\sigma^2_{\log(IM)}$ variance Gaussian RV. In fact, $\varepsilon$ (*epsilon*) is also referred to as the *standardized residual*, as it measures the number of standard deviations that the log of $IM$ is distant from its mean conditional to $\{M = y, R = z\}$.

Because, in most GMPEs, the distribution of $\varepsilon$ is independent of $M$ and $R$, Equation (1) can be rewritten as:

$$\lambda_{IM>x} = \nu \cdot \int_{m_{min}}^{m_{max}} \int_{r_{min}}^{r_{max}} \int_{-\infty}^{+\infty} I\left[IM > x|y, z, w\right] \cdot f_{M,R}(y, z) \cdot f_{\varepsilon}(w) \cdot dy \cdot dz \cdot dw, \tag{3}$$

where $f_{\varepsilon}$ is the PDF of $\varepsilon$, that is, a zero mean and unit variance Gaussian RV, computed when $\varepsilon = w$ (i.e., $w$ represents a realization of the standardized residual), and $I[IM > x|y, z, w]$ is an indicator function equaling one in the case the event in the square brackets occurs, and zero otherwise.

It is important, for the following discussion, to recall that, usually, there is no closed-form solution for the integral so that it is numerically calculated by replacing the integrals in Equation (3) by summations, discretizing the $M$, $R$ and $\varepsilon$ domains via intervals with finite width, that is, with $\Delta y$, $\Delta z$, and $\Delta w$:

$$\lambda_{IM>x} \approx \nu \cdot \sum_{i=1}^{n} \sum_{j=1}^{h} \sum_{k=1}^{l} I\left[IM > x|y_i, z_j, w_k\right] \cdot f_{M,R}\left(y_i, z_j\right) \cdot f_{\varepsilon}(w_k) \cdot \Delta y \cdot \Delta z \cdot \Delta w, \tag{4}$$

where $y_i = m_{min} + [0.5 + (i-1)] \cdot \Delta y$, $n = (m_{max} - m_{min})/\Delta y$, $z_j = r_{min} + [0.5 + (j-1)] \cdot \Delta z$, $h = (r_{max} - r_{min})/\Delta z$, $w_k = \varepsilon_{min} + [0.5 + (k-1)] \cdot \Delta w$, $l = (\varepsilon_{max} - \varepsilon_{min})/\Delta w$ being $\varepsilon_{\mathrm{min}}$ and $\varepsilon_{\mathrm{max}}$ the limiting values, considered in the analysis, for epsilon. Mapping $\lambda_{IM>x}$ versus the corresponding $x$ values leads building a function, whose diagram is referred to as the hazard curve for the site.

Disaggregation of probabilistic seismic hazard[2,3] enables to calculate how much an earthquake scenario, among those considered in the PSHA (a realization of the RVs involved in the hazard integral), contributes to the occurrence or exceedance of $x$. It is required where the most relevant scenarios must be identified. For example, to aid ground motion record selection for dynamic seismic structural analysis.[4]

Considering Equation (3), the most detailed disaggregation result is the JPDF of $\{M, R, \varepsilon\}$ conditional to the occurrence [exceedance] of $x$, that is, $f_{M,R,\varepsilon|IM=x}$ $[f_{M,R,\varepsilon|IM>x}]$. However, disaggregation can be, in principle, performed with respect to any subset of the RVs involved in the hazard integral. For example, $f_{M,R|IM=x}$ $[f_{M,R|IM>x}]$ is also a typical product of disaggregation; this latter case is referred to as disaggregation in terms of $\{M, R\}$, while the former is referred to as disaggregation in terms of $\{M, R, \varepsilon\}$. The two are related by the following marginalization operation:

$$f_{M,R|IM=x}(y, z) = \int_{-\infty}^{+\infty} f_{M,R,\varepsilon|IM=x}(y, z, w) \cdot dw. \tag{5}$$

Whether disaggregation should be performed with respect to occurrence or exceedance of $x$ depends on the purposes of the analysis. The *conditional hazard*[5] or the *conditional mean spectrum*[6] methods require occurrence disaggregation. Tothong et al.[7] consider occurrence disaggregation to compute the probability that the occurrence of a $IM$ intensity level is caused by a pulse-like ground motion, in the case of hazard analysis adjusted for the near-source case.[*] Bradley[8]

[*] In fact, other RVs are involved in the hazard integral explicitly considering near-source pulse-like ground motion.[21]

discusses that the occurrence disaggregation is appropriate for record selection for dynamic analysis of structures at fixed $IM$ levels. Nevertheless, some record selection procedures use the exceedance disaggregation,[9,10] arguing that occurrence and exceedance disaggregation can be similar.

Disaggregation of seismic hazard relies on the *conditional probability rule*, which is applicable only in the case where the conditioning event has non-zero probability, something that does not hold true for the occurrence of any given realization of a continuous RVs (which typical $IMs$ are), being $P[IM = x] = 0 \forall x$. In fact, Fox et al.[11] state that *since IM is a continuous RV, [...] it is not possible to disaggregate hazard for the occurrence of IM = x but instead one must consider a range or 'band' of intensities about the intensity level of interest [...]*. Tothong et al.[7] and Bradley[8] seem to suggest approximating occurrence disaggregation via the replacement of $IM = x$ with $IM \in (x, x + \Delta x)$, where $\Delta x$ is finite, leading to:

$$\begin{cases} f_{M,R|IM=x}(y,z) \approx f_{M,R|IM\in(x,x+\Delta x)}(y,z) = \dfrac{\nu \cdot \{P[IM > x|y,z] - P[IM > x+\Delta x|y,z]\} \cdot f_{M,R}(y,z)}{\lambda_{IM>x} - \lambda_{IM>x+\Delta x}} \\ f_{M,R,\varepsilon|IM=x}(y,z,w) \approx f_{M,R,\varepsilon|IM\in(x,x+\Delta x)}(y,z,w) = \dfrac{\nu \cdot \{I[IM > x|y,z,w] - I[IM > x+\Delta x|y,z,w]\} \cdot f_{M,R}(y,z) \cdot f_{\varepsilon}(w)}{\lambda_{IM>x} - \lambda_{IM>x+\Delta x}} \end{cases}. \quad (6)$$

Needless to say, $\lambda_{IM>x+\Delta x}$, appearing in the equation is the rate of earthquakes causing $IM > x + \Delta x$, and $I[IM > x + \Delta x|y, z, w]$ is an indicator function equal to one if the event in the brackets occurs, and zero otherwise.

Because, as mentioned, occurrence disaggregation is important for a number of earthquake engineering applications, this short communication intends to aid practice by clarifying some issues. It is discussed that occurrence disaggregation in terms of only magnitude and distance, that is, when $f_{M,R|IM=x}$ is sought, is possible without finite interval approximation. In fact, recognizing that the calculus of probability conventionally replaces the occurrence event with $IM$ belonging to an interval of infinitesimal width, it is shown that all terms necessary to compute the disaggregation are already available without the need for additional approximations, other than those required for calculating the hazard integral. Then, it is discussed that, when disaggregation includes $\varepsilon$, $f_{M,R,\varepsilon|IM=x}$ is theoretically a degenerate function. This stems from the fact that, given $\{M, R, \varepsilon\}$, the corresponding $IM$ value is deterministically known according to classical GMPEs. As a consequence of this issue, $f_{M,R,\varepsilon|IM\in(x,x+\Delta x)}$ in Equation (6) may lead to misleading results if $\Delta x$ is chosen arbitrarily. Finally, this note addresses that the discussed approximation works if $\Delta x$ is chosen coherently with $\Delta y$, $\Delta z$ and $\Delta w$ used to compute the hazard integral; however, in this case, the resulting $\{M, R, \varepsilon\}$ occurrence disaggregation practically leads to the same results as that in terms of $\{M, R\}$, that is, $f_{M,R|IM\in(x,x+\Delta x)}$, showing that $\{M, R, \varepsilon\}$ occurrence disaggregation is ultimately pointless, as it does not add further information. The note addresses these issues in the order that they are listed, followed by a discussion of the approximations via a PSHA application, and ends with some conclusive remarks.

## 2 | OCCURRENCE DISAGGREGATION IN TERMS OF MAGNITUDE AND DISTANCE

Occurrence disaggregation in terms of $\{M, R\}$ results in the $f_{M,R|IM=x}$ conditional JPDF. However, acknowledging that for continuous RVs it is $P[IM = x] = 0 \ \forall x$, the $f_{M,R|IM=x}$ symbol actually indicates $f_{M,R|IM\in(x,x+dx)}$, where $dx$ is infinitesimal. This observation allows to apply the conditional probability rule as:[12]

$$f_{M,R|IM=x}(y,z) = \frac{f_{IM|M=y,R=z}(x) \cdot f_{M,R}(y,z)}{f_{IM}(x)}. \quad (7)$$

In the equation, $f_{M,R}$ represents the JPDF of magnitude and distance already introduced, while, according to Equation (2), $f_{IM|M=y,R=z}$ is the PDF of the $IM$ lognormal RV, conditional to the earthquake's features, that is:

$$f_{IM|M=y,R=z}(x) = \frac{1}{x \cdot \sigma_{\log(IM)}} \cdot f_{\varepsilon}\left[\frac{\log(x) - \mu_{\log(IM)}(y,z,\underline{\theta})}{\sigma_{\log(IM)}}\right]. \quad (8)$$

(The dependence of the left-hand side on $\underline{\theta}$ is neglected for simplicity). Finally, $f_{IM}$ is the PDF of $IM$, conditional to the occurrence of one *generic* earthquake[13] at the site, that is, an earthquake of unspecified magnitude and source-to-site-distance. This PDF is also available, and there are at least two equivalent ways to get it. The first one requires recognizing

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that the exceedance rate in Equation (1) can be written as $\lambda_x = \nu \cdot P[IM > x]$, where $P[IM > x]$ is the probability of exceedance of $x$ upon the occurrence of one generic earthquake. Therefore, $f_{IM}$ can be obtained by taking the derivative of the hazard curve with respect to $x$ and dividing it by the $\nu$ rate:

$$f_{IM}(x) = \frac{dP[IM \leq x]}{dx} = -\frac{dP[IM > x]}{dx} = \frac{1}{\nu} \cdot \left| \frac{d\lambda_{IM>x}}{dx} \right|. \tag{9}$$

Because there is, usually, no closed-form solution for the hazard integral, this derivative must be numerically approximated, something that can be avoided by recognizing that $f_{IM}$ can also be computed via the total probability theorem:

$$f_{IM}(x) = \int_{m_{min}}^{m_{max}} \int_{r_{min}}^{r_{max}} f_{IM|M=y,R=z}(x) \cdot f_{M,R}(y,z) \cdot dy \cdot dz. \tag{10}$$

This equation, which is equivalent to the integral in Equation (3), may need the same discretization required to compute the hazard integral, that is, that of $M$ and $R$:

$$f_{IM}(x) \approx \sum_{i=1}^{n} \sum_{j=1}^{h} f_{IM|M=y_i,R=z_j}(x) \cdot f_{M,R}\left(y_i, z_j\right) \cdot \Delta y \cdot \Delta z, \tag{11}$$

but not any other further. It can be anticipated here that $f_{M,R,\varepsilon|IM=x}$ does not, in fact, add information with respect to $f_{M,R|IM=x}$, because, for any $\{M = y, R = z\}$ pair, Equation (2) provides the unique $w$ such that $IM = x$. In other words, once the relevant $\{M, R\}$ scenarios are identified, also the corresponding $\{M, R, \varepsilon\}$ scenarios are identified, due to the GMPE.

## 3 | OCCURRENCE DISAGGREGATION IN TERMS OF MAGNITUDE, DISTANCE, AND EPSILON

Following the same reasoning as in the previous section, the computation of $f_{M,R,\varepsilon|IM=x}$ can be approached as:

$$f_{M,R,\varepsilon|IM=x}(y,z,w) = \frac{f_{IM|M=y,R=z,\varepsilon=w}(x) \cdot f_{M,R}(y,z) \cdot f_{\varepsilon}(w)}{f_{IM}(x)} = \frac{\delta(u-x) \cdot f_{M,R}(y,z) \cdot f_{\varepsilon}(w)}{f_{IM}(x)}, \tag{12}$$

where $u = \mu_{\log(IM)}(y, z, \underline{\theta}) + \sigma_{\log(IM)} \cdot w$, and $f_{IM|M=y,R=z,\varepsilon=w}(x) = \delta(u - x)$, that is, the Dirac's delta function centered on $x$, which reflects the fact that, given $\{M = y, R = z, \varepsilon = w\}$, there is no uncertainty left in $IM$, as already recalled. Therefore, it is demonstrated that only $f_{M,R|IM=x}$ can be meaningfully defined, while the $f_{M,R,\varepsilon|IM=x}$ distribution is degenerate. Once again, this is because there is only one $\varepsilon$ value corresponding to the $IM$ value the hazard of which is disaggregated.

It is easy to show that this conclusion still applies to the PSHA carried out via a logic tree; i.e., with multiple GMPEs and/or source models. Also note that only $\{M, R, \varepsilon\}$ are treated herein as RVs in the hazard integral; if also $\underline{\theta}$ were to contain elements that are also treated as RVs, this result would hold true if disaggregation considers $\varepsilon$ and all the other RVs altogether.

It is also beneficial to make the analytical relationship between $f_{M,R,\varepsilon|IM=x}$ and $f_{M,R|IM=x}$ explicit, that is, to apply the marginalization of Equation (5), recalling the definition of Dirac's delta function:

$$f_{M,R|IM=x}(y,z) = \int_{-\infty}^{+\infty} \frac{\delta(u-x) \cdot f_{M,R}(y,z) \cdot f_{\varepsilon}(w)}{f_{IM}(x)} \cdot dw = \int_{-\infty}^{+\infty} \frac{\delta(u-x) \cdot f_{M,R}(y,z) \cdot f_{\varepsilon}[w(u)]}{f_{IM}(x)} \cdot \frac{dw(u)}{du} \cdot du = \frac{f_{M,R}(y,z)}{f_{IM}(x)} \cdot \int_{-\infty}^{+\infty} \delta(u-x) \cdot f_{\varepsilon}[w(u)] \cdot \frac{1}{u \cdot \sigma_{\log(IM)}} \cdot du = \frac{f_{M,R}(y,z)}{f_{IM}(x)} \cdot f_{IM|M=y,R=z}(x). \tag{13}$$

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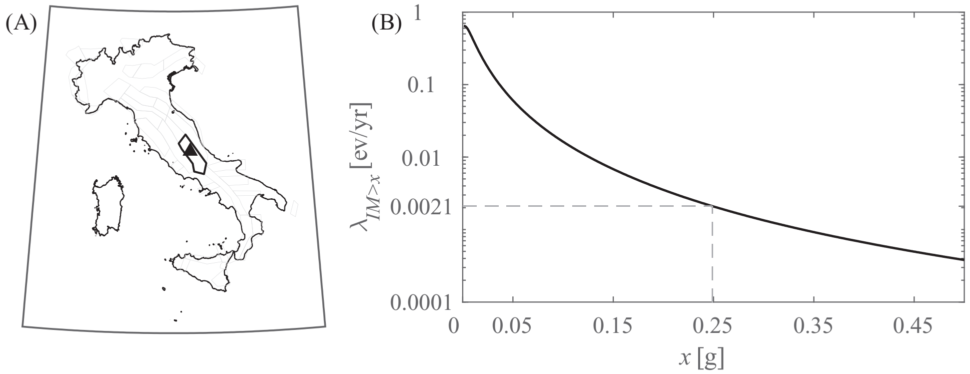


**FIGURE 1** (A) Site (triangle) and seismic sources (polygons); (B) hazard curve for the site.

In the equation it is clearly $w(u) = [u - \mu_{\log(IM)}(y, z, \underline{\theta})]/\sigma_{\log(IM)}$. Equation (13) will be used in the next section to show that the only meaningful approximation of $f_{M,R,\varepsilon|IM=x}$ is the one that ultimately leads to an approximation of $f_{M,R|IM=x}$.

## 4 | APPROXIMATED DISAGGREGATION DISTRIBUTIONS

Having established the occurrence disaggregation format for $f_{M,R|IM=x}$, the effects of the finite $IM$ interval approximation are explored. To this aim, the peak ground acceleration (PGA) hazard of the site of L'Aquila (central Italy) is considered. PSHA for this site is carried out, via REASSESS,[14] using the source model from Meletti et al.,[15] which consists of 36 source zones, pictured in Figure 1A together with the site. The sources are characterized in terms of annual rates of earthquakes associated with surface-wave magnitude bins with width equal to 0.3 magnitude units.[16] The GMPE is that of Ambraseys et al.,[17] which is applicable within the 4.0-7.6 magnitude range and for distances, in terms of the *Joyner & Boore* metric,[18] of up to 200 km. However, the illustrative case presented herein is developed, for simplicity, only considering the source zone the site falls into, as it has already been demonstrated that this contributes more than 90% to the PGA hazard with 475 years exceedance return period at L'Aquila.[19] For this source, the minimum and maximum magnitude considered in PSHA are equal to 4.15 and 7.45, respectively, while the dominant rupture mechanism is normal, which was accounted for in the analyses as suggested by Bommer et al.[20] In the calculations, it is assumed $\Delta y = 0.1$ magnitude units and $\Delta z = 2$ km. For $\varepsilon$: $\varepsilon_{min} = -5$, $\varepsilon_{max} = 5$ and $\Delta w = 0.05$. The resulting hazard curve for rock site conditions is given in Figure 1B.

Occurrence disaggregation in terms of magnitude and distance is computed, following Equation (7), for the PGA with 475 years exceedance return period, that is 0.25g.[†] The resulting disaggregation distribution is shown in Figure 2A, where the representation is obtained by multiplying the $f_{M,R|IM=im}$ by $\Delta y$ and $\Delta z$ (the width of bins in the figure is the same as the analysis). The color of the bars represents the unique epsilon value for which $IM = x$ given $\{M, R\}$. Still in Figure 2 (panels from B to D), the approximated $f_{M,R|IM=im}$ disaggregation distributions from Equation (6) are provided for three $\Delta x$ values. One of these values, $\Delta x = 0.007$g, is the one corresponding to $\Delta w = 0.05$ as per $\Delta x = x \cdot (10^{\sigma_{\log(IM)} \cdot \Delta w} - 1)$, when $x = 0.25g$. This relationship is readily obtained writing, for the same $\{M, R\}$, Equation (2) twice, for $IM = x$ and $IM = x + \Delta x$, and then subtracting one from the other (in the case that multiple GMPEs are involved in PSHA, one $\Delta x$ per ground motion model could be selected). The other two $\Delta x$ values considered are 0.001g and 0.1g. It can be observed that, although the approximation is not needed, it provides (at least in this case) accurate results that, as expected, even improve with the reducing of $\Delta x$.

It was shown that occurrence disaggregation is a degenerate function, while assuming $IM \in (x, x + \Delta x)$ as the conditioning event enables calculations. Therefore, the approximated $\{M, R, \varepsilon\}$ disaggregation, according to Equation (6), has been computed for the same PGA value and using the three $\Delta x$ values just considered, as shown in Figure 3.

Once again, the $f_{M,R,\varepsilon|IM=x}$ values obtained are multiplied by $\Delta y$, $\Delta z$ and $\Delta w$. It appears that, when $\Delta x$ is coherent with $\Delta w$ (Figure 3B), the resulting distribution is, by virtue of Equation (13), the same as the approximated $f_{M,R|IM=im}$ obtained with the same $\Delta x$ (Figure 2C). Conversely, when $\Delta x$ is relatively large, that is 0.1g, for any magnitude-distance pair, more than one $\varepsilon$ value contributes to the disaggregation, although – as extensively discussed – there is only one

[†] It was demonstrated that, if the GMPE is the one of the type in Equation (2), the soil term is additive with respect to the mean of the GMPE, and it does not affect the standard deviation, then disaggregation is invariant with respect to the soil type.[22]

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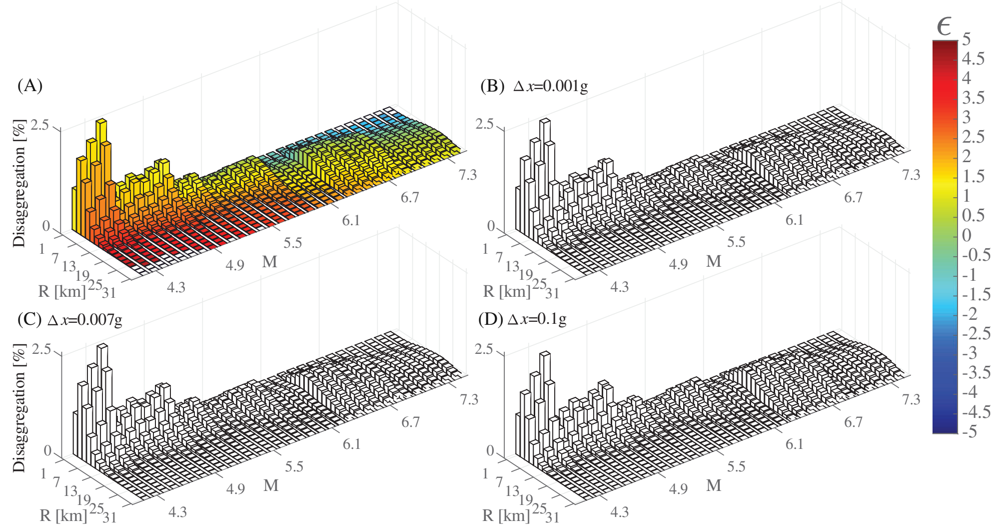


**FIGURE 2** Occurrence disaggregation and approximations, in terms of $M$ and $R$, for the PGA with 475 years exceedance return period on rock in L'Aquila. Distance is epicentral.

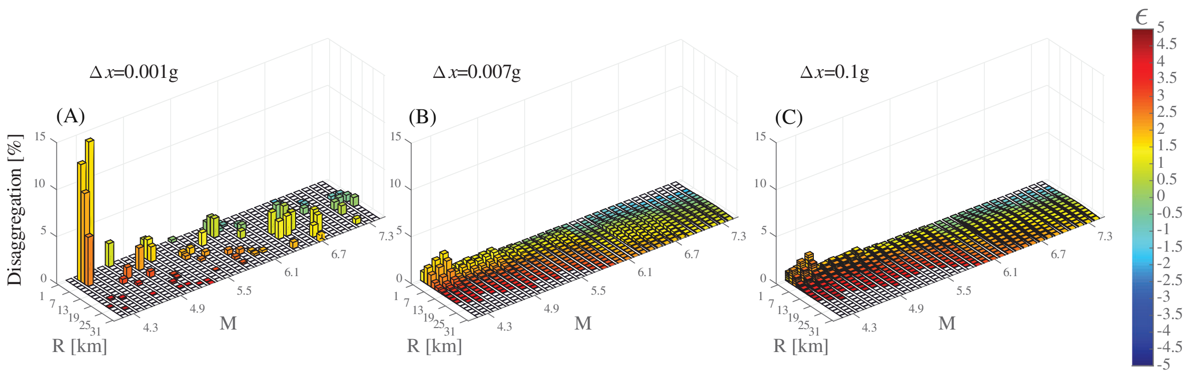


**FIGURE 3** Approximated occurrence disaggregation, in terms of $\{M, R, \varepsilon\}$, for the PGA with 475 years exceedance return period on rock in L'Aquila. Distance is epicentral. (The bin width for $\varepsilon$ in the figure is 0.1.)

standardized residual corresponding to $IM = x$. Reducing the interval width to $\Delta x = 0.001g$ leads to misleading results, as the disaggregation distribution shows sparse $\{M, R, \varepsilon\}$ scenarios with non-zero contributions. This is because the assumed $\Delta w$ (i.e., 0.05) is larger than that corresponding to $\Delta x = 0.001g$ (according to the relationship between $\Delta x$ and $\Delta w$ introduced above). Thus, the calculations do not allow the epsilon values corresponding to $IM \in (x, x + \Delta x)$ to be captured, for many magnitude-distance pairs.

## 5 | FINAL REMARKS

Occurrence disaggregation of probabilistic seismic hazard is typically addressed, in an approximated manner, considering a finite interval around the ground motion intensity measure value of interest. With the purpose of clarifying some issues pertaining to such an approximation, this short communication addressed that:

1. occurrence disaggregation in terms of magnitude and distance is possible without finite $IM$ interval approximation; moreover, disaggregation in terms of $\{M, R\}$ already provides all information about the relevant scenarios for the $IM$ occurrence, including epsilon;

2. occurrence disaggregation is a degenerate function when it is carried out in terms of magnitude, distance (and the other RVs possibly considered in the hazard integral) along with epsilon; consequently, approximating it via a finite $IM$ interval can lead to misleading results;
3. a meaningful approximated occurrence disaggregation that includes epsilon is the one where the $IM$ interval width matches the discretization of the variables' domain used in the calculations; however, in this case it is perfectly equivalent to the approximated disaggregation in terms of $\{M, R\}$.

It is believed that this note may help earthquake engineering practice for the cases where occurrence disaggregation is needed to identify the earthquake scenarios relevant for the analysis in question.

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## DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article, as no datasets were generated or analyzed during the current study.

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