Improving the static pushover analysis in the Italian seismic code by proper piecewise-linear fitting of capacity curves

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ABSTRACT

An improvement of the Italian Code bilinear fit for static pushover (SPO) curves is put forward aimed at significantly decreasing the error introduced in the conventional SPO analysis by the piecewise linear fitting of the capacity curve. While other issues, such as the relationship to pass from the response of multi-degree-of-freedom to single-degree-of-freedom systems (MDOF – SDOF), have been heavily examined in the last decades and improvements and changes to the original method have been proposed and introduced in codes and guidelines, the piecewise linear fit assumed has not yet been systematically investigated. The determination of an optimal multilinear fit has now become a more pressing issue, since new generation modeling approaches lead to highly curved pushover shapes with significant stiffness changes, especially when explicitly incorporating the initial uncracked stiffness of sections. In such cases, even determining the code-standard equivalent elastic stiffness and yield strength of the simple elastic-plastic approximation can be greatly improved upon.

In the approach proposed herein, the error introduced by the piecewise multilinear fit of the force-deformation relationship is quantified by studying it at the SDOF level, away from any interference from MDOF effects. Incremental Dynamic Analysis (IDA) is employed to enable a direct comparison of the actual curved backbones versus their piecewise linear approximations in terms of the spectral acceleration capacity for a continuum of limit-states, allowing an accurate interpretation of the results in terms of performance. An optimized elastic-plastic bilinear fit is the first enhanced solution to decrease systematically the error introduced in the SPO analysis if compared to the hybrid fit approach provided by the Italian seismic provisions. Moreover this fit allows employing the same R-μ-T (strength reduction factor-ductility-period) relationship already prescribed by the Italian Code.

1 INTRODUCTION

The Italian building code (CS.LL.PP., 2008) and its explicative document (CS.LL.PP., 2009) have been updated recently according to modern design standards, changing significantly the specific provisions for design, assessment and retrofitting in seismic zone. This currently enforced code is the last step of a process that began in 2003 after the San Giuliano earthquake (M_w 5.9), in southern Italy. The enhancements involved different parts of the code and most of the improvements followed the recommendations already provided in Eurocode 8 (CEN, 2004; CEN 2005). On the other hand, some aspects, especially those concerning the nonlinear static procedure, have diverged from the consolidated version adopted by Eurocode 8 and based on the original N2 approach (Fajfar and Fischinger, 1988). In fact, some of the provisions of US guidelines (FEMA 356, 2000; FEMA 440, 2005), concerning the, so called, *coefficient approach* in the nonlinear static procedure, have also been adopted in the new Italian Code, resulting, in some cases, in hybrid prescriptions.

In the main framework of a wide investigation aimed at singling out the best fit approach to be adopted for the definition of the equivalent SDOF in nonlinear static analysis, a focused study has been carried out to assess the performance of the fitting rules suggested in the explicative document (CS. LL. PP., 2009) of the recent Italian building code. The first part of this investigation focused on fitting non-negative (without softening branch) capacity curves as described by De Luca et al., (2011), where the methodology that will be presented in the next section was firstly employed.

The need for a quantitative definition of the error introduced by the fit of capacity curves in nonlinear static analysis arose since the nonlinear static procedure (NSP), based on static pushover analysis (SPO), is becoming a routine approach for the assessment of the seismic capacity of existing buildings. All NSP approaches consist of the same five basic steps: (a) perform static pushover analysis of the multi-degree-of-freedom (MDOF) system to determine the base shear versus (e.g., roof) displacement response curve; (b) fit a piecewise linear function (typically bilinear) to define the period and backbone of an equivalent single degree of freedom system (SDOF); (c) use a pre-calibrated R-μ-T (reduction factor – ductility – period) relationship for the extracted piecewise linear backbone to obtain SDOF seismic demand for a given spectrum; (d) use the static pushover curve to extract MDOF response demands; (e) compare demands to capacities; see for example (Fajfar and Fischinger, 1988). NSP is a conventional method without a rigorous theoretical foundation for application on MDOF structures (Krawinkler and Seneviratna, 1998), as several approximations are involved in each of the above steps. On the other hand, its main strength is providing nonlinear structural demand and capacity in a simple and straightforward way. Although several improvements and enhancements have been proposed since its introduction, any increase in the accuracy of the method is worth only if the corresponding computational effort does not increase disproportionately. Extensively investigated issues are the choice of the pattern considered to progressively load the structure and the implication of switching from the nonlinear analysis of a multiple degree of freedom (MDOF) system to the analysis of the equivalent SDOF sharing the same (or similar) capacity curve. Regarding the shape of the force distributions, it was observed that an adaptive load pattern could account for the differences between the initial elastic modal shape and the shape at the collapse mechanism (e.g. Antoniou and Pinho, 2004). Contemporarily, other enhanced analysis methodologies were proposed to account for higher mode effects and to improve the original MDOF-to-SDOF approximation (e.g., Chopra and Goel, 2002). Regarding the demand side, efforts have been put to provide improved relationships between strength reduction factor, ductility, and period (R-μ-T relationships), to better evaluate the inelastic seismic performance at the SDOF level (Vidic et al., 1994; Miranda and Bertero, 1994).

One of the issues that have not yet been systemically investigated is the approximation introduced by the imperfect piecewise linear fit of the capacity curve for the equivalent SDOF. The necessity to employ a *multilinear* fit (an inexact, yet common, expression to describe a piecewise linear function) arises due to the use of pre-determined R-μ-T relationships that have been obtained for idealized systems with those piecewise linear backbones. This has become even more important recently, since nonlinear modeling practice has progressed towards realistic multi-member models, which often accurately capture the initial stiffness using uncracked section properties. The gradual plasticization of such realistic elements and models introduces a high curvature into the SPO curve that cannot be easily represented by one or two linear segments. It is an important issue whose true effect is often blurred, being lumped within the wider implications of using an equivalent SDOF approximation.

The choice of the piecewise linear fit is typically restricted by the availability of R-μ-T relationships that can account for the equivalent SDOF backbones employed when fitting the capacity curve. Even if R-μ-T relationships that can capture far more complex backbones have recently appeared (e.g. Vamvatsikos and Cornell, 2005), the bilinear approach is by far the most widely employed in guidelines and literature, such as Eurocode 8 and Italian code that shares the same bilinear R-μ-T relationship (Vidic et al. 1994). The study presented herein deals with the bilinear approximation of the capacity curves, also addressing a specific comparison with the approach followed by the Italian guidelines regarding this issue.

The approach presented herein will be based on the accurate assessment of the effect of the equivalent SDOF fit on the nonlinear static procedure results. The latter can be achieved by proper quantification of the bias introduced into the estimate of the seismic response at the level of the SDOF itself. Incremental dynamic analysis, IDA (Vamvatsikos and Cornell, 2002), will be used as benchmark method to quantify the error introduced by a bilinear fit with respect to the exact capacity curve of the SDOF. Figure 1a shows a typical example, where an elastic-plastic backbone fit is used according to FEMA-440. While this fit approach is meant to result to an unbiased approximation in terms of seismic performance, the median IDA results of Figure 1b show the actual error that is introduced by such code-mandated fitting rules. In most cases, they lead to an unintended and hidden bias that is generally conservative. On the other hand, this bias can become unreasonably high in many situations.

Therefore different issues come out: first, develop a methodology aimed at quantifying the bias introduced by the fitting of a capacity curve (De Luca et al., 2011); second, perform a systematical investigation aimed at providing fits that can reduce the error, thus providing a comparison with approaches followed in codes and guidelines, that can function as a benchmark to evaluate the improvements achieved.

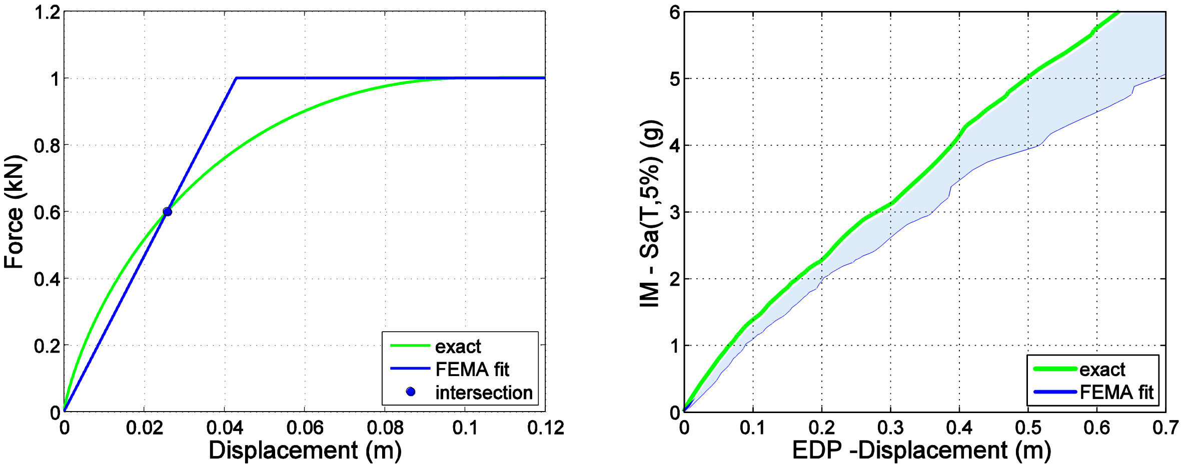

Figure 1. (a) Example of exact capacity curve versus its elastic-plastic bilinear fit according to FEMA-440 and (b) the corresponding median IDA curves showing the negative (conservative) bias due to fitting for T=0.5 sec.

2 METHODOLOGY

The first main target is the quantification of the error introduced in the NSP-based seismic performance assessment by the replacement of the original capacity curve of the system, termed the “exact” or “curved” backbone, with a piecewise linear approximation, i.e., the “fitted” or “approximate” curve (e.g., Figure 1a). This will enable a reliable comparison between different fitting schemes in an attempt to minimize the observed discrepancy between actual and estimated performance. In all cases, to achieve an accurate and focused comparison of the effect of fitting only, it is necessary to disaggregate the error generated by the fit from the effect of approximating an MDOF structure via an SDOF system. Thus, all the investigations are carried out entirely at the SDOF level, using a variety of capacity curve shapes, different periods and hysteresis rules and using IDA as the method of choice for assessing the actual performance of the different alternatives.

An ensemble of SDOF oscillators is considered with varying curved shapes of force-deformation backbones. They are all fitted accordingly with bilinear elastic-plastic shapes. For each considered curved backbone shape, 5% viscous damping was used and appropriate masses were employed to obtain periods of 0.2, 0.5, 1 and 2 sec. In all cases, both the exact and the approximate system share the same mass, thus replicating the approach followed in the conventional NSP methodology.

When comparing an original system with its approximate, having a piecewise linear backbone, the same hysteretic rules are always employed, so that both systems display the same characteristics when unloading and reloading in time-history analyses. In other words, all differences observed in the comparison can be attributed to the fitted shape of the approximate backbone, obviously also capturing any corresponding differences in the oscillator period.

For each exact shape of the SDOF's capacity curve and for each period value, several piecewise linear fit approximations have been considered according to different fitting rules. To enable a precise comparison that will allow distinguishing among relatively similar backbones in consistent performance terms, as it was previously stated, IDA will be employed. Median IDA curves of the exact capacity curves and their backbones are compared according to the IM given EDP approach, see De Luca et al. 2011 for details.

To perform IDA for each exact and approximate oscillator considered, a suite of sixty ground motion records was used, comprised of both horizontal components from thirty ordinary records from the PEER NGA database. They represent a large magnitude, short distance bin having no near-source directivity or soft-soil effects. Using the hunt & fill algorithm (Vamvatsikos and Cornell, 2004), 34 runs were performed per record to capture each IDA curve with excellent accuracy. The IM of choice was the 5%-damped spectral acceleration at the period T of the oscillator, $S_a(T)$, while the oscillator displacement δ was used as the corresponding EDP, being the only SDOF response of interest when applying the NSP.

Once the IM and EDP are decided, interpolation techniques allow the generation of a continuous IDA curve from the discrete points obtained by the 34 dynamic analyses for each ground motion record. The resulting sixty IDA curves can then be employed to estimate the summarized IDA curves for each exact and approximate pair of systems considered. Still, in order to be able to compare an exact system with reference period T with its approximation, having an equivalent period T_{eq}, it was necessary to have their summarized IDA curves expressed in the same IM. In this case it is chosen to be $S_a(T)$, i.e. the spectral ordinate at the period of the exact backbone oscillator. Thus, while the approximate system IDA curves are first estimated as curves in the $S_a(T_{eq}) - \delta$ plane, they are then transformed to appear on $S_a(T) - \delta$ axes. This is achieved on a record-by-record basis by multiplying all the $S_a(T_{eq})$ values, from the runs that comprise the i-th IDA curve, by the constant spectral ratio $[S_a(T) / S_a(T_{eq})]_i$ that characterizes the i-th record (Fragiadakis et al., 2006).

The error due to the fitting is evaluated for every value of displacement in terms of the relative difference between the two system median S_a-capacities, both evaluated at the reference period T of the exact system, as it is shown in equation (1).

$$e_{50}(\delta) = \frac{S_{a,50}^{fit}(\delta) - S_{a,50}^{exact}(\delta)}{S_{a,50}^{exact}(\delta)} \tag{1}$$

3 INVESTIGATION OF FITS

Bilinear elastic-plastic or elastic-hardening fits are the fundamental force-deformation approximations employed in all NSP guidelines. The simplicity of the bilinear shape means that the only need is to estimate the position of the nominal "yield point" and select a value for the constant post-elastic stiffness. Eurocode 8 (CEN, 2004) suggests a piecewise bilinear fit based on an equal area criterion. The objective is to balance the areas of the mismatch regions where the fitted elastic-plastic idealized backbone lies above and below the capacity curve. This approach is similar to the original N2 method (Fajfar and Fischinger, 1988). As a consequence, Eurocode 8 prescribes an R-μ-T relationship (Vidic et al., 1994) based on the elastic-perfectly-plastic fit. FEMA 356 employs a bilinear idealized relationship with an initial slope and a post yield slope evaluated by balancing the area above and below the capacity curve up to the target displacement and setting the initial effective slope at a base shear force equal to 60% of the nominal yield strength. The proposed graphical procedure is iterative.

Following the spirit of such guidelines, the methodology described in the previous section and applied in the following has been designed to strike at the core of the fitting problem. By assuming the same hysteretic rules in both the exact and the approximate system, the piecewise linear approximation will be linked only to the shape of the capacity curve. At the same time, improved fitting techniques will be investigated avoiding any iterative procedure, thus assuming that the curve itself allows capturing, with its shape, all the characteristics.

As it has been shown by De Luca et al. (2011), both the equal area rule, employed in Eurocode 8, and the 60% rule, employed in all FEMA documents, can be very conservative. Capturing the initial stiffness by means of a fit at 10% of the maximum shear of the exact backbone can provide a significant improvement. An example of such a conservative effect is shown in Figure 2 where two backbones with non-softening behavior are fitted according to Eurocode 8 fit (*equal area*), FEMA 60% rule (*FEMA*) and finally according to a fit that captures the initial stiffness (*10% fit*). Figures 2a and 2b represent a system with mild and high initial backbone-curvature, respectively. Figure 3 shows the errors introduced by the three fits, presented in Figure 2, for a period equal to 0.5 seconds. These errors obviously increase significantly in the case of non-trivial changes in the initial stiffness of the curved backbone (see Figure 3b), that can be typical of modeling approaches that account for uncracked stiffness. A fitting rule that captures the initial stiffness (*10% fit*) can lead to errors that seldom exceeds 20% compared to errors, even if conservative, that can get to over 50%, if Eurocode 8 or FEMA fit are applied in the case of "highly curved" systems.

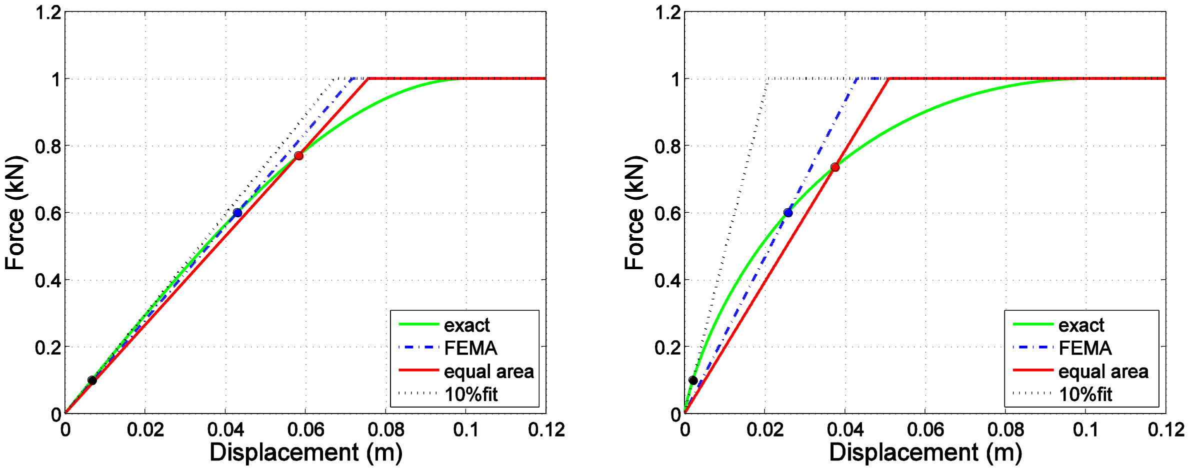

Figure 2. Comparison of non-softening capacity curves and their corresponding fits having (a) insignificant versus (b) significant changes in initial stiffness.

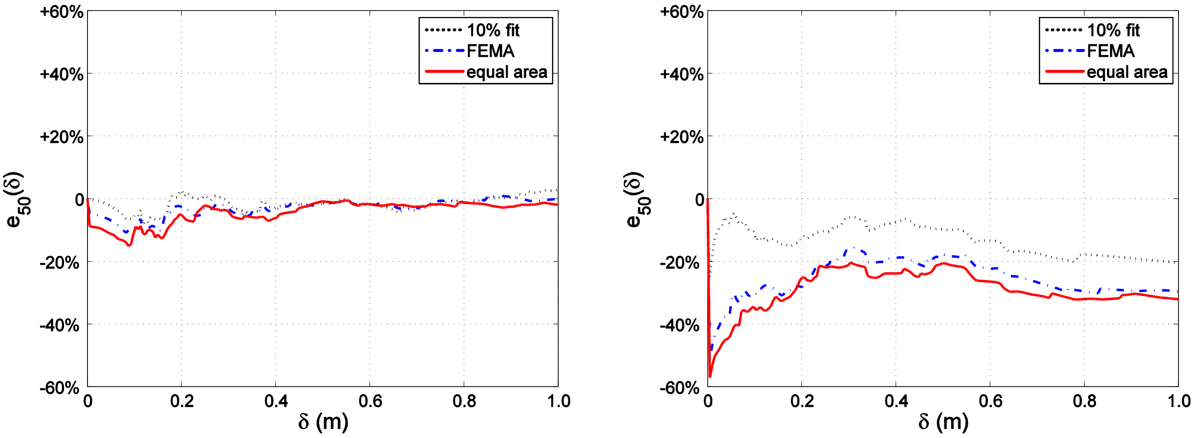

Figure 3. Median relative error comparison between the 10%, FEMA and equal area fits for T = 0.5 sec, when applied to the capacity curves of Figure 2: (a) insignificant versus (b) significant changes in initial stiffness.

Having assessed that standard bilinear elastic-plastic and elastic-hardening fitting rules can be improved, it is important to investigate, as well, whether improved fits can also be devised to reduce the error in backbones that also show some mild negative stiffness, characterized by softening. The latter represents the general aim of the fitting approach provided in the Italian provisions (CS.LL.PP, 2009).

Italian guidelines suggest accounting for softening behavior up to the point of a 15% degradation of maximum base shear in the capacity curve. The fit is based on 60% rule for the initial stiffness, in analogy with all the FEMA documents. Then an equal area criterion is applied to derive the plateau of the bilinear fit; the latter can be extended until the point where a 15% degradation of the maximum base shear is reached (Figure 4). The result of such an approach is that the plateau of the elastic-plastic fit is always lower than the shear at the maximum point of the exact backbone. Obviously, if the structural model used cannot display any negative stiffness, such fitting criteria simply become equivalent to the FEMA provisions. The obtained bilinear fit according to the Italian code provisions is then accompanied by the classical Vidic et al (1994) R-μ-T relationship employed in Eurocode 8.

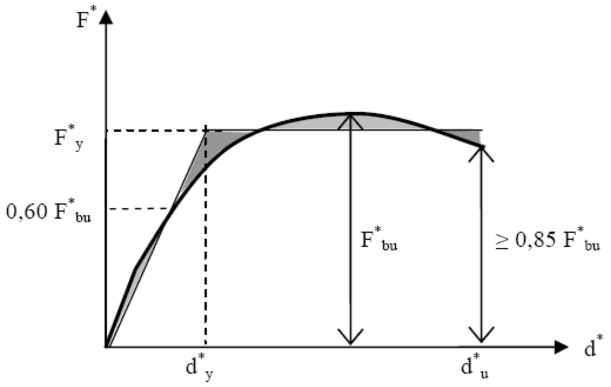

Figure 4. Italian fit of mildly softening capacity curves according to CS. LL. PP. 2009.

The Italian hybrid fitting rule is worth to be investigated, especially to check whether considerations regarding the best approach for non-softening behaviors (De Luca et al, 2011) are still reliable to capture early negative response. In other words, the general principle of getting the initial stiffness (10% rule) is going to be checked for a backbone family of softening curves and consequently compared with the 60% rule.

In addition, the merits of balancing the mismatch areas above and below the capacity curve will also be discussed. While the Italian Code, as most similar guidelines, is concerned with providing a specialized fit for a given target point on the capacity curve, following the methodology described in section 2, herein it will be sought to provide near-optimal fits for a continuum of limit states. Thus, it will be intentionally avoided any dependence on any specific target point on the backbone, in essence checking at the same time all possible such points. This is achieved by employing a simple direct search approach, where the height of the plastic plateau for the fit is gradually moved from 80% to 100% of the peak shear in the exact capacity curve. Consequentially, while checking for a broadly optimal fit, all possible fits that could arise from applying the Italian Code approach to different target points will be efficiently captured.

Two example backbones from the numerous tests conducted are shown in Figure 5. They were chosen to emphasize two different softening trends, combined with different curvature changes in the initial part of the backbones: insignificant (see Figure 5a) versus significant (see Figure 5b). Four different fits are displayed out of the large number that has been checked: the initial stiffness is fixed at 10% or 60% of the maximum shear combined with two plateau levels at 80% (L) and 100% (P) of peak shear, thus obtaining four fitting approaches named 10%L, 10%P, 60%L and 60%P respectively. The performances of each of the four fits considered are shown in Figure 6 and Figure 7 for T=0.2, 0.5 sec. Results show how the changes in curvature in the initial part still play an important role for fitting performances. The 10% fit improves results as long as there are significant changes in the curvature of the exact backbone. Furthermore, catching the peak point (P versus L fits) seems to have better performances than a lower plateau value, regardless of any area-balancing rules. The latter results are partially confirmed by the fits suggested in other studies where the maximum shear value is selected as one of the criteria for fitting softening backbones (Han et al., 2010). For low frequencies and significant changes in the initial stiffness, the 10% fit, in both its versions showed herein, P and L, can lead to slightly non conservative results at the beginning of the backbone. The same effect was observed also in the case of non-softening backbones, see De Luca et al. (2011) for details. Still, for conventional limit states of interest and for most practical applications for which a static pushover is used, the target points will not be located in this early near-elastic part of the backbone.

The fit approaches showed have been tested for a sample family of exact backbones in which curvature at the beginning and the slope of the softening have been varied. The sample family of backbones considered and their hysteresis loops are presented in Figure 8. The median error trends of the 60%L and the 100%P fits are compared in Figure 9 to Figure 12, respectively for the four periods considered (0.2, 0.5, 1.0 and 2.0 seconds). The error is mapped according to the characteristic points of the exact backbone; the peak point “p”, where the maximum shear is attained, and the ultimate point “u”, where the exact capacity curve attains the residual branch. Figure 9 to Figure 12 are meant to represent the median error to be expected when fitting a generic capacity curve with a specific rule, (in this case 60%L and 100%P rules respectively). The 10%P fit is found to be an unbiased fit approach that can be extended to the mildly softening part of the backbone at each of the periods investigated, showing robust performances for this portion of backbone in a wide range of frequencies.

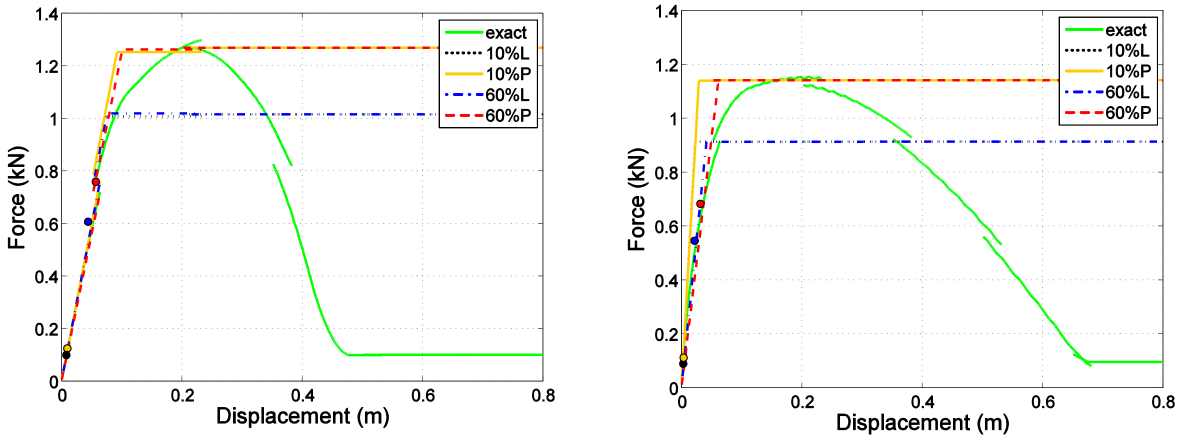

Figure 5. Comparison of two capacity curves with softening branch and their corresponding fits having (a) insignificant versus (b) significant changes in initial stiffness.

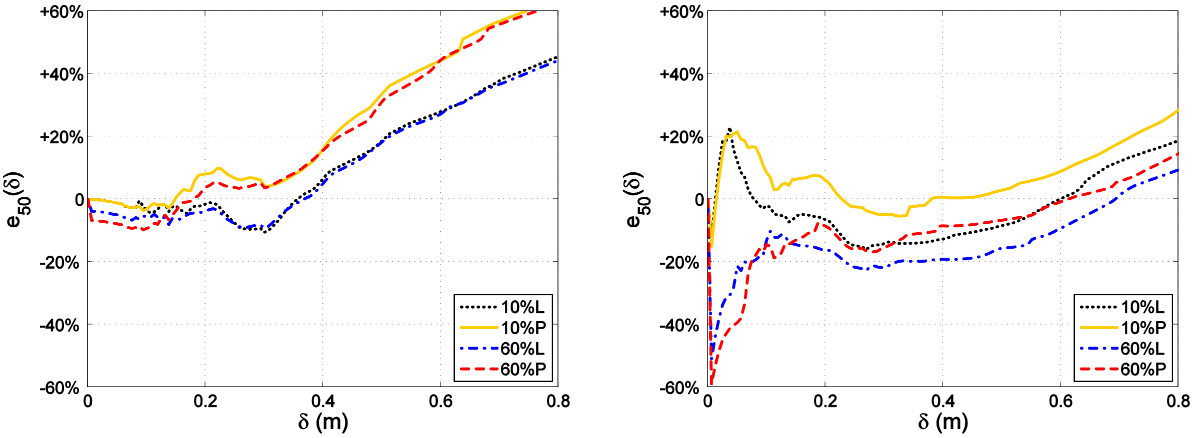

Figure 6. Median relative error comparison between the 10%L, 10%P, 60%L and 60%P fits for $T = 0.2$ sec, when applied to the capacity curves of Figure 5: (a) insignificant versus (b) significant changes in initial stiffness and different softening slopes.

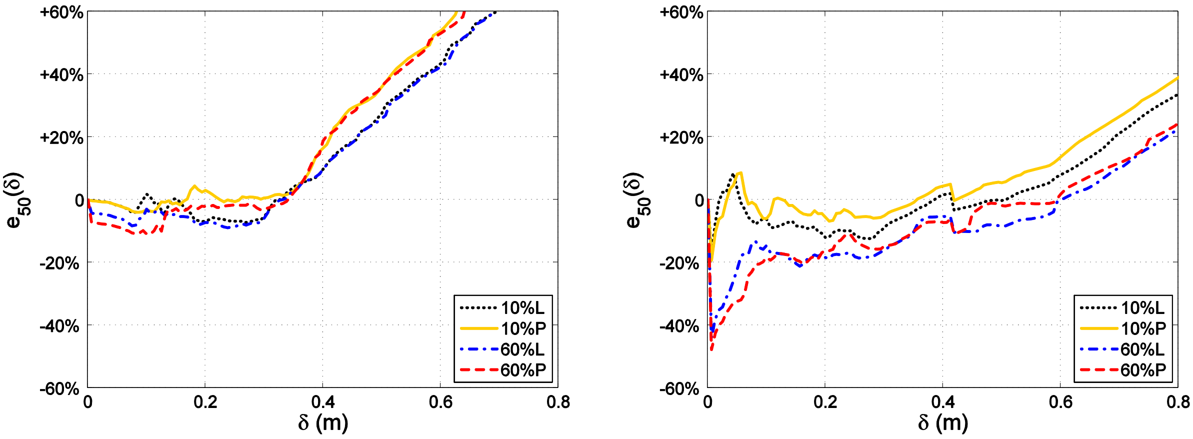

Figure 7. Median relative error comparison between the 10%L, 10%P, 60%L and 60%P fits for $T = 0.5$ sec, when applied to the capacity curves of Figure 5: (a) insignificant versus (b) significant changes in initial stiffness and different softening slopes.

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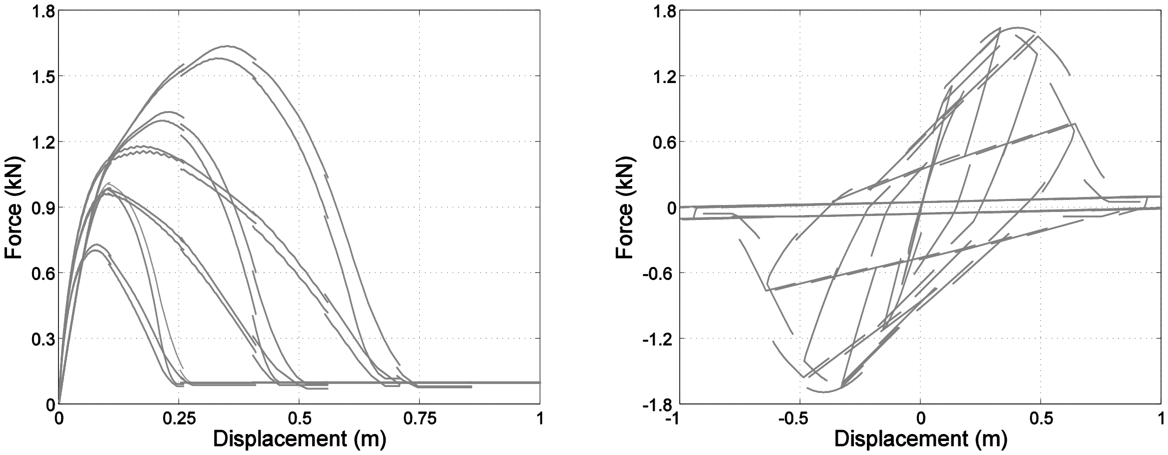

Figure 8. Backbones (a) and example hysteretic behavior according to pinching hysteresis rule (b) of the family of capacity curves considered.

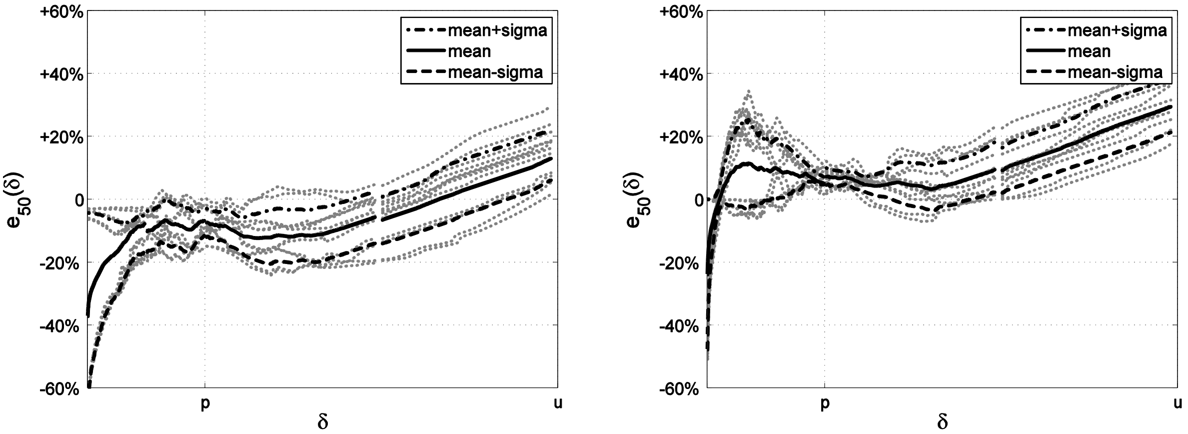

Figure 9. The median relative error at T = 0.2 sec if (a) the 60%L and (b) 10%P fits are employed, respectively, for the family of capacity curves in Figure 8 (grey dotted lines).

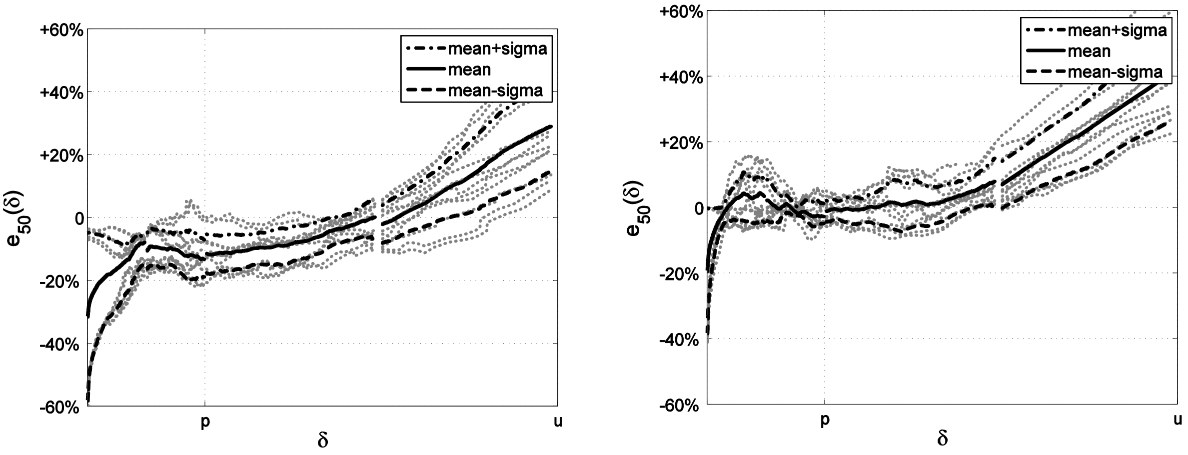

Figure 10. The median relative error at T = 0.5 sec if (a) the 60%L and (b) 10%P fits are employed, respectively, for the family of capacity curves in Figure 8 (grey dotted lines).

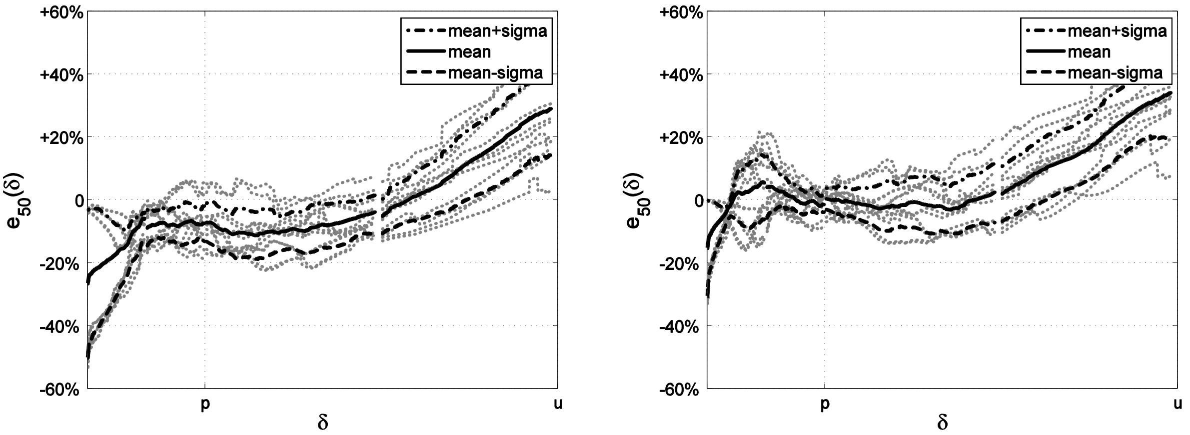

Figure 11. The median relative error at T = 1.0 sec if (a) the 60%L and (b) 10%P fits are employed, respectively, for the family of capacity curves in Figure 8 (grey dotted lines).

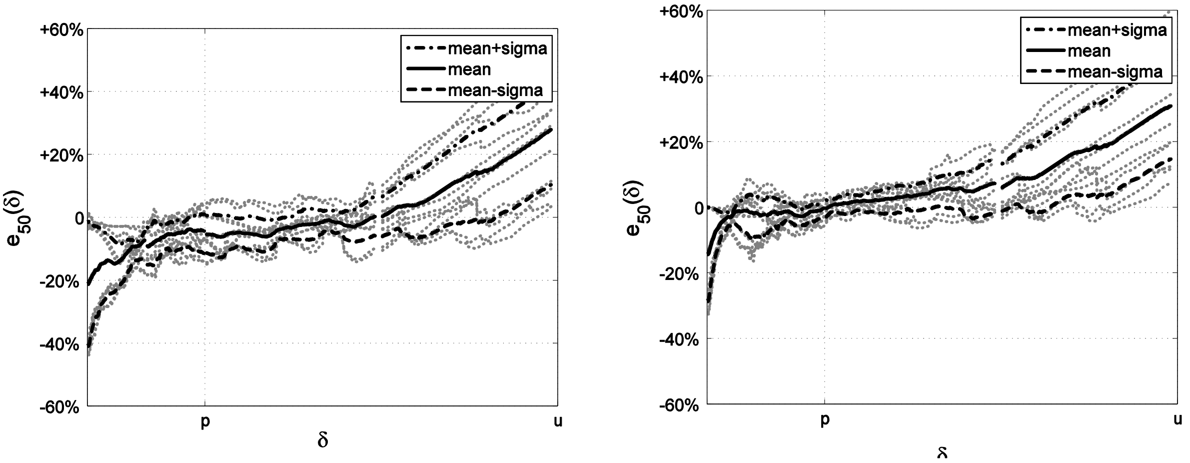

Figure 12. The median relative error at T = 2.0 sec if (a) the 60%L and (b) 10%P fits are employed, respectively, for the family of capacity curves in Figure 8 (grey dotted lines).

4 CONCLUSIONS

Structural seismic assessment based on the nonlinear static procedure is founded on the general assumption that the behavior of an MDOF system can be interpreted by the response of an equivalent SDOF. This necessitates a number of specific approximations at various stages of the procedure. An IDA-based methodology, already employed for the investigation of non-softening capacity curves, has been employed in this study in the case of backbone with softening to assess the reliability of the fit approach suggested by recent Italian seismic code.

The Italian Code fit rule that is meant to extend the FEMA guidelines to also capture mildly softening behaviors in the capacity curve, is studied to check if such hybrid approach can provide improved accuracy in the fit. This fit conjugates the 60% rule, basic approach of US guidelines, with the equal area criterion, typical of European guidelines and, additionally, allows accounting for softening up to a 15% degradation of the peak base shear.

The Italian fit rule is found to be conservative in analogy with all the other code fits (Eurocode 8 and FEMA). It is to be noted that the limit to the softening branch imposed in the Italian code (up to 85% of the maximum shear) was found to be reliable. The investigations of this study, in fact, showed that the error introduced by any bilinear fit increases dramatically when softening branch goes down to values lower than 85% of the maximum shear and bilinear fits could not catch at all the final part of the softening behavior, ending up with a systematical overestimation of the response; while within code-mandated softening values (up to 15% degradation of the maximum base shear) the error can be still considered acceptable and at least comparable to the one introduced in the initial part of the backbone.

Besides the check of the Italian code fit, the investigation of this family of backbones shows a systematical result regarding the plastic plateau fit at the maximum shear value of the exact capacity curve. This seems to be the better solution rather than any reduced conservative value. Such an elastic-plastic fit that captures the maximum base shear and has an initial stiffness fitted at 10% of it, termed a 10%P fit, can be reliably adopted in nonlinear static analysis procedures to improve the fitting and reduce the error introduced by it.

Furthermore, the latter result represents a step towards the definition of a fully optimized fit rule that can represent a further enhancement to be considered as an upgrade in current seismic provisions. While any enhanced three-segment piecewise linear fit that incorporates a softening branch would further improve the accuracy in the equivalent SDOF backbone, it would also require changes in the R-μ-T relationship. On the other hand, the improved rules presented herein can fit seamlessly in current seismic codes without requiring any further changes.

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