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Discussion of "Areal exceedance of ground motion as a characteristic of multiple-site seismic hazard: Sensitivity analysis" by V. Sokolov, F. Wenzel [Soil Dyn. Earthq. Eng. 126 (2019), Article 105752]

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1. Introduction

The discussed paper deals with the fraction of sites, in a region of interest, in which ground motion from an earthquake exceeds some predefined level, as a metric for seismic hazard assessment. This metric is called Fraction of Areal Exceedance (FAE). The paper revolves around the effect of hazard modelling, especially of ground motion prediction equations, on the probabilities associated to different FAE values in time intervals. The study is interesting and useful; therefore, the discussers believe that, in the benefit of the reader interested in this matter, it could be worthwhile to clarify two issues, which are related each other:

(i) under the assumption of the discussed paper, regardless of the considered ground motion correlation model, the process of occurrence of earthquakes exceeding a certain fractional area value is certainly Poisson; moreover, its rate only depends on the rates of earthquakes on the seismic sources and on the probability that the considered fraction is exceeded in a generic earthquake, that is an earthquake of unspecified magnitude and location;
(ii) because of (i), the simulation does not need to generate stochastic catalogs (i.e., it is not needed to simulate earthquakes in time), but an event-based approach, in which simulations are carried out for events of given magnitude and location, suffices; in fact, the event-based simulation is also certainly more efficient computationally.

The two issues are demonstrated and discussed in the following two sections.

2. Poisson process with random selection

In this section we refer to the following part of the discussed paper, a few lines above of equation (5), where the text reads:

"*Obviously, the questions whether the FAE does follow a Poisson process, and if not, what may be consequences of using the assumption, require special evaluation … in practical estimations. In this study we assume tentatively that FAE is Poissonian …*"

We want to reassure that, under the hypotheses of the paper, those of classical hazard, if the number of earthquakes on the sources occur according to a homogeneous Poisson process (HPP), say $\{N(t), t \geq 0\}$, also the number of earthquakes that cause a fractional exceedance region (Fr) larger than a threshold, fr, occur according to an HPP, say $\{N_{fr}(t), t \geq 0\}$. This is because the process of earthquakes causing $Fr > fr$ originates from an HPP with r*andom selection*, then it is also an HPP (e.g. Refs. [1]).[1]

The rate of earthquakes causing $Fr > fr$, say λ_{fr},[2] can be computed by the rate of occurrence of earthquakes on the sources affecting the

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[1] This kind of processes are referred also as *filtered* or *thinned* Poisson processes [5].
[2] This rate is indicated as γ in the discussed paper.

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region of interest, let us call it ν, multiplied by the probability that, given the occurrence of a generic earthquake (E), it causes $Fr > fr$. In fact, indicating this latter probability as $p_{fr} = P[Fr > fr|E]$, the sought rate can be computed as:

$$\lambda_{fr} = \nu \cdot p_{fr}. \tag{1}$$

therefore, because λ_{fr} is the rate of a HPP, the probability of exceeding fr in the $(0, t)$ interval is necessarily equal to: $P[Fr(0, t) > fr] = 1 - e^{\lambda_{fr} \cdot t}$. In other words, equation (5) of the discussed paper is exact without further assumptions or approximations. For completeness, hereafter, the demonstration that an HPP with random selection is still an HPP is briefly given.

The number of exceedances of the threshold fr in $N(t) = n$ earthquakes, of unknown magnitude and location, is the sum of n independent and identically distributed Bernoulli random variables (RVs) characterized by the p_{fr} parameter. These Bernoulli RVs, in each event, assume value 1 when Fr exceeds the threshold fr, and 0 otherwise. Consequently, given $N(t) = n$, the conditional probability of $N_{fr}(t) = k$, that is $P[N_{fr}(t) = k|N(t) = n]$, is computed via the binomial probability mass function: $C_k^n \cdot p_{fr}^k \cdot (1 - p_{fr})^{n-k}$, where C_k^n is the binomial coefficient. Therefore, because $N(t)$ is Poisson distributed also $N_{fr}(t)$ is Poisson distributed, with rate λ_{fr}:

$$\begin{aligned} P[N_{fr}(t) = k] &= \sum_{n=0}^{+\infty} P[N_{fr}(t) = k|N(t) = n] \cdot P[N(t) = n] = \sum_{n=k}^{+\infty} C_k^n \cdot p_{fr}^k \cdot (1 - p_{fr})^{n-k} \cdot \frac{(\nu \cdot t)^n}{n!} \cdot e^{-\nu \cdot t} = \\ &= \frac{e^{-\nu \cdot t} \cdot p_{fr}^k \cdot (\nu \cdot t)^k}{k!} \cdot \sum_{n=k}^{+\infty} \frac{[\nu \cdot t \cdot (1 - p_{fr})]^{n-k}}{(n-k)!} = \frac{e^{-\nu \cdot t} \cdot p_{fr}^k \cdot (\nu \cdot t)^k}{k!} \cdot \sum_{j=0}^{+\infty} \frac{[\nu \cdot t \cdot (1 - p_{fr})]^j}{j!} \\ &= \frac{e^{-\nu \cdot t} \cdot p_{fr}^k \cdot (\nu \cdot t)^k}{k!} \cdot e^{\nu \cdot t \cdot (1-p)} = \frac{(p_{fr} \cdot \nu \cdot t)^k}{k!} \cdot e^{-p_{fr} \cdot \nu \cdot t} = \frac{(\lambda_{fr} \cdot \nu \cdot t)^k}{k!} \cdot e^{-\lambda_{fr} \cdot t},\ k = \{0,1,2, \ldots\}. \end{aligned} \tag{2}$$

Once this demonstration is acknowledged, it can be recognized that it applies to derive the counting process of earthquakes showing any (specific) effect with the following characteristics: (a) the probability of observing the effect is the same in different earthquakes of unspecified magnitude and location; (b) the effects of interest, produced by different earthquakes, are stochastically independent.

It is also to note that, as discussed in Ref. [2], this result is obtained using the same arguments which allow to demonstrate that the total number of exceedances (collectively observed in the reference area) may significantly depart from the Poisson distribution, depending on source-sites configuration. This is because the total number of exceedances, in general, do not match the conditions (a-b).

3. Event-based simulation

To evaluate the rate to be used in equation (5), named λ_{fr} herein, the authors generate a stochastic catalog lasting several thousand years. This strategy does not fully exploit the properties of the Poisson process with random selection. Acknowledging equation (1), given that in the discussed paper ν is used to generate the synthetic catalog (i.e., its treated as known), the simulation is only needed to compute the probability that the fractional area is exceeded given the occurrence of one event, that is, p_{fr}. Because in probabilistic seismic hazard analysis, for each single source, the distribution of earthquake magnitude (M) and location $\{X, Y\}$ are known, the sought probability can be computed as an application of the total probability theorem:

$$p_{fr} = P[Fr > fr|E] = \int_M \int_{X,Y} P[Fr > fr|M = m, R = r] \cdot f_M(m) \cdot f_{X,Y}(x, y) \cdot dr \cdot dm, \tag{3}$$

where, f_M and $f_{X,Y}$ are the distribution of M and $\{X, Y\}$ respectively, and $P[Fr > fr|M = m, X = x, Y = y]$ is the (time-invariant) probability that $Fr > fr$, given earthquake's magnitude and location. (Simple adjustments are needed in the case of multiple sources; see for example [3].)

To solve the integral in equation (3) via simulation the following steps are only needed:

1. Sample an earthquake magnitude according to f_M;
2. Sample an earthquake location according to $f_{X,Y}$;
3. Simulate ground motions at the cells by means of which the region of interest is discretized[3];
4. Count 1 if $Fr > fr$ and 0 otherwise;
5. Repeat steps 1–4 an arbitrary number of times (say k) and evaluate p_{fr} as the total number of counts from step 4 divided by k;
6. Compute λ_{fr} via equation (1).

This procedure, besides being simpler, also avoids introducing useless sources of variability. Indeed, ν is known a priori, while to simulate earthquake catalogs will introduce a spurious inferential uncertainty, as the ν resulting from a simulated catalog will not be equal to the assigned one, because of the finite number of runs. Thus, the event-based simulation is certainly more efficient as discussed in papers dealing with software implementations of multi-site probabilistic seismic hazard analysis (e.g. Ref. [4]).

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[3] At this step any ground motion correlation model can be applied. The chosen model will only affect the p_{fr} value determined in step 5.