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Short Note
Soil-Invariant Seismic Hazard and Disaggregation

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Abstract Results of probabilistic seismic hazard analysis (PSHA) depend on the soil conditions of the site investigated. Consequently, it is generally expected that disaggregation, usually employed to gather further information about hazard levels of interest, changes with the soil class. This short note discusses the relationship between hazard curves and disaggregations computed for different soil conditions at the same site. In particular, it is analytically proven that there are cases, depending on the structure of the ground-motion prediction equations employed, in which disaggregations for different soil conditions are necessarily invariant. It is also demonstrated that, in these situations, hazard curves for different soil conditions can be immediately obtained from a curve developed for a reference soil class. The analytical proofs derived do not require any further testing or validation; nevertheless, the results are illustrated via simple case studies to show how they may imply applicative advantages both in the cases of single-model and logic tree PSHA.

Introduction

Probabilistic seismic hazard analysis (PSHA; e.g., Cornell, 1968; McGuire, 2004), in its classical format, is based on the combination of statistical characterizations of source, path, and site conditions to provide the annual rate of seismic events exceeding a ground-motion intensity measure (IM) threshold im. Equation (1), assuming for simplicity one seismic source, reports a typical hazard integral, in which ν is the rate of occurrence of earthquakes above a magnitude of interest, $f_{M,R}(m,r)$ is the distribution of magnitude (M) and source-to-site distance (R) in one event, and $P(\mathrm{IM} > \mathrm{im}|m,r,\theta)$, typically provided by a ground-motion prediction equation (GMPE), is the probability of exceeding the im threshold conditional to M, R, and one or more other covariates θ, representing, for example, a specific soil condition:

$$\lambda_{\mathrm{im}} = \nu \cdot \iint_{m,r} P(\mathrm{IM} > \mathrm{im}|m,r,\theta) \cdot f_{M,R}(m,r) \cdot dr \cdot dm. \quad (1)$$

For earthquake engineering and engineering seismology applications, once PSHA is carried out, the so-called hazard disaggregation (e.g., Bazzurro and Cornell, 1999) is employed to gather further insights on the earthquakes most threatening for the site of interest, for example, in terms of magnitude and distance pairs most causative for the exceedance of a specific IM value.

PSHA results change, for a specific site, if site conditions are changed, and this is trivially evident from equation (1), as at least one of the covariates of the GMPE changes. Consequently, hazard disaggregation is expected, in general, to change at the same site if soil-site conditions are changed.

It is often believed and recommended that PSHA computations are repeated to account for different soil-site conditions at the same site (e.g., Bazzurro and Cornell, 2004). Nevertheless, it is worthwhile to recall that there are cases, which could be not infrequent, depending on the structure of the GMPE(s) employed in PSHA, in which disaggregation for different soil conditions are invariant at the same site. Moreover, in these situations, hazard curves for different soil conditions can be obtained rigorously from translation (i.e., horizontal rigid-body motion) of the curve developed for a reference soil class (e.g., rock). How much the curves displace each other can be anticipated *a priori* from the GMPE.

This short note intends to discuss the relationship between hazard results for different soil conditions at the same site, analytically demonstrating cases for invariance. To this aim, the following starts from hazard disaggregation, in the case of one or multiple seismic sources, computed via a single-model hazard. Subsequently, the relationships between hazard curves and ordinates of uniform hazard spectra (UHS) for different soil conditions are obtained. Finally, the case of logic-tree-based hazard analysis is addressed.

The core of the study presented is purely analytical, because all the arguments are simple mathematical proofs, which by definition do not require any further testing or validation. Nevertheless, some case-study examples, for a real site in Italy, are presented to show that the results obtained apply to several recent GMPEs and, consequently, they may have practical value in the context of PSHA.

Single-Model Disaggregation

One Seismic Source

Assume one wants to compute disaggregation in terms of M and R for the im that has a specific rate of exceedance λ_{im_1}, at the site for a soil condition indicated by θ_1. Assume also the same rate, indicated by λ_{im_2}, has been computed for the site considering a different soil class, indicated by θ_2, and that disaggregation is computed also for this soil hazard.

The two disaggregations are indicated in equation (2), in which all the terms have been already defined, except $f_{M,R|\mathrm{IM}>\mathrm{im},\theta}(m,r)$, which represents the result of disaggregation; that is, the distribution of M and R conditional to the exceedance of the IM value corresponding to the rate the denominator refers to:

$$\begin{cases} f_{M,R|\mathrm{IM}>\mathrm{im}_1,\theta_1}(m,r) = \frac{\nu \cdot P(\mathrm{IM}>\mathrm{im}_1|m,r,\theta_1)\cdot f_{M,R}(m,r)}{\lambda_{\mathrm{im}_1}} \\ f_{M,R|\mathrm{IM}>\mathrm{im}_2,\theta_2}(m,r) = \frac{\nu \cdot P(\mathrm{IM}>\mathrm{im}_2|m,r,\theta_2)\cdot f_{M,R}(m,r)}{\lambda_{\mathrm{im}_2}} \end{cases} . \quad (2)$$

Because the objective is to compare the two disaggregations, the hypothesis is that the two rates of exceedance are the same $\lambda_{\mathrm{im}_1} = \lambda_{\mathrm{im}_2}$ (of course it is expected that the two corresponding IM values on the different soils classes $(\mathrm{im}_1,\mathrm{im}_2)$ are different). If the rates being disaggregated are the same, then equation (3) applies. The latter, recalling equation (1) and the distributive property of integrals, may be rewritten as in equation (4):

$$\lambda_{\mathrm{im}_1} - \lambda_{\mathrm{im}_2} = 0 \quad (3)$$

$$\begin{aligned} &\iint_{m,r} P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) \cdot f_{M,R}(m,r) \cdot dr \cdot dm \\ &\quad - \iint_{m,r} P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) \cdot f_{M,R}(m,r) \cdot dr \cdot dm \\ &= \iint_{m,r} [P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)] \\ &\quad \cdot f_{M,R}(m,r) \cdot dr \cdot dm = 0. \end{aligned} \quad (4)$$

We shall now focus on the last line of the equation. Because the $f_{M,R}(m,r)$ terms are certainly nonnegative, the $P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)$ terms need to have different signs for the integral to be null. In other words, it has to be that $P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)$ is negative for some (m,r) and positive for others, such that the integral is equal to zero. Alternatively, the integral is null also if $P(\mathrm{IM}>\mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM}>\mathrm{im}_2|m,r,\theta_2) = 0 \ \ \forall(m,r)$. This is the case if the GMPE has a structure of the type in equation (5), in which there is a term $g(m,r)$ that depends on magnitude and distance (and possibly other parameters), θ is a coefficient depending on soil conditions, and ε is the residual; that is, a zero-mean Gaussian random variable (RV) with standard deviation σ (the residual may possibly be split in different components such as inter- and intraevent, yet it does not affect the results discussed here):

$$\log(\mathrm{IM}) = g(m,r) + \theta + \varepsilon = \mu_{m,r} + \theta + \varepsilon. \quad (5)$$

In fact, the GMPE provides the distribution of the $\log(\mathrm{IM})$ RV, conditional on (m,r,θ), which in this case is a Gaussian RV with mean $\mu_{m,r} + \theta$ and standard deviation σ. Thus, as mentioned, the GMPE allows retrieval of $P(\mathrm{IM} > \mathrm{im}|m,r,\theta)$ needed in the hazard integral. In the case of equation (5), θ only affects the mean of the distribution of $\log(\mathrm{IM})$, whereas it does not affect the standard deviation σ.

At this point, it is worth recalling that this situation is frequent. It is often assumed in GMPEs that the soil changes the mean of the model, not the residual, which in turn may depend on other covariates such as magnitude (to follow). This may be verified via the comprehensive study of Douglas (2014) and the related online repository (see Data and Resources), which proves that GMPEs of the type in equation (5) are, to date, the majority. (See also Bazzurro and Cornell, 2004, and Goulet *et al.*, 2007, for discussions on how variability of ground-motion models can be adjusted for site conditions.)

Consequent to the postulated structure of the GMPE, it results, for each M and R pair, $P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) = 0$. To recognize this, let us first write the difference under investigation as in equation (6), in which $\Phi(\cdot)$ is the Gauss function, and it is assumed, for simplicity of notation, that $\theta_1 = 0$ and $\theta_2 = \theta$.

$$\begin{aligned} &P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) \\ &\quad = 1 - \Phi\left[\frac{\log(\mathrm{im}_1) - \mu_{m,r}}{\sigma}\right] \\ &\quad - \left\{1 - \Phi\left[\frac{\log(\mathrm{im}_2) - (\mu_{m,r} + \theta)}{\sigma}\right]\right\} \\ &\quad = \Phi\left[\frac{\log(\mathrm{im}_2) - (\mu_{m,r} + \theta)}{\sigma}\right] - \Phi\left[\frac{\log(\mathrm{im}_1) - \mu_{m,r}}{\sigma}\right] \\ &\quad = \Phi\left\{\frac{[\log(\mathrm{im}_2) - \theta] - \mu_{m,r}}{\sigma}\right\} - \Phi\left[\frac{\log(\mathrm{im}_1) - \mu_{m,r}}{\sigma}\right]. \end{aligned} \quad (6)$$

Equation (6) suggests that the same Gaussian distribution may be used to compute both $P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1)$ and $P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)$. Therefore, because the Gauss function is strictly monotonic (e.g., Benjamin and Cornell, 1970), the order of $\log(\mathrm{im}_1)$ and $\log(\mathrm{im}_2) - \theta$ reflects on the order of the corresponding probabilities (see equation 7 and Fig. 1a).

$$\begin{cases} \log(\mathrm{im}_1) > \log(\mathrm{im}_2) - \theta \Rightarrow P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) < P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) & \forall(m,r) \\ \log(\mathrm{im}_1) \le \log(\mathrm{im}_2) - \theta \Rightarrow P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) \ge P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) & \forall(m,r) \end{cases} . \quad (7)$$

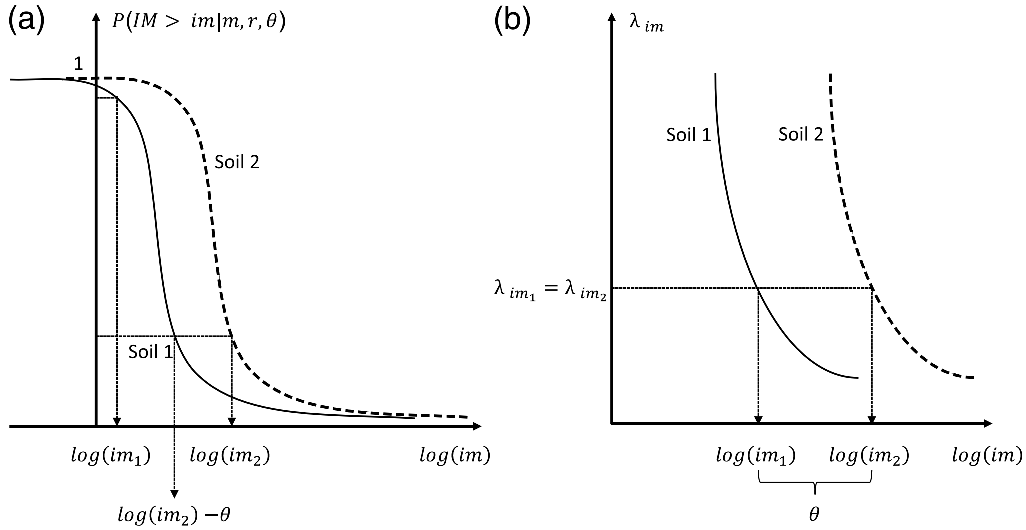

Figure 1. (a) Monotonic nature of the Gaussian complementary cumulative distribution function and order of probabilities; and (b) single-model hazard curves for two soil classes at the same site. (Figure not to scale.)

As a consequence, if the probabilities in equation (7) were different, then the $P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)$ terms in equation (4) would have all the same sign. This prevents equation (4) to be null, thus equation (8) necessarily holds:

$$P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) = P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) \quad \forall(m,r). \tag{8}$$

The direct consequence is that disaggregations on the two soils coincide. Indeed, as it may be argued from equation (9), the disaggregation equations for the two sites have the same denominator by hypothesis, the same $f_{M,R}(m,r)$ and ν terms, and finally equation (8) demonstrated equality of the last term:

$$\begin{aligned} f_{M,R|\mathrm{IM}>\mathrm{im}_1,\theta_1}(m,r) &= \frac{\nu \cdot P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) \cdot f_{M,R}(m,r)}{\lambda_{\mathrm{im}_1}} \\ &= \frac{\nu \cdot P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) \cdot f_{M,R}(m,r)}{\lambda_{\mathrm{im}_2}} \\ &= f_{M,R|\mathrm{IM}>\mathrm{im}_2,\theta_2}(m,r). \end{aligned} \tag{9}$$

Multiple Seismic Sources

The results obtained in this section hold also in the case of multiples sources, if a single GMPE of the type in equation (5) is employed in the hazard analysis. Indeed, equation (10) applies, in which the j subscript refers to the generic source among those considered, which are k in number.

$$\begin{aligned} \lambda_{\mathrm{im}_1} - \lambda_{\mathrm{im}_2} &= \sum_{j=1}^{k} \nu_j \cdot \iint_{m,r} [P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)] \cdot f_{M,R,j}(m,r) \cdot dr \cdot dm \\ &= \iint_{m,r} [P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2)] \cdot \left[\sum_{j=1}^{k} \nu_j \cdot f_{M,R,j}(m,r)\right] \cdot dr \cdot dm = 0. \end{aligned} \tag{10}$$

At this point, the same reasoning, starting from equation (4), can be applied to get the same result of equation (9).

Magnitude-Dependent GMPE Residuals

To recognize that the illustrated result holds also in the case of magnitude-dependent standard deviation of the residuals, indicated by σ_m, it is sufficient to rewrite equation (6) as equation (11):

$$\begin{aligned} &P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_1) - P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_2) \\ &\quad = \Phi\left\{\frac{[\log(\mathrm{im}_2) - \theta] - \mu_{m,r}}{\sigma_m}\right\} - \Phi\left[\frac{\log(\mathrm{im}_1) - \mu_{m,r}}{\sigma_m}\right]. \end{aligned} \tag{11}$$

This equation shows, once again, that if σ_m is the same for the two soil conditions, then the same Gaussian function may be used to retrieve the exceedance probabilities of the two IM thresholds. Then, equation (7) holds for any magnitude, meaning that the order of probabilities depends on the order of the thresholds, and not on the standard deviations, provided that these do not change with the soil class. This proves that equation (8) is also retained, and therefore all conclusions following it.

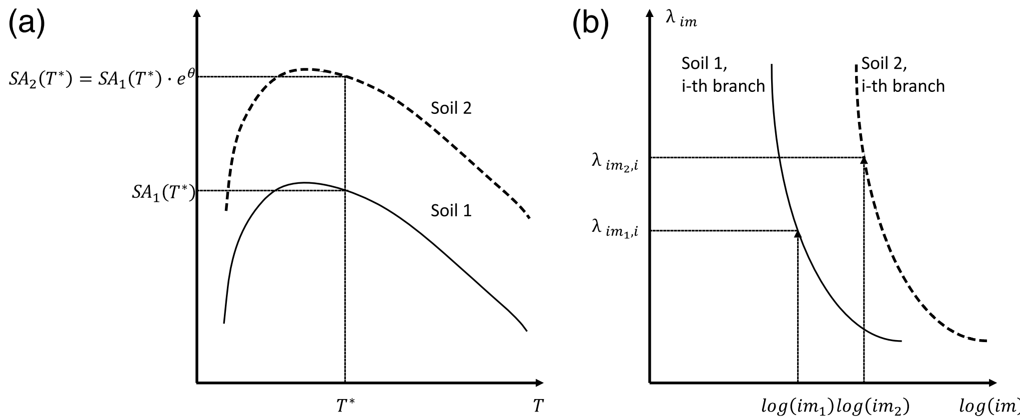

Figure 2. (a) Relationship between uniform hazard spectra (UHS) for two different soil conditions; and (b) rates of exceedance of disaggregated thresholds in one branch of a logic tree.

Single-Model Hazard Curves and Uniform Hazard Spectra

It is now worthwhile to focus on an interesting implication of the results of the previous section. In fact, given two hazard curves for the same site, yet for different soil conditions, if the abscissa is represented in the logarithmic scale, for each rate of exceedance of im, the corresponding values of intensity from the two curves are separated by a factor θ (assuming again that $\theta_1 = 0$ and $\theta_2 = \theta$). In other words, the two curves are, actually, the same curve, just horizontally moved by the soil coefficient of the GMPE (Fig. 1b).

This is immediately consequent from equation (8) because it was shown with equation (6) that the thresholds for the two soils im_1 and im_2, corresponding to the same rate of exceedance, are the same percentile of the IM distribution from the GMPE, then their distance in logarithmic scale is θ. Consequently, because the $f_{M,R}(m,r)$ terms in the hazard integrals are the same for the two soils, such a difference holds in the resulting hazard curves:

$$\log(\mathrm{im}_2) = \log(\mathrm{im}_1) + \theta \Rightarrow \lambda_{\mathrm{im}_2} = \lambda_{\mathrm{im}_1}. \tag{12}$$

This has important earthquake engineering implications, beyond the possibility of quickly retrieving soil hazard from rock or stiff-soil hazard. For example, if the IM is the spectral acceleration $\mathrm{SA}(T)$ and the factor θ is constant across a range of oscillation period periods T, then, given a return period (Tr), the absolute value of the difference, in logarithmic scale, of the UHS for the two site conditions is θ, whereas the ratio in linear scale is equal to e^{θ}. This means that one UHS can be readily retrieved from the other one (Fig. 2a). It is more frequent that in GMPEs the soil coefficient θ changes with the spectral ordinate. In this case, the e^{θ} ratio still applies for every specific spectral ordinate of the UHS.

The Case of Logic Tree

In the case of hazard assessment based on logic tree (e.g., Bommer *et al.*, 2005), n models of the type in equation (1) are employed, and the rate of exceedance of an IM threshold is provided by equation (13), in which $\lambda_{\mathrm{im},i}$ is the rate of exceedance according to the ith branch and P_i is its nonnegative weight, such that $\sum_{i=1}^{n} P_i = 1$:

$$\lambda_{\mathrm{im}} = \sum_{i=1}^{n} \lambda_{\mathrm{im},i} \cdot P_i. \tag{13}$$

Considering now, again, the case of two soil conditions for the same site, and the IM values corresponding to the same rate of exceedance, equation (14) holds, in analogy with equation (4). In equation (14), the i subscript indicates the GMPE and source model of the ith branch of the logic tree:

$$\begin{aligned}\lambda_{\mathrm{im}_1} - \lambda_{\mathrm{im}_2} &= \sum_{i=1}^{n} \lambda_{\mathrm{im}_1,i} \cdot P_i - \sum_{i=1}^{n} \lambda_{\mathrm{im}_2,i} \cdot P_i = 0 \\ &\Rightarrow \sum_{i=1}^{n} (\lambda_{\mathrm{im}_1,i} - \lambda_{\mathrm{im}_2,i}) \cdot P_i \\ &= \sum_{i=1}^{n} \Big\{ \nu_i \cdot \iint_{m,r} [P(\mathrm{IM} > \mathrm{im}_1 | m,r,\theta_{1,i}) \\ &\quad - P(\mathrm{IM} > \mathrm{im}_2 | m,r,\theta_{2,i})] \cdot f_{M,R,i}(m,r) \cdot dr \cdot dm \Big\} \\ &\quad \cdot P_i = 0. \end{aligned} \tag{14}$$

Nevertheless, it is not granted that each $(\lambda_{\mathrm{im}_1,i} - \lambda_{\mathrm{im}_2,i})$ is null. In fact, the thresholds $(\mathrm{im}_1, \mathrm{im}_2)$ corresponding to the total rates $(\lambda_{\mathrm{im}_1}, \lambda_{\mathrm{im}_2})$, in general, correspond to different rates for the two soils in each branch of the logic tree (Fig. 2b). As a consequence, $(\lambda_{\mathrm{im}_1,i} - \lambda_{\mathrm{im}_2,i})$ in equation (14) may have different signs for different values of i. This does not warrant $P(\mathrm{IM} > \mathrm{im}_1 | m,r,\theta_{1,i}) = P(\mathrm{IM} > \mathrm{im}_2 | m,r,\theta_{2,i}) \quad \forall (m,r,i)$. It cannot be concluded that disaggregation is necessarily retained when changing the site class, even if all GMPEs employed in the logic tree are of the form of equation (5), as it can be argued from the following equation:

$$f_{M,R|\mathrm{IM}>\mathrm{im}_1,\theta_1}(m,r) = \frac{\sum_{i=1}^{n} \nu_i \cdot P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_{1,i}) \cdot f_{M,R,i}(m,r) \cdot P_i}{\lambda_{\mathrm{im}_1}} \neq \frac{\sum_{i=1}^{n} \nu_i \cdot P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_{2,i}) \cdot f_{M,R,i}(m,r) \cdot P_i}{\lambda_{\mathrm{im}_2}} = f_{M,R|\mathrm{IM}>\mathrm{im}_2,\theta_2}(m,r). \quad (15)$$

However, there is a case in which disaggregation is preserved between two soil conditions even if logic tree is employed. It happens when all GMPEs in the logic tree have the same soil coefficient. Indeed, assuming for simplicity $\theta_{1,i} = 0\ \forall i$ and $\theta_{2,i} = \theta\ \forall i$, then the two thresholds im_1 and im_2, in which $\log(\mathrm{im}_2) = \log(\mathrm{im}_1) + \theta$, have the same probability of exceedance $P(\mathrm{IM} > \mathrm{im}_1|m,r,\theta_{1,i} = 0) = P(\mathrm{IM} > \mathrm{im}_2|m,r,\theta_{2,i} = \theta)$ on the first and second soil condition according to each GMPE. Then, they also have the same rate of exceedance according to the whole logic tree, exactly as in equation (12). This result also implies that the more the soil coefficients are similar among the GMPEs, the more disaggregations tend to be similar between the two soils (see also the Illustrative Cases section).

Finally, when the logic tree is used, it may be worth mentioning that a useful consequence for hazard analyses also applies if the soil coefficients of GMPEs are different. In fact, because the results of the section referring to the single-model disaggregation hold for each of the branches of the logic tree, the hazard curves for a soil condition may be computed first by moving, via the factor θ_i, each curve computed for another soil condition in the same branch of the logic tree. Then each of these curves may be weighted by P_i and the total rates of exceedance for the target soil may be readily obtained:

$$\log(\mathrm{im}_{2,i}) = \log(\mathrm{im}_{1,i}) + \theta_i \Rightarrow \lambda_{\mathrm{im}_2} = \sum_{i=1}^{n} \lambda_{\mathrm{im}_2,i} \cdot P_i = \sum_{i=1}^{n} \lambda_{\mathrm{im}_1,i} \cdot P_i. \quad (16)$$

Illustrative Cases

All the results derived in this study are analytical proofs and they do not require any validation. On the other hand, it may be beneficial for the reader to develop real case studies that help to pin the main conclusions and to acknowledge that they apply in several practical applications. To this goal, a site in the district of Naples (southern Italy) is considered. Its location is shown in Figure 3a, along with the seismic source zones that surround it according to the model of Meletti *et al.* (2008), which lies at the basis of the Italian seismic hazard map (Stucchi *et al.*, 2011). These zones are used to compute the hazard for the site in question in three cases. All of them are carried out using, for the zones, the parameters of the Gutenberg–Richter relationships (Gutenberg and Richter, 1944) provided in Barani *et al.* (2009, 2010). Earthquake locations are assumed to be uniformly distributed within the zones. Hazard is expressed in terms of rate of exceedance of IMs corresponding to nine return periods.

Single-Model Hazard Case

The first case is that of a single-model hazard (equation 1) featuring a GMPE model of the type in equation (5), which as discussed represents the majority of prediction equations nowadays. In the first example, the peak ground acceleration (PGA) hazard is computed with the model of Bommer *et al.* (2012) extending the Akkar and Bommer (2010) GMPE. Two soil conditions are considered: (1) rock and (2) soft soil. The θ_1 and θ_2 coefficients for this GMPE are 0 and 0.08320 for rock and soft soil, respectively. Figure 3c shows the resulting hazard curves, whereas Figure 3d reports the difference in terms of logarithms of im for any return period. It is evident that the soil curve is distant from the rock curve by a term equal to the soil coefficient, as equation (12) indicates. This necessarily implies conservation of disaggregation between the soil conditions for any return period (equation 9). The disaggregation, which indeed coincides for the two soil conditions, is reported in Figure 3b. In particular, the 475 yr hazard disaggregation is represented in the figure (the reader interested in the relationship between the shape of disaggregation distributions and the source model for Italy can refer to Iervolino *et al.*, 2011, for further discussion).

Standard Deviation of Residuals Changing with Soil Conditions

The second example is to illustrate that, contrary to the first case, if the standard deviation of the residuals changes with soil conditions, that is, the GMPE is not of the type in equation (5), disaggregation is not retained between two different soil types. To this aim, the Bommer *et al.* (2012) GMPE is fictitiously forced to have a 20% increased standard deviation of the residuals for soft soil with respect to the rock case (this artifact was necessary because the author could not find a GMPE in which residuals change with the soil condition). In fact, the original standard deviation of the residuals used for rock is equal to 0.28, whereas it is increased to 0.34 for the soft soil case. The rest of this example is the same as the previous one.

Figure 4 shows the disaggregation of 2475 yr PGA hazard, where it is apparent that, if the soil condition implies a change in the standard deviation of the GMPEs residuals, disaggregation is not retained passing from one soil condition to another.

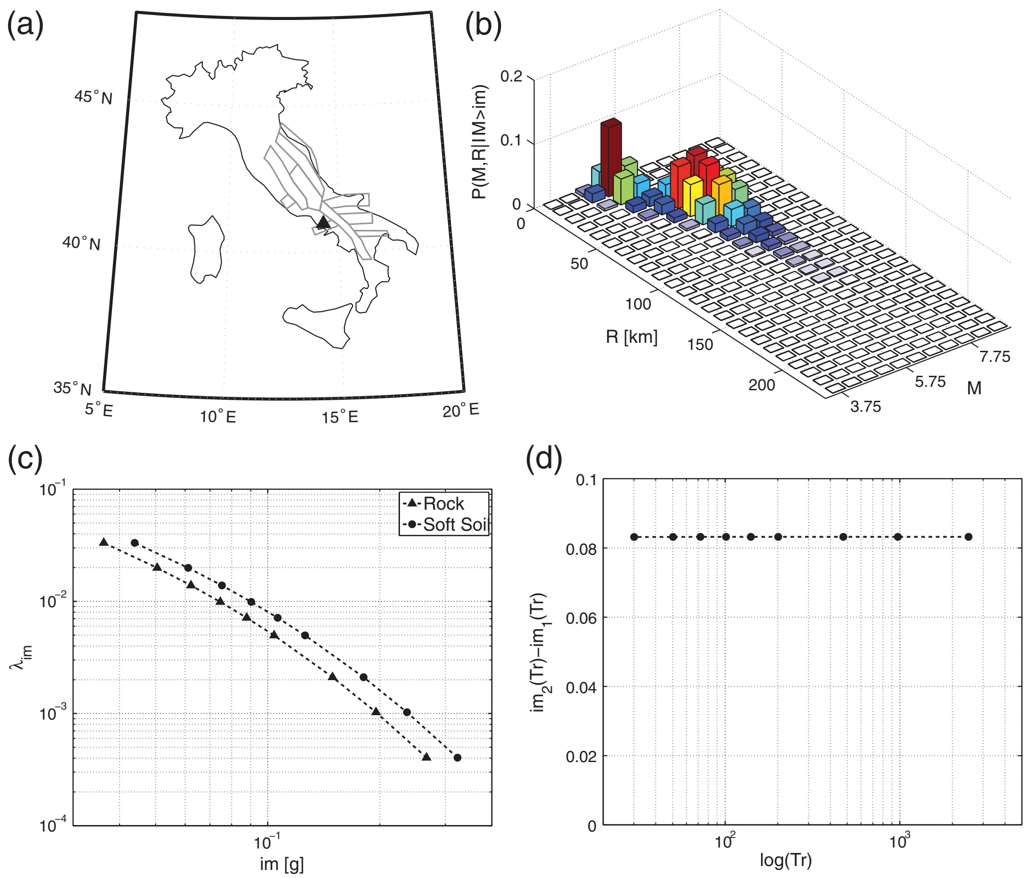

Figure 3. (a) Considered site (triangle) and seismic source zones of Meletti *et al.* (2008) within 200 km; (b) disaggregation in terms of magnitude and distance of the hazard for the peak ground acceleration (PGA) with return period of 475 yr; (c) PGA hazard curves for rock and soft soil conditions using the Bommer *et al.* (2012) ground-motion prediction equation (GMPE); and (d) difference of the hazard curves in terms of PGA given the return period.

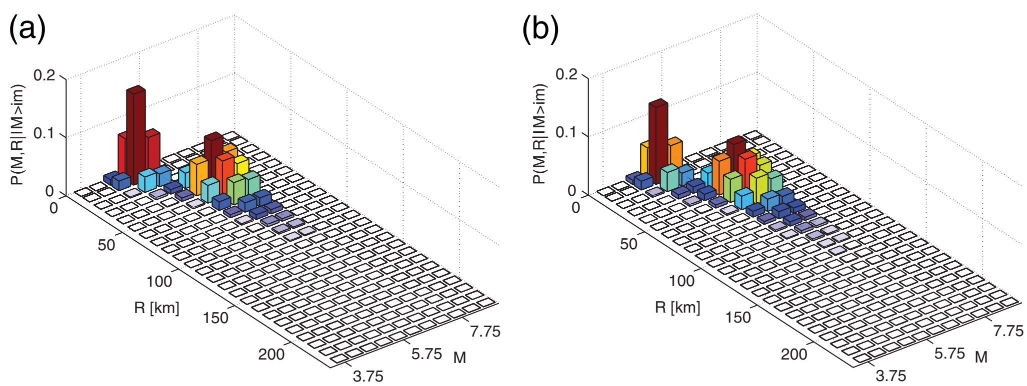

Figure 4. Disaggregation for the PGA hazard with 2475 yr return period: (a) rock versus (b) soft soil fictitiously increasing the standard deviation of the residuals by 20% with respect to rock.

Logic Tree Case

The final example refers to the logic tree and illustrates that, as proven, even if all GMPEs employed are of the type in equation (5), disaggregation is not preserved among soil classes; however, if the site coefficients are similar among GMPEs, then disaggregation tends to be similar. For the purposes of the example, the Bommer *et al.* (2012) and the Ambraseys *et al.* (1996) GMPEs are considered, with weights both equal to 1/2. Figure 5a shows the PGA hazard curves computed for the rock condition and for the soft soil condition (the soil coefficients in the Ambraseys *et al.*, 1996, GMPE for rock and soft soil are 0 and 0.124, respectively).

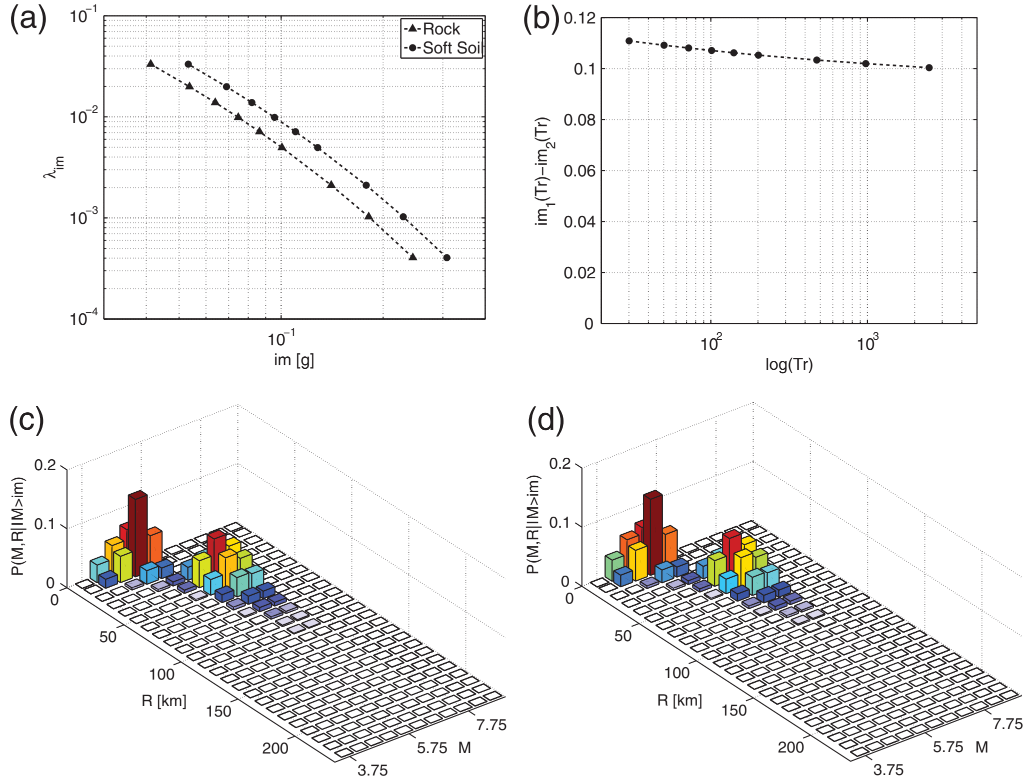

Figure 5. (a) Logic tree PGA hazard curves using Ambraseys *et al.* (1996) and Bommer *et al.* (2012) GMPEs; (b) difference of the hazard curves in terms of PGA given the return period; (c) disaggregation of the 2475 yr return period rock hazard; and (d) disaggregation of the 2475 yr return period soft soil hazard.

These two models are both of the type in equation (5), yet, as it may be argued from Figure 5b, the difference between im thresholds for any return period is not immediately related to the soil coefficients, as in the case of the single-model hazard. However, coefficients for soft soil in the two GMPEs are similar, both being around 0.1. Therefore, in the case of logic tree, disaggregations for rock and soft soil hazards are not the same; however, they should be similar, as discussed. Indeed, Figure 5c,d shows the disaggregations of the 2475 yr hazard for the two soil conditions, which closely resemble each other, yet are not perfectly identical.

Summary

This short note recalled some implications of the structure of GMPEs on the relationship of hazard curves and consequent disaggregation results, when two different soil classes at the same site are considered. The hypothesis, not infrequent, is that the soil class affects only the mean in the GMPE, yet leaves unchanged the standard deviation of the residuals. In particular, it was shown that:

1. in the case of a single-model hazard (often used to approximate a full logic tree):
   (a) hazard disaggregation for the two soil conditions is the same for a given annual rate being disaggregated;
   (b) the two hazard curves are factually the same curve moved by the factor that in the GMPE differentiates the two soil classes;
   (c) results (a) and (b) hold even in the case that the standard deviation of the residuals is a function of magnitude, as it could happen;
2. in the case of a logic tree:
   (d) hazard disaggregation is not expected to be invariant with the site class even if the GMPEs are all of the type postulated, except if the site coefficient is the same for all of them (which implies similarity for close coefficients);
   (e) for each branch, point (b) above holds, then the hazard curves for a soil class may be retrieved quickly by moving each curve computed in the logic tree for another site class by a factor depending on the GMPE of the branch, and then weighting the results via the weights of the branches.

These simple yet analytically rigorous results were finally illustrated using recent GMPEs. This was to show that they may be helpful for earthquake engineering applications where one is often in need of evaluating hazard curves and design earthquakes from disaggregation for a specific site class starting from a reference soil condition hazard.

Data and Resources

All resources of this short note came from the listed references. The ground-motion prediction equation (GMPE) repository of Douglas (2014) may be found at http://www.gmpe.org.uk/ (last accessed February 2016).

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