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Generalized Earthquake Counting Processes for Sequence-Based Hazard

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Abstract Sequence-based probabilistic seismic hazard analysis (SPSHA) allows us to account for the effect of aftershocks in the assessment of seismic structural-design actions (Iervolino *et al.*, 2014, 2018). In fact, it generalizes classical probabilistic seismic hazard analysis (PSHA; Cornell, 1968), combining it with aftershock-PSHA (Yeo and Cornell, 2009). SPSHA associates in time aftershocks to mainshocks and, therefore, retains a desirable property of classical PSHA; that is, events (earthquakes in PSHA and mainshock–aftershock sequences in SPSHA) occur according to homogeneous Poisson processes (HPPs). Nevertheless, the number of earthquakes in SPSHA is not Poisson-distributed. This is addressed herein, in which the probability distribution is formulated and discussed for the following random variables: (1) the count of all earthquakes pertaining to sequences originating in any time interval; (2) the count of all earthquakes occurring in any time interval; (3) the count of all earthquakes that cause exceedance of an arbitrary ground-motion intensity threshold at the site of interest, generated by sequences originating in any time interval. An application referring to central Italy is also developed to help the discussion. The three main findings are that: (1) the formulated SPSHA counting processes further generalize PSHA; that is, they degenerate in the corresponding mainshock HPPs, if aftershocks are neglected; (2) to associate the aftershocks to the corresponding mainshocks in time is fit for hazard assessment purposes; and (3) the variance-to-mean ratio of the counting distributions is significantly larger than one; consequently, the occurrence processes cannot be approximated by Poisson processes. These results, which complete the SPSHA framework, can be a reference for model calibration exercises when SPSHA is computed via simulation and in those cases in which the probability of an exact number of exceedances is of interest, rather than that of observing at least one exceedance (e.g., for seismic damage accumulation studies).

Introduction

Probabilistic seismic hazard analysis (PSHA) determines the seismic threat at a site of interest by means of the (annual) rate of exceedance of a ground-motion intensity measure (IM) threshold (Cornell, 1968). In fact, PSHA computes the average number of earthquakes, per unit of time, producing ground motion at the site that exceeds im. For a number of reasons discussed in the following, although it is acknowledged that earthquakes occur in clusters (i.e., they are concentrated in space and time), PSHA is typically formulated referring only to the prominent events in each cluster, the so-called mainshocks. A classical definition of a mainshock is "the largest magnitude earthquake in an identified cluster." Mainshocks are isolated in earthquake catalogs via techniques collectively known as catalog declustering (e.g., Gardner and Knopoff, 1974). Consequently, the calculated exceedance rate neglects the effect of the other events possibly occurring in the cluster, that is, foreshocks and/or aftershocks, that may imply systematic underestimation of seismic hazard.

Several research attempts have been carried out to overcome the issue; among those most recent, it is beneficial to the discussion to recall Boyd (2012) and Marzocchi and Taroni (2014). The latter approach aims at performing PSHA based on a nondeclustered catalog and considering the occurrence of all earthquakes and their effects in analogy to mainshocks. The former, acknowledging that clusters are located in time around the mainshock, aims at including, via Monte Carlo simulation, the effects of foreshocks and aftershocks in each cluster.

Developing further from Boyd (2012), and taking advantage of the aftershock-PSHA (APSHA) developed by Yeo and Cornell (2009), the so-called sequence-based probabilistic seismic hazard analysis (SPSHA) has been introduced (Iervolino *et al.*, 2014). With respect to the mentioned approaches, SPSHA still relies on a declustered catalog and generalizes the PSHA integral in an analytical framework. In fact, the SPSHA integral precisely degenerates in the classical hazard integral if the effect of aftershocks is neglected. It is also mentioned that SPSHA neglects foreshocks, which are, however, generally considered of minor importance with respect to aftershocks from the engineering point of view.

Under classical PSHA hypotheses, earthquakes causing exceedance of any IM threshold at a site of interest follow a homogeneous Poisson process (HPP). Analogously, in SPSHA, mainshock–aftershock sequences also occur according to an HPP. Consequently, the exceedance probability of an IM threshold in a time interval is computed via the exponential distribution, which is the major advantage of SPSHA. Nevertheless, because SPSHA combines PSHA with APSHA, in which the occurrence of aftershocks follows a non-homogeneous Poisson process (NHPP) conditional to the mainshock features, the random variables (RVs) counting earthquakes in time are not Poisson-distributed.

To characterize SPSHA in terms of earthquake-counting processes is the goal of this stochastic modeling study, in which the probability distributions of the following RVs are formulated: (1) the count of all earthquakes generated by sequences originating in any time interval; (2) the count of all earthquakes occurring in any time interval, which also allows us to address how appropriate it is to associate aftershocks to mainshocks in time; and (3) the count of all earthquakes generated by sequences originating in any time interval and that cause exceedance of an arbitrary IM threshold at the site of interest.

The remainder of this article is structured such that the probabilistic essentials of PSHA, APSHA, and SPSHA are briefly recalled first. Subsequently, the counting process of all earthquakes in SPSHA from sequences starting in a given time interval is derived. Then, a more refined formulation is given and compared with the previous one, that is, the count of all earthquakes occurring in a given time interval. Finally, the occurrence process of earthquakes causing exceedance of an IM threshold is addressed. Direct formulas for the mean and variance of the processes are also provided. Illustrative applications, developed for central Italy, allow a discussion with respect to the corresponding counting processes of mainshocks (i.e., Poisson). Final remarks close the study.

PSHA, APSHA, and SPSHA

In the following, the essentials of PSHA, APSHA, and their combination to get SPSHA, are briefly recounted.

PSHA

PSHA provides, for a site of interest, the average number of earthquakes in one year causing exceedance of a given IM threshold, say im. The rate of exceedance of im, indicated as $\lambda_{im,E}$ in equation (1), is typically obtained via the so-called hazard integral, which is written, for simplicity, for the case of a single-seismic source:

$$\lambda_{im,E} = \nu_E \cdot \int_{r_{E,\min}}^{r_{E,\max}} \int_{m_{E,\min}}^{m_{E,\max}} P[IM_E > im|m_E, r_E] \times f_{M_E,R_E}(m_E, r_E) \cdot dm_E \cdot dr_E. \quad (1)$$

In the equation, ν_E is the rate of earthquakes above a minimum magnitude of interest ($m_{E,\min}$) and below the maximum magnitude ($m_{E,\max}$) deemed possible for a considered seismic source, and IM_E indicates the mainshock IM. The term $P[IM_E > im|m_E, r_E]$, usually provided by a ground-motion prediction equation (GMPE), is the probability that the intensity threshold is exceeded given an earthquake of magnitude $M_E = m_E$, from which the site is separated by a distance $R_E = r_E \in (r_{E,\min}, r_{E,\max})$. The function $f_{M_E,R_E}(m_E, r_E)$ is the joint probability distribution of earthquake's magnitude and distance RVs. For each source, these two RVs are usually considered stochastically independent. More specifically, $f_{M_E}(m_E)$ is often described by a truncated exponential distribution, derived by the Gutenberg–Richter (GR) relationship (Gutenberg and Richter, 1944), and $f_{R_E}(r_E)$ is obtained on the basis of the source–site geometrical configuration.

It is useful for the following discussion to recall that the integral in equation (1) is nothing else than the probability that the ground-motion threshold is exceeded, given the occurrence of a generic event, that is, an earthquake of unknown magnitude and distance from the site. In other words, it is the probability of exceeding im conditional to the occurrence of one event, $P[IM_E > im|E]$:

$$P[IM_E > im|E] = \int_{r_{E,\min}}^{r_{E,\max}} \int_{m_{E,\min}}^{m_{E,\max}} P[IM_E > im|m_E, r_E] \times f_{M_E,R_E}(m_E, r_E) \cdot dm_E \cdot dr_E. \quad (2)$$

PSHA basically results in the computation of the rate via the hazard integral because the typical assumption is that the earthquakes of interest, that is, those characterized by the ν_E rate, occur according to an HPP. In other words, as it is well known, the RV counting the number of events (N_E) within an arbitrary time interval $(t, t + \Delta t)$ is Poisson-distributed:

$$P[N_E(t, t + \Delta t) = n] = P[N_E(\Delta t) = n] = \frac{(\nu_E \cdot \Delta t)^n}{n!} \cdot e^{-\nu_E \cdot \Delta t}, \quad \forall n \in \{1, 2, \ldots\}, \quad (3)$$

in which ν_E is the parameter of the model and $\nu_E \cdot \Delta t$ is the mean of the RV. The HPP is an independent and stationary increment process (e.g., Ross, 1996). Roughly speaking, the RVs counting the occurrences in nonoverlapping intervals are independent, and they are also identically distributed for intervals of the same width; that is, $P[N_E(t, t + \Delta t) = n] = P[N_E(\Delta t) = n]$. This also implies that the HPP is a memoryless process; that is, the probability of observing a given number of earthquakes in a future interval is independent of the occurrence history up to the interval's starting time. Finally, if multiple (independent) seismic sources all follow an HPP, then they also globally follow an HPP. In other words, in the case of s sources, equation (3) still applies, and it is characterized by the global rate $\nu_E = \sum_{i=1}^{s} \nu_{E,i}$, in which $\nu_{E,i}$, $i = \{1, 2, \ldots, s\}$, are the rates of the sources.

In fact, it is possible to show that, if the earthquakes occur according to an HPP, the earthquakes causing exceedance of *im* also follow an HPP. In other words, the RV counting the number of earthquakes causing exceedance of the IM threshold at the site, say $N_{E,im}(t, t+\Delta T)$, is distributed according to the following equation:

$$P[N_{E,im}(t,t+\Delta t)=n]=P[N_{E,im}(\Delta t)=n] \\ =\frac{(\lambda_{im,E}\cdot\Delta t)^n}{n!}\cdot e^{-\lambda_{im,E}\cdot\Delta t},\quad \forall n\in\{1,2,\ldots\}, \tag{4}$$

requiring, again, only one parameter, which is precisely the rate computed via the hazard integral in equation (1). In the case of s sources, such a rate is computed one source at a time and the results summed up, that is, $\lambda_{im,E}=\sum_{i=1}^{s}\lambda_{im,E,i}$.

Applicability of equation (4) is highly desirable, because a single time-invariant parameter completely describes the process, which simply provides further results. For example, as it is well known, it follows that the RV measuring the time between two consecutive events causing exceedance of *im* at the site ($T_{E,im}$) is exponentially distributed. In fact, the probability of exceedance of *im* in $(t, t+\Delta t)$ is given by the complement to the probability that no earthquakes cause exceedance:

$$P[T_{E,im}\le\Delta t]=1-P[N_{E,im}(\Delta t)=0]=1-e^{-\lambda_{im,E}\cdot\Delta t}. \tag{5}$$

$$E[N_{A,im|m_E}(t,t+\Delta t)]=E[N_{A|m_E}(t,t+\Delta t)]\cdot\int_{r_{A,\min}}^{r_{A,\max}}\int_{m_{A,\min}}^{m_E}P[IM_A>im|m_A,r_A]\cdot f_{M_A,R_A|M_E,R_E}(m_A,r_A|m_E,r_E)\cdot dm_A\cdot dr_A, \tag{8}$$

In more general terms, the interarrival times of earthquakes are independent and identically (and exponentially) distributed RVs. The, common, mean of these random variables is $(\lambda_{im,E})^{-1}$, that is, the return period of the ground motions causing exceedance at the site.

In summary, if the earthquake catalog, on the basis of which ν_E is computed, allows us to maintain the assumption in equation (3), PSHA is very easy to manage probabilistically. This is a meaningful reason to decluster catalogs, which basically means that one event per identified earthquake cluster (i.e., the mainshock) is retained. Moreover, declustering is also needed for catalog-completeness issues. Many catalogs are still dominated by events the characteristics of which are inferred with the tools of historical seismology and for which foreshock and aftershocks are not represented well enough. On the other hand, probabilistic seismic hazard based on equation (1) inherently neglects the probability that exceedance of *im* is caused by an aftershock (or foreshock) rather than by a mainshock, which is not necessarily negligible (Iervolino *et al.*, 2018).

APSHA

Yeo and Cornell (2009) developed APSHA for short-term risk assessment, that is, to manage the engineering risk after a significant earthquake at the day time scale. APSHA starts from the modified Omori law (Utsu, 1961), in which, given that a mainshock of magnitude $M_E=m_E$ occurred at $t=0$, the daily rate of aftershocks $\nu_{A|m_E}(t)$ is given by the following equation:

$$\nu_{A|m_E}(t)=\frac{10^{a+b\cdot(m_E-m_{\min})}-10^a}{(t+c)^p}, \tag{6}$$

in which $\{a,b,c,p\}$ are parameters; $m_{\min}$ is the minimum considered aftershock magnitude, which cannot be larger than that of the mainshock. Note that equation (6) is the daily rate of aftershocks in the range $(m_{\min}, m_E)$. From $\nu_{A|m_E}(t)$, it follows that the average number of aftershocks in $(t, t+\Delta t)$, say $E[N_{A|m_E}(t,t+\Delta t)]$, is given by the following equation:

$$E[N_{A|m_E}(t,t+\Delta t)]=\int_t^{t+\Delta t}\nu_{A|m_E}(\tau)\cdot d\tau \\ =\frac{10^{a+b\cdot(m_E-m_{\min})}-10^a}{p-1}\cdot[(t+c)^{1-p}-(t+\Delta t+c)^{1-p}]. \tag{7}$$

The average number of aftershocks causing exceedance of the threshold at the site $E[N_{A,im|m_E}(t,t+\Delta t)]$ can be computed via the hazard integral in the following equation:

in which $P[IM_A>im|m_A,r_A]$ is still from a GMPE (IM_A is the aftershock IM), and $f_{M_A,R_A|M_E,R_E}(m_A,r_A|m_E,r_E)$ is the magnitude–distance distribution of the generic aftershock. (Note that it is indicated that this distribution is conditional on the mainshock distance; in fact, it should be, more precisely, mainshock location. Similarly, $E[N_{A,im|m_E}(t,t+\Delta t)]$ is also dependent on mainshock location, even if the symbol only indicates dependence on the mainshock magnitude. The symbols are retained for consistency with previous studies.)

Once again, the integral in equation (8) is the probability that a generic aftershock causes exceedance upon occurrence, which can be indicated as $P[IM_A>im|m_E,r_E]$:

$$P[IM_A>im|m_E,r_E]=\int_{r_{A,\min}}^{r_{A,\max}}\int_{m_{A,\min}}^{m_E}P[IM_A>im|m_A,r_A] \\ \times f_{M_A,R_A|M_E,R_E}(m_A,r_A|m_E,r_E)\cdot dm_A\cdot dr_A. \tag{9}$$

According to APSHA, the occurrence of the aftershocks follows an NHPP, conditional on the features of the mainshock (note that the literature also discusses other processes for aftershock occurrence; e.g., Ogata, 1988; Marsan and Helmstetter, 2017). Under the NHPP hypothesis, for any time interval, the RV counting the aftershocks exceeding the *im* threshold at the site, $N_{A,im|m_E}(t,t+\Delta t)$, is still Poisson-distributed but characterized by a time-variant mean:

$$P[N_{A,im|m_E}(t,t+\Delta t)=n]=\frac{(E[N_{A,im|m_E}(t,t+\Delta t)])^n}{n!}\times e^{-E[N_{A,im|m_E}(t,t+\Delta t)]},\quad \forall n\in\{1,2,\ldots\}. \tag{10}$$

The NHPP is an independent increments process; however, it has nonstationary increments. This means that the RVs counting earthquakes in nonoverlapping time intervals are independent; however, their distribution also depends on when the time interval begins. This is because the mean function $E[N_{A,im|m_E}(t,t+\Delta t)]$ is, owing to the modified Omori law, decreasing as the time elapsed since the mainshock progresses. In the NHPP, the interarrival times are neither exponential nor independent and identically distributed. In fact, the interarrival times tend to become larger as the time since the mainshock passes by, and the average time between the events is not constant. It is finally important to note that, according to APSHA, one tends to observe a smaller maximum magnitude of aftershocks as time increases. This is because the mean of the counting process is decreasing, and then it is less probable for extreme magnitudes to occur.

SPSHA

Assuming that the mainshocks occur according to an HPP, as per classical PSHA, it can be acknowledged that mainshock–aftershock sequences occur according to the same process; and not only that, it must also be that they occur with the same rate of the mainshocks that is ν_E, that is, $1/\nu_E$ is also the average interarrival time of the clusters, as depicted in Figure 1. This practically means to associate the aftershocks to the corresponding mainshocks in time.

Indicating as $IM_{\cup A}$ the largest IM at the site caused by the aftershock sequence, the probability that exceedance is observed, given the occurrence of a sequence (i.e., of a mainshock), can be indicated as $P[IM_E > im \cup IM_{\cup A} > im|E]$, that is, the probability that exceedance is caused by a generic mainshock (of unknown characteristics) or by its aftershocks. Multiplying such a probability by the rate of occurrence of mainshocks allows us to compute the annual rate, say λ_{im}, of mainshock–aftershock sequences exceeding im: $\lambda_{im} = \nu_E \cdot P[IM_E > im \cup IM_{\cup A} > im|E]$. It was demonstrated in Iervolino *et al.* (2014) that, under the hypotheses for aftershock hazard discussed in the previous section, λ_{im} can be computed via equation (11), that is, a generalization of equation (1):

$$\lambda_{im}=\nu_E\cdot\left\{1-\int_{r_{E,\min}}^{r_{E,\max}}\int_{m_{E,\min}}^{m_{E,\max}}P[IM_E\le im|m_E,r_E]\cdot e^{-E[N_{A,im|m_E}(0,\Delta T_A)]}\cdot f_{M_E,R_E}(m_E,r_E)\cdot dm_E\cdot dr_E\right\}, \tag{11}$$

in which ν_E, $P[IM_E \le im|m_E,r_E] = 1 - P[IM_E > im|m_E,r_E]$, $f_{M_E,R_E}(m_E,r_E)$, and the integrals' domain are the same as defined in equation (1). The exponential term is the probability that, within the assumed duration of the sequence $(0,\Delta T_A)$, none of the aftershocks from a mainshock of

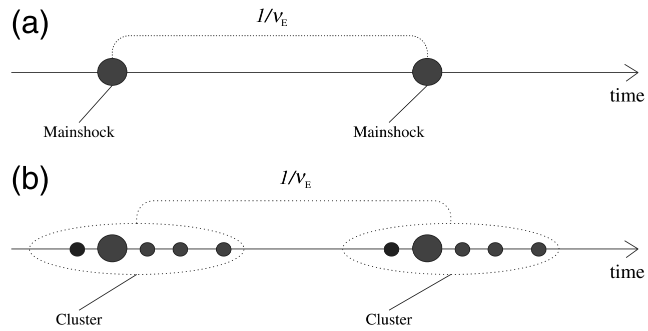

Figure 1. The assumption behind sequence-based probabilistic seismic hazard analysis (SPSHA). Earthquake sequences (b) occur at the same rate of mainshocks (a).

features $\{m_E, r_E\}$ cause exceedance of im. In fact, equation (11) represents a hazard integral for aftershocks (the exponential term) conditional to $M_E = m_E$ and $R_E = r_E$, nested in a classical PSHA integral. It is easy to recognize that it must be $\lambda_{im} \ge \lambda_{im,E}$; that is, accounting for aftershocks increases the hazard. Moreover, equation (11) precisely degenerates in equation (1) in case aftershocks are neglected; that is, $E[N_{A|m_E}(0,\Delta T_A)] = 0,\ \forall m_E \in (m_{E,\min}, m_{E,\max})$. Finally, it should be noted that the term within the braces represents the mentioned probability $P[IM_E > im \cup IM_{\cup A} > im|E]$. (Note that ΔT_A could be, in principle, mainshock-magnitude dependent.)

SPSHA has three main characteristics: (1) it provides a solution for the underestimation issue of seismic hazard because of neglecting aftershocks; (2) it still considers rates from a declustered catalog, avoiding the issues of aftershock completeness; (3) most importantly, λ_{im} is still the rate characterizing an HPP. In other words, the counting process of sequences causing exceedance is still of the type in equation (4), yet it is obtained replacing $\lambda_{im,E}$ with λ_{im}. This is a relevant advantage from the earthquake engineering point of view. In seismic design, one is typically interested in the probability that at least one exceedance of a critical ground-motion IM is observed during a time interval, say Δt, for example, the design-life of the structure. In SPSHA, such a probability can be computed as $1 - e^{-\lambda_{im}\cdot\Delta t}$, because the interarrival time of exceedance-causing sequences is still exponentially distributed, exactly as in PSHA. Thus, design seismic actions can be updated accounting for the aftershock effect within the classical probabilistic framework.

Earthquakes from Seismic Sequences Originating in a Given Time Interval

Having established that sequences able to cause exceedance of im occur according to an HPP, individual earthquakes do not; this is because mainshocks occur according to an HPP, whereas aftershocks occur according to NHPPs, conditional to the mainshock magnitude. In this section, the distribution of the total number of earthquakes generated by mainshock–aftershock seismic sequences occurring in the interval $(t, t+\Delta t)$ is derived. Such an RV is named $N(t, t+\Delta t)$, and to define its distribution the probability $P[N(t, t+\Delta t)=n], \forall n \in \{0,1,2,\ldots\}$ is needed. This distribution is also the distribution of the total number of earthquakes occurring in $(t, t+\Delta t)$, if aftershocks are associated in time to the corresponding mainshocks.

For $n=0$, because possible earthquakes from sequences occurring prior to t are not considered, the probability that no earthquakes are generated at all equals the probability that no mainshocks occur in $(t, t+\Delta t)$; thus $P[N(t, t+\Delta t)=0]=e^{-\nu_E \cdot \Delta t}$. For $n \geq 1$, i mainshocks $(i \leq n)$ and $n-i$ aftershocks could occur. Therefore, applying the total probability theorem, equation (12) results:

$$P[N(t, t+\Delta t)=n]=\sum_{i=1}^{n} P[N_A=n-i | N_E(t, t+\Delta t)=i] \cdot P[N_E(t, t+\Delta t)=i], \tag{12}$$

in which $P[N_E(t, t+\Delta t)=i]$ can be computed according to equation (3). $P[N_A=n-i | N_E(t, t+\Delta t)=i]$, being the probability that the aftershocks are $n-i$ in number, can be computed recalling that, according to APSHA, the counting process of aftershocks is an NHPP conditional to the mainshock's magnitude. Therefore, it is worthwhile to rewrite equation (12) reapplying the total probability theorem and conditioning it to the mainshock magnitudes:

$$\begin{aligned} P[N(t, t+\Delta t)=n]=&\sum_{i=1}^{n} P[N_E(t, t+\Delta t)=i] \cdot \int_{m_{E,1}} \int_{m_{E,2}} \cdots \int_{m_{E,i}} P[N_A=n-i | m_{E,1}, m_{E,2}, \ldots, m_{E,i}] \\ &\times f_{M_{E,1}, M_{E,2}, \ldots, M_{E,i}}(m_{E,1}, m_{E,2}, \ldots, m_{E,i}) \cdot dm_{E,1} \cdot dm_{E,2} \ldots dm_{E,i}. \end{aligned} \tag{13}$$

in which $\{m_{E,1}, m_{E,2}, \ldots, m_{E,i}\}$ are the magnitudes of the mainshocks, $f_{M_{E,1}, M_{E,2}, \ldots, M_{E,i}}$ is their joint distribution, and $P[N_A=n-i | m_{E,1}, m_{E,2}, \ldots, m_{E,i}]$ is the probability that $n-i$ aftershocks are generated by mainshocks of given magnitudes. In the equation, each integral over the mainshock magnitude is intended to be extended on $(m_{E,\min}, m_{E,\max})$.

Regarding the joint magnitude distribution, according to classical PSHA, mainshock magnitudes are independent and identically distributed (i.i.d.) RVs, then

$$f_{M_{E,1}, M_{E,2}, \ldots, M_{E,i}}(m_{E,1}, m_{E,2}, \ldots, m_{E,i}) = \prod_{j=1}^{i} f_{M_E}(m_{E,j}). \tag{14}$$

In particular, if the mainshock magnitude follows a GR relationship, then $f_{M_E}(m_E)=\beta \cdot e^{-\beta \cdot m_E} \cdot (e^{-\beta \cdot m_{\min}}-e^{-\beta \cdot m_{\max}})^{-1}$, in which β is a parameter. This hypothesis, although not strictly needed for what follows, will be retained because it is a common case, thus

$$f_{M_{E,1}, M_{E,2}, \ldots, M_{E,i}}(m_{E,1}, m_{E,2}, \ldots, m_{E,i})=\frac{\beta^i \cdot e^{-\beta \cdot \sum_{j=1}^{i} m_{E,j}}}{(e^{-\beta \cdot m_{\min}}-e^{-\beta \cdot m_{\max}})^i}. \tag{15}$$

Regarding $P[N_A=n-i | m_{E,1}, m_{E,2}, \ldots, m_{E,i}]$, according to APSHA, the number of aftershocks following a mainshock of given magnitude is Poisson distributed with mean from equation (7). Moreover, given the mainshock magnitudes, the counts of aftershocks pertaining to different sequences are independent RVs. Then, N_A is the sum of independent Poisson RVs, which is still a Poisson RV with the mean equal to the sum of the means of all sequences:

$$P[N_A=n-i | m_{E,1}, m_{E,2}, \ldots, m_{E,i}] = \frac{\left\{\sum_{j=1}^{i} E[N_{A|m_{E,j}}(0, \Delta T_A)]\right\}^{n-i}}{(n-i)!} \cdot e^{-\sum_{j=1}^{i} E[N_{A|m_{E,j}}(0, \Delta T_A)]}. \tag{16}$$

Therefore, equation (13) can be rewritten plugging in equations (3), (15), and (16). After that, equation (7) is used to express $E[N_{A|m_{E,j}}(0, \Delta T_A)]$ in it, that is:

$$\begin{aligned} P[N(t, t+\Delta t)=n]=&\sum_{i=1}^{n} \frac{(\nu_E \cdot \Delta t)^i}{i!} \cdot e^{-\nu_E \cdot \Delta t} \cdot \int_{m_{E,1}} \int_{m_{E,2}} \cdots \int_{m_{E,i}} \frac{\left\{\sum_{j=1}^{i} \frac{10^{a+b \cdot (m_{E,j}-m_{\min})}-10^a}{p-1} \cdot [c^{1-p}-(\Delta T_A+c)^{1-p}]\right\}^{n-i}}{(n-i)!} \\ &\times e^{-\sum_{j=1}^{i} \frac{10^{a+b \cdot (m_{E,j}-m_{\min})}-10^a}{p-1} \cdot [c^{1-p}-(\Delta T_A+c)^{1-p}]} \cdot \frac{\beta^i \cdot e^{-\beta \cdot \sum_{j=1}^{i} m_{E,j}}}{(e^{-\beta \cdot m_{\min}}-e^{-\beta \cdot m_{\max}})^i} \cdot dm_{E,1} \cdot dm_{E,2} \ldots dm_{E,i}. \end{aligned} \tag{17}$$

In fact, it is possible to write a single equation working for any $n \geq 0$. It is given in equation (18), in which $I_{\{n=0\}}$ is an indicator function that equals one if the condition in the subscript is met and zero otherwise:

$$P[N(t,t+\Delta t)=n]=$$
$$I_{\{n=0\}} \cdot e^{-\nu_E \cdot \Delta t} + (1-I_{\{n=0\}}) \cdot \sum_{i=1}^{n} \frac{(\nu_E \cdot \Delta t)^i}{i!} \cdot e^{-\nu_E \cdot \Delta t} \cdot \int_{m_{E,1}} \int_{m_{E,2}} \ldots \int_{m_{E,i}} \frac{\left\{\sum_{j=1}^{i} \frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1} \cdot [c^{1-p}-(\Delta T_A+c)^{1-p}]\right\}^{n-i}}{(n-i)!} \times$$
$$e^{-\sum_{j=1}^{i} \frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1} \cdot [c^{1-p}-(\Delta T_A+c)^{1-p}]} \cdot \frac{\beta^i \cdot e^{-\beta \cdot \sum_{j=1}^{i} m_{E,j}}}{(e^{-\beta \cdot m_{\min}}-e^{-\beta \cdot m_{\max}})^i} \cdot dm_{E,1} \cdot dm_{E,2} \ldots dm_{E,i}. \tag{18}$$

From this latter equation, it appears that, if aftershocks are disregarded, then the counting process of earthquakes degenerates in the HPP of mainshocks in equation (3). This is because if $E[N_{A|m_{E,j}}(0,\Delta T_A)]=0\ \forall m_{E,j}$, then $P[N_A=n-i|m_{E,1},m_{E,2},\ldots,m_{E,i}]$ equals one for $i=n$ and zero otherwise; thus equation (13) reduces to $P[N(t,t+\Delta t)=n]=P[N_E(t,t+\Delta t)=n]$.

Mean and Variance of the Number of Earthquakes

It is interesting to directly determine the average number of earthquakes produced by sequences originating in the $(t,t+\Delta t)$ interval, that is, $E[N(t,t+\Delta t)]$. It can be computed easily as the mean number of mainshocks plus the average number of aftershocks per sequence. Because the latter is known conditional to the mainshock magnitude, it is worthwhile to compute such a mean using the conditional-expectation theorem. This is given in equation (19), in which all the symbols have been defined already:

$$\begin{aligned} E[N(t,t+\Delta t)] &= \nu_E \cdot \Delta t \cdot \left\{1+\int_{m_{E,\min}}^{m_{E,\max}} E[N_{A|m_E}(0,\Delta T_A)] \cdot f_{M_E}(m_E) \cdot dm_E\right\} \\ &= \nu_E \cdot \Delta t \cdot E[N|E]. \end{aligned} \tag{19}$$

Note that $E[N(t,t+\Delta t)]$ only depends on Δt. Moreover, the term in the braces represents the expected number of earthquakes in the sequence, given the occurrence of a mainshock of any magnitude $E[N|E]$. (The E symbol after the conditioning bar, which indicates the mainshock occurrence, must not be confused with $E[\cdot]$, which indicates the expected value.)

The variance of the total number of earthquakes can also be computed. The first step is to compute the variance of the number of earthquakes, given the occurrence of a mainshock of any magnitude or $\mathrm{VAR}[N|E]$:

$$\begin{aligned} \mathrm{VAR}[N|E] &= \mathrm{VAR}[E[N|m_E]] + E[\mathrm{VAR}[N|m_E]] \\ &= \int_{m_{E,\min}}^{m_{E,\max}} \{1+E[N_{A|m_E}(0,\Delta T_A)]-E[N|E]\}^2 \cdot f_{M_E}(m_E) \cdot dm_E \\ &+ \int_{m_{E,\min}}^{m_{E,\max}} E[N_{A|m_E}(0,\Delta T_A)] \cdot f_{M_E}(m_E) \cdot dm_E. \end{aligned} \tag{20}$$

The equation, which is an application of the law of total variance, reflects the fact that the distribution of the number of aftershocks is Poisson conditional to the mainshock magnitude and that, in the Poisson distribution, the mean equals the variance. At this point, to compute the variance in $(t,t+\Delta t)$, the law of total variance can be applied again, considering that when i sequences occur in the interval, the mean and variance of the number of events are equal to $i \cdot E[N|E]$ and $i \cdot \mathrm{VAR}[N|E]$, respectively. Then, because an infinite number of sequences can possibly occur in the interval:

$$\mathrm{VAR}[N(t,t+\Delta t)] = \mathrm{VAR}[i \cdot E[N|E]] + E[i \cdot \mathrm{VAR}[N|E]] =$$
$$\{E[N|E]\}^2 \cdot \sum_{i=0}^{+\infty} i^2 \cdot \frac{(\nu_E \cdot \Delta t)^i}{i!} \cdot e^{-\nu_E \cdot \Delta t} - \left\{E[N|E] \cdot \sum_{i=0}^{+\infty} i \cdot \frac{(\nu_E \cdot \Delta t)^i}{i!} \cdot e^{-\nu_E \cdot \Delta t}\right\}^2 + \mathrm{VAR}[N|E] \cdot \sum_{i=0}^{+\infty} i \cdot \frac{(\nu_E \cdot \Delta t)^i}{i!} \cdot e^{-\nu_E \cdot \Delta t} =$$
$$\nu_E \cdot \Delta t \cdot (\{E[N|E]\}^2 + \mathrm{VAR}[N|E]). \tag{21}$$

Finally, a formulation for the mean of earthquakes within the (m_1,m_2) magnitude bin, $E[N_{m\in(m_1,m_2)}(t,t+\Delta t)]$, can also be obtained. It is given by the number of mainshocks with magnitudes within (m_1,m_2) plus the number of aftershocks with magnitudes within (m_1,m_2), $E[N_{A,m\in(m_1,m_2)}|m_E]$, for each of the possible mainshock magnitudes that can occur, weighed by its probability:

$$\begin{aligned} E[N_{m\in(m_1,m_2)}(t,t+\Delta t)] = \nu_E \cdot \Delta t \cdot \Bigg\{ & \int_{m_1}^{m_2} f_{M_E}(m_E) \cdot dm_E \\ & + \int_{m_{E,\min}}^{m_{E,\max}} E[N_{A,m\in(m_1,m_2)}|m_E] \cdot f_{M_E}(m_E) \cdot dm_E \Bigg\}. \end{aligned} \tag{22}$$

The $E[N_{A,m\in(m_1,m_2)}|m_E]$ term can be obtained from equation (7), in the way described in Yeo and Cornell (2009).

Multiple Sources

In the case of multiple seismic sources, say s in number, if ν_E is the sum of mainshock rates for all the seismic sources, and the magnitude distribution is the mixture of all magnitude distributions for all the sources:

$$\begin{cases} \nu_E = \sum_{i=1}^{s} \nu_{E,i} \\ f_{M_E}(m_E) = \sum_{i=1}^{s} f_{M_{E,i}}(m_E) \cdot \frac{\nu_{E,i}}{\nu_E} \end{cases}, \tag{23}$$

then equation (18) still applies if the modified Omori law parameters are common to all the sources. (In equation 23, $\nu_{E,i}/\nu_E$ is the probability that if a mainshock occurs, it has been generated by the ith source.)

Earthquakes Occurring in a Given Time Interval

The subject of the previous section was the distribution of earthquakes belonging to sequences originating in $(t, t+\Delta t)$. This section targets the distribution of the number of all earthquakes occurring in the time interval of interest. The former and the latter are, in principle, different because: (1) there may be aftershocks that occur in $(t, t+\Delta t)$ yet pertain to sequences staring prior to t and then should be considered; (2) there may be aftershocks pertaining to sequences starting in $(t, t+\Delta t)$ yet occur outside the interval (i.e., after $t+\Delta t$) and then that should not be considered. A representation of this issue is given in Figure 2.

Under the hypotheses of SPSHA, the distribution of the number of earthquakes $(t, t+\Delta t)$ can also be formulated. To this aim, it is convenient to consider three intervals: $(t-\Delta T_A, t)$, $(t, t+\Delta t-\Delta T_A)$, and $(t+\Delta t-\Delta T_A, t+\Delta t)$, in which it is assumed $\Delta t > \Delta T_A$. The sought probability is given by:

$$\begin{aligned} P[N(t,t+\Delta t)=n] = P[&i \text{ aftershocks in } (t,t+\Delta t) \text{ from sequences originated in } (t-\Delta T_A, t) \\ &\cap k \text{ earthquakes in } (t,t+\Delta t) \text{ from sequences originated in } (t, t+\Delta t-\Delta T_A) \\ &\cap n-k-i \text{ earthquakes in } (t,t+\Delta t) \text{ from sequences originated in } (t+\Delta t-\Delta T_A, t+\Delta t)]. \end{aligned} \tag{24}$$

Equation (24) helps in recognizing that the earthquakes occurring in $(t, t+\Delta t)$ are the sum of three groups $N(t,t+\Delta t) = N^{(a)}(t,t+\Delta t) + N^{(b)}(t,t+\Delta t) + N^{(c)}(t,t+\Delta t)$. $N^{(a)}(t,t+\Delta t)$ are those aftershocks, occurring in $(t,t+\Delta t)$, from sequences originating prior to t; $N^{(b)}(t,t+\Delta t)$ are the mainshocks and aftershocks pertaining to sequences originating within $(t, t+\Delta t-\Delta T_A)$; and $N^{(c)}(t,t+\Delta t)$ are those mainshocks and aftershocks from sequences originating in $(t+\Delta t-\Delta T_A, t+\Delta t)$; see Figure 2. It helps to divide $N(t,t+\Delta t)$ in these contributions because $N^{(b)}(t,t+\Delta t)$ are earthquakes from sequences that will certainly end within $(t,t+\Delta t)$, whereas $N^{(a)}(t,t+\Delta t)$ and $N^{(c)}(t,t+\Delta t)$ are earthquakes from sequences that only partly develop within $(t,t+\Delta t)$; nevertheless, it is possible to locate in time the mainshocks and to count how many earthquakes fall in the considered interval.

The exact distribution of the count of the total number of earthquakes can be formulated starting from equation (24). The contributions $\{N^{(a)}(t,t+\Delta t), N^{(b)}(t,t+\Delta t), N^{(c)}(t,t+\Delta t)\}$ depend on sequences generated in nonoverlapping time intervals. Therefore, owing to the properties of the HPP, they are stochastically independent, and the following equation results:

$$\begin{aligned} P[N(t,t+\Delta t)=n] = &\sum_{i=0}^{n}\sum_{k=0}^{n-i} P[N^{(a)}(t,t+\Delta t)=i] \\ &\times P[N^{(b)}(t,t+\Delta t)=k] \cdot P[N^{(c)}(t,t+\Delta t)=n-k-i]. \end{aligned} \tag{25}$$

It is worthwhile to start with $N^{(b)}(t,t+\Delta t)$. As mentioned, these are events from sequences originating in $(t, t+\Delta t-\Delta T_A)$; these sequences will necessarily fully develop in $(t,t+\Delta t)$, because even if one mainshock occurs at the end of the interval, that is at $t+\Delta t-\Delta T_A$, all of aftershocks will occur prior to $t+\Delta t$. Therefore, $P[N^{(b)}(t,t+\Delta t)]$ descends from the same steps given in the Earthquakes from Seismic Sequences Originating in a Given Time Interval section:

$$\begin{aligned} P[N^{(b)}(t,t+\Delta t)=k] = {} & I_{\{k=0\}} \cdot e^{-\nu_E\cdot(\Delta t-\Delta T_A)} + (1-I_{\{k=0\}}) \cdot \sum_{l=1}^{k} \frac{(\nu_E\cdot\Delta t-\nu_E\cdot\Delta T_A)^l}{l!} \cdot e^{-\nu_E\cdot(\Delta t-\Delta T_A)} \\ & \times \int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,l}} \frac{\left\{\sum_{j=1}^{l} \frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[c^{1-p}-(\Delta T_A+c)^{1-p}]\right\}^{k-l}}{(k-l)!} \cdot e^{-\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[c^{1-p}-(\Delta T_A+c)^{1-p}]} \\ & \times \frac{\beta^l\cdot e^{-\beta\cdot\sum_{j=1}^{l} m_{E,j}}}{(e^{-\beta\cdot m_{\min}}-e^{-\beta\cdot m_{\max}})^l} \cdot dm_{E,1}\cdot dm_{E,2}\ldots dm_{E,l}. \end{aligned} \tag{26}$$

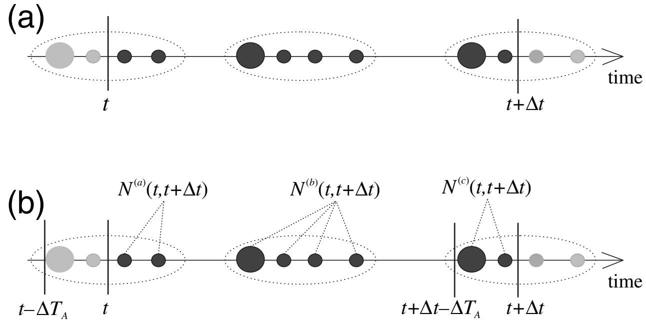

Figure 2. (a) Events occurring in $(t, t+\Delta t)$ and associated seismic sequences and (b) originating intervals for the events occurring in $(t, t+\Delta t)$.

Regarding $N^{(a)}(t, t+\Delta t)$, these are aftershocks from sequences originating prior to t but occurring in $(t, t+\Delta t)$; to count them it is not necessary to look back further than $t-\Delta T_A$, because sequences starting before will end earlier than t. Because in $(t-\Delta T_A, t)$ a number of mainshocks equal to l can occur, with $l = \{0, 1, 2, \ldots\}$, $P[N^{(a)}(t, t+\Delta t) = i]$ has to be computed conditional to this possibility, as in equation (27), in which it is acknowledged, once again, that mainshocks occur according to an HPP:

$$P[N^{(a)}(t,t+\Delta t)=i]=\sum_{l=0}^{+\infty}P[N^{(a)}(t,t+\Delta t)=i|N_E(t-\Delta T_A,t)=l]$$
$$\times P[N_E(t-\Delta T_A,t)=l]=\sum_{l=0}^{+\infty}P[N^{(a)}(t,t+\Delta t)=i|N_E(t-\Delta T_A,t)=l]$$
$$\times\frac{(\nu_E\cdot\Delta T_A)^l}{l!}\cdot e^{-\nu_E\cdot\Delta T_A}. \tag{27}$$

At this point, the probability that these mainshocks collectively produce i aftershocks falling in $(t, t+\Delta t)$ is needed. If $l = 0$, this probability is one when $i = 0$ and it is zero otherwise. If $l \geq 1$, it can be computed via the NHPP of equation (16), given that the magnitude and the time of occurrence of these mainshock is known. In this case equation (28) applies, in which $t_{E,j}$ is the occurrence time of the jth mainshock, $j = \{1, 2, \ldots, l\}$. (Note that the equation 28 counts the aftershocks occurring in the $(t-t_{E,j}, \Delta T_A)$ interval that is the portion of the sequences after t in the timescale in which each mainshock occurred at zero.)

$$P[N^{(a)}(t,t+\Delta t)=i|N_E(t-\Delta T_A,t)=l]$$
$$=\frac{\left\{\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[(t-t_{E,j}+c)^{1-p}-(\Delta T_A+c)^{1-p}]\right\}^i}{i!}$$
$$\times e^{-\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[(t-t_{E,j}+c)^{1-p}-(\Delta T_A+c)^{1-p}]}. \tag{28}$$

According to the HPP, the occurrence times of mainshocks, $T_{E,j}, j = \{1, 2, \ldots, l\}$, are i.i.d. uniformly distributed RVs, so that their joint probability density is equal to $(\Delta T_A)^{-l}$. Then, applying the total probability theorem to equation (27) and plugging it in equation (28), the following equation results:

$$P[N^{(a)}(t,t+\Delta t)=i]=I_{\{i=0\}}\cdot e^{-\nu_E\cdot\Delta T_A}+\sum_{l=1}^{+\infty}\frac{(\nu_E\cdot\Delta T_A)^l}{l!}\cdot e^{-\nu_E\cdot\Delta T_A}\times$$
$$\int_{t_{E,1}}\int_{t_{E,2}}\cdots\int_{t_{E,l}}P[N_A(t,t+\Delta t)=i|t_{E,1},t_{E,2},\ldots,t_{E,l}]\cdot f_{T_{E,1},T_{E,2},\ldots,T_{E,l}}(t_{E,1},t_{E,2},\ldots,t_{E,l})\cdot dt_{E,1}\cdot dt_{E,2}\ldots dt_{E,l}=$$
$$I_{\{i=0\}}\cdot e^{-\nu_E\cdot\Delta T_A}+\sum_{l=1}^{+\infty}\frac{(\nu_E\cdot\Delta T_A)^l}{l!}\cdot e^{-\nu_E\cdot\Delta T_A}\cdot\int_{t-\Delta T_A}^{t}\int_{t-\Delta T_A}^{t}\cdots\int_{t-\Delta T_A}^{t}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,l}}P[N_A(t,t+\Delta t)=i|t_{E,1},t_{E,2},\ldots,t_{E,l},m_{E,1},m_{E,2}\ldots,m_{E,l}]\times$$
$$\frac{\beta^l\cdot e^{-\beta\cdot\sum_{j=1}^{l}m_{E,j}}}{(e^{-\beta\cdot m_{\min}}-e^{-\beta\cdot m_{\max}})^l}\cdot dm_{E,1}\cdot dm_{E,2}\ldots dm_{E,l}\cdot\frac{1}{(\Delta T_A)^l}\cdot dt_{E,1}\cdot dt_{E,2}\ldots dt_{E,l}=$$
$$I_{\{i=0\}}\cdot e^{-\nu_E\cdot\Delta T_A}+\sum_{l=1}^{+\infty}\frac{(\nu_E\cdot\Delta T_A)^l}{l!}\cdot e^{-\nu_E\cdot\Delta T_A}\cdot\int_{t-\Delta T_A}^{t}\int_{t-\Delta T_A}^{t}\cdots\int_{t-\Delta T_A}^{t}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,l}}\frac{\left\{\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[(t-t_{E,j}+c)^{1-p}-(\Delta T_A+c)^{1-p}]\right\}^i}{i!}\times$$
$$e^{-\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[(t-t_{E,j}+c)^{1-p}-(\Delta T_A+c)^{1-p}]}\cdot\frac{\beta^l\cdot e^{-\beta\cdot\sum_{j=1}^{l}m_{E,j}}}{(e^{-\beta\cdot m_{\min}}-e^{-\beta\cdot m_{\max}})^l}\cdot dm_{E,1}\cdot dm_{E,2}\ldots dm_{E,l}\cdot\frac{1}{(\Delta T_A)^l}\cdot dt_{E,1}\cdot dt_{E,2}\ldots dt_{E,l}. \tag{29}$$

Note that for $i = 0$, the sought probability is the probability of zero mainshocks prior to t plus the probability of zero aftershocks for any number of mainshocks prior to t.

Also $P[N^{(c)}(t, t+\Delta t) = n-k-i]$ can be computed via an analogous reasoning. The only adjustments needed are that: (1) the mainshocks occurring $(t+\Delta t-\Delta T_A, t+\Delta t)$ contribute to the total number of earthquakes occurring in $(t, t+\Delta t)$; thus they can be $n-k-i$ at the most; (2) this time, the aftershocks produced by these sequences and occurring prior to $t+\Delta t$ are of interest; thus the interval $(0, t+\Delta t - t_{E,j})$ is the one to compute the NHPP of each sequence. Based on these considerations, the following equation results:

$$P[N^{(c)}(t,t+\Delta t)=n-k-i]=I_{\{n-k-i=0\}}\cdot e^{-\nu_E\cdot\Delta T_A}+(1-I_{\{n-k-i=0\}})\cdot\sum_{l=1}^{n-k-i}\frac{(\nu_E\cdot\Delta T_A)^l}{l!}\cdot e^{-\nu_E\cdot\Delta T_A}\times$$
$$\int_{t+\Delta t-\Delta T_A}^{t+\Delta t}\int_{t+\Delta t-\Delta T_A}^{t+\Delta t}\cdots\int_{t+\Delta t-\Delta T_A}^{t+\Delta t}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,l}}\frac{\left\{\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[c^{1-p}-(t+\Delta t-t_{E,j}+c)^{1-p}]\right\}^{n-k-i-l}}{(n-k-i-l)!}\times$$
$$e^{-\sum_{j=1}^{l}\frac{10^{a+b\cdot(m_{E,j}-m_{\min})}-10^a}{p-1}\cdot[c^{1-p}-(t+\Delta t-t_{E,j}+c)^{1-p}]}\cdot\frac{\beta^l\cdot e^{-\beta\cdot\sum_{j=1}^{l}m_{E,j}}}{(e^{-\beta\cdot m_{\min}}-e^{-\beta\cdot m_{\max}})^l}\cdot dm_{E,1}\cdot dm_{E,2}\ldots dm_{E,l}\cdot\frac{1}{(\Delta T_A)^l}\cdot dt_{E,1}\cdot dt_{E,2}\ldots dt_{E,l}. \quad (30)$$

At this point, the formulation of $P[N(t,t+\Delta t)]$ is completed, in the sense that all the terms of equation (25) are now specified. For a clearer understanding of the stochastic process formulated in this section, Figure 3 provides a flowchart for the solution of equation (25) via simulation of $\{N^{(a)}(t,t+\Delta t), N^{(b)}(t,t+\Delta t), N^{(c)}(t,t+\Delta t)\}$. It requires the location in time and attribution of magnitude to the mainshocks of the sequences generating $N^{(a)}(t,t+\Delta t)$ and $N^{(c)}(t,t+\Delta t)$, whereas for $N^{(b)}(t,t+\Delta t)$ it is only needed to assign a magnitude to each mainshock. The steps to simulate $N^{(b)}(t,t+\Delta t)$ correspond to the solution, via simulation, of equation (18).

Counting Process of Earthquakes Exceeding a Threshold

It is now useful to derive the equation of the total number of earthquakes exceeding an IM threshold at a site of interest. This distribution follows equation (4) according to classical PSHA, that is, for mainshocks. The same distribution also applies for seismic sequences causing at least one exceedance; that is, equation (4) can be applied using the rate computed as per equation (11). However, as it happens for the variable counting all earthquakes, the RV counting those causing exceedance at a site, say $N_{im}(t,t+\Delta t)$, is also not Poisson. Nevertheless, its equation can be derived, and it is the objective of this section. In fact, $P[N_{im}(t,t+\Delta t)=n]$, $\forall n$ can be formulated distinguishing seismic sequences in which the mainshock causes exceedance of im with respect to sequences starting from a mainshock that does not cause exceedance:

$$P[N_{im}(t,t+\Delta t)=n]=P[k\,\text{earthquakes from sequences with exceeding mainshock in}\,(t,t+\Delta t)$$
$$\cap\, n-k\,\text{earthquakes from sequences with nonexceeding mainshock in}\,(t,t+\Delta t)]. \quad (31)$$

Naming $N_{E,im}(t,t+\Delta t)$ the count of mainshocks exceeding threshold and $N_{E,\overline{im}}(t,t+\Delta t)$ the count of those not exceeding, it is possible to write, applying the total probability theorem, the sought probability as:

$$P[N_{im}(t,t+\Delta t)=n]=\sum_{k=0}^{n}\left\{\sum_{i=0}^{k}P[N_{A,im}=k-i|N_{E,im}(t,t+\Delta t)=i]\right.$$
$$\times P[N_{E,im}(t,t+\Delta t)=i]\cdot\sum_{l=0}^{+\infty}P[N_{A,im}=n-k|N_{E,\overline{im}}(t,t+\Delta t)=l]$$
$$\left.\times P[N_{E,\overline{im}}(t,t+\Delta t)=l]\right\}. \quad (32)$$

The equation, in which $N_{A,im}$ is the number of aftershocks exceeding the threshold and the other terms have been defined already, expresses the probability that to have n exceeding earthquakes: there might be k earthquakes from sequences with i exceeding mainshocks and $k-i$ exceeding aftershocks and $n-k$ exceeding aftershocks from l sequences (possibly infinite in number) in which mainshocks do not cause exceedance.

Note that not only do the former earthquakes occur according to the HPP in equation (4), but also the process counting the mainshocks not causing exceedance is an HPP with rate $\nu_E-\lambda_{im}$; moreover, the two HPPs are stochastically independent, so:

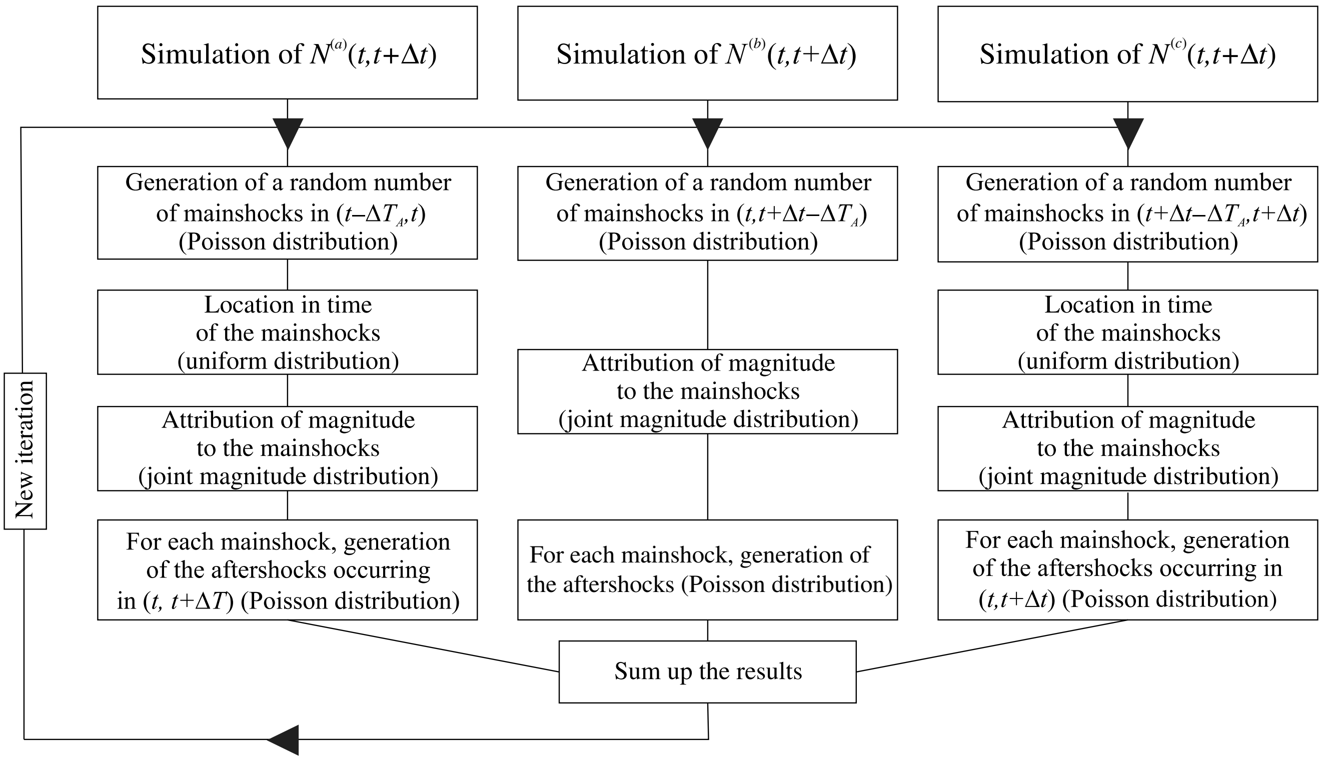

Figure 3. Flowchart of the simulation to compute the total number of earthquakes occurring in $(t, t+\Delta t)$.

$$P[N_{im}(t,t+\Delta t)=n]=\sum_{k=0}^{n}\left\{\sum_{i=0}^{k}P[N_{A,im}=k-i|N_{E,im}(t,t+\Delta t)=i]\cdot\frac{(\lambda_{E,im}\cdot\Delta t)^i}{i!}\cdot e^{-\nu_{E,im}\cdot\Delta t}\right.$$
$$\left.\times\sum_{l=0}^{+\infty}P[N_{A,im}=n-k|N_{E,\overline{im}}(t,t+\Delta t)=l]\cdot\frac{[(\nu_E-\lambda_{E,im})\cdot\Delta t]^l}{l!}\cdot e^{-(\nu_E-\lambda_{E,im})\cdot\Delta t}\right\}. \tag{33}$$

$P[N_{A,im}=k-i|N_{E,im}(t,t+\Delta t)=0]$ is equal to one if $k-i=0$ and is equal to zero otherwise. Similarly, $P[N_{A,im}=n-k|N_{E,\overline{im}}(t,t+\Delta t)=0]$ is equal to one if $n-k=0$ and is equal to zero otherwise. At this point, the missing terms in equation (33), $P[N_{A,im}=k-i|N_{E,im}(t,t+\Delta t)=i]$ and $P[N_{A,im}=n-k|N_{E,\overline{im}}(t,t+\Delta t)=l]$, can be computed via the Poisson distributions with the mean from equation (8). However, because the $P[IM_A>im|m_A,r_A]$ term in equation (8) depends on the aftershock magnitude and distance that, in turn, depends on the magnitude and distance of the mainshock (in fact, it is the mainshock location, as mentioned in the APSHA section), it is necessary to condition first on these RVs:

in which the two distributions $f_{M_{E,1},M_{E,2},\ldots,M_{E,i},R_{E,1},R_{E,2},\ldots,R_{E,i}|im}$ and $f_{M_{E,1},M_{E,2},\ldots,M_{E,l},R_{E,1},R_{E,2},\ldots,R_{E,l}|\overline{im}}$ are the joint distributions of magnitude and source-to-site-distance of exceeding and nonexceeding mainshocks, respectively. It is easy to recognize that these distributions can both be obtained via disaggregation of mainshock hazard (e.g., Iervolino *et al.*, 2011), that is, from classical PSHA. In fact, because the effects of multiple mainshocks, in terms of IM, are independent of each other, it results that the joint distributions are just the product of the disaggregation distributions of each mainshock, which are also identically distributed:

$$P[N_{im}(t,t+\Delta t)=n]=$$
$$\sum_{k=0}^{n}\left\{\sum_{i=0}^{k}\frac{(\lambda_{E,im}\cdot\Delta t)^i}{i!}\cdot e^{-\lambda_{E,im}\cdot\Delta t}\cdot\int_{r_{E,1}}\int_{r_{E,2}}\cdots\int_{r_{E,i}}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,i}}P[N_{A,im}=k-i|m_{E,1},m_{E,2},\ldots,m_{E,i},r_{E,1},r_{E,2},\ldots,r_{E,i}]\times\right.$$
$$f_{M_{E,1},M_{E,2},\ldots,M_{E,i},R_{E,1},R_{E,2},\ldots,R_{E,i}|im}(m_{E,1},m_{E,2},\ldots,m_{E,i},r_{E,1},r_{E,2},\ldots,r_{E,i})\cdot dm_{E,1}\cdot dm_{E,2}\cdot\ldots dm_{E,i}\cdot dr_{E,1}\cdot dr_{E,2}\cdot\ldots dr_{E,i}\times$$
$$\sum_{l=0}^{+\infty}\frac{[(\nu_E-\lambda_{E,im})\cdot\Delta t]^l}{l!}\cdot e^{-(\nu_E-\lambda_{E,im})\cdot\Delta t}\cdot\int_{r_{E,1}}\int_{r_{E,2}}\cdots\int_{r_{E,l}}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,l}}P[N_{A,im}=n-k|m_{E,1},m_{E,2},\ldots,m_{E,l},r_{E,1},r_{E,2},\ldots,r_{E,l}]\times$$
$$\left.f_{M_{E,1},M_{E,2},\ldots,M_{E,l},R_{E,1},R_{E,2},\ldots,R_{E,l}|\overline{im}}(m_{E,1},m_{E,2},\ldots,m_{E,l},r_{E,1},r_{E,2},\ldots,r_{E,l})\cdot dm_{E,1}\cdot dm_{E,2}\cdot\ldots dm_{E,l}\cdot dr_{E,1}\cdot dr_{E,2}\cdot\ldots dr_{E,l}\right\}. \tag{34}$$

$$\begin{cases} f_{M_{E,1},M_{E,2},\ldots,M_{E,i},R_{E,1},R_{E,2},\ldots,R_{E,i}|im} = \prod_{j=1}^{i} f_{M_E,R_E|im}(m_{E,j},r_{E,j}) \\ f_{M_{E,1},M_{E,2},\ldots,M_{E,l},R_{E,1},R_{E,2},\ldots,R_{E,l}|\overline{im}} = \prod_{j=1}^{l} f_{M_E,R_E|\overline{im}}(m_{E,j},r_{E,j}) \end{cases}. \quad (35)$$

At this point, the count of aftershocks exceeding the threshold can be computed considering the NHPP of aftershocks exceeding the threshold instead of that of all aftershocks:

$$P[N_{im}(t,t+\Delta t)=n]=\sum_{k=0}^{n}\Bigg(\Bigg\{I_{\{k=0\}}\cdot e^{-\lambda_{E,im}\cdot\Delta t}+(1-I_{\{k=0\}})\cdot\sum_{i=1}^{k}\frac{(\lambda_{E,im}\cdot\Delta t)^i}{i!}\cdot e^{-\lambda_{E,im}\cdot\Delta t}\times$$
$$\int_{r_{E,1}}\int_{r_{E,2}}\cdots\int_{r_{E,i}}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,i}}\frac{\left\{\sum_{j=1}^{i}E[N_{A,im|m_{E,j}}(0,\Delta T_A)]\right\}^{k-i}}{(k-i)!}\cdot e^{-\sum_{j=1}^{i}E[N_{A,im|m_{E,j}}(0,\Delta T_A)]}\times$$
$$\left[\prod_{j=1}^{i}f_{M_E,R_E|im}(m_{E,j},r_{E,j})\right]\cdot dm_{E,1}\cdot dm_{E,2}\ldots dm_{E,i}\cdot dr_{E,1}\cdot dr_{E,2}\ldots dr_{E,i}\Bigg\}\times$$
$$\Bigg\{I_{\{n-k=0\}}\cdot e^{-(\nu_E-\lambda_{E,im})\cdot\Delta t}+\sum_{l=1}^{+\infty}\frac{[(\nu_E-\lambda_{E,im})\cdot\Delta t]^l}{l!}\cdot e^{-(\nu_E-\lambda_{E,im})\cdot\Delta t}\cdot\int_{r_{E,1}}\int_{r_{E,2}}\cdots\int_{r_{E,l}}\int_{m_{E,1}}\int_{m_{E,2}}\cdots\int_{m_{E,l}}\frac{\left\{\sum_{j=1}^{l}E[N_{A,im|m_{E,j}}(0,\Delta T_A)]\right\}^{n-k}}{(n-k)!}\times$$
$$e^{-\sum_{j=1}^{l}E[N_{A,im|m_{E,j}}(0,\Delta T_A)]}\cdot\left[\prod_{j=1}^{l}f_{M_E,R_E|\overline{im}}(m_{E,j},r_{E,j})\right]\cdot dm_{E,1}\cdot dm_{E,2}\ldots dm_{E,l}\cdot dr_{E,1}\cdot dr_{E,2}\ldots dr_{E,l}\Bigg\}\Bigg). \quad (36)$$

Again, if aftershocks are neglected, that is $E[N_{A,im|m_{E,j}}(0,\Delta T_A)] = 0\,\forall m_{E,j}$, then the process degenerates in the HPP of exceeding mainshocks characterizing PSHA. This can be acknowledged recognizing that the two probabilities $P[N_{A,im}=k-i|m_{E,1},m_{E,2},\ldots,m_{E,i},r_{E,1},r_{E,2},\ldots,r_{E,i}]$ and $P[N_{A,im}=n-k|m_{E,1},m_{E,2},\ldots,m_{E,l},r_{E,1},r_{E,2},\ldots,r_{E,l}]$ are equal to one for $k=i$ and $n=k$, respectively; otherwise, they are both equal to zero. Moreover, $\sum_{l=0}^{+\infty}(l!)^{-1}\cdot[(\nu_E-\lambda_{E,im})\cdot\Delta t]^l\cdot e^{-(\nu_E-\lambda_{E,im})\cdot\Delta t}=1$. Thus this process is also a generalization of the one PSHA refers to, that is, it degenerates in the HHP of equation (4) if there are no exceeding aftershocks.

To aid the understanding of the way this model has been derived, Figure 4a provides a flowchart for the solution of equation (36). As mentioned, it requires the magnitude and location in space of the mainshocks, distinguishing between those causing and those that do not cause exceedance, and the conditional Poisson distribution for the exceeding aftershocks. However, Figure 4b provides an alternative strategy for simulation that does not require distinguishing between exceeding and nonexceeding mainshocks. It starts with the simulation of a number of mainshocks in the interval, then magnitude and location are attributed to each mainshock (usually the mainshock distribution is uniform over the source), and then the mainshock IM at the site is simulated using a lognormal distribution with parameters, conditional to mainshock magnitude and distance, provided by the GMPE. Finally the number of exceeding aftershocks is generated, considering the Poisson distribution, conditional to mainshock magnitude and location.

In analogy with equation (19), a relatively simple relationship for the mean of earthquakes exceeding a threshold, $E[N_{im}(t,t+\Delta t)]$, can be derived. It can be obtained from the expected number of exceeding earthquakes, given the occurrence of a mainshock, $E[N_{im}|E]$, which is given by the probability of the mainshock exceeding plus the number of exceeding aftershocks for any possible mainshock magnitude and distance:

$$E[N_{im}|E]=P[IM_E>im|E]+\int_{r_{E,\min}}^{r_{E,\max}}\int_{m_{E,\min}}^{m_{E,\max}}E[N_{A,im|m_E}(0,\Delta T_A)]\times f_{M_E,R_E}(m_E,r_E)\cdot dm_E\cdot dr_E. \quad (37)$$

At this point, the expected number of exceeding earthquakes in an interval can be obtained as $E[N_{im}(t,t+\Delta t)]=\nu_E\cdot\Delta t\cdot E[N_{im}|E]$.

Application

For the illustration of the results, one of the source zones of the Italian source model of Meletti *et al.* (2008) lying at the basis of the hazard map used to define seismic actions for structural design in the country (Stucchi *et al.*, 2011) is considered. The model features 36 zones, numbered from 901 to 936 (see Fig. 5a). Herein, zone 923 is considered (Fig. 5b); it is in central Italy, one of the most seismically hazardous areas in the country. The magnitude distribution for this region is bounded between $m_{E,\min}=4.3$ and $m_{E,\max}=7.3$. The annual rate of the mainshocks within these events is 0.645

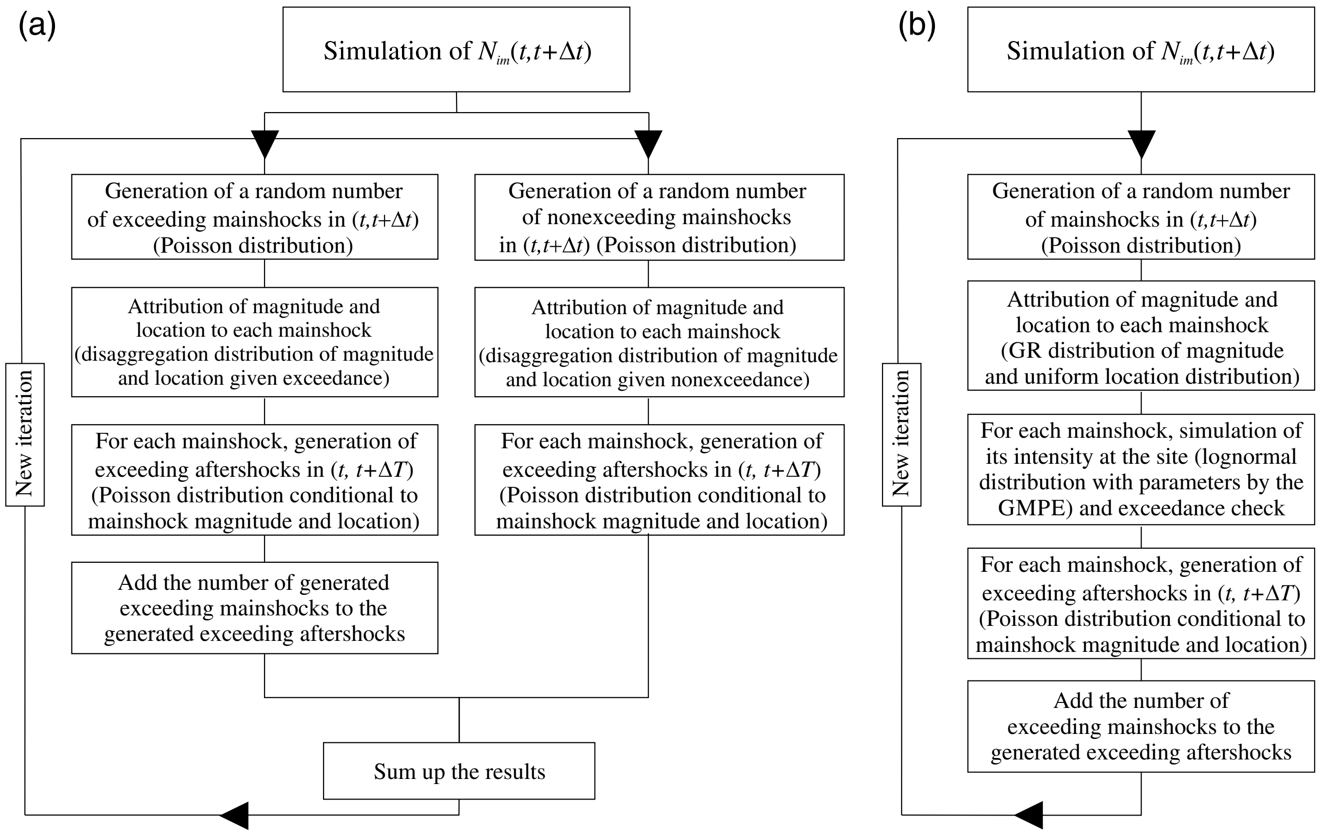

Figure 4. Flowchart of the (a) simulation to compute the total number of exceeding earthquakes from sequences originating in $(t, t + \Delta t)$, distinguishing between exceeding and nonexceeding mainshocks and (b) simulation that does not require distinguishing between exceeding and nonexceeding mainshocks. GMPE, ground-motion prediction equation; GR, Gutenberg–Richter.

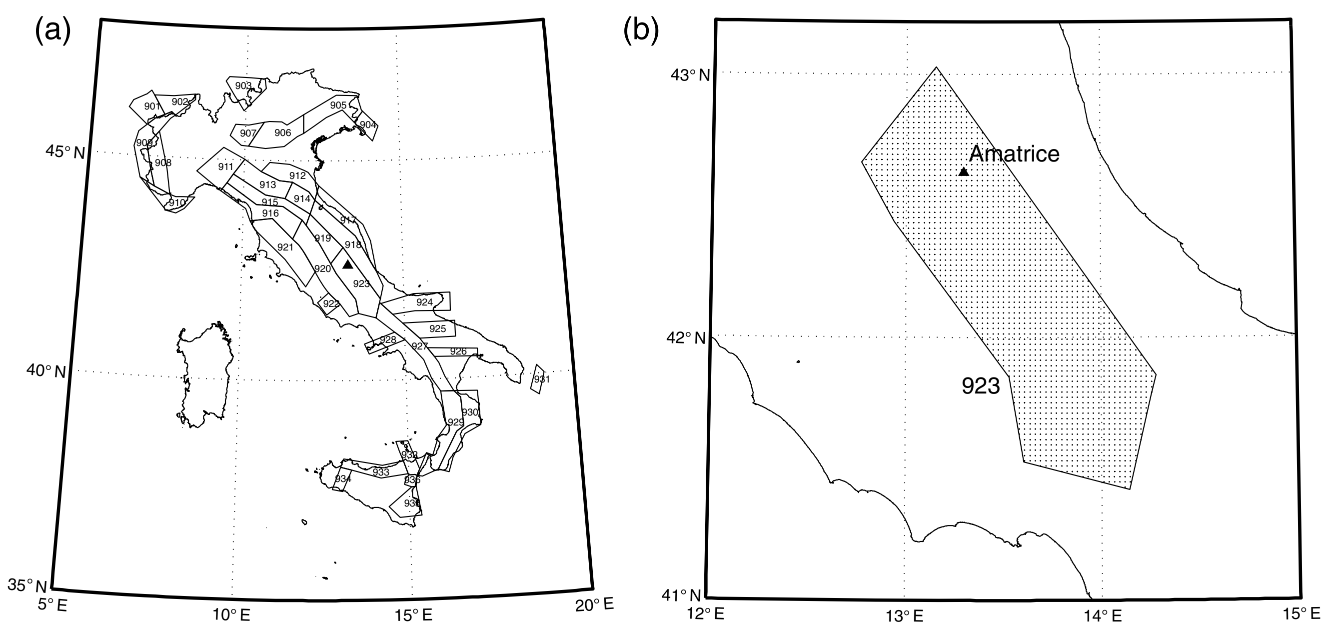

Figure 5. (a) Italian seismic source model as per Meletti *et al.* (2008). (b) Zone 923 considered in the application. The black triangle is the Amatrice site.

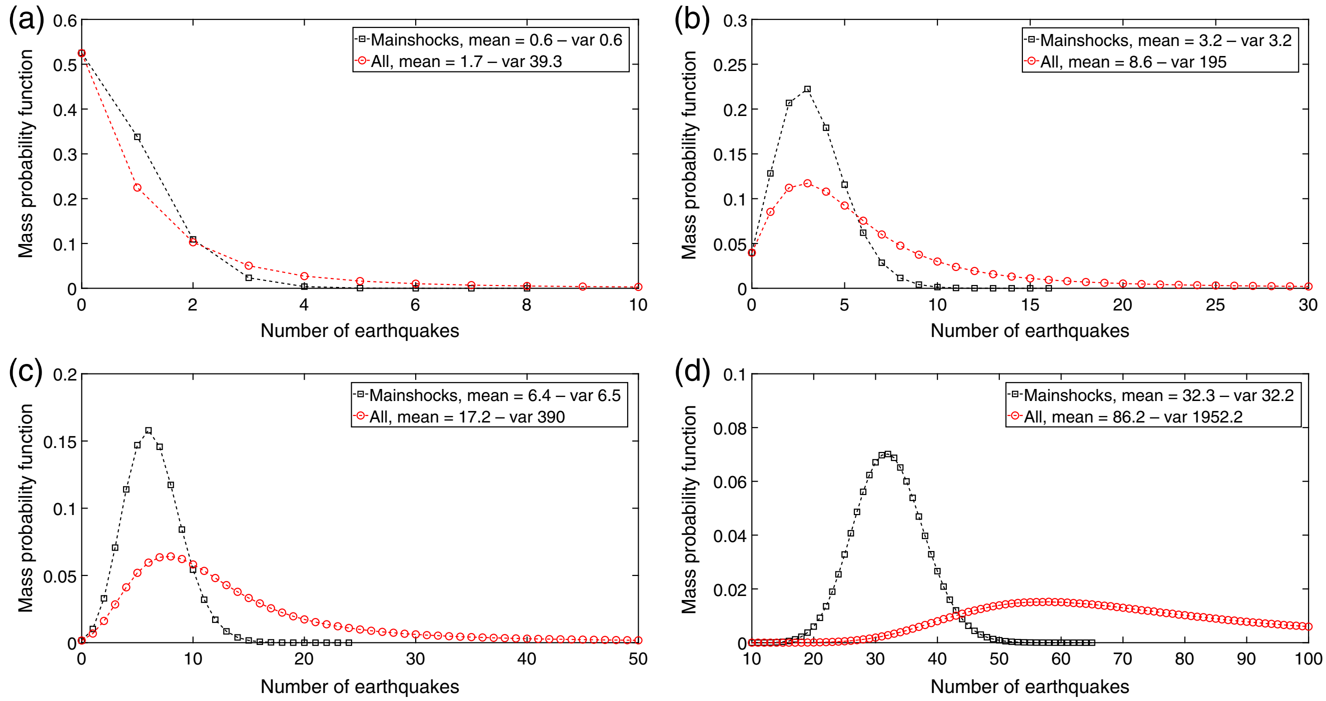

Figure 6. Distribution of the mainshocks (squares) and of the total number of earthquakes (circles) from sequences originating in: (a) 1, (b) 5, (c) 10, and (d) 50 yr. The color version of this figure is available only in the electronic edition.

events/yr, and the parameter of the magnitude distribution is $\beta = 1.85$ (Barani *et al.*, 2009). The parameters of the modified Omori law considered are $\{a = -1.66, b = 0.96, c = 0.03, p = 0.93\}$ and are calibrated on Italian sequences by Lolli and Gasperini (2003). The minimum aftershock magnitude is taken equal to the minimum mainshock magnitude, that is, $m_{\min} = m_{E,\min} = 4.3$. The sequences' duration is set equal to $\Delta T_A = 90$ days, because it was verified that larger values do not affect the results when $\Delta t > \Delta T_A$ (see Iervolino *et al.*, 2018).

Earthquakes Generated from Sequences Originating in an Interval

To start, equation (18) was numerically solved for four Δt values, 1, 5, 10, and 50 yr, and the results are given in Figure 6. In the figure, the corresponding (Poisson) distributions of the mainshocks only are also given. It first emerges from the calculations that the mean of the total number of earthquakes is equal to 1.7, 8.6, 17.2, and 86.2 for 1, 5, 10, and 50 yr, respectively. Comparing with the mainshock count, the two distributions look similar when the time interval is relatively small. However, as equation (19) shows, the ratio of the mean of the total number of earthquakes to that of the mainshocks alone is independent of Δt, and it is about 2.7. Moreover, in the Poisson distribution, the variance-to-mean ratio is invariably equal to one because the mean equals the variance, which is a substantially smaller value than what is found in this example, that is, about 23 independently of Δt. Finally, as expected, the two probabilities of zero earthquakes coincide between the two distributions.

Earthquakes Occurring in an Interval

As a second relevant application, the counting process of equation (25) can be compared with that of equation (18) to evaluate the differences, if any, between the total number of earthquakes occurring in $(t, t + \Delta t)$ and the total number of earthquakes from sequences originating in $(t, t + \Delta t)$. An example of such a comparison has been developed for the same seismic source zone discussed in the previous section and under the same working hypotheses, considering one and 50 yr intervals. The results are given in Figure 7a,c, in which it can be observed that the two distributions are not distinguishable, providing a sound basis in favor of equation (18) for practical purposes. This is justified by Figure 7b,d, in which the distributions of the three contributions to $N(t, t + \Delta t)$ are given. It can be observed that the dominating contribution is $N^{(b)}(t, t + \Delta t)$, which is very similar to equation (18).

Earthquakes Exceeding a Threshold

To evaluate the counting process of equation (36), a site and an *im* threshold have to be chosen. To this aim, the site of Amatrice village is considered (Fig. 5). The considered IM is the peak ground acceleration (PGA) on rock; its threshold is arbitrarily set at $im = 0.05g$. The GMPE of Ambraseys *et al.* (1996) is considered for both mainshocks and aftershocks. Consistent with applications in Iervolino *et al.* (2014,

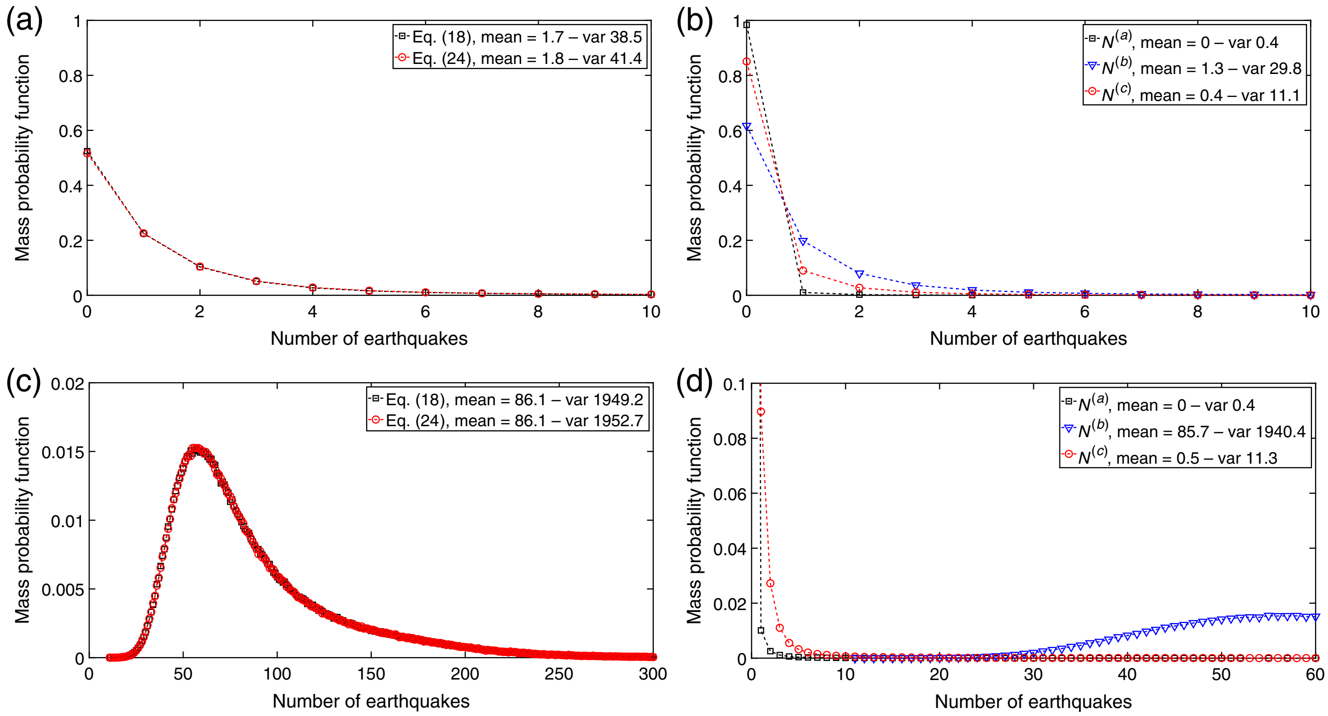

Figure 7. Comparison of the distribution of the total number of earthquakes from sequences originating in $(t, t + \Delta t)$ and the total number of earthquakes occurring in $(t, t + \Delta t)$ for: (a) 1 and (c) 50 yr time intervals. Distributions of the contributions of the total number of earthquakes occurring in for (b) 1 and (d) 50 yr time intervals. The color version of this figure is available only in the electronic edition.

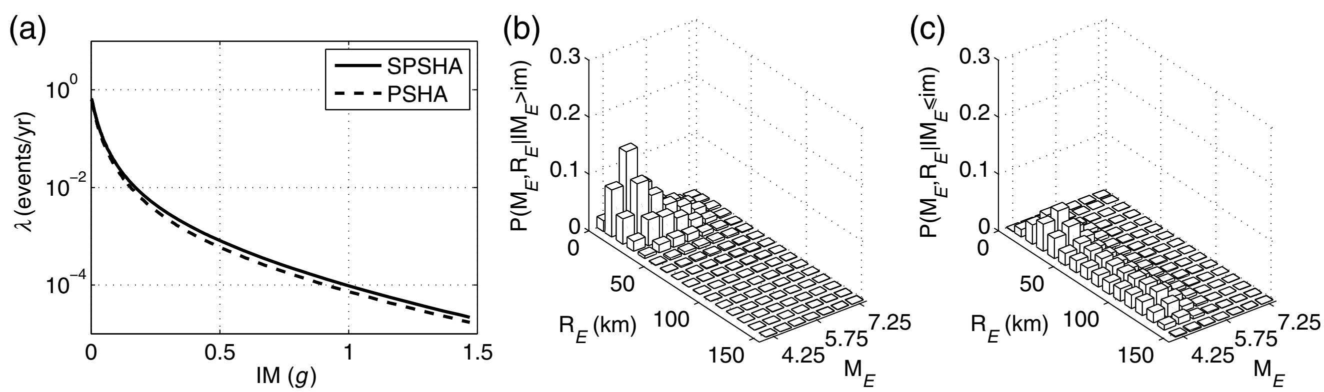

Figure 8. (a) Probabilistic seismic hazard analysis (PSHA) and SPSHA curves for peak ground acceleration (PGA) on rock at Amatrice considering zone 923 only. (b) PSHA disaggregation for mainshocks exceeding the $0.05g$ at the site. (c) PSHA disaggregation for mainshocks not exceeding $0.05g$ at the site.

2018), it is assumed that aftershocks can occur within a circle around the mainshock location. The area of the aftershock source zone (S_A) is proportional to the mainshock magnitude $S_A = 10^{m_E - 4.1}$ (km^2) (Utsu, 1961). Considering only zone 923, such a PGA value has an annual exceedance rate equal to $\lambda_{im} = 0.09$ and $\lambda_{im,E} = 0.08$ in the case of SPSHA and PSHA, respectively, which can be seen from the hazard curves in Figure 8a. Figure 8b,c provides the (discretized) disaggregation distributions for the exceeding and nonexceeding mainshocks at Amatrice. (Calculations are carried out with the REASSESS software; Chioccarelli *et al.*, 2019.)

In this framework, $P[N_{im}(t, t + \Delta t) = n]\,\forall n$ is computed for 1, 5, 10, and 50 yr. The results are given in Figure 9, in which, for comparison, the corresponding (Poisson) distributions of the exceeding mainshocks are also given. It emerges from the results that the mean of the total number of earthquakes exceeding the threshold is equal to 0.2, 1, 2, and 10 for 1, 5, 10, and 50 yr, respectively. The distributions of the total number of earthquakes are more similar to those of mainshocks with respect to what is observed in Figure 6. However, the ratio of the variance to the mean is still independent of Δt and equal to about 9 in this case, which

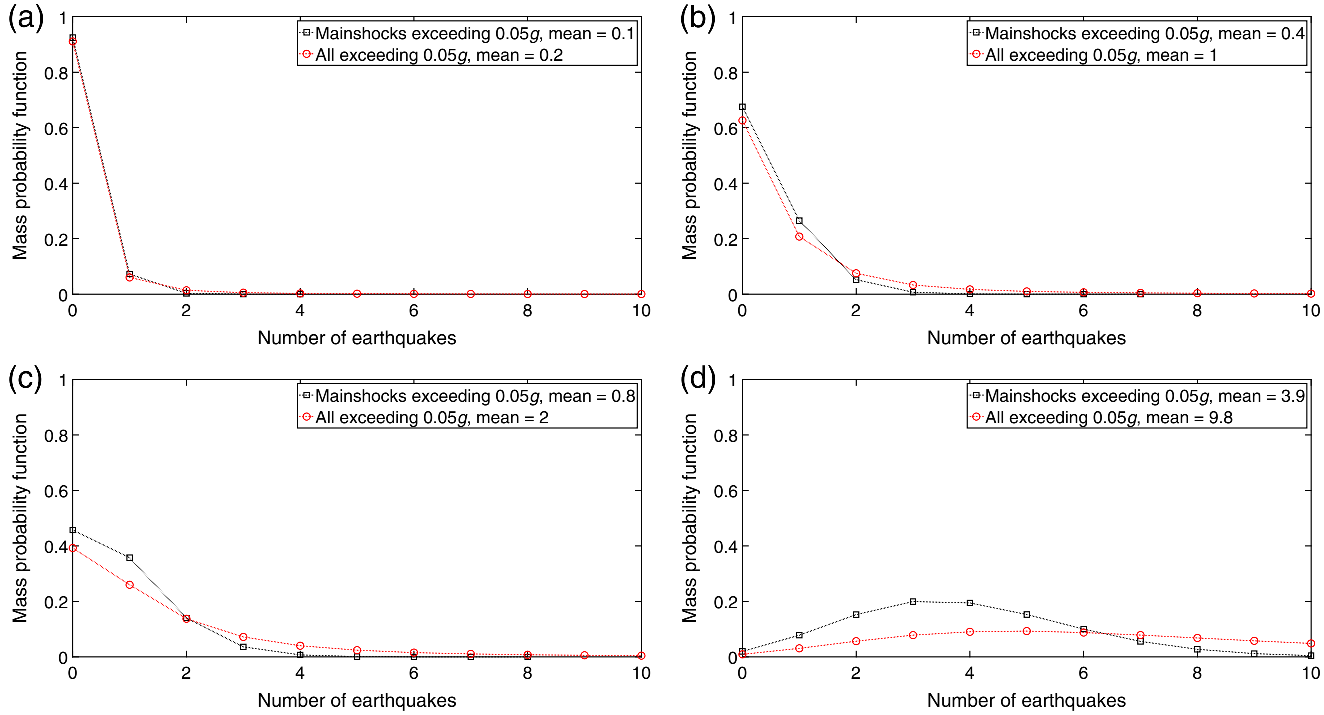

Figure 9. Distribution of the mainshocks (square markers) and of the total number of earthquakes (round markers) exceeding $0.05g$ at the site of Amatrice from sequences originating in: (a) 1, (b) 5, (c) 10, and (d) 50 yr. The color version of this figure is available only in the electronic edition.

is substantially larger than one. Notice that the difference between the values of observing zero exceedances between the two distributions gets larger as the interval width increases. In fact, the probability of observing zero exceedances is not the same between the two distributions (conversely with respect to what happens when counting all earthquakes), and it is systematically lower when aftershocks are accounted for. This is intuitive, in fact, because the larger the interval the larger the expected number of sequences than the larger the number of aftershocks potentially causing exceedance. Furthermore, note that the probability of zero exceedances in these distributions could be computed easily from the hazard curve in Figure 8. This is because, as extensively discussed, the probability of observing at least one exceedance can be computed as $1 - e^{\lambda_{im} \cdot \Delta t}$, which is precisely one minus the probability of observing zero earthquakes occurring from the distributions in Figure 9.

Conclusions

SPSHA allows us to include back the effects of aftershocks on the rate of exceedance of ground-motion IMs, retaining some major advantages of classical PSHA, such as exponential interarrival time distribution of sequences causing exceedance and the use of declustered catalogs for which completeness is more easily achieved. Nevertheless, the counting processes of earthquakes in SPSHA follow different distributions that derive from the classical assumption of HPP for mainshocks and NHPP for aftershocks, conditional to mainshock magnitude. In this study, some of these processes have been discussed. In particular, the distribution of the total number of earthquakes generated by sequences originating in an arbitrary time interval was derived first, along with formulations for the mean and the variance. Moreover, the process counting the earthquakes occurring in an arbitrary time interval was also formulated. This process is different from the previous one because it considers the aftershocks falling in the interval yet pertaining to sequences occurring prior to it and discards those aftershocks that pertain to sequences originating in the interval yet falling outside it. The process counting all earthquakes that occur in sequences and cause the exceedance of an arbitrary ground-motion intensity threshold at a site of interest was also addressed.

The illustrative application to a seismic source zone among those of the official Italian model used to determine seismic design actions for structures allowed us to conclude the following:

- all the discussed earthquake count distributions, although deriving from combinations of Poisson processes, substantially depart from Poisson, having variance-to-mean-ratio significantly different than one;
- the distribution of the number of earthquakes from sequences originating in a given interval closely approximates the distribution of the number of all earthquakes occurring in the same interval, which indicates that it is appropriate to associate aftershocks to the corresponding mainshocks for hazard assessment purposes;

- the derived processes generalize PSHA; that is, they degenerate in the homogenous Poisson processes from classical hazard if aftershocks are neglected.

The provided equations complete the framework of sequence-based hazard and can be useful for model calibration exercises when SPSHA is computed via simulation and in those cases in which the probability of an exact number of exceedances is of concern rather than the probability of at least one exceedance, for example, for seismic damage accumulation studies.

Data and Resources

All data and resources used in this study come from the listed references.

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