The effect of dependence of observations on hazard validation studies

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SUMMARY

In countries with an advanced seismic technical culture, where best-practice probabilistic hazard studies are available along with dense seismic networks, there is an increasing interest in validation of hazard maps. This, basically, means trying to quantitatively understand whether probabilities estimated via hazard analysis are consistent with observed frequencies of exceedance of ground motion intensity thresholds. Because the exceedance events of interest are typically rare with respect to the time span covered by data from seismic monitoring networks, a common approach underlying these studies is to pool observations from different sites. The main reason for this is to collect a number of data large enough to convincingly perform a statistical analysis. However, this is often done neglecting the intrinsic stochastic dependence affecting observations at different sites in the same earthquake. On these premises, the presented study demonstrates how this may lead to potentially fallacious conclusions about inadequateness of probabilistic seismic hazard assessment. The study refers, as an example, to an ideal seismic source zone and some recording sites. It is shown, how accounting for the dependence of intensity on magnitude and source to site distance, may change the results of validation from fail to pass. Some considerations with respect to other studies, attempting to validate Italian data via thirty years of seismic observations all across the country, are also made.

Keywords: *Probabilistic seismic hazard analysis, Validation, Disaggregation, Binomial distribution.*

1. INTRODUCTION

Due to their underlying predictive meaning, probabilistic seismic hazard analysis or PSHA (e.g., Cornell, 1968; Reiter, 1990) studies are debated and often questioned (e.g., Hanks et al., 2012, Kossobokov and Nekrasova, 2012, Stein et al., 2011 and 2012, Stirling, 2012). Italy is not an exception in this sense; indeed, in the country there is a constant debate on the consistency and adequacy of the national hazard map (Stucchi et al., 2011), which serves as a basis for the definition of seismic actions for structural design according to the current building code (CS.LL.PP., 2008).

A number of studies tried to quantitatively confirm or disprove probabilistic seismic hazard estimates via observed ground motions over the years (e.g., Albarello and D'amico, 2008). The soundest studies, attempting to validate hazard maps, are based on the theory of hypothesis testing or confidence intervals (e.g., Mood et al., 1974). In fact, these studies recognize that validating hazard at a single site requires a large number of earthquake observations, which is seldom available due to very long time (on average) required to collect those (e.g., Iervolino, 2013). Therefore, they tend to pool together seismic records at different sites, in the same time span, to create a sample of sufficiently large size to make the formal comparison with PSHA. However, it seems that, in these exercises, the effect of stochastic dependency of observations at different sites, yet in the same earthquake, is often overlooked. The consequent risk is that of being led to fallacious conclusions, labelling seismic hazard estimates from PSHA as erroneous (often claimed not conservative).

Hazard maps are usually a collection of ground motion intensity measures (IMs) values, corresponding to percentiles of site-specific marginal IM distributions. This is because the civil structures are typically point-like, and therefore codes require location-specific SPHA. The aim of this paper is to recall that, due to certain basic aspects of PSHA, in the case the same earthquake affects more than one site, recordings are not independent and therefore the observed IM exceedances should be cautiously compared to hazard maps. The cause for stochastic dependency is twofold: (i) there's stochastic dependency carried by the ground motion prediction equation or GMPE, as hazard *disaggregation* shows (e.g., Iervolino et al., 2011); (ii) there may be also spatial correlation of GMPE's intraevent residuals (e.g., Esposito and Iervolino, 2011). This study will focus on (i) as it is sufficient to prove the argument that spatial dependency of observation in a single seismic event must be take into account in PSHA validation attempts. Other forms of dependency, such as spatial clustering of exceedances, are also neglected.

To this end the remainder of the paper is organized such that a brief review of site-specific and regional PSHA, is initially given. Then, simple examples of how hazard validation would quantitatively change if the dependency of observations were accounted for, are discussed. Finally, some recommendations for comparison of hazard and observed ground motions are addressed with respect to one of the approach to PSHA validation found in literature.

2. SITE-SPECIFIC AND REGIONAL PSHA ESSENTIALS

In its standard form, PSHA consists of the estimate of the mean rate (e.g., annual) of exceedance of a given value of an IM, for example peak ground acceleration or PGA, at a site of interest (e.g., the location where a building under design is to be constructed). The computation of this rate, which can be represented as λ_{IM}, is often carried out considering: at first the rate of earthquake occurrence on the source, ν ; then the conditional probability of IM exceedance given event magnitude (M) and source-to-site distance (R), as well as other parameters; and finally by averaging over all possible events via the joint distribution of M and R, as in Equation (1).[1]

This articulation, is only for convenience, because the $P\left[IM \geq im \mid m \cap r\right]$ term is obtained from GMPEs, while ν and $P\left[M = m \cap R = r\right]$, the latter being the joint probability of M and R, are provided based on seismicity – historical or instrumental – and geological information about the source.

$$\lambda_{IM} = \nu \cdot P\left[IM > im\right] = \nu \cdot \sum_{m,r} P\left[IM > im \middle| m \cap r\right] \cdot P\left[M = m \cap R = r\right] \tag{1}$$

[1] For the sake of simplicity in this illustration, the probabilities are expressed for discrete random variables while, strictly speaking, these should be considered continuous. In fact, sums and probabilities should be replaced by integrals and probability density function, respectively.

In fact, it is possible to show that, if the occurrence of earthquakes on the source follows a homogeneous Poisson processes (HPP) with rate ν, then also the process describing the occurrence of events determining exceedance of the IM at the site of interest, follows a HPP. Furthermore, the rate of the latter depends on that of the former as per Equation (1). It is a *filtered* process; the occurrence of earthquakes on the source is filtered by the probability that the resulting ground motion will cause the exceedance of the *im* intensity level in question at the site. In other words, among all the earthquakes occurring on the fault, retaining only those causing the considered effect at the site, the occurrence of events belonging to this *random selection* is still described by a HPP.

If the analysis as per Equation (1) is repeated for all IM-values in a range of interest, a curve for λ_{IM}, as a function of *im*, is obtained. It is termed *hazard curve*, and for each IM-value provides the rate of the specific HPP regulating its exceedance at the site of interest.

One important consequence of the HPP assumption for earthquake occurrence[2] is that the random time elapsed between two consecutive events (i.e., the *interarrival* time), is characterized by the *exponential* distribution. Therefore, the probability that the time between two events causing the exceedance of the IM-value of interest at the site, $T(im)$, is lower than *t*, is given by Equation (2). The same distribution also provides the probability to observe at least one exceedance of *im* during *t* years.

$$P[\text{at least one exceedance of } im \text{ during } t] = P\left[T(im) \le t\right] = 1 - e^{-\lambda_{IM} \cdot t} \tag{2}$$

In the case of *regional* seismic hazard, one may want to calculate, for example, the ground motion intensity, which has a specific *annual rate of exceedance in at least one of several* sites of interest. Let the objective of *regional* probabilistic seismic hazard analysis (e.g., Esposito and Iervolino, 2011) be to compute the annual rate of the event, which causes the exceedance of a certain IM-value in at least one of two sites, $\{1,2\}$, in the same region. Such a calculation could be carried out by implementing Equation (3). In the equation $\{R_1, R_2\}$ are the earthquake distances from sites 1 and 2 respectively.

$$\begin{aligned}\lambda_{IM_1 \cup IM_2} &= \\ &= \nu \cdot \left\{ 1 - \sum_{m, r_1, r_2} P\left[IM_1 \le im_1 \cap IM_2 \le im_2 \mid m \cap r_1 \cap r_2\right] \cdot P\left[M = m \cap R_1 = r_1 \cap R_2 = r_2\right] \right\}\end{aligned} \tag{3}$$

The need for the joint probability, $P\left[IM_1 \le im_1 \cap IM_2 \le im_2 \mid m \cap r_1 \cap r_2\right]$, in Equation (3), recalls that GMPEs always imply stochastic dependency of IMs at different sites. This is because the mean of IM at the two sites changes with the value of M and with the earthquake location, which affects the distances, and also because there could be spatial dependency of intraevent residuals of the ground motion prediction model (neglected in the rest of this paper, as mentioned earlier on).

To understand this issue, in Figure 1 an ideal, $20 \times 80\, km^2$, seismic source is considered. It is discretized in sixty-eight possible earthquake source locations. It is also imagined that four recording stations are located in the sites indicated by triangles labeled 1-4 in the figure. It is assumed that event rate of earthquakes is $\nu = 1\left[events/yr\right]$, globally over the source zone. The distribution of magnitude is a truncated exponential one, defined in the $\left[4.5, 7\right]$ range. The *b*-

[2] In this work, considerations regarding the choice – however frequent – of the HPP to model earthquake occurrence are omitted, as well as any discussion of possible alternatives.

value of the Gutenberg-Richter relationship is equal to one. The considered GMPE is that of Ambraseys et al. (1996).[3]

For each (equally likely) possible earthquake location in the picture, 10^4 values of magnitude were generated via a montecarlo simulation. These simulations where used to compute the site-specific hazard curves for sites $\{1,2\}$ via Equation (1), and the joint hazard via Equation (3). Resulting hazard curves are shown in Figure 2. It is to note that the joint hazard may not be lower than those corresponding to hazard for each individual site.

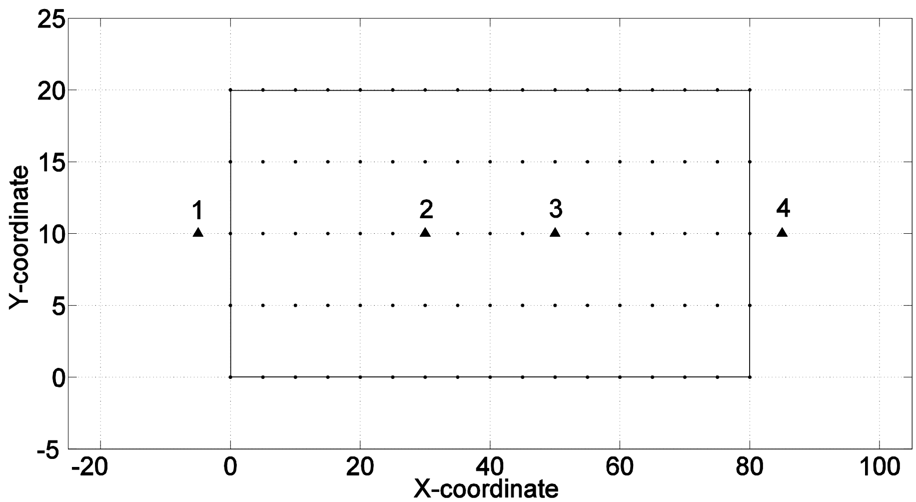

Figure 1. Ideal seismic source zone and considered sites (distances in km).

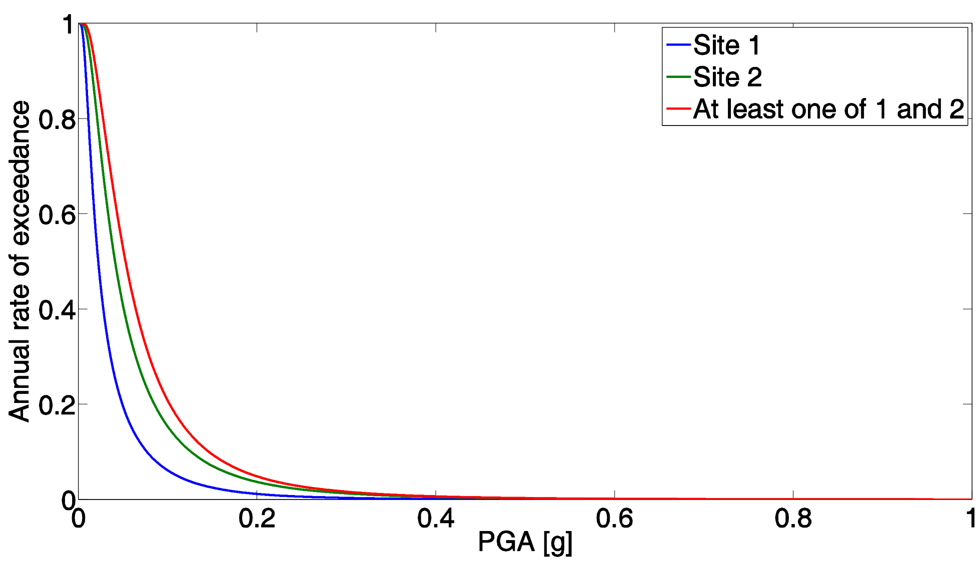

Figure 2. Site-specific (marginal) and regional (joint) hazard for sites 1 and 2 in Figure 1.

Another, even more straightforward way to recall that IM observations at different sites, yet in the same earthquake event, cannot be considered independent random variables, is readily provided by the well-known tool of hazard disaggregation. Given a hazard curve for a specific site and a

[3] In fact, the considered GMPE uses fault distance, while herein it is used as if it was epicentral distance.

threshold in terms of IM, disaggregation results in a distribution that, given exceedance of the considered IM-level, provides the probability, for example, of each possible magnitude-distance being the causative event for such an exceedance at the considered location, $P\left[M=m\cap R=r\middle|IM>im\right]$. Such a distribution may be obtained via the Bayes' theorem (Mood et al., 1974) as in Equation (4). As an example, Figure 3 shows disaggregation of the PGA with 10% exceedance probability in thirty years, $PGA(10/30)$, for sites 1 and 2.

$$P\left[M=m\cap R=r\middle|IM>im\right]=\frac{\nu\cdot P\left[IM>im\middle|m\cap r\right]\cdot P\left[M=m\cap R=r\right]}{\lambda_{IM}} \tag{4}$$

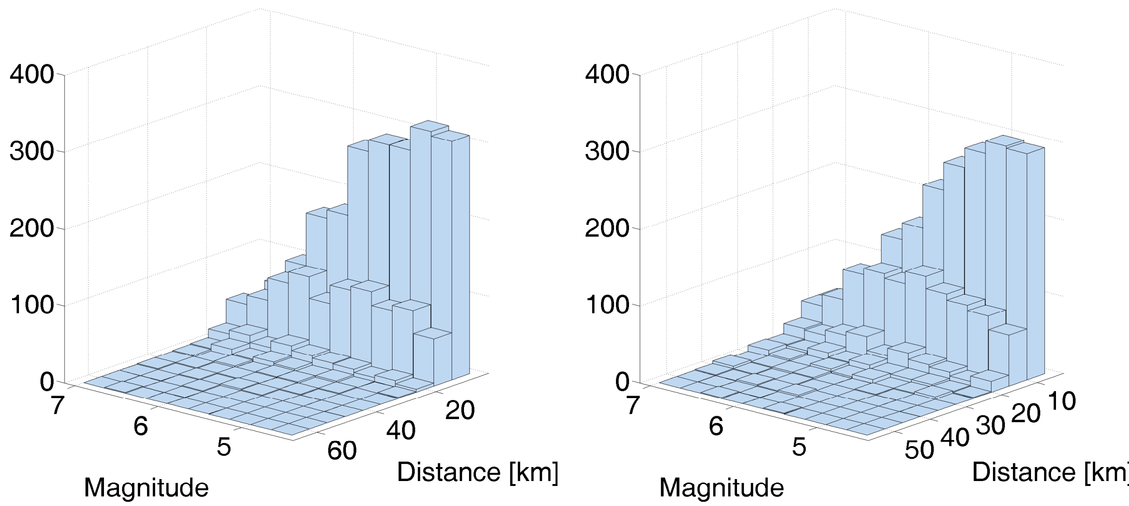

Figure 3. 10% in 30 yr PGA hazard disaggregation for site 1 (left) and site 2 (right). Vertical axis is the probability of the *M-R* pair being causative for the exceedance.

It follows from disaggregation, that once exceedance is observed at site 1 in one earthquake, then the probability of exceedance of site 2 changes with respect to the hazard curve for the site, which is the definition of stochastic dependency, Equation (5). As it will be clarified in the following, this has important reflections on validation of a hazard map, which includes both sites 1 and 2.

$$\begin{aligned}
&P\left[IM_2>im_2\middle|IM_1>im_1\right]=\\
&=\sum_{m,r}P\left[IM_2>im_2\middle|IM_1>im_1\cap m\cap r\right]\cdot P\left[M=m,R=r\middle|IM_1>im_1\right]\neq\\
&\neq P\left[IM_2>im_2\right]=\sum_{m,r}P\left[IM_2>im_2\middle|m\cap r\right]\cdot P\left[M=m\cap R=r\right]
\end{aligned} \tag{5}$$

In fact, on the basis of results obtained via the montecarlo simulation described above, it is possible to calculate the probability that a generic earthquake causes exceedance of specific values of intensity, im_1 and im_2, at the sites 1 and 2. For example, im_1 and im_2 may be set equal to $PGA_1(10/30)$ and $PGA_2(10/30)$, respectively; i.e., the values of PGA which corresponds a

10% exceedance probability at each of the sites.[4] More specifically, it is possible to compute: the probability, P_0, that in one (generic) event none of the two sites experiences exceedance; the probability, P_1, to observe exceedance in (exactly) one of the two sites; and the probability, P_2, of observing an exceedance in both the sites. Then, it is easy to verify that the simulation leads to different results with respect to those one obtains in the case it is assumed that exceedances at the two sites (in a generic event) are stochastically independent. In fact, under this hypothesis, the number of exceedances of $PGA_1(10/30)$ and $PGA_2(10/30)$, being the sum of independent and equally distributed Bernoulli random variables, can be considered (by definition) a binomial, $B(n,p)$, random variable with $n=2$ and $p=0.003512$. In fact, the mean and the variance of the total number of exceedances in t years of $PGA_i(10/30)$, for both the dependent and the independent case, may be computed as in Equations (6).

$$\begin{cases} \mu(t) = \nu \cdot t \cdot \sum_{i=0}^{2} i \cdot P_i = \nu \cdot t \cdot \left(0 \cdot P_0 + 1 \cdot P_1 + 2 \cdot P_2\right) \\ \sigma^2(t) = \nu \cdot t \cdot \left(\sum_{i=0}^{2} i^2 \cdot P_i\right) = \nu \cdot t \cdot \left(0^2 \cdot P_0 + 1^2 \cdot P_1 + 2^2 \cdot P_2\right) \end{cases} \tag{6}$$

Means and variances obtained for $t=1$ and $\nu=1$ using the distribution obtained via montecarlo simulation, for different pairs of sites in Figure 1, and those obtained using the binomial distribution, are reported in the following Table 1. For the pairs in the second and third columns, the values considered for the $PGA_3(10/30)$ and $PGA_4(10/30)$ as well as the values of probability P_0, P_1, and P_2 are obtained adopting the same approach used for sites 1 and 2 in the example above.

Table 1. Mean and variance of the random variable counting the number of exceedances in one year for pairs of sites.

	Sites 1,2	Sites 1,4	Sites 2,3	Any two sites considered independent
Mean	0.0070	0.0070	0.0070	0.0070
Stand. Dev.	0.0839	0.0838	0.0853	0.0840

Results show that, as expected, the binomial model allows to calculate correctly the mean number of exceedances (which is the same for sites $\{1,2\}$, $\{1,4\}$, and $\{2,3\}$), yet it does not allow to calculate the *exact* (i.e., according to the considered assumptions) value of the variance, which from simulation results larger considering the pair of sites $\{2,3\}$ and smaller for $\{1,4\}$ and about equal for $\{1,2\}$. This result is due to the fact that the binomial model is not able to account for the negative correlation that exist between exceedances in sites $\{1,4\}$, which are relatively far from each other, and the positive correlation between exceedances in sites relatively close, $\{2,3\}$. It only approximates the results for sites $\{1,2\}$, at an intermediate distance.

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[4] $PGA(10/30)$ implies probability of exceedance in one event equal to 0.003512. Indeed, from Equation (2): $P\left\{T\left[PGA(10/30)\right] \leq 30\right\} = 1 - e^{-1 \cdot P\left[PGA > PGA(10/30)\right] \cdot 30} = 0.1$, from which $P\left[PGA > PGA(10/30)\right] = 0.003512$ derives.

3. THE EFFECT OF REGIONAL HAZARD ON SITE-SPECIFIC HAZARD TESTING

In order better clarify the importance of the arguments discussed so far, it can be worthwhile to illustrate the implications they can have in practical applications. To do so, in the next section, mathematical and conceptual details of the validation study discussed in Albarello and D'amico (2008) are recalled. Subsequently, it is finally shown how neglecting stochastic dependence can lead to erroneous conclusions.

3.1. An Italian Hazard Validation Study

In Albarello and D'amico (2008) interesting validation problem is discussed. The aim of the study was to validate the 10% in 30 yr PGA hazard from the official nationwide hazard map. To this aim, the authors gathered data from sixty-eight seismic station operating during a thirty years period across the entire Italian territory. In fact, for these stations, which recorded thirty-eight earthquakes, according to the Italian accelerometric archive or ITACA (http://itaca.mi.ingv.it/), it was observed that thirteen times (collectively) the PGA with a 10% in 30 years exceeding probability according to the hazard map of Stucchi et al. (2011), was actually exceeded. In Figure 4 the map with the stations as well as the location and year of earthquake occurrence and/or of exceedance is given.

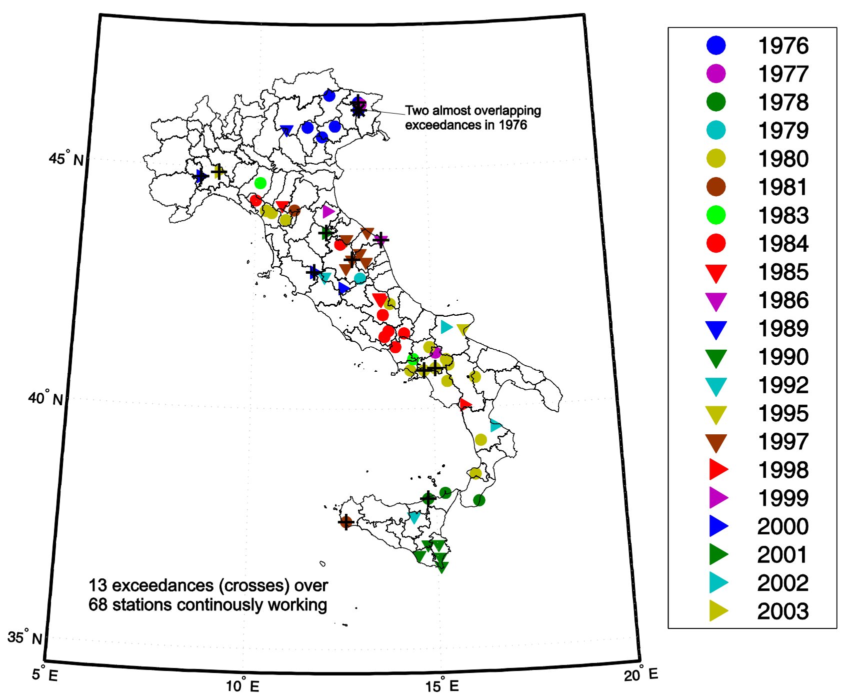

Figure 4. Seismic stations operating in a thirty years time frame, earthquake date and exceedance of PGA(10/30) indicated with crosses.

In the cited study, in order to perform the requested statistical test, authors adopt two approaches, both based on the hypothesis that exceedances in different sites, given that these are *sufficiently far away from each other,* can be considered stochastically independent. In particular, in the first approach, termed the *counting approach*, the event of exceedance of PGA with 10% in 30 years exceeding probability at each site *i*, is modeled as a

Bernoulli random variable which assumes value 0 if exceedance is not observed, and value 1 if $PGA_i(10/30)$ is exceeded *at least once* in thirty years at site *i*.[5] This random variable is characterized by 0.1 probability of observing the exceedance, which is a direct consequence of definition of $PGA_i(10/30)$.[6] Indeed, Bernoulli variables counting the exceedance of $PGA_i(10/30)$ at least once in in thirty years at different sites are equally distributed with $p=0.1$. Then, under the independence hypothesis, the probability to observe *k* exceedances in thirty years over the sixty-eight stations is given by the binomial distribution $B(n,p)$ in Equation (7), where the number of trials, *n,* is 68 and $p=0.1$.

$$P\left[k\ exceedances\ of\ PGA(10/30)\ across\ 68\ stations\ in\ 30\ yr\right]=\binom{68}{k}\cdot 0.1^k\cdot 0.9^{68-k} \quad (7)$$

Consequently, they computed the mean and the variance of the number of sites in which at least an exceedances in thirty years is observed as $n\cdot p=68\cdot 0.1=6.8$ and $n\cdot p\cdot(1-p)=68\cdot 0.1\cdot(1-0.1)=6.12$ respectively. Finally, they performed a formal statistical test to check the (null) hypothesis that the exceedances probability at the generic site is 0.1, as suggested by the Italian hazard map, against the (alternative) hypothesis that this probability is probability larger than 0.1. In fact, considered that from available data it results that in 30 years the discussed exceedance has been observed in thirteen of the sixty-eight sites and noted that for the central limit theorem it can assumed that:

$$P\left[\left|number\ of\ exceedances-6.8\right|>1.96\cdot\sqrt{6.12}\right]\cong 0.05 \quad (8)$$

they concluded that, being $13-6.8=6.2>1.96\cdot\sqrt{6.12}=4.85$, the observed number of exceedances, give evidence (at 0.05 significance level) that the real value of the exceedances probability at the generic site is larger than 0.1.

3.2. Results obtained accounting for stochastic dependence

In this section it is shown how the presence of stochastic dependency among exceedances at different sites can invalidate decisions taken on the basis of Equation (8). In fact, the use of the binomial model in presence of the discussed form of stochastic dependency, which, ultimately, depends on the spatial distribution of the considered sites with respect to the sources zone, can give a value of the variance that can differ from the *exact* one in a way to change the result of a hypothesis test.

To further illustrate this issue, let consider the ideal seismic source introduced in Figure 1. In this case it is supposed that sixty-eight sites exist (Figure 5). The simulation in this case is performed adopting the same numerical approach and source features previously listed. For each of the 10^4 value of M and for all possible event locations (which are also sixty-eight), the PGAs at each of the sites were simulated. The obtained set of $68\cdot 10^4$ observations was used to compute the values of $PGA_i(10/30)$, $\{i=1,2,...,68\}$, as well as the values of probabilities $\{P_0,P_1,...,P_{68}\}$

[5] Note that, obviously, each of the sites has different values of $PGA(10/30)$ corresponding to the same exceedance probability.

[6] This result, that rigorously applies under the common hypotheses of PSHA, neglects the case of exceedance in aftershock sequences; e.g., Iervolino et al., 2014).

that a single, generic, event causes exceedances of $PGA_i(10/30)$ at any given number of sites simultaneously.

In Figure 6 the probabilities obtained from the simulations are compared to the case it is assumed independence of exceedance events; i.e., in the case number of exceedance in a single event is modeled as a binomial $B(n,p)$ random variable with $n=68$ and $p=0.003512$.

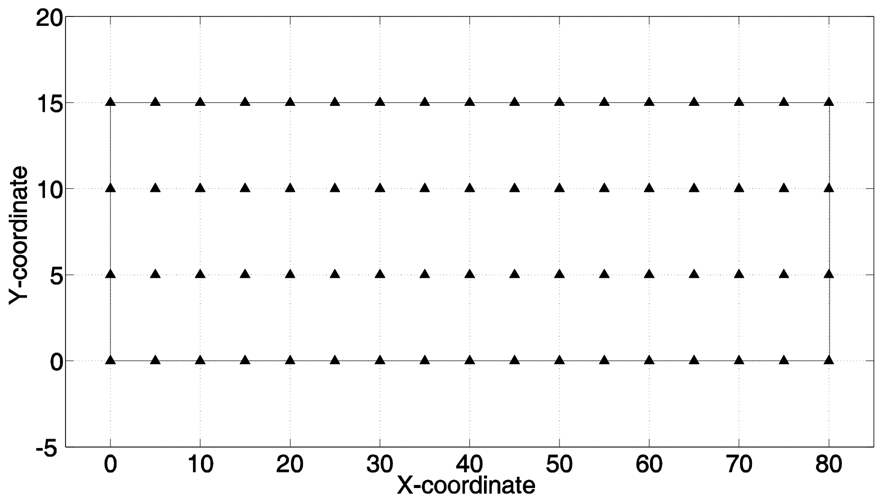

Figure 5. Ideal seismic source zone and sites (distances in km).

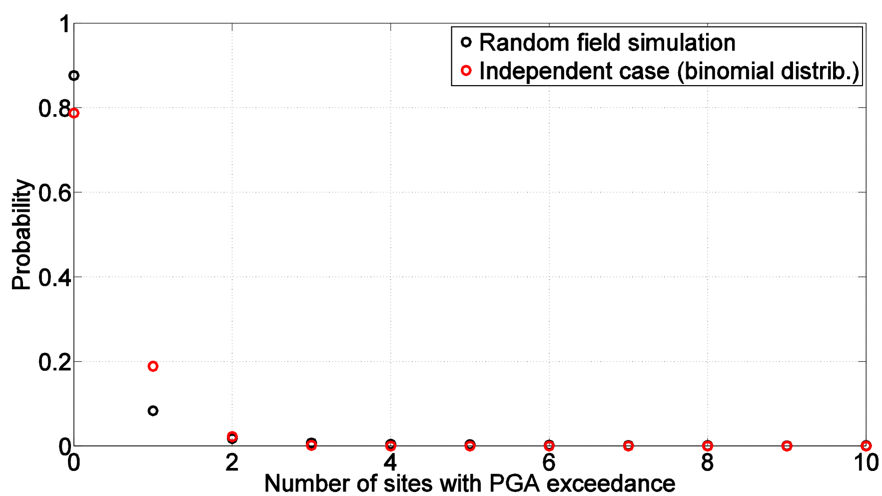

Figure 6. Distributions, in one year, of the number of sites with contemporary exceedance of the PGA with 10% exceedance probability in 30 years.

It may be seen that the spatial dependency of ground motions significantly affects the distributions. Indeed, from Equation (6), Equations (9) results. The mean computed adopting the binomial model (i.e., $68 \cdot 0.003512 \cdot 30 = 7.16$) coincides, as expected, with the mean calculated using the simulation, whereas a difference exists between the variances, which result equal to 8.85 and 26.54 for the binomial model and for the model obtained via simulation, respectively.

$$\begin{cases} \mu(t) = \nu \cdot t \cdot \left(0 \cdot P_0 + 1 \cdot P_1 + ... + 68 \cdot P_{68}\right) \\ \sigma^2(t) = \nu \cdot t \cdot \left(0^2 \cdot P_0 + 1^2 \cdot P_1 + ... + 68^2 \cdot P_{68}\right) \end{cases} \tag{9}$$

At this point, re-applying Equation (8), on the basis of these results, the decision rule in Equation (10) is obtained.

$$\begin{cases} Simulation: P\left[\left|number\ of\ \ exceedances - 7.16\right| > 1.96 \cdot \sqrt{26.54}\,\right] \cong 0.05 \\ Independent: P\left[\left|number\ of\ \ exceedances - 7.16\right| > 1.96 \cdot \sqrt{8.85}\,\right] \cong 0.05 \end{cases} \tag{10}$$

Supposing that in thirty years thirteen exceedances have been observed over sixty-eight sites, because $13 - 7.16 = 5.84$ and $1.96 \cdot \sqrt{8.85} = 5.83$, adopting the binomial model would lead to conclude, that at the significance level $\alpha = 0.05$, data give evidence that the true p is different from 0.1. On the other hand, because $1.96 \cdot \sqrt{26.54} = 10.1$, the simulations allow to verify that in the considered case the observed number of exceedance is consistent (at the same significance level) with the (null) hypothesis that the exceedance probability at the generic site is 0.1, as suggested by PSHA.[7]

4. CONCLUSIONS

The paper discussed some arguments, which should be taken into consideration when attempting to validate probabilistic seismic hazard studies versus observed earthquakes. In particular, it was discussed that ground motion intensity records at different seismic stations in the same earthquake are not independent. Such a form of spatial stochastic dependence arises, primarily but not only, from the ground motion prediction equation, and is confirmed by seismic hazard disaggregation. Indeed, given that the exceedance at one site is observed, the probability of exceedance at another site changes with respect to the hazard curve. As a consequence, the test statistic to validate hazard cannot rely on models that are not able to account for these form of dependence.

To quantitatively evaluate the effect of such a dependence on possible observed samples of ground motion exceedances, some simple examples were set up. They consisted of an ideal seismic source and some sites affected by its seismicity. It was shown that the variance of the number of exceedances may results larger than that obtained under the hypothesis that exceedance in different sites are independent. It is also shown, how accounting for this dependence may change the results of statistical tests adopted in validation study from reject to not-reject the hypothesis that observations are consistent with the probabilistic seismic hazard map.

It is believed that these arguments, very simple from the statistical point of view, can help the future validations of hazard studies, a field of commendable effort for earthquake engineers and engineering seismologists.

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[7] Here attention is focused on the total number of exceedance in thirty years. Hence a little difference is obtained in terms of both mean and variance with respect to results obtained by Albarello e D'amico (2008), where it is considered the number of sites in which at least one exceedance is observed. This little difference doesn't affect the validity of the results obtained in this section.

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