

Estimation uncertainty for some common seismic fragility curve fitting methods

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ABSTRACT

This technical note illustrates and makes available some simple procedures to assess the estimation uncertainty for the parameters of seismic fragility curves. The considered fragility fitting methods refer to the lognormal assumption and are supposed to be based on the results of multi-stripe dynamic analysis of a deterministic non-linear structural model, so that the uncertainty in the fragility parameters arises from the so-called record-to-record variability. The discussed procedures are based on the statistics approach of resampling with substitution, which is commonly referred to as bootstrap. It is also briefly discussed how the estimation uncertainty depends on the maximum value of the probability of failure given seismic intensity that is observed from structural analysis. This work may aid earthquake engineering practice because, both the curve fitting and estimation uncertainty algorithms are implemented in a major update of an application-ready software tool made available at <https://www.reluis.it/it/progettazione/software/r2r-eu.html>.

1. Introduction

Although not strictly required to evaluate the seismic reliability of structures (e.g., Ref. [1]), fitting a parametric fragility curve to the results of structural analysis might be convenient for several reasons, the most important of which is completely defining a surrogate vulnerability model for the structure in question via a few parameters. Among the different approaches possible, seismic fragility analysis is often based on simulations of the dynamic behavior of a non-linear structural model. These simulations are typically performed for some pre-defined values, hereafter referred to as *stripes*, of a ground motion intensity measure or IM. At each stripe the structural model is excited by a number of ground motions (GMs) sharing the same IM value. Based on the way GMs are selected and manipulated this kind of analysis is referred to as incremental dynamic analysis (IDA) or multi-stripe analysis (MSA), the latter generally considered being more advanced than the former [2,3]. The measured structural response to each GM, which is expressed in terms of a so-called engineering demand parameter (EDP), is used to evaluate the seismic vulnerability at the investigated IM stripes and to eventually fit a parametric fragility curve. Literature provides a great deal of fitting methods, which however typically refer to the lognormal shape of the obtained curve.

It has been discussed in literature that the sample of GMs used for the structural analysis and the limited explicative power of common IMs

with respect to typical EDPs, let to arise an issue known as to *record-to-record variability*; i.e., records sharing the same IM value, determine different structural responses (EDP values). In statistical terms, this is referred to as *sample-to-sample variability* of response, which leads to estimation uncertainty in the fragility, and ultimately in the seismic structural reliability obtained after integrating the fragility with the hazard curve for the construction site; i.e., within the performance-based earthquake engineering framework (e.g., Refs. [4–6]). Possibly, this uncertainty can be considered large, and therefore should be quantified to gather meaningful insights on the derived fragility model.

The objective of this technical note is to discuss simple procedures to get a glance of the estimation uncertainty for some possible fragility fitting methods referring to the lognormal assumption. It is assumed that the structural analysis is conducted within the MSA framework, given that other methods are somewhat addressed in the literature. Moreover, it is assumed that the fragility fitting approach is *EDP-based*, according to the terminology of [2]. Three procedures, based on: (i) maximum likelihood or ML (partly addressed in Ref. [7]); (ii) Gaussian probability plot or GPP; (iii) and minimum least squares or MLS, are considered. The estimation uncertainty algorithms for these fitting procedures are based on the statistics' theory of resampling with substitution, or bootstrapping [8], of structural analysis results. The contribution to earthquake engineering practice of this work is that the discussed fragility fitting and estimation uncertainty assessment procedures have been

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coded in a major upgrade of the application-ready R2R-EU software [7], in particular (ii) and (iii) which are new with respect to implementation (see section 3).

The remainder of the note is structured such that in the next section the EDP-based fragility assessment framework is recalled first. Then the equations for the lognormal fragility parameters are given for the three considered methods. Subsequently the resampling approaches to quantify the estimation uncertainty are illustrated. It is also briefly discussed how the parameters' uncertainty depends on the lumped fragility directly calculated from structural response data. Some final remarks conclude the note.

2. Failures, collapse cases, and fragility, in the EDP-based approach

In the EDP-based framework, for an IM stripe identified by im , the goal of fragility analysis is to evaluate the probability of structural failure given im , which can be indicated as $P[f|IM = im]$. In turn, failure is assumed to occur because an EDP exceeds a limit value, say edp_f . It is also possible that failure is identified by convergence issues of the structural analysis or *numerical instabilities*, which are referred to as *collapse cases* [9]. In such cases an EDP value (i.e., a measure of structural response) is not available from the analysis, yet failure is assumed certain. An example of MSA, featuring ten stripes (i.e., ten IM-values) and twenty records per stripe, is given in Fig. 1 (left). In the figure the

$$\{\hat{\eta}, \hat{\beta}\} = \operatorname{argmax}_{\eta, \beta} \left[\sum_{i=1}^n \left(\ln \left(\frac{m}{m_{f,im_i}} \right) + m_{f,im_i} \cdot \ln \left\{ \Phi \left[\frac{\ln(im_i) - \eta}{\beta} \right] \right\} + (m - m_{f,im_i}) \cdot \ln \left\{ 1 - \Phi \left[\frac{\ln(im_i) - \eta}{\beta} \right] \right\} \right) \right], \quad (3)$$

EDP is the demand-to-capacity ratio, so that $edp_f = 1$, and the number above the plot indicates the number of collapse cases occurring at some stripes.

If the collapse cases are indicated as C , fragility can be evaluated via an application of the total probability theorem:

$$P[f|IM = im] = P[C|IM = im] + P[EDP \geq edp_f | \bar{C}, IM = im] \cdot \{1 - P[C|IM = im]\}, \quad (1)$$

where $P[EDP \geq edp_f | \bar{C}, IM = im]$ is the probability of failure given im when non-collapse occurs, while $P[C|IM = im]$ is the probability of collapse given $IM = im$. If, at the stripe in question, structural simulation is carried out via a sample of GMs of size m , the terms appearing in equation (1) can be estimated, for example, via the frequentist approach:

$$\begin{cases} P[C|IM = im] \approx \frac{m_C}{m} \\ P[EDP \geq edp_f | \bar{C}, IM = im] \approx \frac{1}{m - m_C} \cdot \sum_{j=1}^{m - m_C} I_{edp_j \geq edp_f} \end{cases} \quad (2)$$

In this last set of equations, m_C is the number of analyses leading to collapse and $I_{edp_j \geq edp_f}$ is an indicator function that, for each of the analyses providing meaningful structural response values, equals one if the EDP is larger than the failure threshold and zero otherwise. Clearly, replacing equation (2) in equation (1) yields a value of the fragility equal to the total number of failure observed at the stripe in question: $P[f|IM = im] = \left(m_C + \sum_{j=1}^{m - m_C} I_{edp_j \geq edp_f} \right) / m$. The fragility values evaluated in this way for the example of Fig. 1 (left) are given in Fig. 1 (right)

as crosses.

If the structural analyses are repeated for a number of different IM-stripes, n in number, this leads to a set of fragility values $P[f|IM = im_i]$, $i = 1, 2, \dots, n$, which can be collectively referred to as the *lumped fragility* of the structural model in question. These may be used to fit a continuous and parametric fragility curve, as discussed in the next section. At this stage is it to note, however, that equation (2) only provides estimates of the terms in equation (1) as it is expected that changing the sample of records used leads to variations of the estimates (i.e., sample-to-sample variability), which is at the roots of the estimation uncertainty of the fragility parameters discussed in section 4.

3. Some lognormal fragility fitting methods

3.1. Maximum likelihood

Given that in MSA, at each of the n stripes, m structural analyses are conducted (for simplicity it is assumed that m is constant through the stripes), at the end of the analysis, vectors (samples) of the kind $edp_i = \{edp_{i,1}, edp_{i,2}, \dots, edp_{i,m}\}$, $i = \{1, 2, \dots, n\}$, are available. Each of them can be partitioned in two: one with failure cases including numerical instabilities, of size m_{f,im_i} , and one of non-failure cases, of size $(m - m_{f,im_i})$. The parameters of the lognormal fragility function, $\{\hat{\eta}, \hat{\beta}\}$, can be estimated, as developed in Ref. [10], maximizing the likelihood as:

where $\Phi(\cdot)$ is the standard cumulative Gauss function and the hats on the parameters indicate that they are estimates of the *true*, unknown, parameters. The lognormal fragility can be expressed as $P[f|IM = im] = \Phi\{[\ln(im) - \hat{\eta}] / \hat{\beta}\}$. The thick black line in Fig. 1 (right) is an example of fragility fitted via ML based on the data in Fig. 1 (left).

3.2. Gaussian probability plot

The ML approach has the advantage of not distinguishing between structural analysis when the EDP is available and collapse cases. However, it requires a certain number of failure cases be observed across the IM stripes (see Ref. [10] for a discussion). However, there are cases in which only a few failures, if any, are observed at each stripe, so that the ML fragility fitting is difficult (this may happen, for example, when the structural vulnerability is low compared to the seismic hazard at the site of interest; e.g., Ref. [11]). In this case, fragility analysis must be based mostly on the extrapolation of the structural response. In fact, provided that at each of the n stripes an EDP vector is available, the lognormal fragility can be fitted as follows:

1. For each $IM = im_i$, $i = \{1, 2, \dots, n\}$, response data are divided in collapse cases, if any, and non-collapse cases, the count of which are m_{C,im_i} and $m_{\bar{C},im_i}$; therefore, a vector of the kind $edp_i = \{edp_{i,1}, edp_{i,2}, \dots, edp_{i,m_{\bar{C},im_i}}\}$, $i = \{1, 2, \dots, n\}$ is available,
2. The probability of failure based on the non-collapse cases is evaluated, for example, based on the assumptions that EDP is distributed according to a lognormal model given $IM = im_i$, so that:

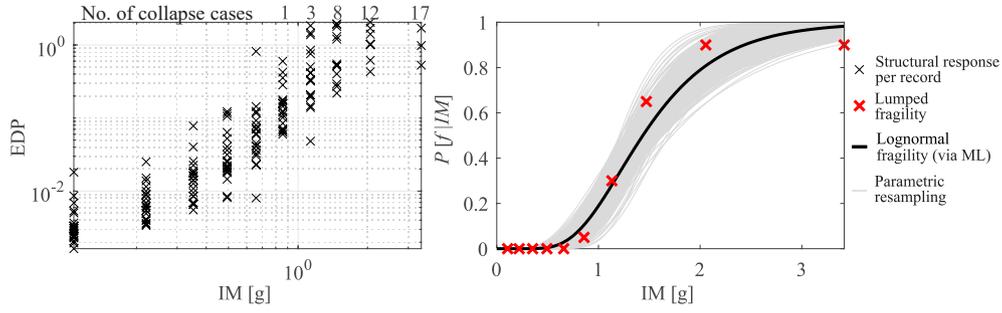


Fig. 1. Left: results of a representative MSA analysis with ten stripes and twenty records per stripe. Right: fragility obtained with ML estimation and distribution of fragilities obtained with five-hundred parametric resampling runs.

$$P[EDP \geq edp_f | \bar{C}, IM = im_i] = 1 - \Phi \left\{ \frac{\ln(edp_f) - \mu_{\ln(EDP)} | \bar{C}, im_i}{\sigma_{\ln(EDP)} | \bar{C}, im_i} \right\}, \quad (4)$$

where $\{\mu_{\ln(EDP)} | \bar{C}, im_i, \sigma_{\ln(EDP)} | \bar{C}, im_i\}$ are the mean and the standard deviation of the logarithms of EDP when $IM = im_i$, which can be frequently estimated as:

$$\{\hat{\eta}, \hat{\beta}\} = \underset{\eta, \beta}{\operatorname{argmin}} \left[\sum_{i=1}^n \left(\Phi^{-1} \left\{ \frac{m_{C, im_i}}{m} + \left[1 - \Phi \left(\frac{\ln(edp_f) - \mu_{\ln(EDP)} | \bar{C}, im_i}{\sigma_{\ln(EDP)} | \bar{C}, im_i} \right) \right] \cdot \left(1 - \frac{m_{C, im_i}}{m} \right) \right\} - \frac{\ln(im_i) + \eta}{\beta} + \frac{\eta}{\beta} \right)^2 \right]. \quad (8)$$

$$\begin{cases} \mu_{\ln(EDP)} | \bar{C}, im_i \approx \frac{1}{m - m_{C, im_i}} \cdot \sum_{j=1}^{m - m_{C, im_i}} \ln(edp_{i,j}) \\ \sigma_{\ln(EDP)} | \bar{C}, im_i \approx \sqrt{\frac{1}{m - m_{C, im_i} - 1} \cdot \sum_{j=1}^{m - m_{C, im_i}} \left[\ln(edp_{i,j}) - \mu_{\ln(EDP)} | \bar{C}, im_i \right]^2} \end{cases}; \quad (5)$$

- The lumped fragility value for $IM = im_i$ is computed via equation (1), where $P[C|IM = im_i] = m_{C, im_i} / m$;
- Alternatively, at each stripe, value equal to one can be assigned to those analyses leading to collapse and zero to the others, and logistic regression (e.g., Ref. [12]) can be performed to obtain the collapse

$$\{\hat{\eta}, \hat{\beta}\} = \underset{\eta, \beta}{\operatorname{argmin}} \left(\sum_{i=1}^n \left\{ \frac{m_{C, im_i}}{m} + \left[1 - \Phi \left(\frac{\ln(edp_f) - \mu_{\ln(EDP)} | \bar{C}, im_i}{\sigma_{\ln(EDP)} | \bar{C}, im_i} \right) \right] \cdot \left(1 - \frac{m_{C, im_i}}{m} \right) - \Phi \left(\frac{\ln(im_i) - \eta}{\beta} \right) \right\}^2 \right) \quad (9)$$

probability as a continuous function of IM:

$$P[C|IM = im] = 1 / [1 + e^{-(\alpha_1 + \alpha_2 \cdot im)}], \quad (6)$$

where $\{\alpha_1, \alpha_2\}$ are the logistic regression coefficients (see the next section for an example);¹

- Once all the terms of equation (1) are computed, n fragility values,

¹ There are alternative functions, other than the logistic, that can be employed to model the collapse probability.

$P[f|IM = im_i]$, $i = 1, 2, \dots, n$, are available, and from them, the corresponding ordinates of a GPP, $\{z_1, z_2, \dots, z_n\}$ can be obtained as in the following equation, where $\Phi^{-1}(\cdot)$ indicates the inverse Gauss function:

$$\hat{z}_i = \Phi^{-1} \{P[f|IM = im_i]\}, \quad i = 1, 2, \dots, n; \quad (7)$$

- At this point, ordinary least square regression of the $\{\ln(im_i), \hat{z}_i\}$, $i = 1, 2, \dots, n$ data, can be performed (e.g., Ref. [13]):

In fact, this yields a line the slope of which is $\hat{\beta}^{-1}$ and the intercept is $-\hat{\eta}/\hat{\beta}$, which are function of the fragility parameters.² Fig. 2 (left) shows the GPP for the data of Fig. 1 (left), while Fig. 2 (right) shows the fitted fragility (thick black line).

3.3. Minimum least-squares

This method is similar to the previous one to the extent that steps 1-2 are the same. However, once the $P[f|IM = im_i]$, $i = 1, 2, \dots, n$, values are available, the fragility parameters are directly obtained minimizing the sum of squared errors [14] as:

Fig. 3 (right) shows the fragility (thick black line), based on MSA of Fig. 1 (left) obtained via MLS.³ In this case, logistic regression was performed to obtain the collapse probabilities; the latter is given in Fig. 3

² This equation features a frequentist estimate of the collapse probability; in the logistic case, $1/[1 + e^{-(\alpha_1 + \alpha_2 \cdot im)}]$ replaces $m_{C, im_i}/m$.

³ The figure shows lumped fragility values somewhat different from the previous example because, in this case, the collapse probability is modelled via the logistic regression.

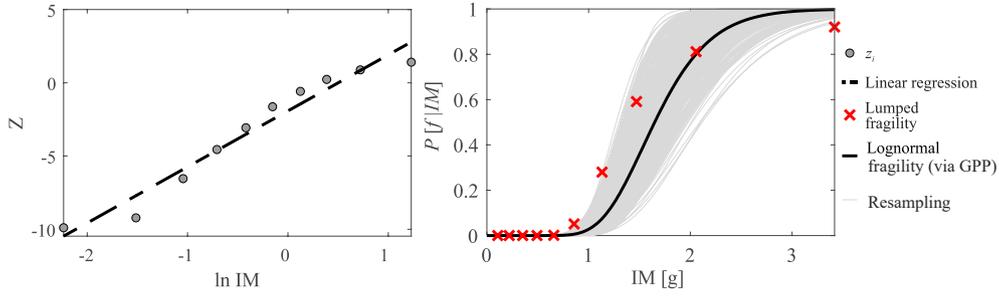


Fig. 2. Left: example of Gaussian probability plot and fitting of data points from MSA. Right: fragility obtained with the GPP method and related distribution of fragilities with five-hundred non-parametric resampling runs.

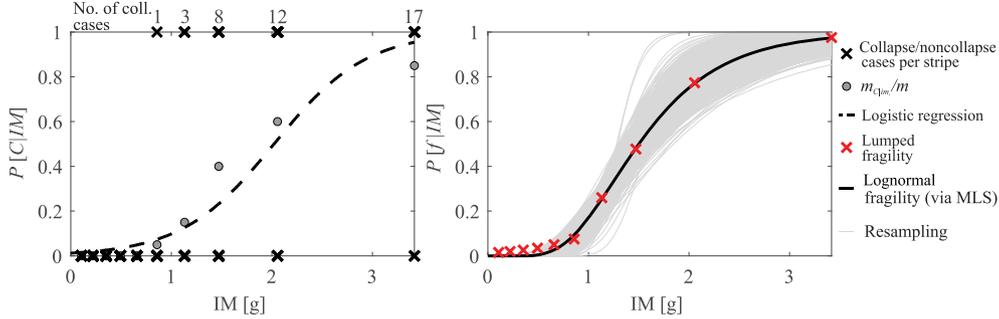


Fig. 3. Left: example of logistic regression to determine the collapse probability as a function of IM. Right: fragility obtained with the MLS method (using logistic regression for the collapse cases) and related distribution of fragilities with five-hundred non-parametric resampling runs.

(left) for completeness.

4. Assessment of estimation uncertainty

4.1. ML

The parameters evaluated in equation (3) (as well as those from GPP and MLS) are indicated as $\{\hat{\eta}, \hat{\beta}\}$ because, due to record-to-record variability, they can be interpreted as only estimates of the true fragility parameters, $\{\eta, \beta\}$, which are unknown. The distribution of $\{\hat{\eta}, \hat{\beta}\}$ deriving from sample-to-sample variability of responses provides a measure of such estimation uncertainty. A possible procedure (also discussed in Ref. [4]) to derive a distribution of $\{\hat{\eta}, \hat{\beta}\}$ and based on parametric resampling, would consist of the following steps:

1. $\{\hat{\eta}, \hat{\beta}\}$ are initially evaluated from equation (3) and are assumed equal to the true fragility parameters;
2. At each stripe $IM = im_i$, $i = 1, 2, \dots, n$, a number of failures, m_{f,im_i}^* , among m cases is simulated (extracted) from a binomial distribution with parameter $p_i = \Phi\{\ln(im_i) - \hat{\eta}\} / \hat{\beta}$;
3. Step 2 also provides non-failure cases, which are necessarily equal to $m - m_{f,im_i}^*$;
4. After steps 2-3 are repeated $\forall i = \{1, 2, \dots, n\}$, that is, for the each of the n stripes at which MSA is performed, a vector of the kind $\{m_{f,im_1}^*, m_{f,im_2}^*, \dots, m_{f,im_n}^*\}$ is obtained and equation (3) can be re-applied to obtain a new estimation of the fragility parameters that can be indicated as $\{\hat{\eta}^*, \hat{\beta}^*\}$;
5. Repeating these steps (2-4) k times (an arbitrary number) gives a distribution of the parameters and then of the structural fragility.

An example of the results of this procedure when $k = 500$ are given in Fig. 1 (right) as thin gray lines.

4.2. GPP and MLS

In this approach, to account for estimation uncertainty:

1. The EDP vector for each stripe, $edp_i = \{edp_{i,1}, edp_{i,2}, \dots, edp_{i,m_{c,im_i}}\}$, $i = \{1, 2, \dots, n\}$, is resampled with substitution to obtain a new vector, $edp_i^* = \{edp_{i,1}^*, edp_{i,2}^*, \dots, edp_{i,m_{c,im_i}}^*\}$;
2. This data vector is used as the input of steps 1-2 of section 3.2 that ultimately lead to a new set of fragility function parameters, via either equation (8) or equation (9), that can be once again indicated as $\{\hat{\eta}^*, \hat{\beta}^*\}$;
3. Repeating the resampling of edp_i data an arbitrary number of times yields the same number of fragility functions, which can help getting a sense of estimation uncertainty involved in the fragility fitting procedure.

Examples of the results when $k = 500$ are given in Fig. 2 (right) and Fig. 3 (right) as gray thin lines for GPP and MLS, respectively. Note that this is, factually, a non-parametric resampling plan, as opposite to the one of the previous section that can be considered parametric [4]. Also note that in this procedure, the number of collapse cases at each stripe is kept fixed to the value of the original data. However, in principle, this number can also be resampled considering the $P[C|IM = im]$ values initially obtained.

5. Estimation uncertainty and maximum lumped fragility

The uncertainty in the fragility parameters shown in the previous section inherently depends on the structural response recorded at the MSA stripes. A proxy for the estimation uncertainty in the fragility parameters can also be the maximum value of the lumped fragility (*lumped fragility maximum* or LFM) estimated to according equations (1) and (2), that is $LFM \triangleq \max\{P[f|IM = im_1], P[f|IM = im_2], \dots, P[f|IM = im_n]\}$. In fact, as also discussed by literature (e.g., Refs. [10,15]), the fragility curve

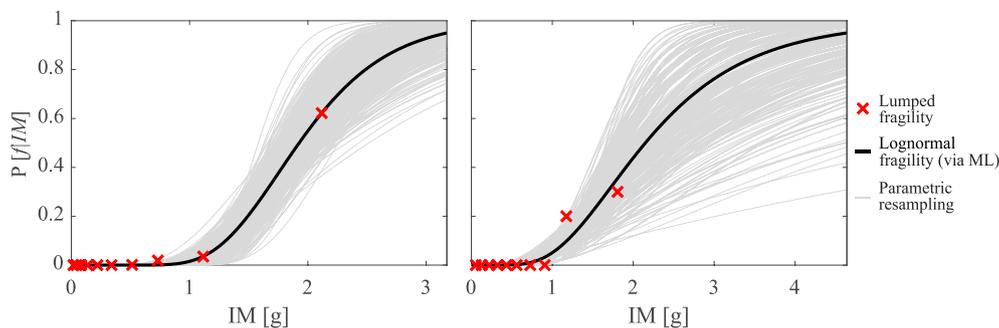


Fig. 4. Fragility curves and resampling results for two cases with different LFM. Left: LFM=0.6. Right LFM=0.3.

estimation is best constrained when the IM discretization at which dynamic analysis is performed gives a large and densely populated range of lumped $P[f|IM = im_i]$ values in the $(0, 1)$ interval. Assuming that the part of the fragility probability interval is well covered by the structural analyses at the lower intensity measures, if the LFM value is relatively small the uncertainty in the fragility parameters is expected to be larger. In this respect, Fig. 4 shows two cases where the lumped fragility values are plotted against the result of the fitting procedure obtained via ML (black solid line). In the shown cases the LFM is within 0.3–0.6 range, while that in Fig. 1 (right) is larger than 0.9.⁴ The figure shows also the results of five-hundred resampling runs and it is apparent the larger variability of the curves as the LFM gets lower.⁵ To quantify this effect, for example, the relative root-mean-square error (RMSE) of the $\hat{\beta}$ parameter of the fragility curve can be used. It is evaluated assuming that the value obtained from equation (3) is the true value (i.e., the first obtained estimate), which is consistent with the parametric resampling approach:

$$RMSE = \sqrt{\frac{1}{k} \sum_{i=1}^k \left(\frac{\hat{\beta}_i^* - \hat{\beta}}{\hat{\beta}} \right)^2} \quad (10)$$

In the equation, k is the number of resampling runs ($k = 500$ in this case). It results that for the fragility in Fig. 1, where $LFM > 0.9$, RMSE is equal to 0.15. $LFM = 0.60$ means that the data from dynamic analysis are available up to IM levels slightly above the median. Fragility fitting continues to represent the trend of the empirical data, but the estimation uncertainty increases with RMSE equal to 0.21. Finally, when $LFM = 0.30$ curve fitting is somewhat getting worse and estimation uncertainty further increases, with a RMSE equal to 0.32, which means that heterogeneity of the parametric resampling around the black solid line significantly increases. The LFM is a candidate for a parameter to be tuned, along with m and n values, to design MSA to gather a desired level of RMSE of the fragility parameters.

6. Final remarks

In this technical note, some estimation uncertainty procedures for some common fragility fitting methods that find their roots in consolidated literature, were discussed. The procedures are based on parametric and non-parametric resampling plans and can be applied to multi-stripe nonlinear dynamic analyses, also considering collapse cases arising from numerical instabilities. Each procedure allows to obtain an arbitrarily large sample of fragility curves providing a sense of

⁴ These cases are from fragility analysis of code-conforming structures in Italy, the details of which can be found in Ref. [16]. In the same study, the effect of LFM on the estimation of the parameters of the fragility curve, also varying the fragility fitting methods, can be appreciated via a large set of case studies.

⁵ However, LFM is expected to be a meaningful proxy when the fragility up to LFM is densely populated and/or when it is close to the median of the fragility.

the uncertainty in estimation of the parameters that, can be used, in turn, to obtain the distribution of the estimator of the structural reliability, after each curve is integrated with the seismic hazard. It is also briefly discussed how, in general, the estimation uncertainty grows for diminishing the maximum values of the lumped fragility frequentist estimates, which therefore can be one of the parameters to design structural analysis to achieve a desired level of estimation uncertainty. The provided algorithms are illustrated in a step-by-step manner to be easily applicable; however, they were coded (including the LFM) within a major update of the R2R-EU software (<https://www.reluis.it/it/progettazione/software/r2r-eu.html>), which is a tool made available for further earthquake engineering applications, along with a dedicated tutorial (https://www.reluis.it/images/stories/R2R/R2R-EU_manual.pdf).

Author statement

Iunio Iervolino is the only author.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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