Bulletin of the Seismological Society of America, Vol. 104, No. 2, pp. 1006–1012, April 2014, doi: 10.1785/0120130207

Short Note

Sequence-Based Probabilistic Seismic Hazard Analysis

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Abstract Earthquakes are typically clustered both in space and time. Only mainshocks, the largest magnitude events within each cluster, are considered by classical seismic hazard, which is expressed in terms of rate of exceedance of a ground-motion intensity measure at a site of interest (Cornell, 1968). This kind of probabilistic seismic hazard analysis (PSHA) is used for structural design or assessment in the long term. Recently, for short-term risk management purposes, a similar approach has been adopted to perform aftershock probabilistic seismic hazard analysis (APSHA), conditional to mainshock occurrence (Yeo and Cornell, 2009). PSHA often refers to a homogeneous Poisson process to describe event occurrence, whereas APSHA models aftershock occurrence via a conditional nonhomogeneous Poisson process, the rate of which depends on the magnitude of the mainshock that has triggered the sequence. On the other hand, the clusters, each of which is composed of the mainshock and the following aftershocks, may be seen as single events occurring at the same rate of the mainshocks. This may allow accounting for aftershocks in hazard analysis in a relatively simple manner, as first argued by Toro and Silva (2001) and further investigated by Boyd (2012). In fact, this short note, focusing on the probabilistic aspects, shows the feasibility of analytically combining results of PSHA and APSHA to get a seismic hazard integral accounting for mainshock–aftershocks seismic sequences, which was still missing from the mentioned studies. The results of the application presented help to preliminarily assess the increase in seismic hazard in terms of rate of occurrence of events causing the exceedance of an acceleration threshold (e.g., that considered for structural design) also considering aftershocks. That is a relevant aspect from the earthquake engineering perspective.

Introduction

The probabilistic seismic hazard analysis (PSHA; e.g., McGuire, 2004) is a consolidated procedure to assess the seismic threat for a specific site. PSHA, in its classical format, refers to the occurrence of mainshocks. These are prominent magnitude earthquakes possibly identified within sequences of events concentrated both in space and time (i.e., clusters).

On the other hand, aftershocks in the sequence may be seen as triggered by the mainshock. The features of each sequence are considered to depend only on the magnitude and location of the triggering event, being conditionally independent (in stochastic sense) of past history. On these premises, Yeo and Cornell (2009) developed aftershock probabilistic seismic hazard analysis (APSHA) to express aftershock hazard similar to the mainshock hazard. Indeed, APSHA results are in terms of rate of exceedance of a ground-motion intensity measure (IM) threshold. This is useful in the postmainshock emergency phase; see Yeo and Cornell (2005) for discussion.

It may be argued that the occurrence of clusters can be probabilistically described by the same stochastic process adopted to count the main events. In this context, it is assumed that the occurrence time for each cluster coincides with that of the triggering earthquake. Indeed, starting from Toro and Silva (2001) and Boyd (2012), it appears possible to extend PSHA multiplying the rate of occurrence of mainshocks by the probability that a ground-motion IM threshold is exceeded at least once during the sequence. This means filtering the rate of occurrence of the clusters retaining only those causing the sought exceedance event.

From the engineering point of view, computing the rate of the event referring to the exceedance of a ground-motion intensity level (e.g., that critical to a structure) during the sequence, factually means also considering the chance that an aftershock causes structural failure, whereas the mainshock did not. This leads to sequence-based PSHA (SPSHA), which may be relevant for performance-based seismic design. It allows determining the exceedance rate of the design intensity accounting for the aftershock potential (Iervolino, Giorgio, and Polidoro, 2013). As per common practice of current seismic codes, damage accumulation on the structure is neglected, whereas it may be of interest for short-term risk management; see Yeo and Cornell (2005) and Iervolino, Giorgio, and Chioccarelli (2013) for some results in this direction.

The study presented in the following, starting from the intuitions of the mentioned studies, derives the analytical formulation of SPSHA, that is including aftershocks in the hazard integral, which was still missing in Toro and Silva (2001) and Boyd (2012). It is built on the hypotheses that occurrence of mainshocks is regulated by a homogeneous Poisson process (HPP), whereas occurrence of aftershocks is regulated by a conditional nonhomogeneous Poisson process (NHPP). It is assumed that the rate of occurrence of the aftershocks pertaining to a given sequence, their magnitude range, and their spatial clustering, only depend on magnitude and location of the triggering mainshock. In the paper, foreshocks are neglected, as they are usually very limited in number (Yeo and Cornell, 2009).

As illustrated in the following, the model for aftershocks is based on the modified Omori's law (Utsu, 1961); therefore, the study may be viewed as modeling primary aftershocks. In fact, other models as epidemic-type aftershock sequences (ETAS; e.g., Ogata, 1988) are virtually able to model clusters in which each event is able to generate its own sequence.

The study will not directly deal with issues related to the declustering of earthquakes, which will appear only in terms of the resulting occurrence rate of mainshocks and the parameters of the modified Omori's law that are input data for the proposed model. On the other hand, it is to recall that results obtained for both mainshocks and aftershocks are model dependent. This is because, given the original catalog, clustering is performed on the basis of conventional rules, which are defined via the model one adopts to describe the occurrence of earthquakes.

The paper is structured such that PSHA and APSHA essentials are briefly reviewed first. Then, the combination of the two is analytically discussed to account for the effect of the whole sequence in a single hazard integral. The merely illustrative application, considering a generic seismogenic source, is carried out to compute the annual rate of exceedance of different IM levels using SPSHA, and to evaluate the significance of differences with respect to classical seismic hazard analysis, in which the effects of aftershocks are neglected.

Mainshock, Aftershocks, and Ground-Motion Intensity

In this section, stochastic processes and analytical formulations used to evaluate mainshock and conditional-aftershock hazard, both expressed in terms of rate of exceedance of a ground-motion intensity threshold, are briefly reviewed.

Mainshock Probabilistic Seismic Hazard Analysis

PSHA usually adopts the HPP to probabilistically model the number of earthquakes the seismic source produces. HPP is an independent and stationary increment (i.e., memoryless) process, entirely described by one parameter, the rate ν_E. According to HPP, the number of events N_E occurring in the time interval of interest $(t, t + \Delta T)$ is independent of the history of earthquakes that occurred in the past and has the Poisson probability mass function in equation (1):

$$P[N_E(t, t + \Delta T) = n] = P[N_E(\Delta T) = n] = \frac{(\nu_E \cdot \Delta T)^n}{n!} \cdot e^{-\nu_E \cdot \Delta T}. \qquad (1)$$

It is also consequent to the HPP that the interarrival time distribution of mainshocks is an exponential distribution, in which the mean time between arrivals is the reciprocal of the rate.

In PSHA, at a site of interest, the exceedance of an IM threshold im is also probabilistically described by a HPP (Cornell, 1968). The rate of exceedance of im, $\lambda_{im,E}$, is obtained from ν_E via equation (2), in which the term $P[\mathrm{IM} > im|x, y]$, provided by a ground-motion prediction equation (GMPE), represents the probability that the intensity threshold is exceeded given an earthquake of magnitude $M_E = x$, from which the site is separated by a distance $R_E = y$:

$$\lambda_{im,E} = \nu_E \cdot \int_{r_{E,\min}}^{r_{E,\max}} \int_{m_{E,\min}}^{m_{E,\max}} P[\mathrm{IM} > im|x,y] \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy. \qquad (2)$$

The term f_{M_E,R_E} is the joint probability density function (PDF) of mainshock magnitude and distance random variables (RVs). In the case of a single source, if these two RVs may be considered stochastically independent, f_{M_E} is often described by a Gutenberg–Richter (GR) relationship (Gutenberg and Richter, 1944), and f_{R_E} is obtained on the basis of the source-site geometrical configuration. The integral limits are the magnitudes bounding the GR relationship and the distances defining the domain of possible R_E values (e.g., Reiter, 1990).

Aftershock Probabilistic Seismic Hazard Analysis

APSHA is also expressed in terms of rate of occurrence of events exceeding a ground-motion IM threshold at a site of interest. The main difference with respect to PSHA is that such a rate is time variant. The expected number of events per unit time decreases as the time elapsed since the triggering mainshock increases. In this sense, the process that describes occurrence of aftershocks is conditional to occurrence and characteristics of the mainshock.

The NHPP process adopted to build APSHA is based on the hypothesis that the daily rate of occurrence of the aftershocks $\nu_{A|m_E}(t)$ can be expressed as in equation (3), in which

t indicates the time elapsed since the occurrence of the triggering mainshock, which according to the adopted time scale, occurred at $t = 0$. The model also assumes that magnitude of aftershocks is bounded between a minimum value of interest $m_{\min}$ and that of the triggering mainshock. Coefficients a and b are from a suitable GR relationship, whereas c and p are from the modified Omori's law (Utsu, 1961) for the considered sequence. Finally, given the intensity of the triggering mainshock, intensities of the aftershocks in the sequence are assumed stochastically independent RVs:

$$\nu_{A|m_E}(t) = \frac{10^{a+b\cdot(m_E-m_{\min})} - 10^a}{(t+c)^p}. \tag{3}$$

From equation (3), it follows that the expected number of aftershocks in the $(t, t+\Delta T_A)$ interval is given by equation (4):

$$\begin{aligned} E[N_{A|m_E}(t, t+\Delta T_A)] &= \int_t^{t+\Delta T_A} \nu_{A|m_E}(\tau)\cdot d\tau \\ &= \frac{10^{a+b\cdot(m_E-m_{\min})} - 10^a}{p-1} \cdot [(t+c)^{1-p} \\ &\quad - (t+\Delta T_A + c)^{1-p}]. \end{aligned} \tag{4}$$

Similar to PSHA, APSHA also filters the intensity of the process reducing the rate of occurrence of the events multiplying it by the (time invariant) probability that the IM at the site of interest exceeds the threshold. This leads to the rate of the NHPP process $\lambda_{im,A|m_E}(t)$ as in equation (5), in which f_{M_A,R_A} is the joint PDF of magnitude and source-to-site distance of the generic aftershock:

$$\begin{aligned} \lambda_{im,A|m_E}(t) = \nu_{A|m_E}(t) \cdot \int_{r_{A,\min}}^{r_{A,\max}} \int_{m_{\min}}^{m_E} P[\mathrm{IM} > im|w,z] \\ \cdot f_{M_A,R_A}(w,z)\cdot dw \cdot dz. \end{aligned} \tag{5}$$

The same considerations given in the previous section for f_{M_E,R_E} also apply to f_{M_A,R_A}. Aftershock location, and then source-to-site distance and its limiting values $\{r_{A,\min}, r_{A,\max}\}$, will be discussed later. Indeed, despite the symbols in equation (5), consistent with those of Yeo and Cornell (2009), the rate of exceedance of IM also depends on mainshock location.

Combining the Stochastic Processes for Mainshocks and Aftershocks

In this section, the PSHA accounting for the effects of both mainshock and aftershocks is formulated. The occurrence of sequences is described by a HPP process, and, within a sequence, occurrence of aftershocks is described by a NHPP, the rate function of which is conditional to the magnitude of the triggering event. The aim is, again, to evaluate the annual rate (λ_{im}) of exceedance of a ground-motion IM. Herein, such a rate accounts for the occurrence of events defined as the exceedance of an IM threshold at least once within a sequence, equation (6):

$$\begin{aligned} \lambda_{im} &= \nu_E \cdot P[\mathrm{IM} > im] \\ &= \nu_E \cdot P[\mathrm{IM}_E > im \cup \mathrm{IM}_{\cup A} > im] \\ &= \nu_E \cdot \{1 - P[\mathrm{IM}_E \le im \cap \mathrm{IM}_{\cup A} \le im]\}, \end{aligned} \tag{6}$$

in which IM is the maximum ground-motion intensity among all events in the cluster, IM_E is the mainshock IM and $\mathrm{IM}_{\cup A}$ indicates the maximum intensity among the aftershocks. Indeed, $\mathrm{IM}_{\cup A}$ exceeds the threshold if and only if at least one aftershock produces intensity above the threshold at the site.

According to APSHA, the features of the aftershock sequence entirely depend on the characteristics of the mainshock. The number of events, their magnitude, and their location are function of the size and location of the sequence-triggering earthquake. Therefore, conditional to magnitude and location of the mainshock, the two events defined as the IM threshold is not exceeded: (1) in the mainshock, and (2) in any of the aftershocks, are stochastically independent see equation (7). (Note that this, which follows from the PSHA and APSHA models, is also consistent with Boyd, 2012.)

$$\begin{aligned} \lambda_{im} &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \le im \cap \mathrm{IM}_{\cup A} \le im|x,y] \\ &\quad \cdot f_{M_E,R_E}(x,y)\cdot dx \cdot dy\} \\ &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \le im|x,y] \\ &\quad \cdot P[\mathrm{IM}_{\cup A} \le im|x,y] \cdot f_{M_E,R_E}(x,y)\cdot dx \cdot dy\}. \end{aligned} \tag{7}$$

The probability of not exceeding the threshold during the aftershock sequence is formulated accounting for the fact that such a sequence is composed of a random number of events N_A. According to the NHPP assumption, such an RV is Poisson distributed, as in equation (1), yet with mean given in equation (4). Therefore, applying the total probability theorem to the $P[\mathrm{IM}_{\cup A} \le im|x,y]$ term in equation (7), equation (8) results:

$$\begin{aligned} \lambda_{im} &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \le im|x,y] \cdot \sum_{i=0}^{+\infty} P[\mathrm{IM}_{\cup A} \\ &\quad \le im|x,y,i] \cdot P[N_A = i|x] \cdot f_{M_E,R_E}(x,y)\cdot dx \cdot dy\} \\ &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \le im|x,y] \cdot \sum_{i=0}^{+\infty} (P[\mathrm{IM}_A \\ &\quad \le im|x,y])^i \cdot P[N_A = i|x] \cdot f_{M_E,R_E}(x,y)\cdot dx \cdot dy\} \\ &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \le im|x,y] \cdot \sum_{i=0}^{+\infty} (P[\mathrm{IM}_A \\ &\quad \le im|x,y])^i \cdot \frac{\left(\int_0^{\Delta T_A} \nu_{A|x}(\tau)\cdot d\tau\right)^i}{i!} \cdot e^{-\int_0^{\Delta T_A} \nu_{A|x}(\tau)\cdot d\tau} \\ &\quad \cdot f_{M_E,R_E}(x,y)\cdot dx \cdot dy\}, \end{aligned} \tag{8}$$

in which $P[\mathrm{IM}_{\cup A} \leq im|x,y,i] = 1$ for $i = 0$. $\nu_{A|x}$ reflects the fact that such a rate depends on the mainshock magnitude, and ΔT_A is the duration of the aftershock sequence (the value assumed for this parameter may affect the result of SPSHA as the larger ΔT_A, the larger the mean of the N_A RV, thus the larger the resulting IM exceedance rate). $P[\mathrm{IM}_A \leq im|x,y]$, equal for all aftershocks as per APSHA (Yeo and Cornell, 2009), is the nonexceedance probability of the intensity threshold in the generic aftershock, marginal with respect to its possible magnitude and location, yet given magnitude and location of the mainshock.

Given magnitude and location of the aftershock, the probability the IM threshold is not exceeded is conditionally independent of the mainshock. Then, reformulating the $P[\mathrm{IM}_A \leq im|x,y]$ term in equation (8) via the total probability theorem, equation (9) results:

$$\lambda_{im} = \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \leq im|x,y] \cdot \sum_{i=0}^{+\infty}(\iint_{M_A,R_A} P[\mathrm{IM}_A \leq im|w,z] \cdot f_{M_A,R_A|M_E,R_E}(w,z|x,y) \cdot dw \cdot dz)^i \cdot \frac{(E[N_{A|x}(0,\Delta T_A)])^i}{i!} \cdot e^{-E[N_{A|x}(0,\Delta T_A)]} \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\}, \tag{9}$$

in which the $P[\mathrm{IM}_A \leq im|w,z]$ term is the nonexceedance probability of im in the generic aftershock of known magnitude and location, and $f_{M_A,R_A|M_E,R_E}$ is the magnitude and distance joint PDF of an aftershock, conditional to the features of the mainshock. This PDF accounts for the dependence, of both magnitude of the aftershocks and size/location of the seismogenic zone for aftershocks, on magnitude and location of the triggering mainshock (to follow). The integration limits are those of equations (2) and (5) for mainshock and aftershocks, respectively.

A more compact expression of the hazard integral is given by equation (10).

$$\begin{aligned}\lambda_{im} &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \leq im|x,y] \cdot P[\mathrm{IM}_{\cup A} \leq im|x,y] \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\} \\ &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \leq im|x,y] \cdot e^{-P[\mathrm{IM}_A > im|x,y] \cdot \int_0^{\Delta T_A} \nu_{A|x}(\tau) \cdot d\tau} \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\} \\ &= \nu_E \cdot \{1 - \iint_{M_E,R_E} P[\mathrm{IM}_E \leq im|x,y] \cdot e^{-E[N_{A|x}(0,\Delta T_A)] \cdot \iint_{M_A,R_A} P[\mathrm{IM}_A > im|w,z] \cdot f_{M_A,R_A|M_E,R_E}(w,z|x,y) \cdot dw \cdot dz} \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\}.\end{aligned} \tag{10}$$

In fact, equation (10) is obtained using the equality in equation (11).

$$\begin{aligned}&\sum_{i=0}^{+\infty}(\iint_{M_A,R_A} P[\mathrm{IM}_A \leq im|w,z] \cdot f_{M_A,R_A|M_E,R_E}(w,z|x,y) \cdot dw \cdot dz)^i \cdot \frac{(E[N_{A|x}(0,\Delta T_A)])^i}{i!} \cdot e^{-E[N_{A|x}(0,\Delta T_A)]} \\ &= e^{-E[N_{A|x}(0,\Delta T_A)] \cdot (1 - \iint_{M_A,R_A} P[\mathrm{IM}_A \leq im|w,z] \cdot f_{M_A,R_A|M_E,R_E}(w,z|x,y) \cdot dw \cdot dz)} \\ &= e^{-E[N_{A|x}(0,\Delta T_A)] \cdot (\iint_{M_A,R_A} P[\mathrm{IM}_A > im|w,z] \cdot f_{M_A,R_A|M_E,R_E}(w,z|x,y) \cdot dw \cdot dz)}.\end{aligned} \tag{11}$$

It is to note that the result in equation (10) could also be directly obtained, computing the probability of zero aftershocks causing the exceedance in $(0, \Delta T_A)$, via a NHPP of rate in equation (5). Nevertheless, derivation given allows deeper insights into the implications of the assumptions on the aftershock process on the hazard integral.

Having formulated the hazard integral for the cluster in the case of a single source, it may be worth it to briefly discuss the common case of multiple (independent) sources contributing to the hazard of the site of interest. In the case for each of these the occurrence of mainshocks is modeled via a HPP, the resulting rate is just the summation, over all the sources, of the rates from equation (9). If the occurrence of mainshocks is probabilistically described by means of other processes, for example a renewal process, then the rate of exceedance may not be time invariant (see Polidoro *et al.*, 2013, for discussion). In such cases, if the modified Omori's law still applies for aftershocks, then it is possible to write the equations for the exceedance probability within the cluster similar to this study, yet the resulting formulation will certainly be different.

The proposed approach could be also extended to the case in which alternate models, such as the ETAS (e.g., Ogata, 1988), accounting for the possibility of any earthquake in the cluster to generate its own sequence, are employed in lieu of the modified Omori's law to describe the seismic sequences. These models lead to changing the rate of occurrence of aftershocks and possibly affect that of mainshocks.

Illustrative Application

As an illustrative application of SPSHA, hazard was computed for a site in the middle of a generic seismic source represented by an area, the size of which is $30 \times 100\ \mathrm{km}^2$ (Fig. 1).

Characteristics of the Mainshock and of the Conditional Aftershock Sequence

Mainshock epicenters were assumed as uniformly distributed in the areal seismic source of Figure 1, which was discretized by means of a $5 \times 5\ \mathrm{km}^2$ lattice for computational purposes. Mainshock rate was, arbitrarily, assumed to be $\nu_E = 0.054$ events/year. The magnitude distribution of mainshock was, arbitrarily again, chosen to be a truncated

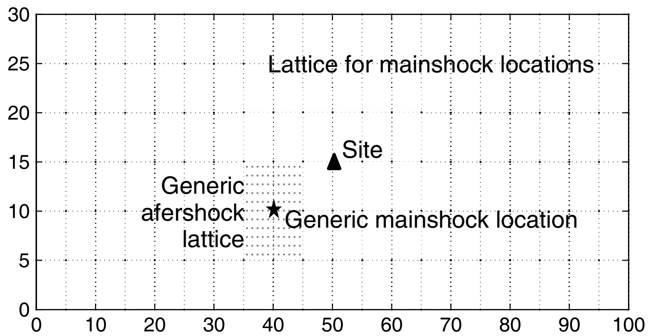

Figure 1. Seismogenic source lattice for mainshocks, generic aftershock lattice around the epicenter of a mainshock, and site of interest.

exponential defined in the [4.3,5.8] range, as illustrated in Figure 2a. The b-value of the GR relationship for mainshocks is 1.056. In the application, magnitude and source-to-site distance were considered independent RVs.

It was assumed that each mainshock has aftershocks constrained in an area around its epicenter. The size of the seismogenic zone for aftershocks in squared kilometers S_A depends on the magnitude of the main event via equation (12); Utsu, 1970; Figure 2b.

$$S_A = 10^{(m_E - 4.1)}. \tag{12}$$

Within this area, arbitrarily assumed a square and discretized by means of a 121 points lattice, epicenters are uniformly distributed (see Fig. 1). In fact, the proposed approach to hazard may deal with any shape of the aftershock source area (e.g., with an ellipsoidal shape, which is often considered). In addition, any function representing the probability of each grid cell of such area could be the location of an aftershock (e.g., PDFs that have a bell-shaped radial decay from the mainshock location, such as in Zhuang *et al.*, 2002). However, this issue does not significantly affect the conclusions of the study, and therefore the uniform distribution in the square was considered for simplicity.

The parameters used in the modified Omori's law and in the GR relationship for aftershocks, that is the parameters of equation (3), were taken from Lolli and Gasperini (2003): $a = -1.66$, $b = 0.96$, $c = 0.03$ (in days), $p = 0.93$, and $m_{\min} = 4.2$. These apply to Italian generic aftershock sequences; Yeo and Cornell (2009), for example, use another set of parameters representing the equivalent California model.

To evaluate both the $P[\mathrm{IM}_E \le im|m, r]$ and the $P[\mathrm{IM}_A \le im|m, r]$ terms, the Ambraseys *et al.* (1996) GMPE was used; therefore, the magnitude scale to be considered is that of this GMPE. Ambraseys *et al.* (1996) use the R_{jb} distance metric, which is the distance to the surface projection of the source (Joyner and Boore, 1981). On the other hand, because the points in Figure 1 are considered epicenters of mainshocks, the relationship in equation (13), by Gruppo di Lavoro (2004), was used to retrieve the value of R_{jb} (in km) to be plugged in the GMPE, converting from the epicentral distance R, which is identified by R_E or R_A in the hazard integrals above.

$$R_{jb} = -3.5525 + 0.8845 \cdot R. \tag{13}$$

Cases and Results

Given the working assumptions taken for the application, SPSHA was computed according to equation (10). In performing this first exercise, the IM considered was the peak ground acceleration (PGA) on rock. Moreover, following Yeo and Cornell (2009), the duration of the aftershock sequence (ΔT_A) was considered arbitrarily (Yeo, personal comm., 2013) equal to 90 days since the mainshock occurrence.

Figure 3a compares the SPSHA results, in terms of annual rate of exceedance of different PGA thresholds, to those obtained via PSHA using equation (2), which is accounting only for mainshocks. Indeed, in Figure 3b the relative difference between the SPSHA and PSHA, in terms of rate, is also depicted. Even if hazard curves appear close, differences up to 30% in rates may be observed.

Because the 5% damped pseudospectral acceleration SA(T) is an IM of general earthquake engineering interest, SPSHA was also computed in terms of this IM, with T (structural period) varying in the 0–2 s range. Results of this further analysis are expressed in terms of uniform hazard spectrum (UHS), that is, a spectrum the ordinates of which all have the same exceedance probability in a given time

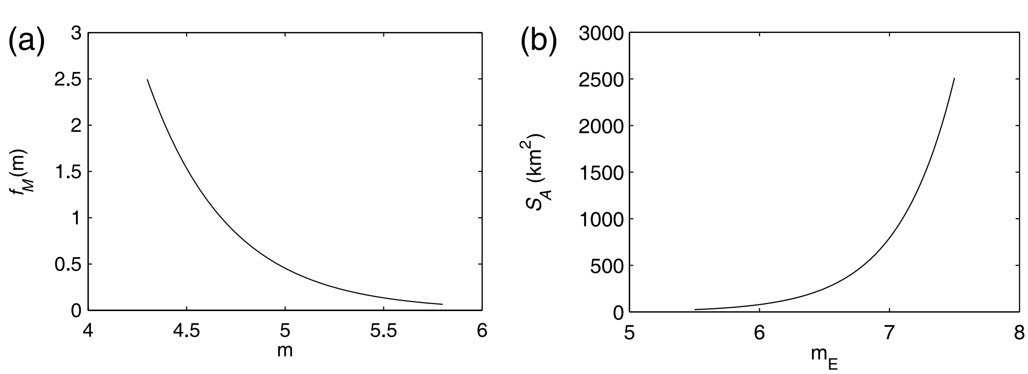

Figure 2. (a) Magnitude distribution for mainshocks. (b) Mainshock magnitude versus aftershock source area.

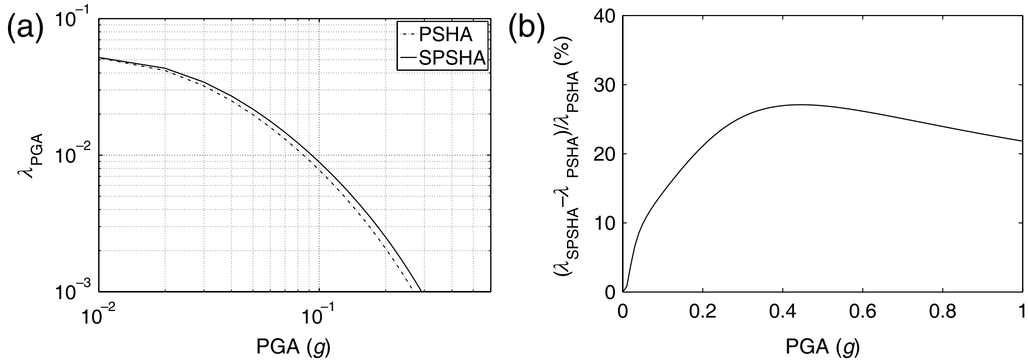

Figure 3. (a) PSHA and SPSHA results in terms of PGA for the illustrative application. (b) PSHA and SPSHA differences in terms of PGA for the illustrative application.

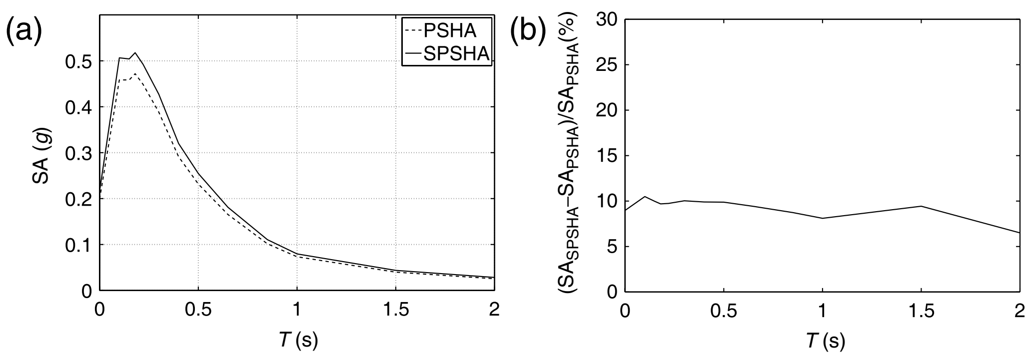

Figure 4. (a) PSHA and SPSHA illustrative application results in terms of 475 year UHS, that is 5% damped pseudospectral acceleration versus oscillation period, in which all ordinates share the same 10% in 50 years exceedance probability. (b) PSHA and SPSHA differences in terms of 475 years UHS for the illustrative application.

frame, or equivalently the same return period (e.g., Reiter, 1990). In Figure 4a, the UHS referring to 475 years, a typical life-safety-related design return period for ordinary structures is compared with its PSHA counterpart. Figure 4b shows the relative differences between the spectra computed via SPSHA and PSHA. Note that in this case comparison is in terms of IM given the return period (rather than rate as in the previous example) and changes up to 10% are observed.

These results, in terms of changes in both rates and accelerations, are comparable to those found by Boyd (2012), even though differences exist in the two studies and applications (Boyd, personal comm., 2013).

Conclusions

The study presented in this short note aimed at contributing to the inclusion of Omori-type aftershocks, to main earthquake events, in the seismic hazard analysis expressed in terms of rate of exceedance of a ground-motion IM. The focus was the probabilistically consistent formalization of the hazard integral, looking at the event of exceeding an intensity threshold at least once during the sequence.

To directly extend seismic hazard including the aftershock potential in the computation of the exceedance rate may be useful for performance-based design, as the intensity critical to the structure of interest could be exceeded in any of the earthquakes of the cluster.

The PSHA for mainshock–aftershocks seismic sequences was built on the HPP assumption for occurrence of mainshocks, and on the conditional NHPP for the occurrence of aftershocks. The latter depends on the features of the mainshock via the modified Omori's law and a semi-empirical relationship between the mainshock characteristics and the aftershock source area.

SPSHA was formulated analytically considering that the effects of aftershocks (i.e., ground-motion intensities) are conditionally independent on everything that happens outside the cluster, given the magnitude and location of the triggering mainshock.

The illustrative application refers to a generic source zone for mainshocks and to a generic aftershock sequence. The SPSHA was compared to the classical PSHA results, both in terms of rates given the IM threshold, and in terms of IM given the return period. Results, at least for the case setup, indicate changes up to about 30% in PGA rate and up to about 10% in pseudospectral acceleration values corresponding to the 475-year return period.

It is believed that the derived formulation may be of earthquake engineering interest, especially with respect to long-term performance-based design and assessment of structures.

Data and Resources

All data used in the study paper came from published sources listed in the references. The report by Toro and Silva (2001) is available at http://www.riskeng.com/downloads/scen_ceus_rept (last accessed July 2013). The report by Gruppo di Lavoro (2004) is available at http://zonesismiche.mi.ingv.it/documenti/rapporto_conclusivo.pdf (in Italian; last accessed July 2013).

Acknowledgments

This study was developed partially in the framework of AMRA—Analisi e Monitoraggio dei Rischi Ambientali scarl, within the Strategies and tools for Real-Time Earthquake Risk Reduction project (REAKT), funded by the European Community via the Seventh Framework Program for Research (FP7); Contract Number 282862; and partially within the ReLUIS-DPC 2010–2013 research program.

The authors want to thank Gee Liek Yeo and Oliver Boyd, for helpful discussions on their aftershock-related work. Finally, the two anonymous reviewers as well as the Associate Editor, Delphine D. Fitzenz, whose comments significantly improved quality and readability of this study, are also acknowledged.

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Manuscript received 31 July 2013;
Published Online 25 March 2014