Proc. of 11th Conf. on Structural Safety and Reliability, ICOSSAR '13, New York, US, June 16-20 2013.

Gamma modeling of continuous deterioration and cumulative damage in life-cycle analysis of earthquake-resistant structures

Iunio Iervolino & Eugenio Chioccarelli
Dipartimento di Strutture per l'Ingegneria e l'Architettura, Università degli Studi di Napoli Federico II, Naples, Italy.

Massimiliano Giorgio
Dipartimento di Ingegneria Industriale e dell'Informazione, Seconda Università di Napoli, Aversa, Italy.

ABSTRACT: Stochastic modeling of structural deterioration at the scale of the life-cycle is the subject of the study. The categories of degradation phenomena considered are those two typical of structures, that is, continuous degradation of mechanical characteristics (i.e., *aging*) and cumulative damage due to point overloads (i.e., earthquakes). The wearing structural parameter is the seismic capacity expressed in terms of kinematic ductility to conventional collapse. The gamma distribution is considered to model damages produced by earthquakes and a closed-form approximation for structural reliability assessment is obtained. Moreover, a gamma stochastic process is considered for continuous deterioration. Finally, the possible transformation of the repeated-shock effect due to earthquakes in an equivalent aging (i.e., *forward virtual age*), leading to a close-form for the superposition of the two processes, is discussed. An example, referring to a simple bilinear elastic-perfectly-plastic system, illustrates potential applicability and limitations of the approach within the performance-based earthquake engineering framework.

1 INTRODUCTION

Dependency on history (e.g., number of occurred earthquakes, time elapsed since the last seismic event, or structural repair, etc.) of seismic structural risk may involve all the three elements constituting the performance-based earthquake engineering framework or PBEE (Cornell and Krawinkler, 2000), that is, loss, vulnerability, and hazard (e.g., Polidoro et al., 2013).

Seismic structural vulnerability is commonly considered affected by two categories of phenomena leading it to vary with time: (1) continuous deterioration of material characteristics, or *aging*, and (2) cumulating damage because of repeated overloading due to earthquake shocks (Sanchez-Silva et al., 2011).

Aging, which in some cases may show an effect in increasing seismic structural fragility (Rao et al., 2010), is often related to aggressive environment which worsens mechanical features of structural elements, for example: corrosion of reinforcing steel due to chloride attack, or carbonation in concrete; e.g., Stewart et al. (2011). To be able to predict the evolution of this kind of degradation may be important in design of maintenance policies (e.g., Frangopol et al., 2004).

Earthquake shocks potentially cumulate damage on the hit structure during its lifetime, unless partial or total restoration; i.e., within a *cycle*. In general, mainly because earthquake occurrences can be considered instantaneous with respect to structural life, it is advantageous to model the cumulative seismic damage process separately from aging. Indeed, to describe earthquakes probabilistically, a *marked point process*, in which each seismic event is represented by its occurrence time and damage it produces, can be adopted. With respect to this model, engineering interest is in the compound point process accounting for the cumulative damage (i.e., the sum of damage increments) produced by all shocks.

If both deterioration effects may be measured in terms of the same parameter expressing the structural capacity, for example the residual ductility to collapse, or $\mu(t)$, then the total degradation may be susceptible of the representation in Figure 1, where an arbitrary path of the process is depicted, as well as a threshold referring to a limit-state of interest.

Formally, the degradation process is that in Equation (1), where μ_0 is the initial capacity in the cycle (e.g., the *as-new* capacity), and $D(t)$ is the cumulated level of deterioration at the time *t*.

$$\mu(t) = \mu_0 - D(t) \tag{1}$$

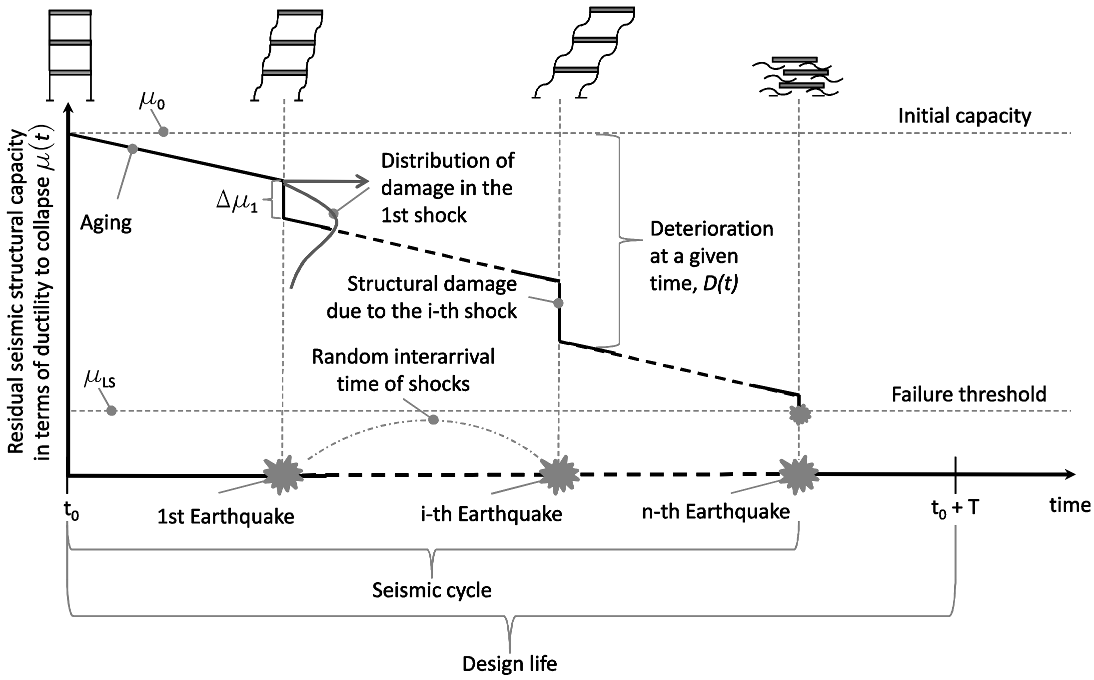

Figure 1. Seismic cycle representation for a structure subjected to aging and repeated earthquake shocks.

As introduced, $D(t)$ can be seen as the sum of two effects, one due to continuous deterioration and one due to accumulation of seismic damage, as in Equation (2), where the first term in the right hand side is the continuous loss of capacity at time *t* due to aging, $\mu_C(t)$, and the second one is the cumulated loss of resistance due to all earthquake events, $N(t)$, occurring until time *t*. Note that $\mu_C(t)$, $\Delta\mu_i$ (damage in a single seismic event), and $N(t)$ all are random variables (RVs).

$$D(t) = \mu_C(t) + \sum_{i=1}^{N(t)} \Delta\mu_i \quad (2)$$

Given this formulation, the probability the structure fails within *t*, $P_f(t)$, is the probability that the structure reaches or passes a threshold related to a certain limit state, μ_{LS}, any time up to *t*, Equation (3). In other words, it is the probability that in $(0,t)$ the capacity travels the distance, $\bar{\mu}$, between the initial value and the threshold. Note that, by definition, Equation (3) also provides the cumulative probability function (CDF) of structural lifetime, $F_T(t)$. To model such a risk is the objective of the study.

$$\begin{aligned} P_f(t) &= F_T(t) = P\left[\mu(t) \le \mu_{LS}\right] = \\ &= P\left[D(t) \ge \mu_0 - \mu_{LS}\right] = P\left[D(t) \ge \bar{\mu}\right] \end{aligned} \quad (3)$$

The following is structured such that cumulative damage only, that is when continuous deterioration is neglected, is investigated first. The developed compound point process assumes that: (1) damage increments, are independent and identically distributed (i.i.d.), (2) the processes regulating earthquake occurrence and seismic damage are independent.

In particular, it is addressed the case damage in which an individual earthquake is susceptible of *gamma* representation and earthquake occurrence follows a homogenous Poisson process (HPP). For this case, approximate-form solutions for the reliability problem is derived. The model also considers that not all earthquakes are necessarily damaging, as not all of them are overloads.

Subsequently, the *gamma process* (e.g., Çinlar, 1980), of acknowledged suitability for probabilistic representation of wear in engineering systems (e.g., Pandey et al., 2004), is considered for continuous deterioration of seismic structural capacity.

Then, the concept of equivalent aging due to earthquake shocks is introduced, which reverting a maintenance principle, is referred to as *forward virtual age*. It is, in fact, the special case in which total degradation can be described via a single gamma process.

Finally, an illustrative application referring to a simple single degree of freedom (SDoF) elastic-perfectly-plastic (EPP) structure, supposed to be located in a comparatively high-seismicity region in central Italy, is developed, also to shed some light on suitability of the hypotheses at the basis of this simple *age-dependent* reliability model.

2 CUMULATIVE EARTHQUAKE DAMAGE

In the case only cumulative damage is considered (i.e., $\mu_C(t)=0, \forall t$), then the deterioration process results as in Equation (4).

$$\mu(t)=\mu_0-D(t)=\mu_0-\sum_{i=1}^{N(t)}\Delta\mu_i \qquad (4)$$

When the occurrence of seismic events is described by a HPP, $N(t)$ has a Poisson distribution with constant λ rate. Thus, considering the distribution of cumulative damage as dependent on the number of occurring earthquakes, and their ground motion intensities measures, $\underline{IM}$, the failure probability may be computed as in Equation (5), where the integral is of *k-th* order.

$$\begin{aligned}
&P_f(t)=P\left[D(t)\geq\bar{\mu}\right]= \\
&=\sum_{k=1}^{+\infty}P\left[D(t)\geq\bar{\mu}\,\middle|\,N(t)=k\right]\cdot\frac{(\lambda\cdot t)^k}{k!}\cdot e^{-\lambda\cdot t}= \\
&=\sum_{k=1}^{+\infty}\int_{\underline{im}}P\left[\sum_{i=1}^{k}\Delta\mu_i\geq\bar{\mu}\,\middle|\,\underline{IM}=\underline{im},N(t)=k\right]\times \\
&\times f_{\underline{IM}}(\underline{im})\cdot d(\underline{im})\cdot\frac{(\lambda\cdot t)^k}{k!}\cdot e^{-\lambda\cdot t}
\end{aligned} \qquad (5)$$

In the classical HPP-based seismic hazard analysis (e.g., McGuire, 2004), IMs, for example first mode spectral acceleration, *Sa*, in different earthquakes are i.i.d. RVs, and $f_{\underline{IM}}(\underline{im})$ is simply the product of k marginal distributions $f_{IM_i}(im)$.

Therefore, the critical issue to solve the reliability problem is to get the probability of cumulated damage exceeding the threshold conditional to ground motion intensities for a given number of earthquakes. This may be addressed in a relatively simple manner if three conditions are met. (1) Damage in the *i-th* earthquake, $\Delta\mu_i$, has always the same CDF in Equation (6), marginal with respect to IM; i.e., $P[\Delta\mu_i\leq\delta\mu]=P[\Delta\mu\leq\delta\mu],\forall i$. (2) Damage produced in different events are independent RVs. In other words, according to conditions (1) and (2), earthquake's structural effects are i.i.d. This, in particular, implies that the structure, in an earthquake, suffers a damage increment independent of its state.

$$F_{\Delta\mu}(\delta\mu)=\int_{im}P\left[\Delta\mu\leq\delta\mu\,|\,IM=x\right]\cdot f_{IM}(x)\cdot dx \qquad (6)$$

Condition (3) is that distribution of sums of damages can be expressed in a simple form. A way to satisfy this condition consists in modeling damage via a RV that enjoys the *reproductive* property.

As an example, in the following the gamma RV is considered to derive solutions for reliability when damage accumulation is due to seismic events only.

2.1 *Gamma-distributed damage increments*

The gamma distribution is shown in Equation (7), where γ_D and α_D are the scale and shape parameters, respectively.

$$f_{\Delta\mu}(\delta\mu)=\frac{\gamma_D\cdot(\gamma_D\cdot\delta\mu)^{\alpha_D-1}}{\Gamma(\alpha_D)}\cdot e^{-\gamma_D\cdot\delta\mu} \qquad (7)$$

The probability density function (PDF) of this continuous and non-negative RV, depending on its shape parameter, can take significantly different appearance. For example, α_D equal to one stretches the distribution to the exponential, a large value of the shape parameter let the PDF be similar to that of a Gaussian RV, while for intermediate values of α_D, it is an alternative to the lognormal one to model skewed non-negative RVs.

Because the sum of k_D i.i.d. gamma-distributed RVs, with scale and shape parameters γ_D and α_D respectively, is still gamma with parameters γ_D and $k_D\cdot\alpha_D$, the probability of cumulative damage exceeding the threshold, conditional to k_D shocks, is given by Equation (8); see the Appendix.

It is to underline that, being continuous and non-negative, the gamma RV is suitable to model only the effect of shocks determining loss of capacity, not those whose intensity is not large enough. In this respect, differently from Equation (5), k_D in Equation (8) refers to the *filtered* HPP of parameter $\lambda_D=\lambda\cdot P[\Delta\mu>0]$, counting damaging events only, $N_D(t)$; to follow.

$$P\left[D(t)\geq\bar{\mu}\,\middle|\,N_D(t)=k_D\right]=\frac{\Gamma_U(k_D\cdot\alpha_D,\gamma_D\cdot\bar{\mu})}{\Gamma(k_D\cdot\alpha_D)} \qquad (8)$$

If the failure probability in Equation (5) can be replaced, using the first moment approximation (e.g., Benjamin and Cornell, 1970), by its value conditional to the expected number of earthquakes until *t*, Equation (9) results.

$$\begin{aligned}
&P_f(t)\approx P\left[D(t)\geq\bar{\mu}\,\middle|\,N_D(t)=E\left[N_D(t)\right]\right]= \\
&=\frac{\Gamma_U(\lambda_D\cdot t\cdot\alpha_D,\gamma_D\cdot\bar{\mu})}{\Gamma(\lambda_D\cdot t\cdot\alpha_D)}
\end{aligned} \qquad (9)$$

3 CONTINUOUS DETERIORATION MODELING

This section refers to the modeling of aging. The key difference with respect to the cumulating damage discussed so far is that its probabilistic representation is a continuous process. An option is the *gamma process* that, if applicable, implies that degradation has independent and stationary gamma-distributed increments, yielding Equation (10) as the marginal distribution of total deterioration up to *t*. In fact, it may prove suitable to model continuously accumulating degradation, such as wear, fatigue, corrosion, crack growth, creep, swell; i.e., typical aging-related structural phenomena (Van Noortwijk et al., 2007).

$$f_{\mu_C(t)}(\mu) = \frac{\gamma_A \cdot (\gamma_A \cdot \mu)^{s_A \cdot t - 1}}{\Gamma(s_A \cdot t)} \cdot e^{-\gamma_A \cdot \mu} \quad (10)$$

In Equation (10) the deteriorating structural parameter is still ductility to collapse. As a consequence, if degradation is due to aging only, the failure probability is given by Equation (11).

$$P_f(t) = P\left[\mu_C(t) > \bar{\mu}\right] = \frac{\Gamma_U(s_A \cdot t, \gamma_A \cdot \bar{\mu})}{\Gamma(s_A \cdot t)} \quad (11)$$

Note that the Equation (10) also implies the mean and the variance of the degradation process are: $E\left[\mu_C(t)\right] = (s_A/\gamma_A) \cdot t$, $Var\left[\mu_C(t)\right] = (s_A/\gamma_A^2) \cdot t$.

4 VIRTUAL AGE

In general, the reliability assessment when both the point and continuous degradation processes affect the structure, requires numerical approach. However, due to the properties of the gamma distribution, in the case aging and seismic damage share the same scale parameter, γ, the probability of failure, conditional to the number of shocks, k_D, allows the closed-form solution of Equation (12).

$$\begin{aligned} &P\left[D(t) \geq \bar{\mu} \mid N_D(t) = k_D\right] = \\ &= \frac{\Gamma_U(s_A \cdot t + k_D \cdot \alpha_D, \gamma \cdot \bar{\mu})}{\Gamma(s_A \cdot t + k_D \cdot \alpha_D)} \end{aligned} \quad (12)$$

The assumption yielding this equation may be restrictive and has to be verified case-by-case. Obviously, when it applies, the model enables evident mathematical advantages. Indeed, Equation (12) allows to derive a handy approximation of reliability considering both degradation phenomena as given in Equation (13), which avails, again, of the first moment approximation. Note that if the *equivalent* shape parameter, $s = s_A + \lambda_D \cdot \alpha_D$, is introduced, Equation (13) formally coincides with that of Equation (11).

$$\begin{aligned} &P_f(t) \approx \int_{\bar{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot t + E\left[N_D(t)\right] \cdot \alpha_D - 1}}{\Gamma\left(s_A \cdot t + E\left[N_D(t)\right] \cdot \alpha_D\right)} \cdot e^{-\gamma \cdot x} \cdot dx = \\ &= \int_{\bar{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s \cdot t - 1}}{\Gamma(s \cdot t)} \cdot e^{-\gamma \cdot x} \cdot dx = \frac{\Gamma_U(s \cdot t, \gamma \cdot \bar{\mu})}{\Gamma(s \cdot t)} \end{aligned} \quad (13)$$

In fact, Equation (12) is also susceptible of an appealing interpretation, in terms of the *virtual age* concept, originally developed to account for the effect of maintenance in the reliability assessment (Kijima, 1989). According to virtual age, repair is seen as rejuvenation, such that, from the reliability assessment point of view, the repaired system is equivalent to the original one, but with an age reduced by the number of years cancelled by repair. In the case under study, the effect of a shock may be seen as producing an instantaneous aging. Indeed, defining the time warp $\tau = \alpha_D / s_A$, Equation (12) can be rewritten as Equation (14), which shows that failure probability of a structure of age *t* and subject to k_D earthquakes, may be computed as that of a structure with age $t + k_D \cdot \tau$, and no shocks. This model may be referred to as *forward* (as opposite to that *backward* of Kijima, 1989) virtual age.

$$\begin{aligned} &P\left[D(t) \geq \bar{\mu} \mid N_D(t) = k_D\right] = \\ &= P\left[D(t + k_D \cdot \tau) \geq \bar{\mu} \mid N(t + k_D \cdot \tau) = 0\right] = \\ &= \frac{\Gamma_U\left[s_A \cdot (t + k_D \cdot \tau), \gamma \cdot \bar{\mu}\right]}{\Gamma\left[s_A \cdot (t + k_D \cdot \tau)\right]} \end{aligned} \quad (14)$$

5 ILLUSTRATIVE APPLICATION

In this section structural modeling is addressed with reference to a simple EPP-SDoF system with reloading/unloading stiffness, which is the same as the initial one. The reason to choose this model is threefold: (i) it is at the basis of earthquake engineering; (ii) earthquake-resistant structures, especially those reflecting modern codes, may be often rendered equivalent to this kind of system; (iii) it shows stable hysteretic cycles that repeat themselves despite of the sequence of excitation it undergoes to. This latter property is especially important with respect to the *age-dependent* reliability models discussed herein, which are based on independent and identically distributed damage increments. In fact, while *state-dependent* approaches to failure probability (e.g., Giorgio et al., 2010 and 2011, Luco et al., 2004) may be required to describe damage cumulating in systems with evolutionary hysteretic behaviour, the results for the EPP system are expected to be of significant generality.

In the following, cumulative damage is addressed first, subsequently continuous deterioration, finally their sum assuming the same scale parameter.

5.1 *Structure and response*

The elastic period of the EPP-SDoF is equal to 0.5 s; weight is 100 kN and the yielding force (Fy) is equal to 19.6 kN, which corresponds to a strength reduction factor equal to 2.5 given a 0.49 g first-mode spectral acceleration.

Chosen engineering demand parameter (EDP) is the kinematic ductility, μ; i.e., the maximum displacement demand, when the yielding displacement is the unit. In fact, choosing such an EDP is equivalent to assume that the collapse of the structure is due to displacements.

If the considered limit state (LS) is *collapse prevention* (CP) derived from FEMA (2000), which assumes conventional collapse of concrete structures at a maximum drift ratio equal to 0.04, the system has an (initial) ductility capacity $\mu_0 = 3.3$, and each damaging shock drains some of this ductility supply, consistent with the collapse criterion in Cosenza and Manfredi (2000).

Once structural system and collapse criterion are defined, it is possible to address the i.i.d. hypothesis of damage increments. Due to its force-displacement relationship, the considered SDoF has a response, which is stable with respect to subsequent earthquakes. It is easy to recognize that this means the maximum displacement reached in the *i-th* earthquake of a sequence is just the same as if the damaging event hit the new structure, plus the residual displacement from the preceding shaking. In other words, *variation* of drained capacity in the *i-th* earthquake is independent both of the age and the state the shock finds the structure in. Thus, different earthquakes produce i.i.d. effects.

5.2 *Calibrating the damage increment distribution*

The CDF of the damage increment in a shock, $\Delta\mu$, may be computed via Equation (6), where $f_{IM}(im)$ is derived from the HPP hazard curve for the site where the structure is supposed to be located. The probabilistic seismic demand term, $f_{\Delta\mu|IM}(\delta\mu \mid IM = im)$, may be computed via *incremental dynamic analysis* (IDA) assuming the spectral acceleration at the elastic period of the SDoF, as an IM (see Iervolino et al., 2012, for details). IDA is developed in terms of structural ductility normalized by μ_0, in a way that the demand is equal to 1 when the CP-LS is attained. Thus, $\Delta\mu$ may be defined as in Equation (15), where μ_{before} and μ_{after} refer to residual capacity before and after the generic shock.

$$\Delta\mu = \frac{\mu_{before} - \mu_{after}}{\mu_0} \tag{15}$$

Marginalization of the distribution of damage increments by $f_{IM}(im)$, as per Equation (6), is site-specific. Considered site is (arbitrarily) Sulmona (13.96 Lon.; 42.05 Lat.), close to L'Aquila in central Italy. Probabilistic seismic hazard analysis for the site was carried out by software specifically developed and described in Iervolino et al. (2011), to which the reader should refer for details. $f_{IM}(im)$ was computed for the pseudo-acceleration spectral ordinate corresponding to the SDoF's elastic period. Note that it is not exactly the hazard curve for the site, while it is the distribution of ground motion intensity *given the occurrence of an earthquake*. In fact, this is required to obtain the marginal distribution of capacity reduction in one shock, and it was obtained from the hazard curve divided by the annual rate of occurrence of events in Sulmona, which is equal to 1.95 (between magnitude 4.3 and 7.3).

In Figure 2, the distribution of damage increment to be used to compute Equation (6) is reported. To comment the plot it has to be recalled that, given the structure, not all earthquakes are strong enough to yield the structure, and $\Delta\mu = 0$ for such shocks. In particular, $\Delta\mu$ is larger than zero only for spectral accelerations larger than about 1.96 m/s^2, which is, in fact, the yielding acceleration of the considered EPP. Thus, damage increment is not a continuous RV and its CDF has the expression in Equation (16).

$$F_{\Delta\mu}(\delta\mu) = \begin{cases} P_0 & \delta\mu = 0 \\ P_0 + \int_0^{\delta\mu} \tilde{f}_{\Delta\mu}(x) \cdot dx & \delta\mu > 0 \end{cases} \tag{16}$$

In other words, the distribution of $\Delta\mu$ is defined by means of a probability density for $\Delta\mu > 0$, and a *probability mass* for $\Delta\mu = 0$. In fact, $P_0 = P[\Delta\mu = 0]$ accounts for the probability that earthquakes are not strong enough to damage the structure. In this application, $P[\Delta\mu = 0]$ is equal to

0.9924. This means that only 0.76% of earthquakes is expected to be damaging.

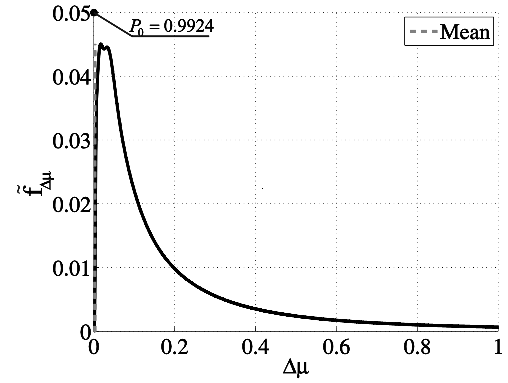

Figure 2. Marginal distribution of $\Delta\mu$ for the structure at the site of interest.

The expected value of $\Delta\mu$, $E[\Delta\mu]=0.0026$, is also reported, yet barely visible, in Figure 2. It means that, for the considered structure at the considered site, a generic earthquake produces a capacity reduction of about 0.26%, on average with respect to both damaging and undamaging events. Thus, referring to the seismic hazard of Sulmona, given that average number of earthquakes in one year is equal to 1.95, the considered SDoF is expected to undergo an average capacity reduction equal to $0.0026 \cdot 1.95 = 0.0051$ or 0.5% per year. Therefore, according to the considered criterion, the structure fails after about 200 years on average.

5.3 *Reliability to cumulated earthquake damage*

The gamma distribution is adopted to model the PDF of shock effect in the case of damage larger than zero, $f_{\Delta\mu}(\delta\mu) = f_{\Delta\mu}(\delta\mu)/(1-P_0)$. Scale, γ_D, and shape, α_D, parameters of the model are set equal to 0.5539 and 0.1916, respectively. These valued were obtained solving the equations $\alpha_D/\gamma_D = E[\Delta\mu|\Delta\mu>0] = 0.3459$ and $\alpha_D/\gamma_D^2 = Var[\Delta\mu|\Delta\mu>0] = 0.6245$; where 0.3459 and 0.6245 are the conditional mean and variance calculated by means of structural analysis.

Failure probability within 50 yr is computed according to Equation (9) and reported in Equation (17), where it also recalled that the expected number of damaging earthquakes is computed filtering the *all earthquakes* HPP.

$$\begin{cases} P_f(50) \approx 0.076 \\ E[N_D(t)] = \lambda_D \cdot t = (1-P_0)\cdot\lambda\cdot t = 0.015\cdot t \end{cases} \tag{17}$$

At this point, it is appropriate to check tolerability of the gamma-assumption for the damage increment. Moreover, it is the occasion also to verify the implications of using the first moment approximation . To this aim, in Figure 3 the CDF of the lifetime of the structure, $F_T(t)$, according to the model in Equation (9), is reported.

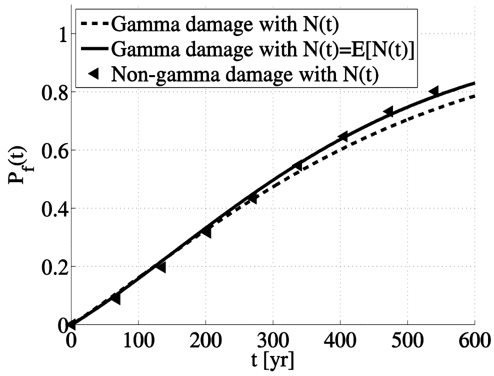

Figure 3. Structural lifetime distribution in the case of earthquake damage according to the approximated model of Equation (9) and removing the approximations.

In the same figure, $P_f(t)$ is also computed: (i) under the assumption damage increment is gamma distributed and explicitly considering the probability associated to any number of shocks, as per Equation (5); and (ii) adopting for $\Delta\mu$ the empirical distribution obtained from structural simulation, and explicitly considering the probability associated to any number of shocks.

The figure shows that, at least up to three hundred years, where failure probability is 0.6 (hardly tolerable for a civil construction), the gamma assumption, even in case of first moment approximation, gives results in agreement with those of the empirical model (on the safe side).

5.4 *Total degradation and virtual age*

This section starts considering the case of a structure subject (only) to continuous deterioration of seismic capacity that may be described via a gamma process with mean and variance function $s_A/\gamma_A \cdot t = 10^{-3}\cdot t$ and $(s_A/\gamma_A^2)\cdot t = 10^{-4}\cdot t$, respectively. A realization of the corresponding process is reported in Figure 4; it emerges that it was assumed continuous deterioration has a mild effect (in accordance to literature; e.g., Vamvatsikos and Dolsek, 2011), and the structure has a median life of about 1000 yr, while P_f is close to one after about 2000 yr, if this is the sole source of degradation.

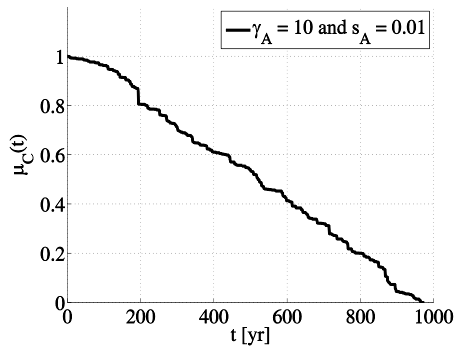

Figure 4. Realization of continuous deterioration process.

Assume now the same scale parameter, for example that of cumulative earthquake damage, can be attributed to both degradation processes. This allows to apply Equation (13) with $\{\gamma = \gamma_D, s = s_A + \lambda_D \cdot \alpha_D\}$ parameters, Equation (18).

$$s = s_A\left(1 + \lambda_D \cdot \frac{\gamma}{s_A} \cdot E\left[\Delta\mu \,|\, \Delta\mu > 0\right]\right) = 0.0034 \qquad (18)$$

It is to recall however, that if $\gamma_A = \gamma_D$, the shape parameter of continuous deterioration process may be reshaped such that the same linear trend is preserved, that is $s_A = \gamma_D \cdot E\left[\mu_C(t)\right]/t$; this implies to force the variance of the process to be $\left(s_A/\gamma_D^2\right)\cdot t$. In this case, if $s_A = \gamma_D \cdot 10^{-3} = 0.5539 \cdot 10^{-3}$, the same mean of aging in Figure 4 is kept, yet the variance results to be $Var\left[\mu_C(t)\right] = 0.0018 \cdot t$. The resulting lifetime CDF, compared with individual and total degradations, is given in Figure 5. It may be deduced that, in this particular case, the approximation provided by the simple model, appears acceptable.

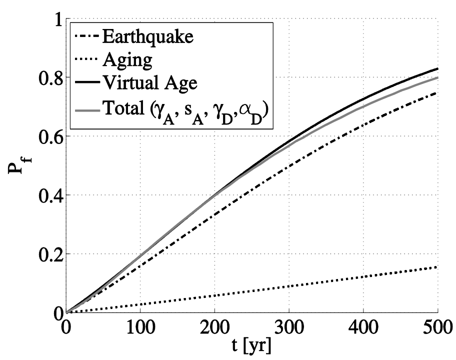

Figure 5. Lifetime CDFs for individual and superimposed process according to the virtual age approximation.

The failure probability in 50 yr referring to this case, that is, plugging Equation (18) in Equation (13) results in $P_f(50) \approx 0.0920$.

It is, finally, interesting to note that, according to the parameters of the application, $\tau = \alpha_D/s_A = 0.1916/\left(0.5539 \cdot 10^{-3}\right) = 346\ yr$, meaning that a generic damaging earthquake produces an instantaneous aging of the structure of more than three hundred years. This illustrates how the forward virtual age concept, if applicable, is attractive: it provides, at a glance, vulnerability of a structure subject to the considered sources of deterioration.

6 CONCLUSIONS

Life-cycle reliability analysis of deteriorating structures was discussed. The structural performance measure considered is the ductility capacity to collapse. First, models for reliability analysis of structures cumulating seismic damage were discussed in the case of gamma distribution for the point process increment. An approximate- yet closed-form solution for reliability was formulated. Second, the gamma-process, especially suitable to represent continuous wear in engineering systems because of its non-negative, independent, and stationary increment characteristics, was adopted to model structural aging. Finally, the computationally attractive forward virtual age option was also introduced for structures subjected to the two degradation phenomena, when both processes may be given the same scale parameter.

The suitability of the discussed reliability model in the performance-based earthquake engineering context was also illustrated via a simple application, which refers to a bilinear SDoF system located in a relatively high seismicity site in central Italy. Conventional collapse prevention limit-state was considered and the gamma distribution's parameters were calibrated based on structural analysis. The results of the models were also discussed with respect to invoked assumptions and approximations.

Results support the conclusion that gamma-process-based stochastic modeling of degrading structures, may be useful in the performance-based earthquake engineering context.

ACKNOWLEDGEMENTS

This study was developed in the framework of *AMRA – Analisi e Monitoraggio dei Rischi Ambientali scarl* (http://www.amracenter.com), within the *Strategies and tools for Real-Time Earthquake Risk Reduction* project (REAKT; http://www.reaktproject.eu), funded by the European Community via the *Seventh Framework Program for Research* (FP7); contract no. 282862.

REFERENCES

Benjamin, J. & Cornell, C.A. 1970. *Probability, statistics and decision for civil engineers*, Mc Graw-Hill.

Çinlar, E. 1980. On a generalization of gamma processes. *J. Appl. Prob.* 17(2): 467-480.

Cornell, C.A. & Krawinkler, H. 2000. *Progress and challenges in seismic performance assessment*. Peer Center Newsletter, 3(2). Pacific Earthquake Engineering Research Center, Berkeley, CA.

Cosenza, E. & Manfredi, G. 2000. Damage indices and damage measueres, *Prog. Struct. Eng. Mat.* 2(1): 50-59.

FEMA 356 2000. *Prestandard and commentary for the seismic rehabilitation of buildings*. Federal Emergency Management Agency. Washington DC, US.

Frangopol, D.M., Kallen, M.-J., van Noortwijk, J.M. 2004. Probabilistic models for life-cycle performance of deteriorating structures: review and future directions. *Prog. Struct. Eng. Mat.* 6(4): 197–212.

Giorgio, M., Guida, M., Pulcini, G. 2010. A state-dependent wear model with an application to marine engine cylinder liners. *Technometrics* 52(2): 172-187.

Giorgio, M., Guida, M., Pulcini, G. 2011. An age- and state-dependent Markov model for degradation processes, *IIE T.* 43(9): 621-632.

Iervolino I., Chioccarelli E., Giorgio M. 2012. Time-Dependent Seismic Reliability of Damage-Cumulating Non-Evolutionary Bilinear Systems. *Proc. of the 15th World Conference on Earthquake Engineering, Lisbon, Portugal*. Paper No. 2893.

Iervolino, I., Chioccarelli, E., Convertito, V. 2011. Engineering design earthquakes from multimodal hazard disaggregation. *Soil. Dyn. Earthq. Eng.* 31(9): 1212–1231.

Kijima, M. 1989. Some results for repairable systems with general repair. *J. Appl. Prob.* 26(1): 89-102.

Luco, N., Bazzurro, P., Cornell, C.A. 2004. Dynamic versus static computation of the residual capacity of a mainshock-damaged building to withstand an aftershock. *Proc. of the 13th World Conference on Earthquake Engineering, Vancouver, Canada*. Paper No. 2405.

McGuire, R.K. 2004. *Seismic Hazard and Risk Analysis, Earthquake Engineering Research Institute*, MNO-10, Oakland, California, 178 pp.

Pandey, M.D., van Noortwijk J.M. 2004. Gamma process model for time-dependent structural reliability analysis. In Watanabe, E., Frangopol, D.M., Utsonomiya, T., (eds.), *Bridge maintenance, safety, management and cost. Proc. of the Second international conference on bridge maintenance, safety and management (IABMAS), Kyoto, Japan.*

Polidoro, B., Iervolino I., Chioccarelli E., Giorgio, M. 2013. Models and issues in history-dependent mainshock hazard. *Proc. of the 11th International Conference on Structural Safety and Reliability (ICOSSAR), New York, NY.*

Rao, A., Lepech, M., Kiremidjian, A., Sun, X.Y. 2010. Time varying risk modeling of deteriorating bridge infrastructure for sustainable infrastructure Design. In Frangopol, D.M., Sause, R, and Kusko, C.S., (eds.), *Bridge Maintenance, Safety, Management, and Life-Cycle Optimization. Proc. of the Fifth International Conference on Bridge Maintenance, Safety and Management (IABMAS), Philadelphia, PA.*

Sanchez-Silva, M., Klutke, G.-A., Rosowsky, D.V. 2011. Life-cycle performance of structures subject to multiple deterioration mechanisms. *Struct. Saf.* 33(3): 206–217.

Stewart, M.G., Wang, X., Nguyen, M.N. 2011. Climate change impact and risks of concrete infrastructure deterioration. *Eng. Struct.* 33(4): 326–1337.

Vamvatsikos, D., Dolsek, M. 2011. Equivalent constant rates for performance-based assessment of aging structures. *Struct. Saf.* 33(1): 8-18.

Van Noortwijk, J.M., van der Weide, J.A.M., Kallen, M.J., Pandey, M.D. 2007. Gamma processes and peaks-over-threshold distributions for time-dependent reliability. *Reliab. Eng. Syst. Saf.* 92(12): 1651-1658.

APPENDIX

Considering the *gamma* and the *upper incomplete gamma* functions given in Equation (19) and Equation (20), respectively, Equation (21) applies.

$$\Gamma(\beta) = \int_0^{+\infty} z^{\beta-1} \cdot e^{-z} \cdot dz \tag{19}$$

$$\Gamma_U(\beta, y) = \int_y^{+\infty} z^{\beta-1} \cdot e^{-z} \cdot dz \tag{20}$$

$$\int_a^{+\infty} \frac{b \cdot (b \cdot z)^{\beta-1}}{\Gamma(\beta)} \cdot e^{-b \cdot z} \cdot dz = \frac{\Gamma_U(\beta, a \cdot b)}{\Gamma(\beta)} \tag{21}$$